Table 6(a) Values of $F_{0.05}$

ν_2 = Degrees of Freedom for Denominator	ν_1 = Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.50	2.47	2.38	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.60	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.51	1.46	1.39	1.32	1.22	1.00

Table 6(b) Values of $F_{0.01}$

ν_2 = Degrees of Freedom for Denominator	ν_1 = Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,240	6,261	6,287	6,313	6,339	6,366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.57	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.93	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.77	1.70	1.59	1.47	1.32	1.00

Hypothesis:

To test whether the two sample are selected from samp population having specified variance or not, we use test of hypothesis concerning two variances. or the variances of the two population sampled are equal. In this section we describe the test of hypothesis concerning two variances.

Let s_1^2 ad s_2^2 are the variances of two samples of sizes n_1 ad n_2 respectively. The population variances are σ_1^2 & σ_2^2. Here the population is normal.

Step 1: Null hypothesis: H_0 $\sigma_1^2 = \sigma_2^2$

Step 2: Alternative hypothesis: $\sigma_1^2 < \sigma_2^2$ or

$\sigma_1^2 > \sigma_2^2$ or

$\sigma_1^2 \neq \sigma_2^2$

Step 3: L.O.S $= \alpha = 5\%$ or 1%.

Step 4: degrees of freedom $\nu_1 = n_1 - 1$, $\nu_2 = n_2 - 1$

Step 5: Test statistic:

$$F = \frac{s_2^2}{s_1^2} \text{ or } \frac{s_1^2}{s_2^2} \text{ or } \frac{s_M^2}{s_m^2}.$$

Step 6: Criterion

Step 7: Conclusion.

Critical Regions for testing $\sigma_1^2 = \sigma_2^2$

Alternative hypothesis	Test Statistic	Rejecting Region for Ho.
$\sigma_1^2 < \sigma_2^2$	$F = \frac{S_2^2}{S_1^2}$	$F_{cal} > F_\alpha(n_2-1, n_1-1)$
$\sigma_1^2 > \sigma_2^2$	$F = \frac{S_1^2}{S_2^2}$	$F_{cal} > F_\alpha(n_1-1, n_2-1)$
$\sigma_1^2 \neq \sigma_2^2$	$F = \frac{S_M^2}{S_m^2}$	$F > F_{\alpha/2}(n_M-1, n_m-1)$

Pb: It is desired to determine whether there is less variability in the silver plating done by Company 1 than in that done by Company 2. If independent random samples of size 12 of the two companies work yield $S_1 = 0.035$ mil and $S_2 = 0.062$ mil, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $\sigma_1^2 < \sigma_2^2$ at the 0.05 LOS.

Sol: + Given that $n_1 = 12$, $n_2 = 12$, $S_1 = 0.035$, $S_2 = 0.062$.

Ho: Null hypothesis : Ho : $\sigma_1^2 = \sigma_2^2$

Alternative hypothesis: H_1 : $\sigma_1^2 < \sigma_2^2$

LOS : $\alpha = 0.05$.

06/05/2020 (T)

d.f: $n_1-1 = 11$, $n_2-1 = 11$.

~~Criterion:~~

Statistic: $F = \frac{S_2^2}{S_1^2} = \frac{(0.062)^2}{(0.035)^2} = 3.14$.

Criterion: $F_{0.05}(11,11) = 2.82$

If $F_{cal} > 2.82$, Reject Ho. ~~oth~~

Otherwise Accept Ho

Conclusion: Since $F_{cal} = 3.14 > 2.82$.

We Reject null hypothesis.

(i.e) We have significance evidence that the plating done by Company 1 is less variance than that by Company 2.

P.

Problem: In two independent samples of size 8 and 10 the sum of squares of deviation of the sample values from the respective sample means were 84.4 and 102.6. Test whether the difference of variances of the population is significant or not. Use 0.05 L.O.S.

Sol: Let σ_1^2 and σ_2^2 be the variances of the two normal populations from which the samples are drawn.

