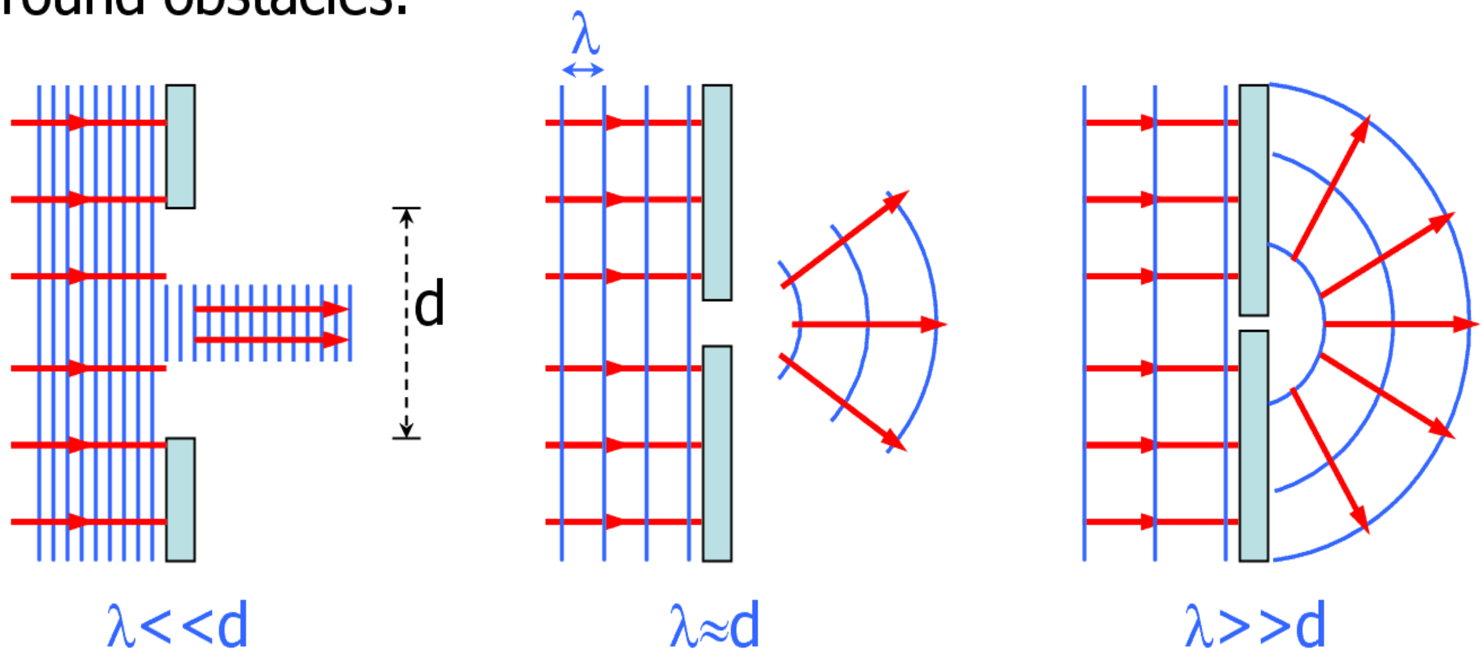
Introduction to Diffraction of Light

Diffraction

Light is an electromagnetic wave, and like all waves, “bends” around obstacles.

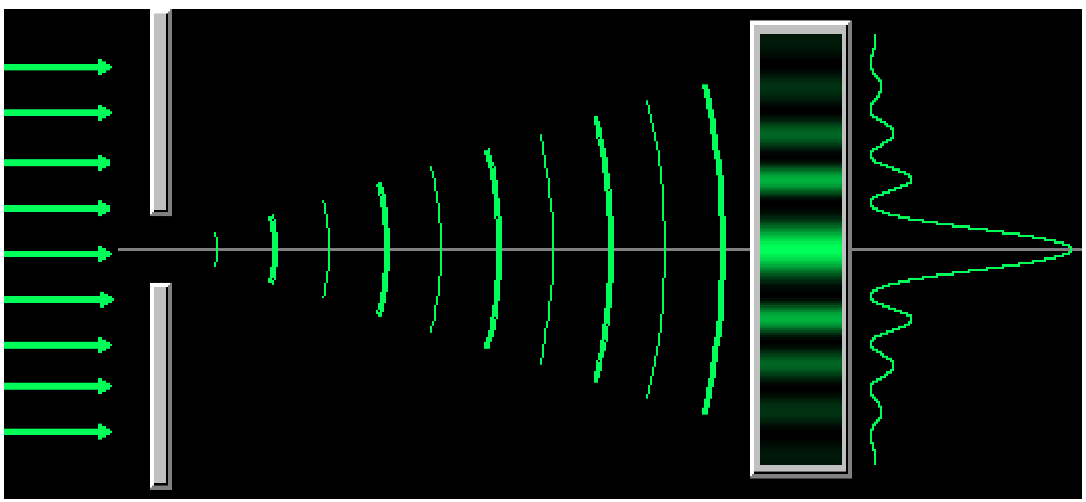


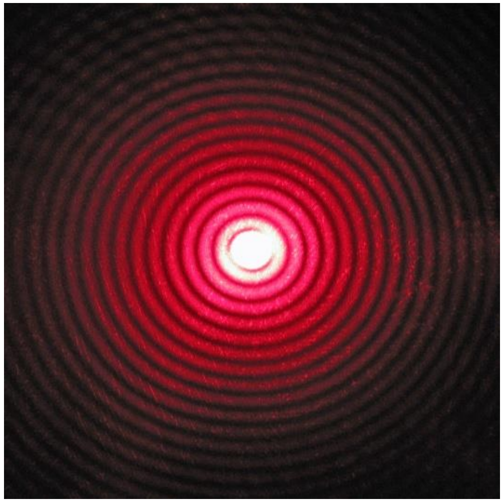
Note: this doesn’t mean light can’t “fit” through the slit!

