ASSIGNMENT - 2

FLAT

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CSE-4
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1. Construct the string aaabbaabbb from the grammar by using Leftmost and Rightmost derivation.

$S \rightarrow aB / bA$

$A \rightarrow a / aS / bAA$

$B \rightarrow b / bS / Abb$

Sol^n:

Left-most derivation for "aaabbaabbb"

$S \rightarrow a\underline{B}$

$\Rightarrow a\underline{A}bb \quad (\because B \rightarrow Abb)$

$\Rightarrow aa\underline{S}bb \quad (\because A \rightarrow aS)$

$\Rightarrow aaa\underline{B}bb \quad (\because S \rightarrow aB)$

$\Rightarrow aaab\underline{S}bb \quad (\because B \rightarrow bS)$

$\Rightarrow aaabb\underline{A}bb \quad (\because S \rightarrow bA)$

$\Rightarrow aaabba\underline{S}bb \quad (\because A \rightarrow aS)$

$\Rightarrow aaabbaa\underline{B}bb \quad (\because S \rightarrow aB)$

$\Rightarrow aaabbaabbb \quad (\because B \rightarrow b)$

Right-most derivation for "aaabbaabbb"

$S \rightarrow a\underline{B}$

$\Rightarrow a\underline{A}bb \quad (\because B \rightarrow Abb)$

$\Rightarrow aa\underline{S}bb \quad (\because A \rightarrow aS)$

$\Rightarrow aaa\underline{B}bb \quad (\because S \rightarrow aB)$

$\Rightarrow aaab\underline{S}bb \quad (\because B \rightarrow bS)$

$\Rightarrow aaabb\underline{A}bb \quad (\because S \rightarrow bA)$

$\Rightarrow aaabba\underline{S}bb \quad (\because A \rightarrow aS)$

$\Rightarrow aaabbaa\underline{B}bb \quad (\because S \rightarrow aB)$

$\Rightarrow aaabbaabbb \quad (\because B \rightarrow b)$

Here the left-most derivation and right-most derivations are same because there is only one variable in every step of derivation.

2: Construct PDA for the language

$$L = \{ WW^R \mid W \text{ is in } \{a, b\}^* \}.$$

Soln: Read the string W and push each symbol onto the stack. After that read each symbol, if it matches with top of the stack then pop off the symbol. When the input is read completly, if stack is empty then the string is acceptable.

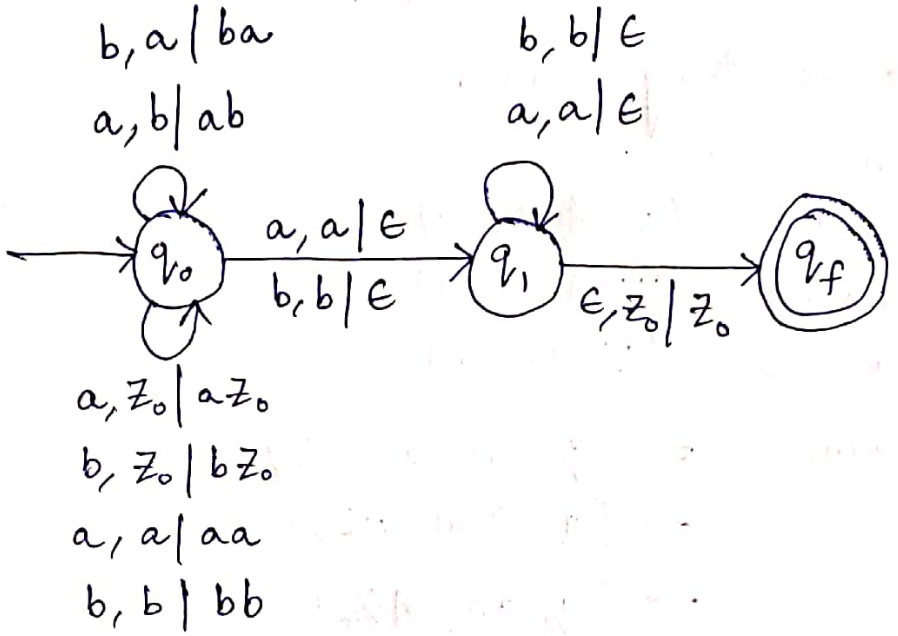
Let q_0 be initial state, q_f be the final state and Z_0 be the initial stack symbol.

$$\delta(q_0, a, Z_0) = (q_0, aZ_0)$$
$$\delta(q_0, a, a) = (q_0, aa)$$
$$\delta(q_0, b, Z_0) = (q_0, bZ_0)$$
$$\delta(q_0, b, b) = (q_0, bb)$$
$$\delta(q_0, a, b) = (q_0, ab)$$
$$\delta(q_0, b, a) = (q_0, ba)$$

$$\delta(q_0, a, a) = (q_1, \epsilon)$$
$$\delta(q_0, b, b) = (q_1, \epsilon)$$
$$\delta(q_1, a, a) = (q_1, \epsilon)$$
$$\delta(q_1, b, b) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$$

∴ The final PDA is,



3: Consider the grammar and convert to PDA.

$S \rightarrow aA / bB$

$A \rightarrow aA / a$

$B \rightarrow bB / b$

Verify whether aaaa is accepted by PDA or not.

Sol^n: The grammar is in GNF, hence we can apply the rule as follows. $\delta(q_0, \epsilon, Z_0) = (q_0, SZ_0)$

- Consider $S \rightarrow aA \mid bB$

  then $S \rightarrow aA$ corresponds to $\delta(q, a, S) \rightarrow (q, A)$

  $S \rightarrow bB$ corresponds to $\delta(q, b, S) \rightarrow (q, B)$.

- Consider $A \rightarrow aA \mid a$

  then $A \rightarrow aA$ corresponds to $\delta(q, a, A) \rightarrow (q, A)$.

  $A \rightarrow a$ corresponds to $\delta(q, a, A) \rightarrow (q, \epsilon)$

- Consider $B \rightarrow bB / b$

  then $B \rightarrow bB$ corresponds to $\delta(q, b, B) \rightarrow (q, B)$

  $B \rightarrow b$ corresponds to $\delta(q, b, B) \rightarrow (q, \epsilon)$.

Finally in state q_0,

$$\delta(q, \epsilon, Z_0) = (q_f, Z_0)$$

Verify for the string aaaa :

$S \rightarrow aA$

$\rightarrow aaA \quad (\because A \rightarrow aA)$

$\rightarrow aaaA \quad (\because A \rightarrow aA)$

$\rightarrow aaaa \quad (\because A \rightarrow a)$.

$\delta(q_0, aaaa, Z_0) \Rightarrow (q, aaaa, SZ_0)$

$\rightarrow (q, aaa, AZ_0)$

$\rightarrow (q, aa, AZ_0)$

$\rightarrow (q, a, AZ_0)$

$\rightarrow (q, \epsilon, Z_0)$

$\rightarrow (q_f, Z_0)$.

As the string has reached the final state, the string is accepted by the PDA.

4. Convert the following grammar into GNF.

$S \rightarrow AB \mid BC$

$A \rightarrow aB \mid bA \mid a$

$B \rightarrow bB \mid cC \mid b$

$C \rightarrow c$

Solⁿ: There are no null and unit productions but still given grammar is not in CNF. So first convert into CNF.

$S \rightarrow AB \mid BC$

$A \rightarrow D_a B \mid D_b A \mid a$

$B \rightarrow D_b B \mid \cancel{D_c C \mid b}\ CC \mid b$

$C \rightarrow c$

$D_a \rightarrow a$

$D_b \rightarrow b.$

Rename the variables as $A_1, A_2, \ldots$

Then the productions are,

$A_1 \rightarrow A_2A_3 \mid A_3A_4$ — ①

$A_2 \rightarrow A_5A_3 \mid A_6A_2 \mid a$ — ②

$A_3 \rightarrow A_6A_3 \mid A_4A_4 \mid b$ — ③

$A_4 \rightarrow c$ — ④

$A_5 \rightarrow a$ — ⑤

$A_6 \rightarrow b$ — ⑥

[Replaced
S with A_1
A with A_2
B with A_3
C with A_4
D_a with A_5
D_b with A_6.

Consider ③ and apply substitution to bring it in GNF.

$A_3 \rightarrow bA_3 \mid cA_4 \mid b$ [$\because$ from ⑥, ④].

Consider ② and apply substitution to bring it in GNF.

