Backward difference operator : (∇)

nabla

$$\boxed{\nabla y_n = y_n - y_{n-1}}$$

$\Delta y_n = y_{n+1} - y_n$

i.e $\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

$$\nabla^2 y_n = \nabla(\nabla y_n) = \nabla(y_n - y_{n-1})$$

$$\boxed{\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}}$$

$$= (y_n - y_{n-1}) - (y_{n-1} - y_{n-2})$$

$$= y_n - 2y_{n-1} + y_{n-2}$$

Newtons backward interpolation formula: Let the function $y = f(x)$ takes the values $y_0, y_1, y_2, \ldots y_n$ corresponding to the values $x_0, x_1, x_2, \ldots, x_n$ i.e $x_0, x_0+h, x_0+2h, \ldots, x_0+nh$ then

$$y(x_n + ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

Note: $\nabla^3 y_n = \nabla^2 y_n - \nabla^2 y_{n-1}$, $\nabla^4 y_n = \nabla^3 y_n - \nabla^3 y_{n-1}$

$$\nabla^5 y_n = \nabla^4 y_n - \nabla^4 y_{n-1}$$

① Find the Cubic Polynomial which takes the following values

x:	0	1	2	3	evaluate $y(4)$.
y:	1	2	1	10.	

Solution

	x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$
x_0	0	1 y_0			
			$\nabla y_1 = y_1 - y_0 = 1$		
x_1	1	2 y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1 = -2$	
			$\nabla y_2 = y_2 - y_1 = -1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2 = +12$
x_2	2	1 y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2 = 10$	
			$\nabla y_3 = y_3 - y_2 = 9$		
x_3	3	10 y_3			

$$y(x_n + ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$h=1$, $n=3$, $x_3=3$, $y_3=10$, $\nabla y_3 = 9$, $\nabla^2 y_3 = 10$, $\nabla^3 y_3 = 12$

$$y(3+p) = 10 + p(9) + \frac{p(p+1)}{2}(10) + \frac{p(p+1)(p+2)}{6}(12)$$

Put $3+p = x \Rightarrow p = x-3$

$$y(x) = 10 + 9(x-3) + \frac{(x-3)(x-2)}{2}(10) + \frac{(x-3)(x-2)(x-1)}{6}(12)$$

Simplify, we get

$$y(x) = 2x^3 - 7x^2 + 6x + 1.$$

$$y(4) = 41.$$

(2) In the table below, the values of y are consecutive terms of a series of which 23.6 is the 6^{th} term. Find the tenth term of the series

x:	3	4	5	6	7	8	9
y:	4.8	8.4	14.5	23.6	36.2	52.8	73.9

Solution

x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$	$\nabla^4 y_i$
x_0 3	y_0 4.8				
x_1 4	y_1 8.4	$\nabla y_1 = 3.6$			
x_2 5	y_2 14.5	$\nabla y_2 = 6.1$	$\nabla^2 y_2 = 2.5$		
x_3 6	y_3 23.6	$\nabla y_3 = 9.1$	$\nabla^2 y_3 = 3$	$\nabla^3 y_3 = 0.5$	
x_4 7	y_4 36.2	$\nabla y_4 = 12.6$	$\nabla^2 y_4 = 3.5$	$\nabla^3 y_4 = 0.5$	$0 = \nabla^4 y_4$
x_5 8	y_5 52.8	$\nabla y_5 = 16.6$	$\nabla^2 y_5 = 4$	$\nabla^3 y_5 = 0.5$	$0 = \nabla^4 y_5$
$\boxed{x_6 \; 9}$	$\boxed{y_6 \; 73.9}$	$\boxed{\nabla y_6 = 21.1}$	$\boxed{\nabla^2 y_6 = 4.5}$	$\boxed{\nabla^3 y_6 = 0.5}$	$\boxed{0 = \nabla^4 y_6}$

$$y(x_n + Ph) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n$$

$n = 6$, $x_n = 9$, $y_n = 73.9$, $\nabla y_n = 21.1$, $\nabla^2 y_n = 4.5$, $\nabla^3 y_n = 0.5$

$h = 1$.

$$y(9+P) = 73.9 + (21.1)P + 4.5\,\frac{P(P+1)}{2} + (0.5)\,\frac{P(P+1)(P+2)}{6}$$

<u>Put $P=1$:</u>

$$y(10) = 73.9 + (21.1) + (4.5) + 0.5$$

$$= 100$$

(3)

x :	0	0.1	0.2	0.3	0.4
$e^x = y$:	1	1.1052	1.2214	1.3499	1.4918

find the value of e^x at $x = 0.38$ using N.B.F.

Solution

x_i		y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$	$\nabla^4 y_i$
x_0	0	1.0000 y_0				
			$\nabla y_1 = 0.1052$			
x_1	0.1	1.1052 y_1		0.0110		
			$\nabla y_2 = 0.1162$		0.0013	
x_2	0.2	1.2214 y_2		0.0123		$\boxed{-0.0002}$ $\nabla^4 y_4$
			$\nabla y_3 = 0.1285$		$\boxed{0.0011}$ $\nabla^3 y_4$	
x_3	0.3	1.3499 y_3		$\boxed{0.0134}$ $\nabla^2 y_4$		
			$\boxed{\nabla y_4 = 0.1419}$			
$\boxed{x_4}$	$\boxed{0.4}$	$\boxed{1.4918}$ y_4				

$$y(x_n + ph) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$+ \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

To find $y(0.38)$, let $x_n + ph = 0.38$

$$0.4 + p(0.1) = 0.38$$

$$p = \frac{0.38 - 0.4}{0.1} = -0.2$$

$$\therefore y(0.38) = 1.4918 + (-0.2)(0.1419) + \frac{(-0.2)(-0.2+1)}{2}(0.0134) + \frac{(-0.2)(-0.2+1)(-0.2+2)}{6}(0.0011)$$

$$= 1.4918 - 0.02838 - 0.001072 - 0.0000528$$

$$= 1.4622952$$

i.e $e^{0.38} \approx 1.46229$

(4) Apply N.B.F and obtain a cubic Polynomial for the data:

x:	3	4	5	6
y:	6	24	60	120

Solution

	x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$
x_0	3	6			
			18		
x_1	4	24		18	
			36		$\boxed{6 \; \nabla^3 y_3}$
x_2	5	60		$\boxed{24 \; \nabla^2 y_3}$	
			$\boxed{60 \; \nabla y_3}$		
$\boxed{x_3}$	$\boxed{6}$	$\boxed{120 \; y_3}$			

$$y(x_n + Ph) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n$$

$+$ ~~[illegible]~~

$x_n = 6, \quad y_n = 120, \quad \nabla y_n = 60, \quad \nabla^2 y_n = 24 \quad \nabla^3 y_n = 6$

$$y(6+P) = 120 + P(60) + \frac{P(P+1)}{2}(24) + \frac{P(P+1)(P+2)}{6}(6)$$

Put $6+P = x \Rightarrow P = x-6.$

$$y(x) = 120 + 60(x-6) + \frac{(x-6)(x-5)}{2}(24) + \frac{(x-6)(x-5)(x-4)}{6}(6)$$

Simplify we get

$$y(x) = x^3 - 3x^2 + 2x$$

$y(10) = 14.8125$

⑤ find $y(10)$ if

x:	5	7	9	11
y:	12	13	14	16

⑥ find $y(0.35)$ if

x:	0	0.1	0.2	0.3	0.4
y:	1	1.095	1.179	1.251	1.310

|| 1.2821

$P = -0.5$