This “bending,” which is most noticeable when the dimension of the obstacle is close to the wavelength of the light, is called “diffraction.” Only waves diffract.

Diffraction

When light falls on an obstacle or small aperture whose size is comparable with wavelength of light, there is a departure or deviation from the straight line propagation, the light bends round the corners of the obstacle or aperture and enters in the geometrical shadow. This bending of light is called diffraction.





The diffraction phenomenon was interpreted by Fresnel. According to Fresnel, the diffraction phenomenon is due to mutual interference of secondary wavelets originating from various points of wave front which are not blocked off by the obstacle. It should be remembered that the diffraction effects are observed only when a portion of wave front is cut off by some obstacle.

Difference between Interference and Diffraction:

Interference	Diffraction
1. Interference is due to the superposition of two different wave trains coming from coherent sources.	1. Diffraction is due to the superposition of secondary wavelets from the different parts of the same wavefront.
2. Fringe width is generally constant.	2. Fringes are of varying width.
3. All the maxima have the same intensity.	3. The maxima are of varying intensities.
4. There is a good contrast between the maxima and minima.	4. There is a poor contrast between the maxima and minima.

Types of Diffraction

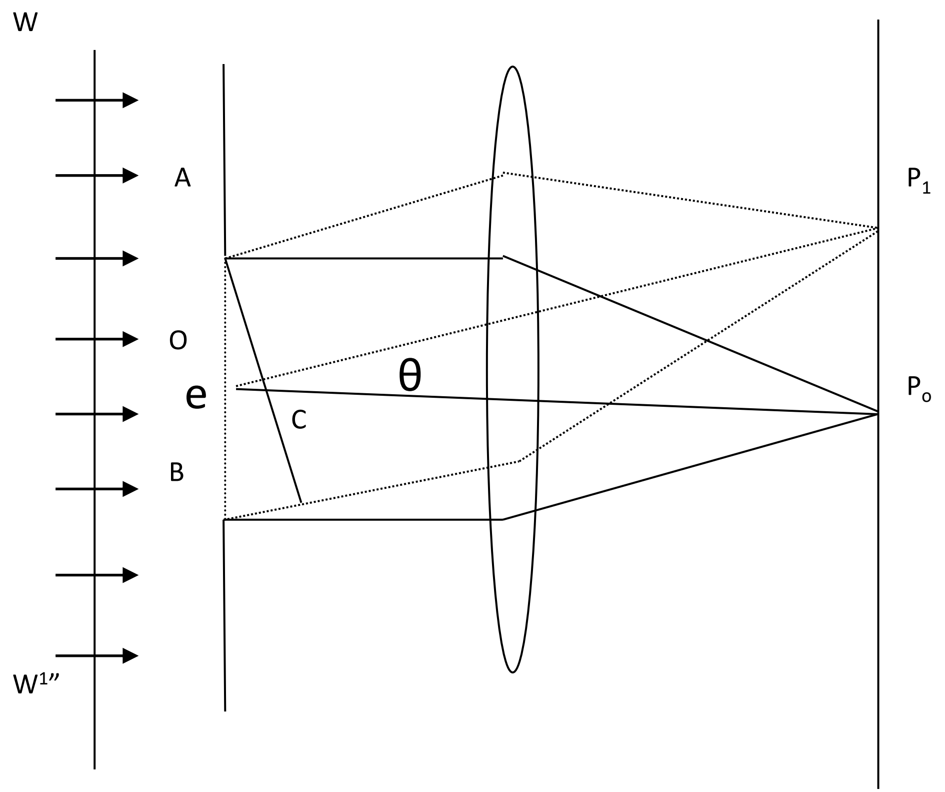
Diffraction phenomena can be divided into the following two general classes

(1) ***Fresnel diffraction***: source and screen are placed at a finite distances from the aperture or obstacle having sharp edges. No lenses are used for making the rays parallel or convergent. The incident wave front is either spherical or cylindrical.

(2) ***Fraunhofer's diffraction***: source and screen are placed at infinity. The wave front which is incident on the aperture or obstacle is plane. Here lenses are used.