$A_2 \rightarrow aA_3 \mid bA_2 \mid a$ [$\because$ from ⑤, ⑥].

Now substitute A_2, A_3 productions in ①.

$A_1 \rightarrow aA_3A_3 \mid bA_2A_3 \mid aA_3 \mid bA_3A_4 \mid cA_4A_4 \mid bA_4$.

$\therefore$ The final productions in GNF are,

$A_1 \rightarrow aA_3A_3 \mid bA_2A_3 \mid aA_3 \mid bA_3A_4 \mid cA_4A_4 \mid bA_4$

$A_2 \rightarrow aA_3 \mid bA_2 \mid a$

$A_3 \rightarrow bA_3 \mid cA_4 \mid b$

$A_4 \rightarrow c$.

$A_5 \rightarrow a$

$A_6 \rightarrow b$.

5. Minimize the given CFG.

$S \to a \mid aA \mid B \mid C$

$A \to aB \mid \epsilon$

$B \to Aa$

$C \to cCD$

$D \to ddd$

Soln: 1) Eliminate the ϵ-productions for given CFG.

$\because A \to \epsilon$, A is a nullable variable.

$S \to a \mid aA \mid B \mid C$

$A \to aB$

$B \to Aa \mid a$

$C \to cCD$

$D \to ddd$

2) Eliminate the unit productions from obtained productions.

$S \to a \mid aA \mid Aa \mid cCD$ $[\because B \to Aa,\ C \to cCD]$

$A \to aB$

$B \to Aa \mid a$

$C \to cCD$

$D \to ddd$

3) Eliminate the useless symbols.

From obtained productions, symbols C and D are useless.

So, after removing productions containing C and D,

$$\left.\begin{aligned} S &\to a \mid aA \mid Aa \\ A &\to aB \\ B &\to a \mid Aa \end{aligned}\right\} \text{Minimised CFG.}$$

6. Derive the Context Free Grammar for the language $N(M)$ where

$M = (\{q_0, q_1\}, \{0, 1\}, \{X, Z_0\}, \delta, q_0, Z_0, \phi)$ and δ given by

$\delta(q_0, 0, Z_0) = (q_0, XZ_0)$ — 1

$\delta(q_1, 1, X) = (q_1, \epsilon)$ — 2

$\delta(q_0, 0, X) = (q_0, XX)$ — 3

$\delta(q_1, \epsilon, X) = (q_1, \epsilon)$ — 4

$\delta(q_0, 1, X) = (q_1, \epsilon)$ — 5

$\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$ — 6

Solⁿ: First add the S transition.

$S \rightarrow [q_0 Z_0 q_0] \mid [q_0 Z_0 q_1]$

① $\delta(q_0, 0, Z_0) = (q_0, XZ_0)$

$[q_0 Z_0 q_0] \rightarrow 0[q_0 X q_0][q_0 Z_0 q_0]$

$[q_0 Z_0 q_0] \rightarrow 0[q_0 X q_1][q_1 Z_0 q_0]$

$[q_0 Z_0 q_1] \rightarrow 0[q_0 X q_0][q_0 Z_0 q_1]$

$[q_0 Z_0 q_1] \rightarrow 0[q_0 X q_1][q_1 Z_0 q_1]$

② $\delta(q_1, 1, X) = (q_1, \epsilon)$

$[q_1 X q_1] \rightarrow 1$

③ $\delta(q_0, 0, X) = (q_0, XX)$

$[q_0 X q_0] \rightarrow 0[q_0 X q_0][q_0 X q_0]$

$[q_0 X q_0] \rightarrow 0[q_0 X q_1][q_1 X q_0]$

$[q_0 X q_1] \rightarrow 0[q_0 X q_0][q_0 X q_1]$

$[q_0 X q_1] \rightarrow 0[q_0 X q_1][q_1 X q_1]$

(4) $\delta(q_1, \epsilon, X) = (q_1, \epsilon)$

$[q_1 X q_1] \rightarrow \epsilon$

(5) $\delta(q_0, 1, X) = (q_1, \epsilon)$

$[q_0 X q_1] \rightarrow 1$

(6) $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$[q_1 Z_0 q_1] \rightarrow \epsilon$

Now rename as variables,

$[q_0 Z_0 q_0]$ to A, $[q_0 Z_0 q_1]$ to B, $[q_0 X q_0]$ to C, $[q_0 X q_1]$ to D, $[q_1 Z_0 q_0]$ to E, $[q_1 Z_0 q_1]$ to F, $[q_1 X q_1]$ to G, $[q_1 X q_0]$ to H

Hence the productions are renamed as :

$S \rightarrow A \mid B$

$A \rightarrow 0CA \mid 0DE$

$B \rightarrow 0CB \mid 0DF$

$C \rightarrow 0CC \mid 0DH$

$D \rightarrow 0CD \mid 0DG \mid 1$

$F \rightarrow \epsilon$

$G \rightarrow 1 \mid \epsilon$

∴ This is the derived CFG.

7. Construct TM for $L = \{ M*N \mid M, N >= 1 \}$.

Sol^n:

Input :

M   N

1	1	0	1	1	1	0	B

2×3

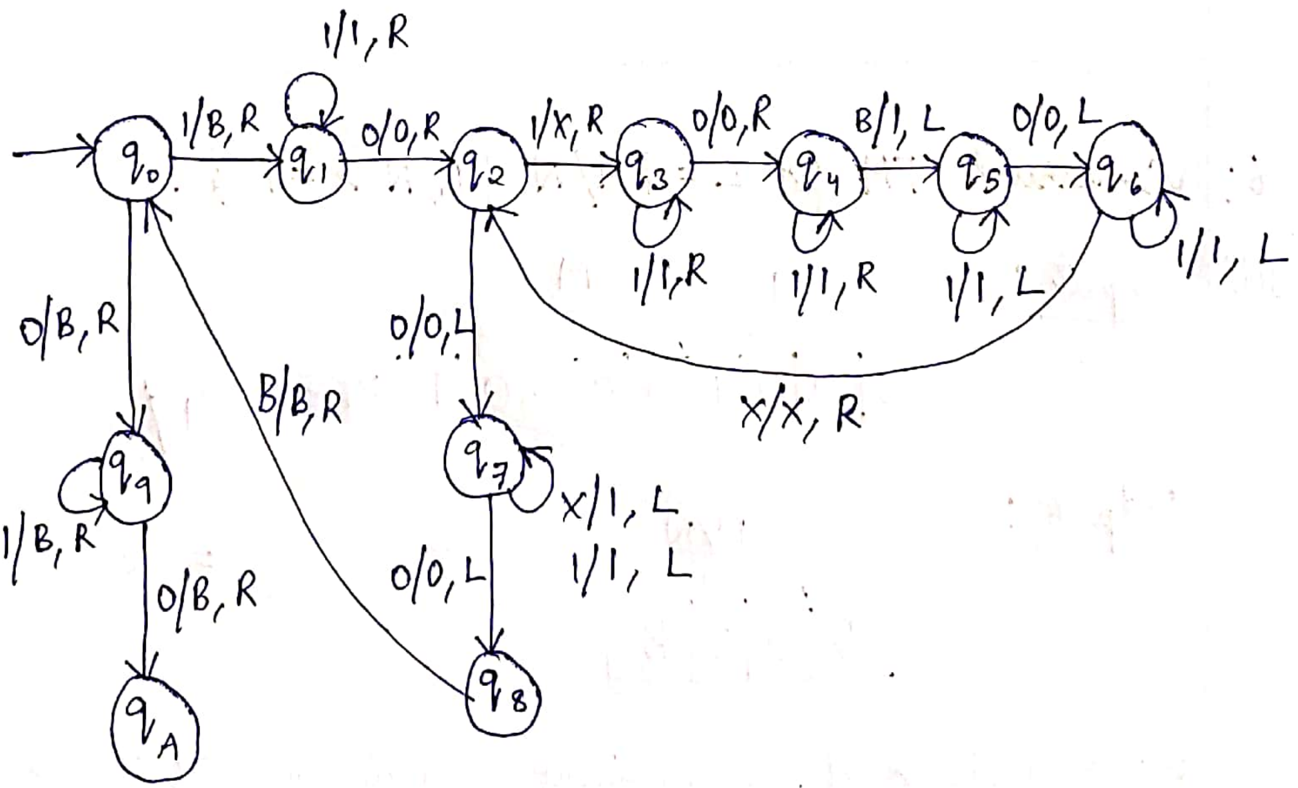
Output :

$M*N$

B	1	1	1	1	1	1	B

$= 6$

For multiplying two numbers $M*N$, place $1^n 0 1^m 0$ on the input tape and design the system such that it first replaces first occurence of '1' by blank and for each occurence of '1' of second integer write '1' at the end. Once all 1's of second integer are replaced with X, now remodify all X's to 1's and move extreme left to find 'B'. Repeat this process until all 1's of first integer are replaced with B. Now replace all 1's of second number by blank and halt.



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	0	1	X	B
q_0	(q_9, B, R)	(q_1, B, R)	—	—
q_1	$(q_2, 0, R)$	$(q_1, 1, R)$	—	—
q_2	$(q_7, 0, L)$	(q_3, X, R)	—	—
q_3	$(q_4, 0, R)$	$(q_3, 1, R)$	—	—
q_4	—	$(q_4, 1, R)$	—	$(q_5, 1, L)$
q_5	$(q_6, 0, L)$	$(q_5, 1, L)$	—	—
q_6	—	$(q_6, 1, L)$	(q_2, X, R)	—
q_7	$(q_8, 0, L)$	$(q_7, 1, L)$	$(q_7, 1, L)$	—
q_8	—	—	—	(q_0, B, R)
q_9	(q_A, B, R)	(q_9, B, R)	—	—
q_A	—	—	—	—

8. Construct TM for $L = \{ M/N \mid M, N >= 1 \}$.

Sol^n: Input :

$$\text{B}\,\underbrace{0\,0}_{N}\,1\,\underbrace{0\,0\,0\,0}_{M}\,1\,\text{B}$$

B	0	0	1	0	0	0	0	1	B

4/2

Output :

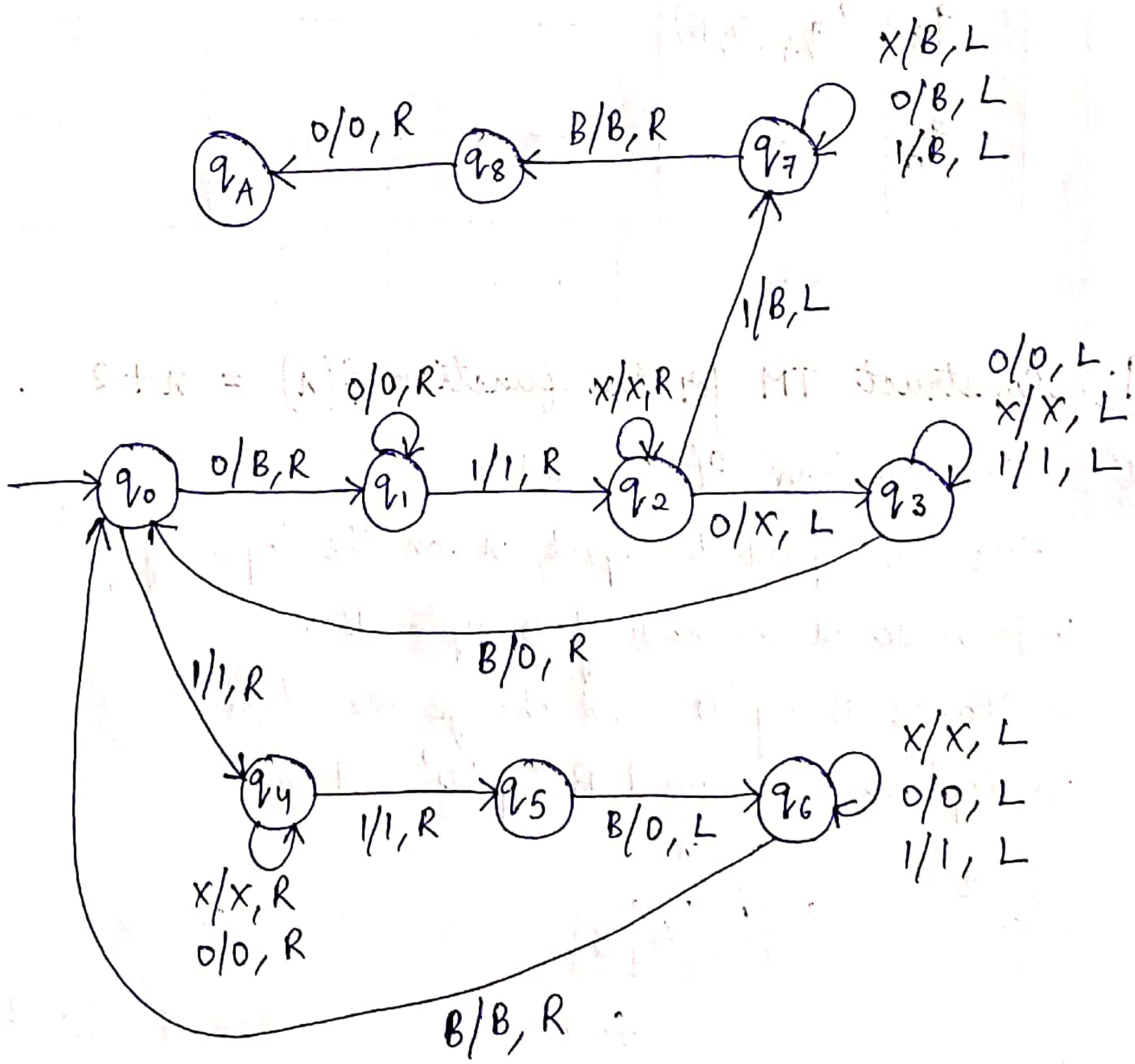
$$\text{B}\,\underbrace{0\,0}_{M/N}\,\text{B}$$

