In [1]:
```
'''
Author: Sameer
Date: 09/11/2018
Read Me:
1 - This code is for building a Gaussian Kernel (RBF) Support Vector Machine
 (SVM) and the optimization
problem (Quadratic Programming) is solved using python cvxopt optimization too
lbox.
Polynomial Kernel SVM can also be build using this code.
2 - Input Samples variable indicates total no. of points present in both Posit
ive and Negative Classes
Gaussian Standard Deviation is a free variable used in the Gaussian Kernel
Order is a free variable used to control the order of the polynomial used in P
olynomial Kernel
Grid Size is controls the number of points to be searched on 1x1 grid to gener
ate decision boundaries
3 - Quadratic Optimization Problem:
        QP: Minimize- 1/2 * X.T * P * X + q.T * X
        ST: G * X <= h and A * X = b
    Matrices were selected based on above descritpion. But for the detailed de
scription go through following link:
    http://cs229.stanford.edu/notes/cs229-notes3.pdf
4 - Solution for the QP problem will not be accurate i.e. lagrange mulitpliers
 will not be absolute zero, so I have
made values below 1.0e-04 to be zero. For Non Support Vectors lagrange mulitpl
iers are zero and for Support Vector they are
greater than zero.
5 - Postive Class is also represented by C_1 and Negative Class is also repres
ented C_-1
'''
```

Out[1]:
```
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epresented C_-1\n'
```

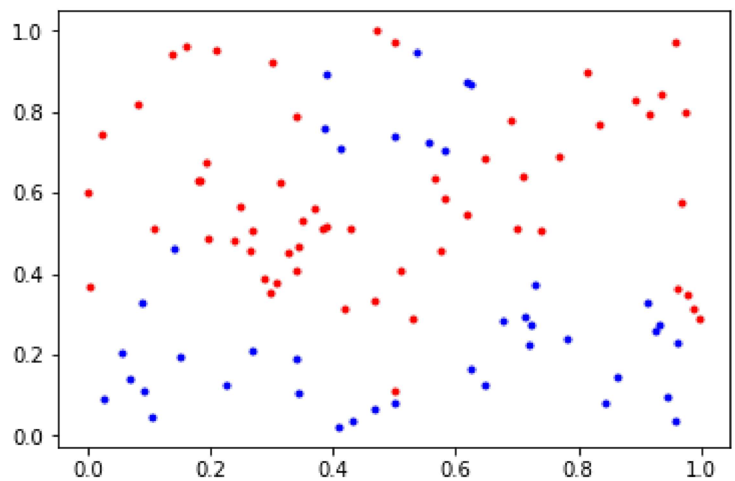
In [2]:
```
# Import Required Libraries
import numpy
import matplotlib.pyplot as plt
import cvxopt
```

In [3]:
```
# Parameters
Input_Samples = 100
Gaussian_Std_Deviation = 0.5
Order = 5
grid_size = 500
```

In [4]:
```
# Check if data exists. If yes load it otherwise create new data and store it
 in text file
if numpy.DataSource().exists('InputPattern.txt'):
    X = numpy.loadtxt('InputPattern.txt')
else:
    X = numpy.random.uniform(0, 1, (Input_Samples, 2))
    numpy.savetxt('InputPattern.txt', X)
```

In [5]:
```
# Create Desired Classification based on some conditions
# Which will give a circle and sine wave boundary and points lying inside them
 will be positive class, egative otherwise
D = numpy.array([])
Pos_Class = numpy.array([[0, 0]])
Neg_Class = numpy.array([[0, 0]])
for i in range(0, Input_Samples):
    if X[i, 1] < 0.2 * numpy.sin(10 * X[i, 0]) + 0.3 or numpy.square(X[i, 0] -
 0.5) + numpy.square(X[i, 1] - 0.8) < numpy.square(0.15):
        D = numpy.concatenate((D, [1]), axis=0)
        Pos_Class = numpy.concatenate((Pos_Class, [X[i]]), axis=0)
    else:
        D = numpy.concatenate((D, [-1]), axis=0)
        Neg_Class = numpy.concatenate((Neg_Class, [X[i]]), axis=0)
```

