23/3/20.

ASSIGNMENT – 3

Electro Magnetic Waves & Fiber Optics.

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CSE – 4
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1. **Poynting Theorem:-**

An electromagnetic wave carries energy when it is propagating. The rate of energy flow is represented by poynting vector $\vec{P}$, proposed by J.H. Poynting.

It states that, " rate of flow of energy per unit area (or) power flow per unit area is equal to the cross product of electric and magnetic vectors at any given point.

$$\vec{P} = \vec{E} \times \vec{H} \quad \text{—— (1)}$$

Consider Maxwell's 4 th equation,

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \quad \text{—— (2)}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{— (3)}$$

$$\Rightarrow \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \quad \text{—— (4)}$$

Using vector identity,

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H})$$

$$\Rightarrow \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) \quad \text{— (5)}$$

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$-\vec{H} \frac{\partial B}{\partial t} - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}.$$

$$[\because B = \mu H] \Rightarrow -\mu \vec{H} \frac{\partial H}{\partial t} - \vec{\nabla}(\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \varepsilon E \cdot \frac{\partial E}{\partial t}$$

$$\Rightarrow -\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \varepsilon E \cdot \frac{\partial E}{\partial t} + \mu H \frac{\partial H}{\partial t}$$

$$= \vec{E} \cdot \vec{J} + \frac{\varepsilon}{2} \cdot \frac{\partial E^2}{\partial t} + \frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

$$= \vec{E} \cdot \vec{J} + \frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2}\right) + \frac{\partial}{\partial t}\left(\frac{\mu H^2}{2}\right).$$

$$\vec{E} \cdot \vec{J} = \frac{-\partial}{\partial t}\left[\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right] - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$$

$$\iiint_V (\vec{E}\cdot\vec{J})\cdot dV = \frac{-\partial}{\partial t}\iiint_V \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dV - \underbrace{\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})}_{G\cdot D\cdot T}$$

$$\boxed{\iiint (\vec{E}\cdot\vec{J})\cdot dV = \frac{-\partial}{\partial t}\iiint \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right) dV - \iint_S (\vec{E}\times\vec{H})\, ds}$$

where G.D.T $\longrightarrow$ Gauss Divergence Theorem.

The rate of dissipation of energy in a volume V is equal to the rate at which stored electromagnetic energy is decreasing plus the rate at which energy flows through the surface of volume V.

Significance of terms:

→ $\iiint(\vec{E}\cdot\vec{J})\, dV$ represents generalisation of Joules law indicating the instantaneous power dissipating in the volume V.

→ $\frac{-\partial}{\partial t}\iiint\left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)\cdot dV$ represents the stored E.M energy per unit second.

The negative sign indicates power is delivered by the field inside the volume to the region outside the volume.

It represents the stored energy is decreasing.

→ $-\iint_S (\vec{E}\times\vec{H})\cdot ds$ represents the rate at which energy flows. This is from law of conservation of energy.

Negative sign indicates energy is entering inside the volume.

Positive sign indicates energy is coming outside the volume.