Null Hypothesis, $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$

$n_1 = 8, \quad n_2 = 10.$

Also given that $\sum'(x_i - \bar{x})^2 = 84.4$

$\sum'(y_i - \bar{y})^2 = 102.6$

If s_1^2 and s_2^2 be the estimates of σ_1^2 and σ_2^2

then $s_1^2 = \frac{1}{n_1 - 1} \sum'(x_i - \bar{x})^2 = 12.057$

$s_2^2 = \frac{1}{n_2 - 1} \sum'(y_i - \bar{y})^2 = 11.4.$

F-statistic $= \frac{s_1^2}{s_2^2} \quad (\because s_1^2 > s_2^2)$

$= \frac{12.057}{11.4} = 1.057$

$F_{cal} = 1.057$

$F_{tab} = F(n_1 - 1, n_2 - 1) = F(7, 9)$ at 5% LOS.

$= 3.29$ (From $F_{0.05}$ table)

If $F_{cal} < F_{tab}$ accept H_0

Since $F_{cal} = 1.057 < F_{tab} = 3.29$

Hence Accept null hypothesis.

Hence The populations have the same Variance.

11/05 (2)

Pb:2) It is known that the mean diameters of rivets produced by two firms A and B are practically the same, but the S.D. may differ. For 22 rivets produced by firm A, the S.D. is 2.9 mm while for 16 rivets manufactured by firm B, the S.D. is 3.8 mm. Compute the statistic you would use to test whether the products of firm A have the same variability as those of firm B and test its significance.

Sol:

Given that $n_1 = 22$, $n_2 = 16$

$S_1 = 2.9$ $S_2 = 3.8$

Null hypothesis : $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative hypothesis : $H_1 : \sigma_1^2 \neq \sigma_2^2$

α : 5% L.O.S.

$$F_{cal} = F = \frac{S_2^2}{S_1^2} = \frac{(3.8)^2}{(2.9)^2} = 1.717.$$

$$F_{tab}(16-1, 22-1) = F(n_2-1, n_1-1) = F(15, 21)$$

$$= 2.18$$

Since $F_{cal} = 1.717 < F_{tab} = 2.18$ at 5% L.O.S.

We Accept the null hypothesis ~~at 5%~~

(i.e) Two samples ~~are~~ from Firm A and Firm B have Same variability.

Scanned with CamScanner

Pb:3) Time taken by the workers in performing a job by method I and method II is given below.

Method - I	20	16	26	27	23	22	—
Method - II	27	33	42	35	32	34	38

Do the data show that the variances of time distribution from population from which these samples are drawn do not differ significantly?

Sol:
$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
20	-2.3	5.29	27	-7.4	54.76
16	-6.3	39.69	33	-1.4	1.96
26	3.7	13.69	42	7.6	57.76
27	4.7	22.09	35	0.6	0.36
23	0.7	0.49	32	-2.4	5.76
22	-0.3	0.09	34	-0.4	0.16
			38	3.6	12.96
134		81.34	241		133.72

$\bar{x} = \frac{\Sigma_i x_i}{n_1} = 22.3$, $\bar{y} = \frac{\Sigma_i y_i}{n_2} = 34.4$

$s_1^2 = \frac{\Sigma_i (x_i - \bar{x})^2}{n_1 - 1} = \frac{81.34}{5} = 16.26$

$s_2^2 = \frac{\Sigma_i (y_i - \bar{y})^2}{n_2 - 1} = \frac{133.72}{6} = 22.29$

Since $s_2^2 > s_1^2$

$$F_{cal} = F = \frac{s_2^2}{s_1^2} = \frac{22.29}{16.268} = 1.3699 \simeq 1.37.$$

$$F_{tab} \text{ at } (n_2-1, n_1-1) = F(6,5) = 4.95$$

Since $F_{cal} = 1.37 < 4.95 = F_{tab}$.

we accept H_0.

There is no significant difference b/w the variances

Pb: The nicotine contents in milligrams in two samples of tobacco were found to be as follows

Sample A	24	27	26	21	25	—
Sample B	27	30	28	31	22	36

Can it be said that the two samples have come from the same normal population?

Sol:

$n_1 =$ $\qquad$ $n_2 =$

$\bar{x} =$ $\qquad$ $\bar{y} =$

$s_1^2 =$ $\qquad$ $s_2^2 =$

$$F = \frac{\quad}{\quad} = 4.075.$$

$F_{cal.} = 4.075$

$F_{tab} =$

Conclusion: