

Time Complexities :

1. GREEDY METHOD :

a) Knapsack problem:

Sorting is the key step to arrange profit/weight ratio in decreasing order.

→ Accept the input, find P_i/W_i , Select highest sort (Quicksort - $O(n \log n)$),
Based on weights ($O(1)$) and then sum

$$\text{Total Time complexity} = O(n \log n)$$

b) Job Sequencing with deadlines :

Each job has a deadline and profit associated to it.

→ Since there exists two nested for loops.

$$\text{Time complexity} = O(n^2)$$

c) Single source Shortest path problem:

Time complexity of Dijkstra's Algorithm is $O(V^2)$. Optimising it by using a min-priority queue reduces to $O(V + E \log V)$. Here V refers to no. of vertices and E is no. of edges.

d) Prim's Algorithm:

Time complexity of Prim's Algorithm using adjacency matrix is $O(n^2)$ where n indicates no. of vertices. It has complexity of $O(E \log V)$ using binary heap and can be improved using Fibonacci Heap to $O(E + \log V)$. Here $E \rightarrow$ no. of Edges, $V \rightarrow$ no. of vertices.

e) Kruskal's Algorithm:

For deletion and reheapify, Kruskal's Algorithm takes $O(E \log E)$ time complexity. The value of E can be almost $O(V^2)$. So the overall time complexity is $O(E \log E)$ or $O(E \log V)$ where E indicates no. of edges and V is no. of vertices.

2. DYNAMIC PROGRAMMING:

a) 0/1 Knapsack problem:

The time complexity of 0/1 knapsack problem is $O(nW)$ where n is the number of items and W is the capacity of the knapsack.

b) Matrix Chain Multiplication:

The time complexity of matrix chain multiplication is $O(n^3)$ due to three nested for loops.

c) All pairs shortest path problem:

The time complexity of all pairs shortest path problem is $O(V^3)$ where V is the number of vertices in the graph.

d) Optimal Binary Search tree:

The time complexity of OBST is $O(n^4)$ which can be optimised to $O(n^3)$ by precalculating sum of frequencies instead of calling `sum()` again and again.

e) Travelling sales person problem:

The time complexity of travelling salesperson algorithm is $O(n^2 \times 2^n)$ where n is the number of nodes. There are at most $n \times 2^n$ subsets. Each subset requires $O(n)$ time complexity.

f) Reliability Design:

The time complexity of reliability design is $O(2^n)$ using multiplication optimisation function).

g) Transitive Closure of graph:

The time complexity of transitive closure of a graph is $O(n^3)$ since there exists three nested for loops.

3. BACK TRACKING:

a) N-Queens Problem:

Time complexity of N-Queens problem is $O(n^n)$ since we check every position on $m \times n$ board where 'n' is no. of queens.

b) Graph Coloring:

Time complexity of Graph coloring problem is $O(n \cdot m^n)$ as we have maximum of m^n combinations with 'm' colours to be placed in 'n' places. Thus it takes maximum of 'mn' time to check each combination.

c) Hamiltonian Cycle:

Time complexity of Hamiltonian cycle is $O(n^n)$ since it tries for $n!$ ways.

d) Sum of subsets:

Time complexity of sum of subsets problem is $O(2^n)$.

4. BRANCH AND BOUND:

a) Travelling Sales Person Problem:

Time complexity of travelling sales problem is $O(n^2)$ because algorithm has two iterations embedded one inside the other for calculation of reduced cost matrix.

b) 0/1 knapsack problem:

Time complexity of 0/1 knapsack is $O(2^n)$. Here 2^n solutions would be generated.