**ME518 - Final project**

Due by 12/15/2017

**Problem 1** (3 points)

Write a code to calculate numerically $u^+(y^+)$ using Van Driest formula:

$$u^+(y^+) = \int_0^{y^+} \frac{2}{1+\sqrt{1+4\,l_m^+(y')^2}}\,dy' \tag{1}$$

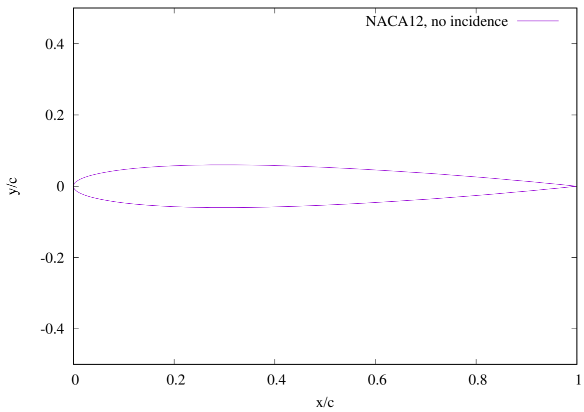
where the mixing length $l_m^+$ varies as

$$l_m^+(y^+) = ky^+[1-\exp(-y^+/A^+)] \quad \text{with } k=0.41,\ A^+=26. \tag{2}$$

Output of problem 1: Plot the graph $u^+(y^+)$ and compare the numerical solution with the approximate solutions valid in the viscous sublayer and in the log-law layer.

**Problem 2** (7 points)

The external velocity distribution $U_e(x)/U_\infty$ for a symmetric NACA0012 airfoil is given in the file naca12_airfoil_0000.dat.




Compute the complete boundary-layer development (laminar and turbulent) on this airfoil for a chord Reynolds number $Re_c = c\,U_\infty/\nu = 6.67\times10^6$. Assume $c = 1\,\mathrm{m}$ and $\nu = 1.5\,10^{-5}\mathrm{m}^2/\mathrm{s}$.

• For the laminar portion use the semi-empirical Thawites's method to calculate the boundary layer integral proprieties: the momentum thickness distribution is modeled as

$$\frac{1}{\nu}\frac{d}{dx}\left(\theta^2 U_e^6\right) = 0.45\,U_e^5 \tag{3}$$

and for a boundary layer starting at a stagnation point on the airfoil ($x_0 \equiv 0$ in this case with no incidence), it can be shown from potential theory and perturbation analysis that

$$\theta_0^2 = \frac{0.075\nu}{(dU_e/dx)_0} \quad \text{for } x=0 \tag{4}$$

$$\theta^2(x) = \frac{0.45\nu}{U_e^6(x)}\int_0^x U_e^5(x')dx' \quad \text{for } x>0 \tag{5}$$

The flatness factor $H \equiv \delta^*/\theta$ is calculated using the empirical correlations

$$
\begin{aligned}
H(x) &= 2.61 - 3.75\lambda(x) + 5.24\lambda^2(x) \quad 0 < \lambda < 0.1 && (6)\\
H(x) &= 2.088 + \frac{0.0731}{\lambda(x) + 0.14} \quad -0.1 < \lambda < 0 && (7)
\end{aligned}
$$

where the Thwaites' parameter is $\lambda(x) = \dfrac{\theta^2}{\nu}\dfrac{dU_e}{dx}$.

• For the turbulent portion use the semi-empirical Head's method to calculate the boundary layer integral proprieties. Define

$$
\begin{aligned}
H &\equiv \frac{\delta^*}{\theta} && (8)\\
H_1 &\equiv \frac{\delta - \delta^*}{\theta} && (9)
\end{aligned}
$$

and assume the following dependence

$$
\frac{1}{U_e}\frac{d}{dx}\left(U_e(\delta - \delta^*)\right) \equiv \frac{1}{U_e}\frac{d}{dx}\left(U_e H_1 \theta\right) = F(H_1) \tag{10}
$$

with

$$
\begin{aligned}
F(H_1) &= \frac{0.0306}{(H_1 - 3.0)^{0.6169}} && (11)\\
H_1 = G(H) &= 3.0445 + \frac{0.8702}{(H - 1.1)^{1.2721}} && (12)
\end{aligned}
$$

Expand Eq. 10 and combine with the Von Karman integral momentum equation,

$$
\frac{d\theta}{dx} + (2 + H)\frac{\theta}{U_e}\frac{dU_e}{dx} = \frac{C_f}{2} \tag{13}
$$

to get

$$
\frac{dH}{dx} = \left( \frac{(1 + H)\dfrac{G(H)}{U_e}\dfrac{dU_e}{dx} - \dfrac{G(H)}{\theta}\dfrac{C_f}{2} + \dfrac{0.0299}{\theta\,(G(H) - 3.0)^{0.6169}}}{\dfrac{dG}{dH}} \right) \tag{14}
$$

The drag coefficient is given by

$$
C_f = \frac{0.246}{10^{0.678\,H}\left(\dfrac{U_e}{U_\infty}R_\theta\right)^{0.268}}\,, \quad R_\theta = \frac{U_\infty \theta}{\nu} \tag{15}
$$

Equations 13, 14 supplemented with Eq. 15 constitue two equations in $\theta$ and $H$ that completely solve the boundary layer: the displacement thickness $\delta^*$ and the boundary layer thicknesses $\delta$ are readily deduced from Eqs. 8-9.

Output of problem 2: Integrate numerically Eqs. 13, 14 and plot $\delta$, $\delta^*$ and $\theta$ as a function of the chord abscissa along the profile.

NOTE 1: Assume that transition to turbulence occurs at $x/c = 0.2$, hence switch the integration of $\theta$ from Thwaites' to Head's method for $x/c > 0.2$. To initialize integration of Eqs. 13, 14 in the turbulent portion of the BL, use the value obtained in the laminar portion: $\theta_{\text{init}} = \theta_{\text{lam}}(x/c = 0.2)$ where $\theta_{\text{lam}}$ is obtained from Eq. 5. Also assume that $H_{\text{init}} = 1.4$.

NOTE 2: Use any integration scheme (but at least second order) for the integration (ideally Runge-Kutta 4 is excellent). Also use any language or numerical package of your preference, e.g. Fortran,C++, or Matlab, etc.