B	0	0	B

= 2

* For division of two numbers, firstly we will be changing the 0 to B and go right neglecting all 0 and moving right.

* When we find 1 go right and change 0 to X and turn left, if X is found we keep X and move right.
* If X, 0, 1 any of these are found don't change and move left. If B is found change to 0 and move right and go to step 1.
* If in step-1, if we got 1 move right then if X move right, if 1 move right. Then if B is found change to 0 and move left.
* If 0, or X or 1 found move left. Else if B found turn right and go to step-1.
* If in step-2, it was found as 0 move ~~right~~ left by changing to blank.
* Then if X or 0 or 1 is found change to B and move right.
* Then if 1 is found do not change and move right.



	0	1	X	B
q_0	(q_1, B, R)	$(q_4, 1, R)$	—	—
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	—	—
q_2	(q_3, X, L)	(q_7, B, L)	(q_2, X, R)	—
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_3, X, L)	$(q_0, 0, R)$
q_4	$(q_4, 0, R)$	$(q_5, 1, R)$	(q_4, X, R)	—
q_5	—	—	—	$(q_6, 0, L)$
q_6	$(q_6, 0, L)$	$(q_6, 1, L)$	(q_6, X, L)	(q_0, B, R)
q_7	(q_7, B, L)	(q_7, B, L)	(q_7, B, L)	(q_8, B, R)
q_8	$(q_A, 0, R)$	—	—	—
q_A	—	—	—	—

9. Construct TM for the function $f(x) = x+2$.

Sol^n: Given function $f(x) = x+2$.

Here we represent input x on the tape by 0^x .

1. Traverse x number of 0 upto B .
2. Change B by '0' and change the state.
3. Replace the second B by '0' and halt.

Input :

$$\overbrace{\quad}^{x}$$

B	0	0	B

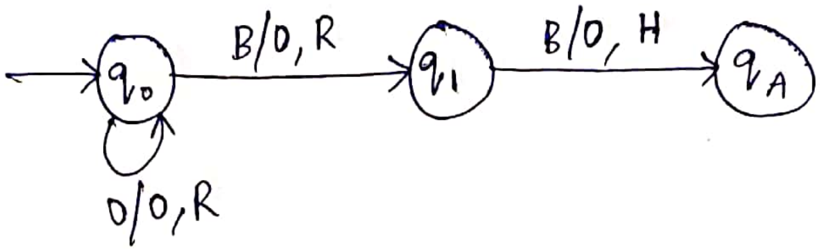
$x = 2$

then $f(x) = 4$

Output :

$$\overbrace{\quad}^{f(x)}$$

B	0	0	0	0	B



	0	B
q_0	$(q_0, 0, R)$	$(q_1, 0, R)$
q_1	—	$(q_A, 0, H)$
q_A	—	—