Fraunhofer's diffraction due to single slit

The path difference between secondary wavelets A and B in direction θ = BC = AB sin θ = e sin θ and corresponding phase difference = $\frac{2\pi}{\lambda} e\sin\theta$



n equal parts and the amplitude of the wave from each part is a. The phase difference between any two consecutive waves from these parts would be =

$\frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right)$ **= d (say)**

Using the vector additional method the resultant amplitude R is given by

$$R = a\,\frac{\sin n\frac{d}{2}}{\sin\frac{d}{2}} = a\,\frac{\sin\frac{(\pi e\sin\theta)}{\lambda}}{\sin\frac{(\pi e\sin\theta)}{n\lambda}}$$

Let $$\frac{(\pi e \sin\theta)}{\lambda} = \alpha$$

$$R = a \frac{\sin \alpha}{\sin \frac{\alpha}{n}}$$

$$R = a \frac{\sin \alpha}{\frac{\alpha}{n}}$$

$$R = na \frac{\sin \alpha}{\alpha}$$

$$R = A \frac{\sin \alpha}{\alpha}$$

$$\boxed{\mathbf{I = R^2 = A^2\left(\frac{\sin \alpha}{\alpha}\right)^2}}$$

Principle maxima: **The expression for the resultant amplitude R can be written in ascending powers of α is as**

$$R = \frac{A}{\alpha}\left[\alpha - \alpha^3/3! + \alpha^5/5! - \alpha^7/7! + \ldots\ldots\ldots\right]$$

$$R = A\left[1 - \alpha^2/3! + \alpha^4/5! - \alpha^6/7! + \ldots\ldots\ldots\right]$$

If the α terms vanish, the value of R will be maximum, i.e.,

$$\alpha = 0$$

$$\alpha = \frac{(\pi e \sin\theta)}{\lambda} = 0, \text{ or } \sin\theta = 0, \theta = 0$$

Now maximum value of R is A and the intensity is proportional to A^2. The condition θ = 0, means that this maximum is formed by those secondary wavelets which travel normal to the slit. This maximum is known as principle maximum.

- ***Minimum intensity positions***: The intensity will be minimum when = 0 but $\alpha \neq 0$.
- The values of α which satisfy this equation are $\alpha = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots\ldots\ldots = \pm m\pi$
- Or $\frac{(\pi e \sin\theta)}{\lambda} = \pm m\pi$, or $e \sin\theta = \pm m\lambda$, where m

  = 1, 2, 3, ...
- In this way we obtained the points of minimum intensity on either side of the principle maximum. The value of m = 0 is not admissible, because for this value θ will be zero and this corresponds to principle maximum.
-

Secondary maxima: **In addition to the principle maximum at α = 0, there are weak secondary maxima between equally spaced minima. The positions can be obtained by differentiating the expression I with respect to α equating to zero. We have,**

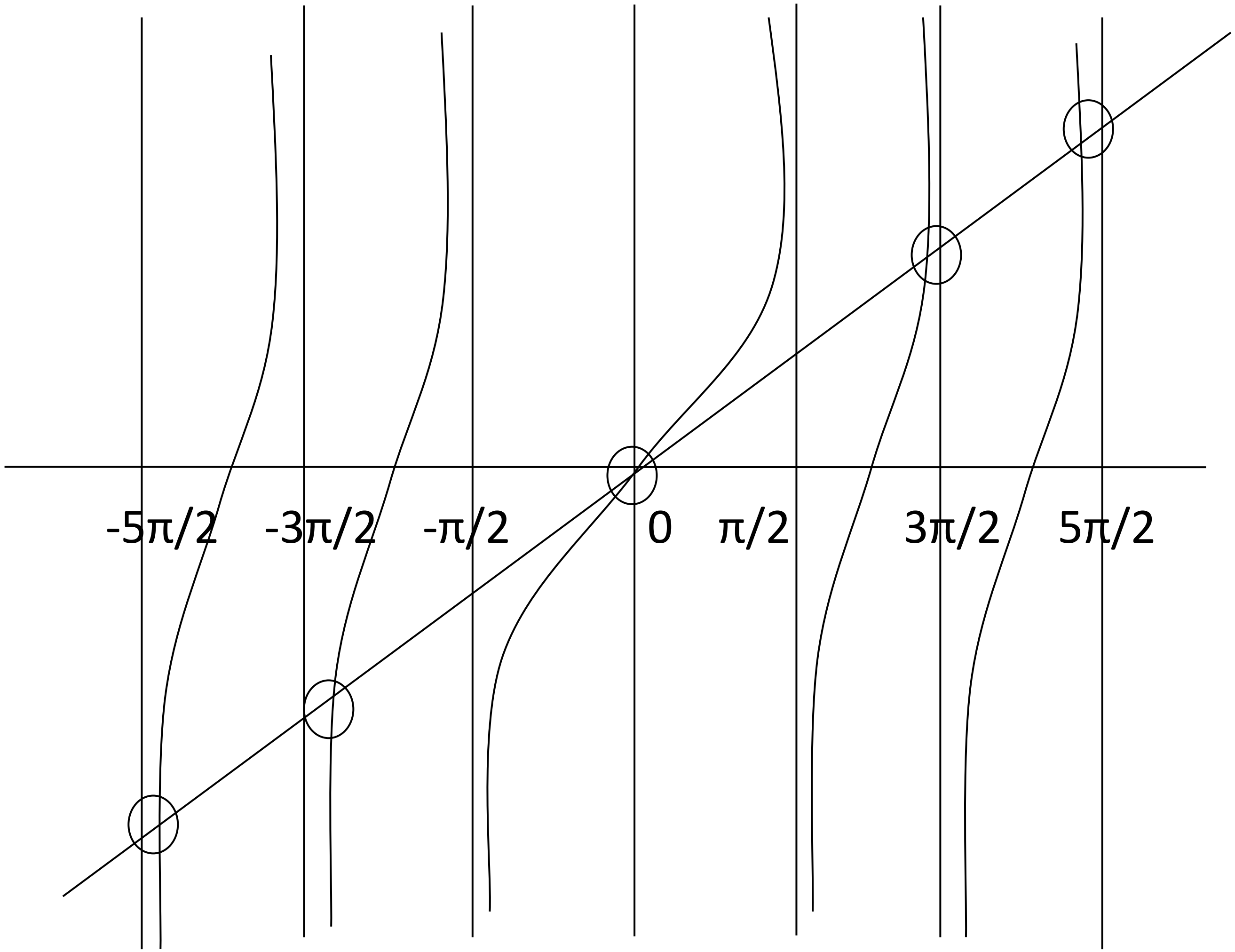
$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2\,\frac{2Sin\alpha}{\alpha}\,\left(\frac{\alpha Cos\alpha - Sin\alpha}{\alpha^2}\right) = 0$$