In [6]:
```
# Plot Results
plt.plot(Pos_Class[1:, 0], Pos_Class[1:, 1], 'b.', label=r'Class $C_1$')
plt.plot(Neg_Class[1:, 0], Neg_Class[1:, 1], 'r.', label=r'Class $C_{-1}$')
plt.show()
```



In [7]:
```
# Required Functions
def Gaussian_Kernel(X1, X2, sigma):
    return numpy.exp(-numpy.square(numpy.linalg.norm(X1 - X2))/(2 * numpy.squa
re(sigma)))
def Polynomial_Kernel(X1, X2, order):
    return (1 + numpy.dot(X1, X2))**order
```

In [8]:
```
# Optimization Problem - Quadratic Programming
P = numpy.empty([Input_Samples, Input_Samples])
for i in range(0, Input_Samples):
    for j in range(0, Input_Samples):
        P[i, j] = D[i] * D[j] * Gaussian_Kernel(X[i], X[j], Gaussian_Std_Devia
tion)
#         P[i, j] = D[i] * D[j] * Polynomial_Kernel(X[i], X[j], Order)
# Refer Documentation for Cvxopt Quadratic Programming for the meaning of the
 matrices
P = cvxopt.matrix(P)
q = cvxopt.matrix(numpy.ones((Input_Samples), dtype='double') * -1)
G = cvxopt.matrix(numpy.diag(numpy.ones((Input_Samples), dtype='double') * -1
))
h = cvxopt.matrix(numpy.zeros(Input_Samples))
A = cvxopt.matrix(D, (1, Input_Samples))
b = cvxopt.matrix(0.0)
sol = cvxopt.solvers.qp(P, q, G, h, A, b)
```

```
     pcost       dcost       gap    pres   dres
 0: -5.6916e+01 -1.6740e+02  1e+02  9e-16  3e+00
 1: -1.0023e+02 -1.4141e+02  4e+01  3e-15  1e+00
 2: -3.4382e+02 -3.9427e+02  5e+01  3e-14  1e+00
 3: -5.2478e+02 -5.9208e+02  7e+01  2e-14  1e+00
 4: -1.8508e+03 -2.0284e+03  2e+02  8e-14  1e+00
 5: -4.2056e+03 -4.5401e+03  3e+02  2e-13  1e+00
 6: -1.0996e+04 -1.1943e+04  9e+02  5e-13  1e+00
 7: -2.5028e+04 -2.8114e+04  3e+03  3e-12  1e+00
 8: -5.1677e+04 -6.1449e+04  1e+04  4e-12  9e-01
 9: -8.8447e+04 -1.0922e+05  2e+04  5e-12  6e-01
10: -1.0995e+05 -1.2477e+05  1e+04  2e-11  2e-01
11: -1.1231e+05 -1.1308e+05  8e+02  3e-11  7e-03
12: -1.1239e+05 -1.1241e+05  2e+01  1e-11  1e-04
13: -1.1240e+05 -1.1240e+05  3e-01  1e-11  1e-06
14: -1.1240e+05 -1.1240e+05  3e-03  2e-11  1e-08
Optimal solution found.
```

In [9]:

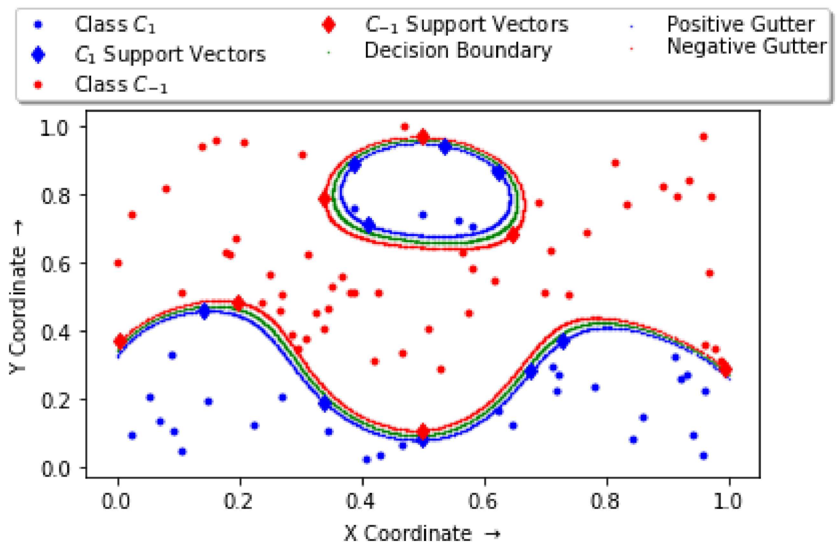
```
# Solution:
Alpha_Prime = numpy.ravel(numpy.array(sol['x']))
Alpha = numpy.array([0 if i < 1.0e-04 else i for i in Alpha_Prime]) # Making a
lpha's perfectly zeroes
SV_Pos_Class = numpy.array([[0, 0]]) # Support Vector for Positive Class
SV_Neg_Class = numpy.array([[0, 0]]) # Support Vecotr for Negative Class
SV_Label = numpy.array([]) # Labels for all Support Vectors, to decrease for l
oop time execution.
for i in range(0, Input_Samples):
    if Alpha[i] != 0:
        if D[i] == 1:
            SV_Pos_Class = numpy.concatenate((SV_Pos_Class, [X[i]]), axis=0)
            SV_Label = numpy.concatenate((SV_Label, [i]), axis=0)
        else:
            SV_Neg_Class = numpy.concatenate((SV_Neg_Class, [X[i]]), axis=0)
            SV_Label = numpy.concatenate((SV_Label, [i]), axis=0)
for k in range(0, Input_Samples):
    if Alpha[k] != 0:
        W = 0
        for i in numpy.nditer(SV_Label):
            W = W + Alpha[int(i)] * D[int(i)] * Gaussian_Kernel(X[int(i)], X[k
], Gaussian_Std_Deviation)
#             W = W + Alpha[int(i)] * D[int(i)] * Polynomial_Kernel(X[int(i)],
 X[k], Order)
        Theta = D[k] - W
        break
```

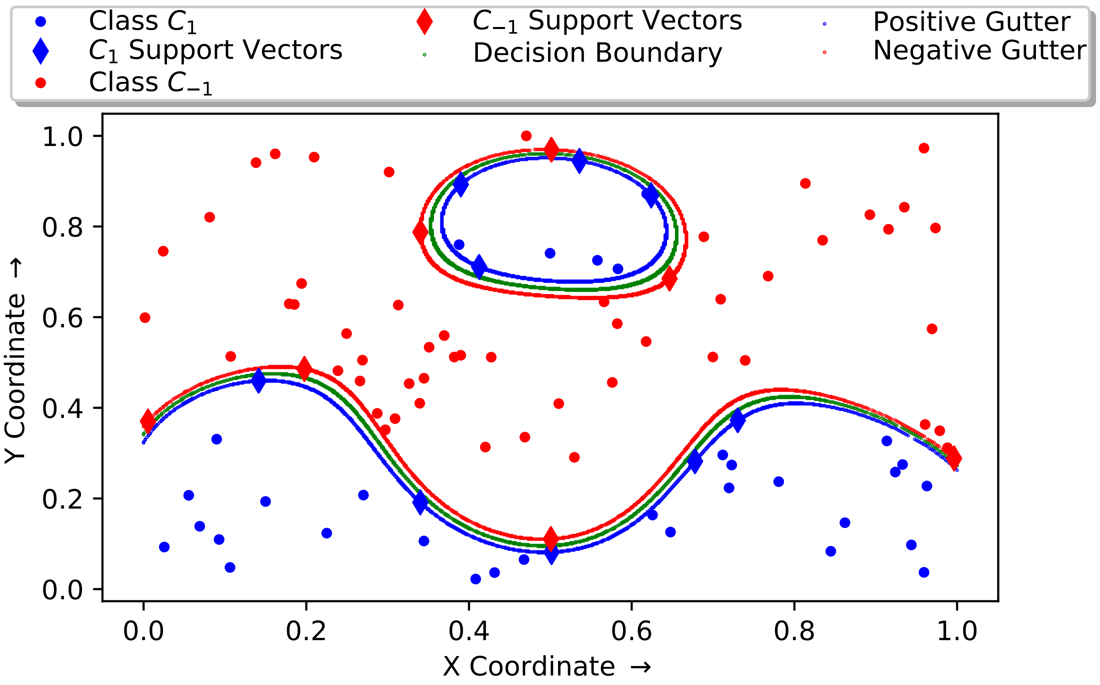
In [10]:

```
# Generate Boundary
x_points = numpy.linspace(0.0, 1.0, grid_size)
y_points = numpy.linspace(0.0, 1.0, grid_size)
H = numpy.array([[0, 0]]) # Decision Boundary
H_Plus = numpy.array([[0, 0]]) # Postivie Gutter
H_Minus = numpy.array([[0, 0]]) # Negative Gutter
for i in range(0, grid_size):
    for j in range(0, grid_size):
        Discriminant = 0
        temp = numpy.array([x_points[i], y_points[j]])
        for k in numpy.nditer(SV_Label):
            Discriminant = Discriminant + Alpha[int(k)] * D[int(k)] * Gaussian
_Kernel(X[int(k)], temp, Gaussian_Std_Deviation)
#             Discriminant = Discriminant + Alpha[int(k)] * D[int(k)] * Polyno
mial_Kernel(X[int(k)], temp, Order)
        Discriminant = Discriminant + Theta
        if -0.1 < Discriminant < 0.1:
            H = numpy.concatenate((H, [temp]), axis=0)
        elif -1.1 < Discriminant < -0.9:
            H_Minus = numpy.concatenate((H_Minus, [temp]), axis=0)
        elif 0.9 < Discriminant < 1.1:
            H_Plus = numpy.concatenate((H_Plus, [temp]), axis=0)
```

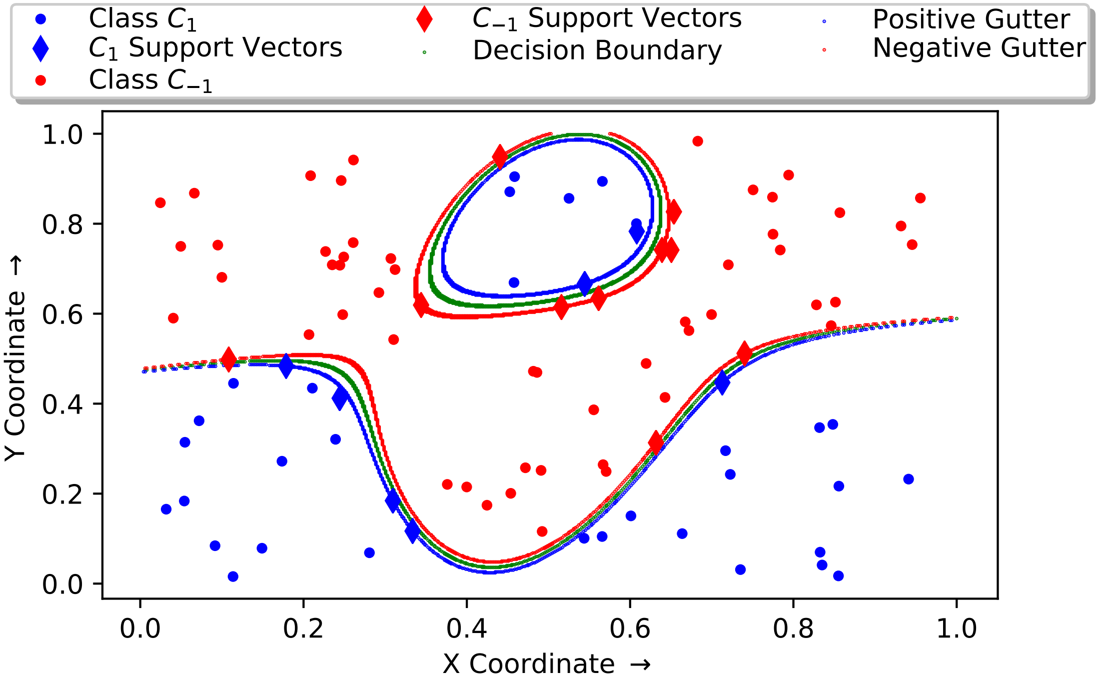
In [11]:

```
# Plot Results
plt.plot(Pos_Class[1:, 0], Pos_Class[1:, 1], 'b.', label=r'Class $C_1$')
plt.plot(SV_Pos_Class[1:, 0], SV_Pos_Class[1:, 1], 'bd', label=r'$C_1$ Support
 Vectors')
plt.plot(Neg_Class[1:, 0], Neg_Class[1:, 1], 'r.', label=r'Class $C_{-1}$')
plt.plot(SV_Neg_Class[1:, 0], SV_Neg_Class[1:, 1], 'rd', label=r'$C_{-1}$ Supp
ort Vectors')
plt.scatter(H[1:, 0], H[1:, 1], s=0.1, c='g', label='Decision Boundary')
plt.scatter(H_Plus[1:, 0], H_Plus[1:, 1], s=0.1, c='b', label='Positive Gutte
r')
plt.scatter(H_Minus[1:, 0], H_Minus[1:, 1], s=0.1, c='r', label='Negative Gutt
er')
plt.xlabel(r'X Coordinate $\rightarrow$')
plt.ylabel(r'Y Coordinate $\rightarrow$')
plt.legend(loc='lower center', bbox_to_anchor=(0.5, 1), fancybox=True, shadow=
True, ncol=3, borderpad=0.1, labelspacing=0.1)
plt.tight_layout()
plt.savefig('Results.pdf')
plt.show()
```





Class C_1
C_1 Support Vectors
Class C_{-1}
C_{-1} Support Vectors
Decision Boundary
Positive Gutter
Negative Gutter
1.0
0.8
0.6
0.4
0.2
0.0
Y Coordinate →
0.0
0.2
0.4
0.6
0.8
1.0
X Coordinate →



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Decision Boundary
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Y Coordinate →
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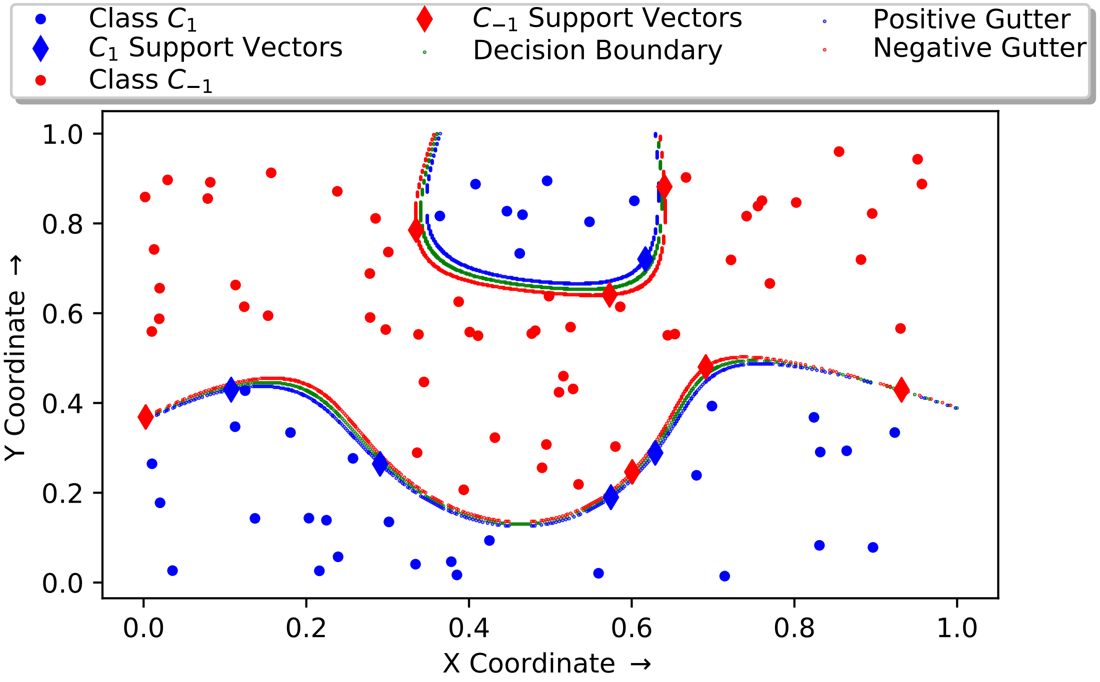
0.6
0.8
1.0
X Coordinate →



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C_1 Support Vectors
Class C_{-1}
C_{-1} Support Vectors
Decision Boundary
Positive Gutter
Negative Gutter
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0.8
0.6
0.4
0.2
0.0
Y Coordinate →
0.0
0.2
0.4
0.6
0.8
1.0
X Coordinate →