Newton's forward interpolation formula:

→ Given a set of $(n+1)$ values namely (x_0, y_0), (x_1, y_1) ... (x_{n-1}, y_{n-1}), (x_n, y_n). Suppose it is required to evalues y at $x_0 + ph$, where p is any real number then

$$y(x_0 + ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

Let $x = x_0 + ph$ then $p = \frac{x - x_0}{h}$

$$\therefore y(x) = y_0 + \Delta y_0 \left(\frac{x - x_0}{h}\right) + \frac{\Delta^2 y_0}{2!} \frac{(x - x_0)(x - x_1)}{h^2} + \frac{\Delta^3 y_0}{3!} \frac{(x - x_0)(x - x_1)(x - x_2)}{h^3} + \cdots$$

Note: $p = \frac{x - x_0}{h}$

$$p - 1 = \frac{x - x_0 - h}{h} = \frac{x - (x_0 + h)}{h} = \frac{x - x_1}{h}$$

$$p - 2 = \frac{x - x_0 - 2h}{h} = \frac{x - (x_0 + 2h)}{h} = \frac{x - x_2}{h}$$

① Estimate a Polynomial for the following table and obtain the function value at 0.5

<u>Solution</u>

	x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
x_0	0	$\boxed{3 \; y_0}$			
			$\boxed{\Delta y_0 = 3}$		
				$\boxed{\Delta^2 y_0 = 2}$	
x_1	1	6 y_1			$\boxed{\Delta^3 y_0 = 0}$
			$\Delta y_1 = 5$		
				$\Delta^2 y_1 = 2$	
x_2	2	11 y_2			$\Delta^3 y_1 = 0$
			$\Delta y_2 = 7$		
x_3	3	18 y_3		$\Delta^2 y_2 = 2$	
			$\Delta y_3 = 9$		
x_4	4	27 y_4			

→ Since $\Delta^2 y$ values are Constant, So we Can get Second degree Polynomial.

$$y(x_0 + ph) = y_0 + \Delta y_0 p + \Delta^2 y_0 \frac{p(p-1)}{2!}$$

$x_0 = 0, \; h = 1.$

$$y(p) = 3 + 3p + 2\frac{p(p-1)}{2!}$$

$$= 3 + 3p + p^2 - p$$

$$y(p) = p^2 + 2p + 3$$

$$\boxed{\begin{aligned} \therefore\; y(x) &= x^2 + 2x + 3 \\ y(0.5) &= 4.25 \end{aligned}}$$

(2) Using Newton's forward interpolation formula, Compute the value of $\sqrt{5.5}$ given $\boxed{\sqrt{5} = 2.236, \sqrt{6} = 2.449, \sqrt{7} = 2.646, \sqrt{8} = 2.828}$

Correct upto 3 decimals.

Solution:

	x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
x_0	5	2.236 y_0			
			0.213 Δy_0		
	6	2.449		-0.016 $\Delta^2 y_0$	
			0.197		0.001 $\Delta^3 y_0$
	7	2.646		-0.015	
			0.182		
	8	2.828			

$$y(x_0+Ph) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

we want $y(5.5)$, so $x_0 + Ph = 5.5$

$x_0 = 5, h = 1 \Rightarrow 5 + P = 5.5$

$$\boxed{P = 0.5}$$

$$y(5.5) = 2.236 + (0.5)(0.213) + \frac{(0.5)(-0.5)}{2!}(-0.016) + \frac{(0.5)(-0.5)(-1.5)}{6}(0.001)$$

$y(5.5) = 2.344565$

Answer is $\boxed{y(\cancel{x}) = 2.3445}$

$y(5.5) =$

(3) From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
No of students	31	42	51	35	31

Solution: Let $y(x)$ denote the number of students whose marks are less than x.

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
x_0 40	$\boxed{y_0\ 31}$	$\boxed{\Delta y_0\ 42}$	$\boxed{\Delta^2 y_0\ 9}$	$\boxed{\Delta^3 y_0\ -25}$	$\boxed{\Delta^4 y_0\ 37}$
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

$$y(x_0+ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

we need $y(45)$, so $45 = x_0 + ph$

$x_0 = 40$ | $p = \frac{45-40}{10} = 0.5$

$h = 10$

$$y(45) = 31 + (0.5)(42) + \frac{(0.5)(-0.5)}{2}(9) + \frac{(0.5)(-0.5)(-1.5)}{6}(-25) + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24}(37)$$

$= 47.87$ (Simplication) ≈ 48

$\therefore$ Number of students whose marks are b/w 40 & 45 $= y(45) - y(40) = 48 - 31 = \underline{\underline{17}}$

(4) Find the cubic Polynomial which takes the following values

x :	0	1	2	3
y :	1	2	1	10 .

Hence obtain the value of $y(4)$.

Solution

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
x_0 0	1 y_0			
		1 Δy_0		
1	2		-2 $\Delta^2 y_0$	
		-1		12 $\Delta^3 y_0$
2	1		10	
		9		
3	10			

$$y(x_0+ph) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$x_0 = 0,\ h = 1.$

$$y(p) = 1 + p(1) + \frac{p(p-1)}{2}(-2) + \frac{p(p-1)(p-2)}{6}(12)$$

Put $p = x$:

$$y(x) = 2x^3 - 7x^2 + 6x + 1$$

$$y(4) = 41$$

(Q) Given $u_1 = 40$, $u_3 = 45$, $u_5 = 54$

find u_2, u_4

Solution:

	x_i	u_i	Δu_i	$\Delta^2 u_i$
x_0	1	40 (u_0)		
			$5^{(\Delta u_0)}$	
				$4^{(\Delta^2 u_0)}$
	3	45		
			9	
	5	54		

$$u(x_0 + ph) = u_0 + p\,\Delta u_0 + \frac{p(p-1)}{2}\Delta^2 u_0$$

$x_0 = 1$, $h = 2$

$$u(1+2p) = 40 + p(5) + \frac{p^2 - p}{2}(4)$$

Let $1 + 2p = x \Rightarrow p = \frac{x-1}{2}$

$$\therefore\ u(x) = 40 + 5\left(\frac{x-1}{2}\right) + \left[\left(\frac{x-1}{2}\right)^2 - \left(\frac{x-1}{2}\right)\right]2$$

Simplify to get $u(x) =$

$$\boxed{\frac{1}{2}\left(x^2 + x + 78\right)}$$

$u(2) = 42$

$u(4) = 49$

(5) Given $\sin 45 = 0.7071$ using N.F. formula
$\sin 50 = 0.7660$ find $\sin 52$
$\sin 55 = 0.8192$
$\sin 60 = 0.8660$

(6) estimate the value of $f(22)$ from the following data:

x:	20	25	30	35	40	45
$f(x)$:	354	332	291	260	231	204

(7) Find the number of men getting wages below Rs 15 from the following data

Wages in Rs (x):	0-10	10-20	20-30	30-40
Frequency:	9	30	35	42

Hint: Let $y(x)$ be the number of men getting wages below x.

then:

x:	10	20	30	40
y:	9	39	74	116

find $y(15)$

(8) find $e^{1.75}$ using N.F.F.

x	1.7	1.8	1.9	2.0
$y = e^x$	5.474	6.050	6.686	7.389