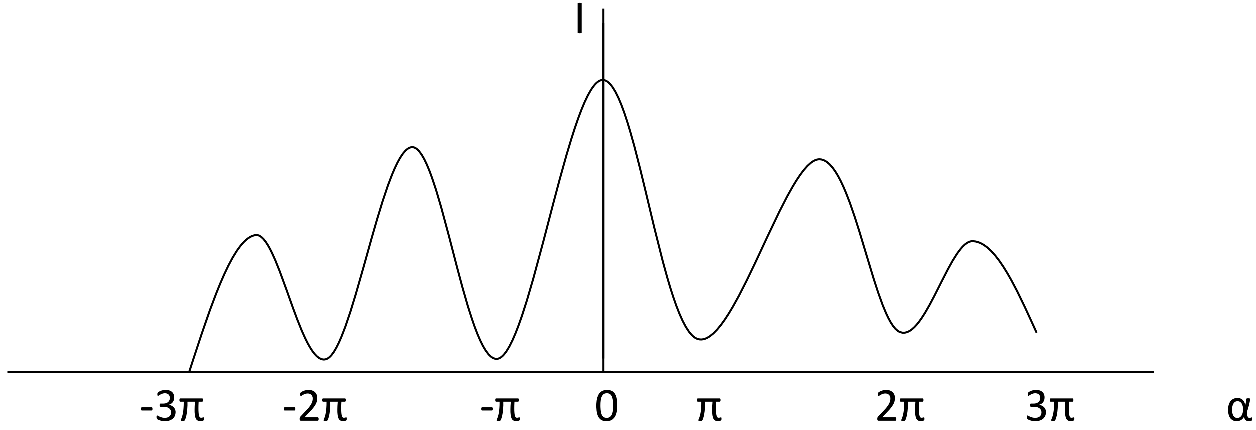
Either sin α = 0 or $\alpha Cos\alpha - Sin\alpha$ **= 0**

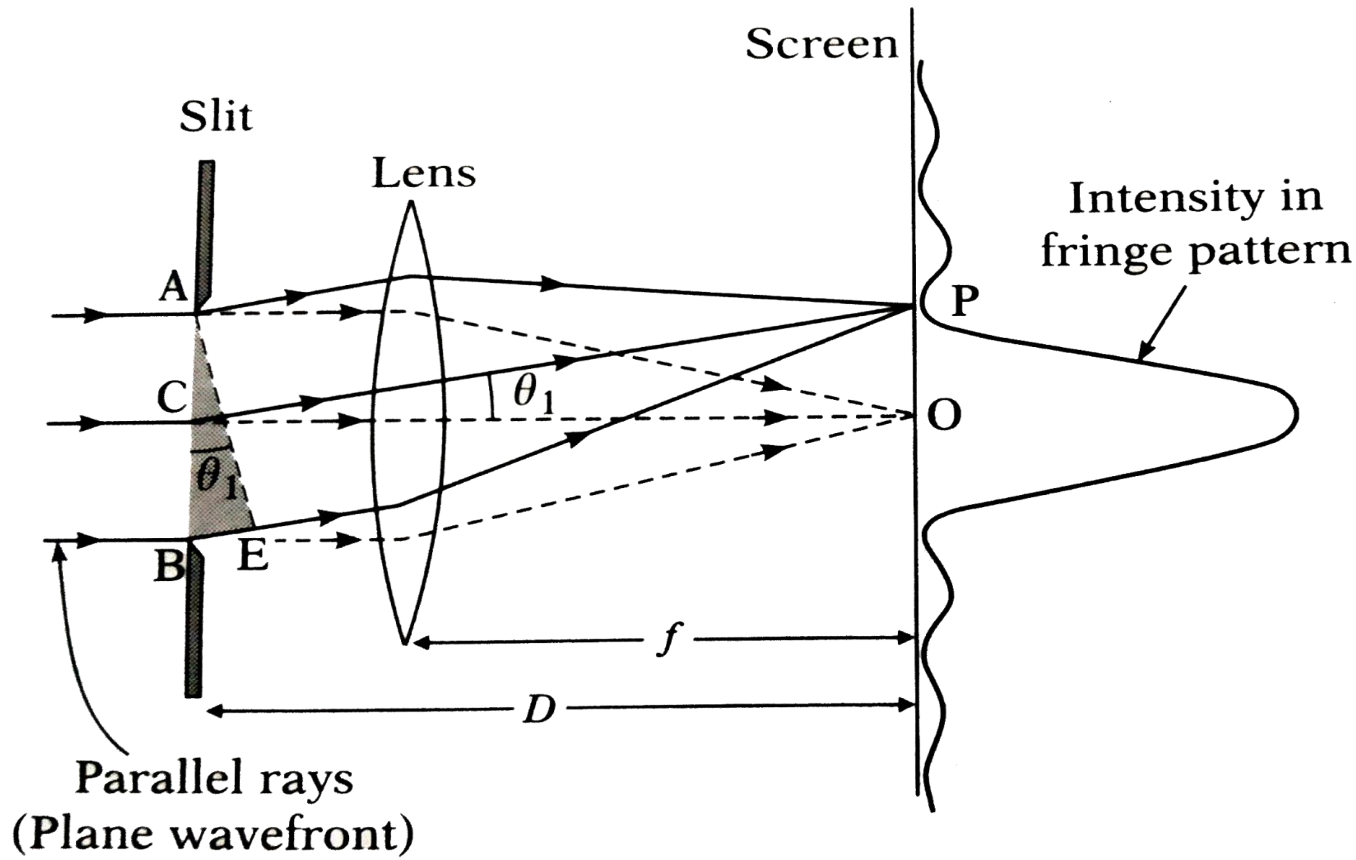
The equation sin α = 0 gives the values of α (except 0) for which the intensity is zero on the screen. Hence the positions of the maxima are given by the roots of the equation = 0 or α = Tanα

The values of α satisfying the above equation are obtained graphically by plotting the curves y = α, y = Tan α on the same graph

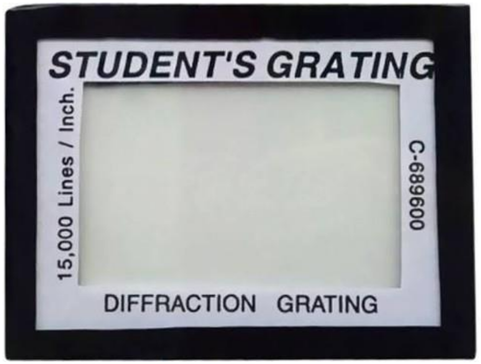


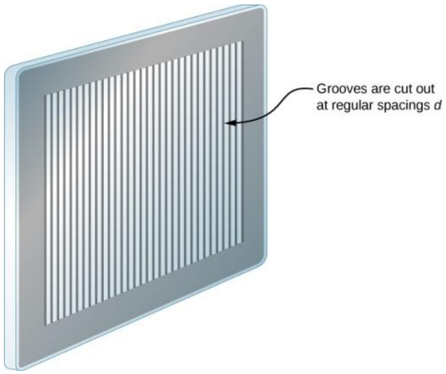
The point of intersection of two curves give the value of α which satisfy the above equation. The plots of y = α and y = Tan α are shown in figure. The points of intersections are α = ± 3π/2, ± 5π/2, ± 7π/2, ...

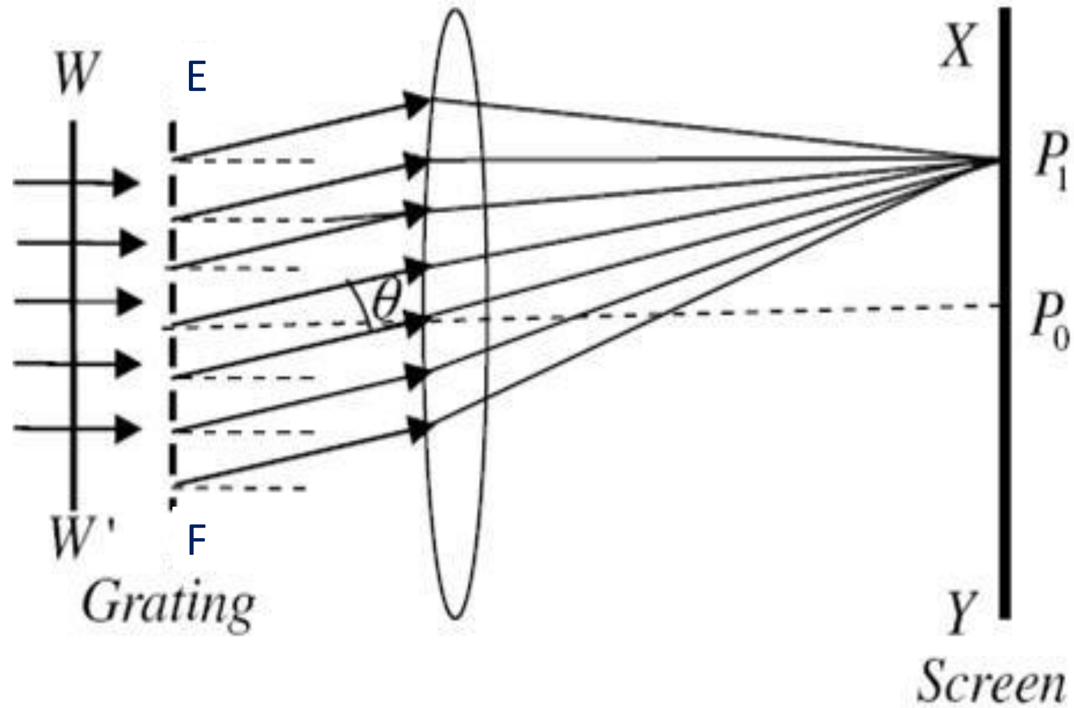

I
-3π
-2π
-π
0
π
2π
3π
α


Screen
Slit
Lens
Intensity in fringe pattern
A
P
C
θ1
O
θ1
B
E
f
D
Parallel rays (Plane wavefront)

Fraunhofer's diffraction due to grating



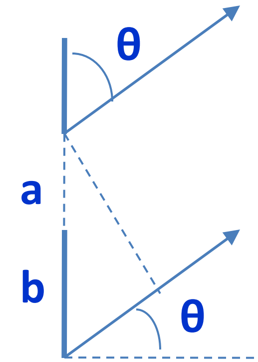


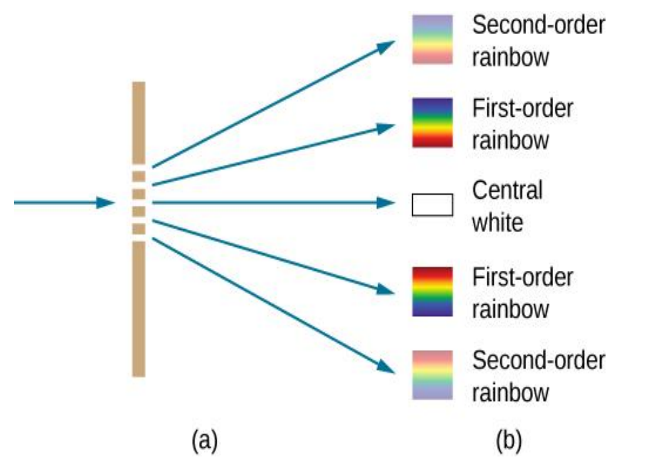


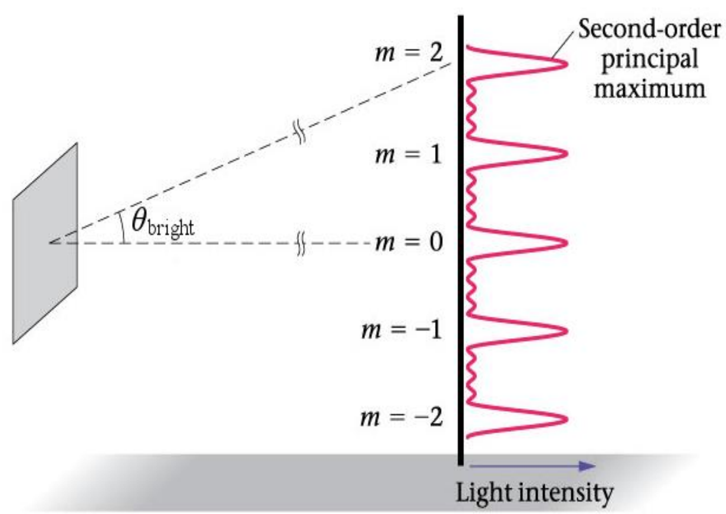
- Consider a plane transmission grating of N slits per cm placed perpendicular to the plane of the paper.
- Let e be the width of each slit and d be the width of each opaque part then (e+d) is known as grating element.
- XY is the screen placed perpendicular to the plane of the paper.
- Suppose a parallel beam of monochromatic light of wavelength λ be incident normally on the grating.
- By Huygen's principle, each of the infinite points in the slits sends secondary wavelets in all directions.

- The secondary wavelets travelling in the same direction of incident light will come to focus at a point P_o of the screen.
- Now considering the secondary wavelets travelling in a direction incident at an angle θ with the direction of the incident light, these waves reach a point P_1 on the passing through the convex lens in different phases.
- As a result dark and bright fringes appear on both sides of the central maximum.
- The intensity at P_1 will depend on the path difference between the secondary waves originating from the respective points A and C of two neighbouring slits.

- In the figure width of slit = a and width of opeque space = b. The path difference between the secondary waves starting from A and C is equal to AC sin θ.
- But AC = AB+BC = a+b; path difference = AC sin θ = (a+b) sin θ
- The point P_1 will be maximum intensity if this path difference is equal to integral multiples of λ.
- In general $(a+b)\sin\theta_m = m\lambda$
- where θ_m is the direction of the m^{th} principal maximum. The minimum is obtained in between two maxima. Similar maxima and minima are obtained on either side of the central maximum. Thus, on each side of the central maximum at P, principal maxima and minimum intensity are observed due to diffracted light.
- The position of the m^{th} minima is given by $(a+b)\sin\theta_m = (2m+1)\lambda/2$







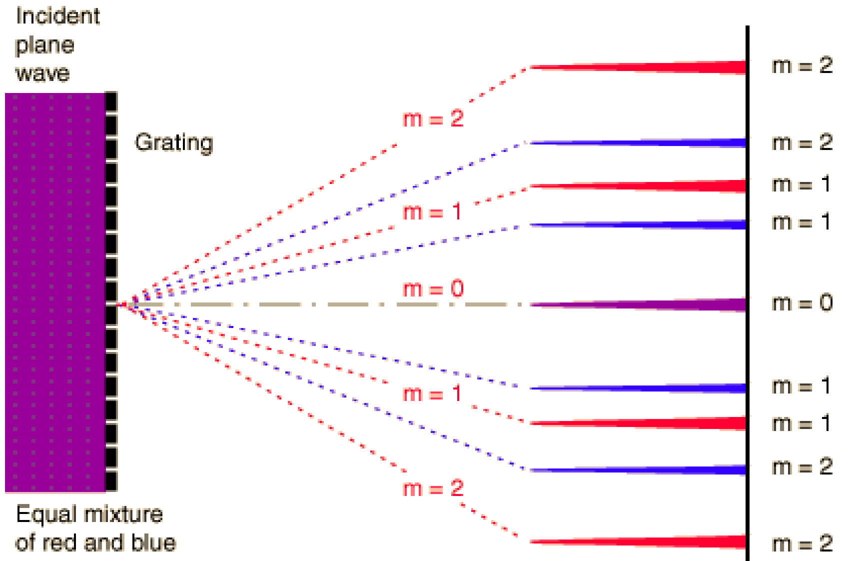
When illuminated by a beam of monochromatic radiation, the system produces N wavelets at an angle θ, each of the amplitude = $A\left(\frac{\sin\alpha}{\alpha}\right)$.

The phase difference between successive wavelets $\delta = 2\pi d\sin\theta/\lambda$

In order to know the resultant intensity at each point along with their positions, the following equation is used.

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

where $\beta = \pi d\sin\theta/\lambda$



Principal maxima: When Sin β = 0; we get the maximum intensity. Therefore $\beta = \pm n\pi$ (n = 0, 1, 2, 3, ...)

After applying the L'Hospital rule, we obtain the resultant intensity for maxima, i.e. $I = A^2(\frac{Sin\ \alpha}{\alpha})^2 N^2$

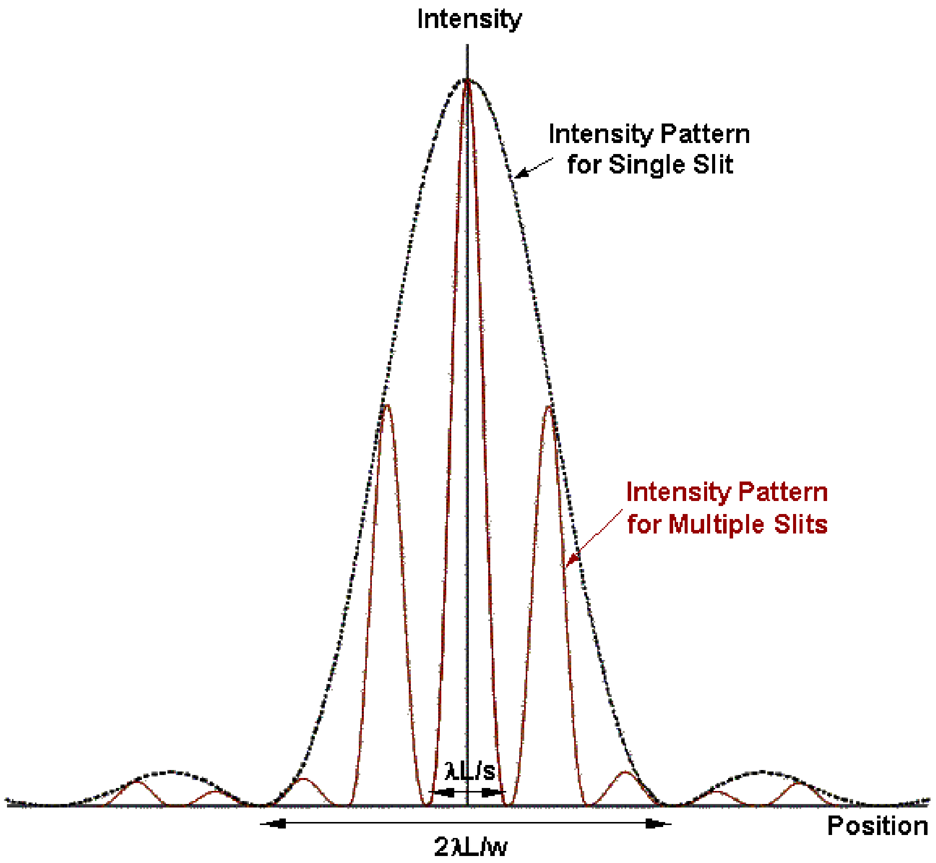
As these maxima are very intense, they are called principle maxima. The equation for the direction of the principal maxima is $\boxed{(a+b)\sin\theta = \pm n\lambda}$ which is known as grating equation.

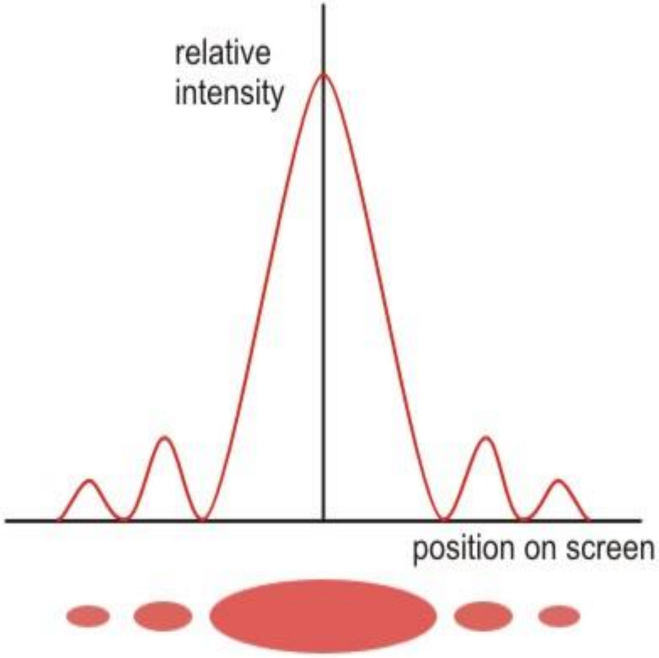
In another words $\sin\theta = \pm nN\lambda$, where $N = 1/(a+b)$, no. of lines per cm

Minima: The intensity is zero when $\sin N\beta = 0$, but $\beta \neq 0$
$N\beta = \pm m\pi$ where m can take all integral values except 0, N, 2N, 3N, ...because these values give the positions of the principal maxima. The positive and negative sign indicate that the minima of a given order lie symmetrically on both the sides of the central principal maxima.

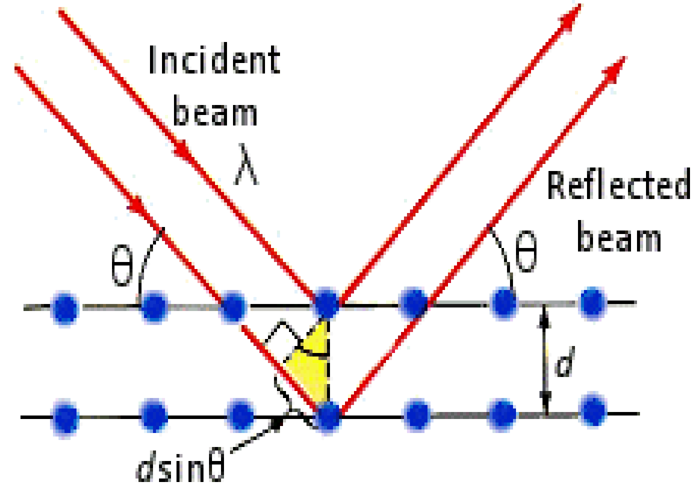
m = 0 gives principal maximum of zero order while m = 1, 2, 3, ... (N-1) give the minima. When m = N, gives the principal maximum of first order. Thus, between zero order and first order principal maxima, we have (N-1) minima. Between two such consecutive minima, the intensity has to be maximum, and these maxima are known as secondary maxima. The secondary maxima are not visible in the grating spectrum as the number of slits is very large.

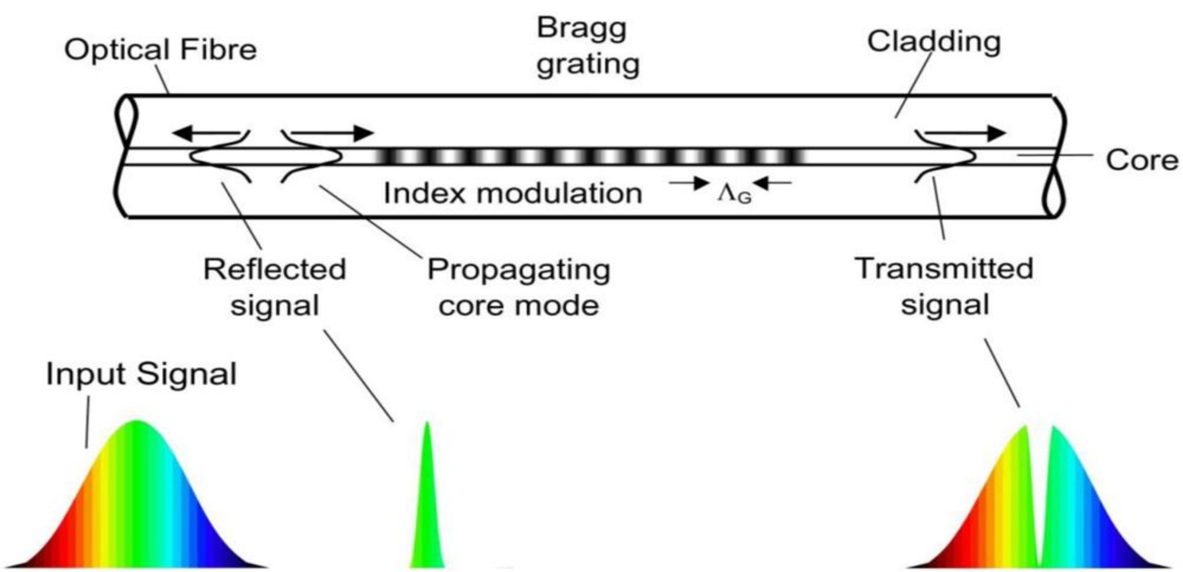
Intensity distribution curve





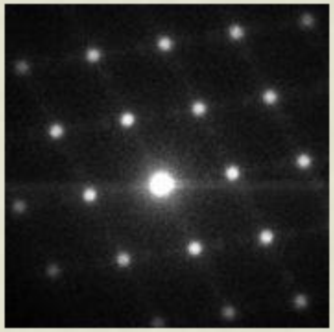
Applications of Diffraction





Diffraction Grating (Contd.)

- Gratings have hundreds of slits per cm.
- Applications in spectroscopy, crystallography etc.



Diffraction pattern from a crystalline solid

Diffraction of light from a CD

Iridescence:
A diffraction phenomenon


Thank You!