we know $\boxed{\Delta y_i = y_{i+1} - y_i}$

$$\Delta^2 y_i = \Delta[\Delta y_i]$$

$$= \Delta[y_{i+1} - y_i]$$

$$\boxed{\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i}$$

$$\Delta^3 y_i = \Delta^2(\Delta y_i)$$

$$= \Delta^2(y_{i+1} - y_i)$$

$$\boxed{\Delta^3 y_i = \Delta^2 y_{i+1} - \Delta^2 y_i}$$

Similarly: $\boxed{\Delta^4 y_i = \Delta^3 y_{i+1} - \Delta^3 y_i}$

Fdward Table

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
x_1	y_1			
		$\Delta y_1 = y_2 - y_1$		
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$
		$\Delta y_2 = y_3 - y_2$		
x_3	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$	$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$
		$\Delta y_3 = y_4 - y_3$		
x_4	y_4		$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$	
		$\Delta y_4 = y_5 - y_4$		
x_5	y_5			

④

x:	0	1	2	3	4
y:	1	3	9	31	81

⑤

x:	1	1.5	2	2.5	3	3.5	4
y:	6	□	10	20	□	1.5	5

⑥

x:	0	1	2	3	4	5	6
y:	5	11	22	40	74	140	261

⑦

x:	1	2	3	4	5
y:	1	2	3	___	5

(3)

x	2	3	4	5	6
y	45	49.9	54.1	—	67.4

Solution

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
2	45				
		4.2			
3	49.9		0.7		
		4.9		a − 59.7	
4	54.1		a − 59		240.2 − 4a
		a − 54.1		180.5 − 3a	
5	a		121.5 − 2a		
		67.4 − a			
6	67.4				

Given 4 data, so we can fit a cubic Polynomial

$\therefore \Delta^4 y_i = 0$

$240.2 - 4a = 0$

$$\boxed{a = 60.05}$$

	x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
x_0	2	y_0 45	$\Delta y_0 = y_1 - y_0 = 4.2$			
x_1	3	y_1 49.9	$\Delta y_1 = y_2 - y_1 = 4.9$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 = 0.7$		
x_2	4	y_2 54.1	$\Delta y_2 = y_3 - y_2 = a - 54.1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1 = a - 59$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0 = a - 59.7$	$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0 = 240.2 - 4a$
x_3	5	y_3 a	$\Delta y_3 = y_4 - y_3 = 67.4 - a$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_2 = 121.5 - 2a$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1 = 180.5 - 3a$	
x_4	6	y_4 67.4				

(2) Find the Missing values

x_i	0	1	2	3	4	5	6	7
y_i	1	-1	1	-1	1	—	—	—

Solution

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$
0	1					
		-2				
1	-1		4			
		2		-8		
2	1		-4		16	
		-2		8		0
3	-1		4		16	
		2		24		0
4	1		28		16	
		30		40		0
5	31		68		16	
		98		56		
6	129		124			
		222				
7	351					

②

	x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
x_0	0	1 y_0				
			$\Delta y_0 = -2$			
x_1	1	-1 y_1		$\Delta^2 y_0 = 4$		
			$\Delta y_1 = 2$		$\Delta^3 y_0 = -8$	
x_2	2	1 y_2		$\Delta^2 y_1 = -4$		$\Delta^4 y_0 = 16$
			$\Delta y_2 = -2$		$\Delta^3 y_1 = 8$	
x_3	3	-1 y_3		$\Delta^2 y_2 = 4$		$\Delta^4 y_1 = \boxed{16}$
			$\Delta y_3 = 2$		$\Delta^3 y_2 = \boxed{24}$	
x_4	4	1 y_4		$\Delta^2 y_3 = \boxed{28}$		$\Delta^4 y_2 = \boxed{16}$
			$\Delta y_4 = \boxed{30}$		$\Delta^3 y_3 = \boxed{40}$	
x_5	5	$\boxed{31}$ y_5		$\Delta^2 y_4 = \boxed{68}$		$\Delta^4 y_3 = \boxed{16}$
			$\Delta y_5 = \boxed{98}$		$\Delta^3 y_4 = \boxed{56}$	
x_6	6	$\boxed{126}$ y_6		$\Delta^2 y_5 = \boxed{124}$		
			$\Delta y_6 = \boxed{225}$			
x_7	7	$\boxed{351}$ y_7				

① Find the Missing Values in the following data:

x	45	50	55	60	65
y	3	☐	2	☐	-2.4

Solution

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
45	3			
		$P-3$		
50	P		$5-2P$	
		$2-P$		$Q+3P-9$
55	2		$Q-4+P$	
		$Q-2$		$3.6-3Q-P$
60	Q		$-0.4-2Q$	
		$-2.4-Q$		
65	-2.4			

For a quadratic ~~Equation~~ Polynomial we know $\Delta^3 y = 0$

$\therefore$ $Q-3P-9=0$ $\Rightarrow$ $Q+3P=9$

$3.6-3Q-P=0$ $\Rightarrow$ $3Q+P=3.6$

$\therefore$ $Q = 0.225$

$P = 2.925$