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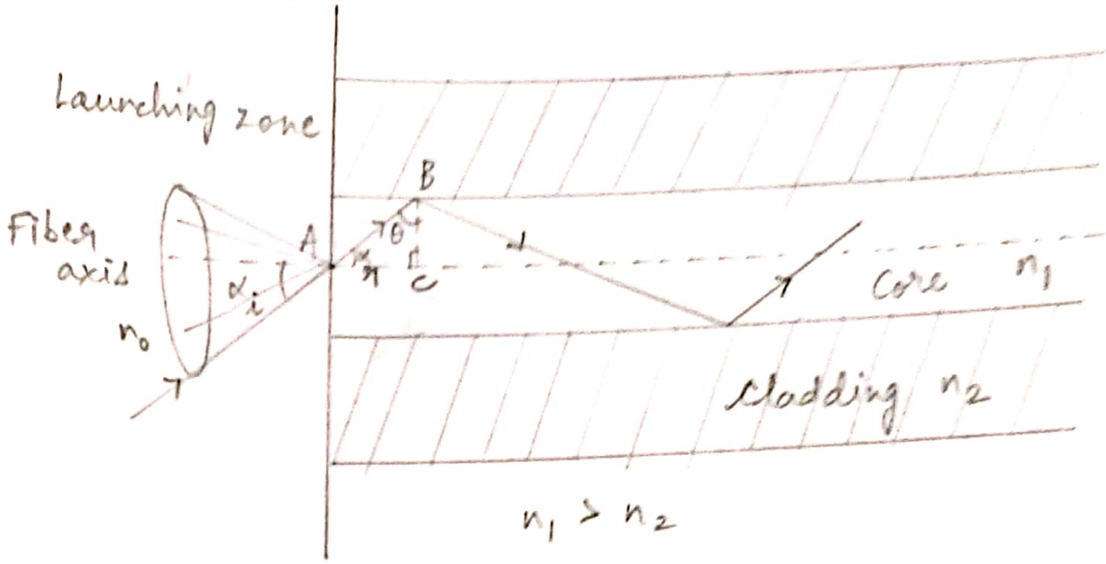
2. Acceptance Angle (Launching Angle) :-

Acceptance angle is the maximum possible angle of launch with which the light can be launched into the fibre to enable the entire light to propogate through fiber by Total Internal Reflection.



From Δ'le ABC,

$\alpha_r = 90 - \theta$ —— ①

By Snell's law, $n_0 \sin\alpha_i = n_1 \sin\alpha_r$

$= n_1 \sin(90-\theta)$

$n_0 \sin\alpha_i = n_1 \cos\theta$

$\Rightarrow \boxed{\sin\alpha_i = \frac{n_1}{n_0}\cos\theta}$ —— ②

Limiting value for θ will be $\theta = \theta_c$, for that let $\alpha_i = \alpha_{i(max)}$.

$$\sin\alpha_{i(max)} = \frac{n_1}{n_0}\cos\theta_c$$

We know that, $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \Rightarrow \sin\theta_c = \frac{n_2}{n_1}$

$$\Rightarrow \cos\theta_c = \sqrt{1 - \frac{n_2^2}{n_1^2}}.$$

$$\therefore \sin\alpha_{i(max)} = \frac{n_1}{n_0}\sqrt{\frac{1-n_2^2}{n_1^2}}$$

$$= \frac{\cancel{n_1}}{n_0}\frac{\sqrt{n_1^2-n_2^2}}{\cancel{n_1}}$$

$$\boxed{\sin\alpha_{i(max)} = \frac{\sqrt{n_1^2-n_2^2}}{n_0}}$$ → Numerical Aperture

$$\Rightarrow \boxed{\alpha_{i(max)} = \sin^{-1}\frac{\sqrt{n_1^2-n_2^2}}{n_0}}$$ → Acceptance angle.

, for air $n_0 = 1$.

ASSIGNMENT-4

Unit 4 : Semi Conductor Physics.

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1) Carrier Concentration of electrons.

Intrinsic carrier concentration:

In case of pure semiconductor, the no. of e^- generated will be equal to same no. of holes created. As the two charge carriers are treated same it can be denoted by n_i, which is intrinsic density or intrinsic concentration.

We can write,
$$n = p = n_i$$
$$n_i^2 = np$$
$$= \left(N_c e^{-(E_c - E_F)/kT}\right)\left(N_v e^{-(E_F - E_v)/kT}\right)$$
$$= (N_c N_v) e^{-(E_c - E_v)/kT}$$

But, $(E_c - E_v) = E_g$

$$\Rightarrow \quad n_i^2 = (N_c N_v) e^{-E_g/kT}$$

By substituting the values of N_c and N_v,
$$= 4\left[\frac{2\pi kT}{h^2}\right]^3 (m_e^* m_h^*)^{3/2} e^{-E_g/kT}$$

$$\boxed{n_i = 2\left[\frac{2\pi kT}{h^2}\right]^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT}}$$

This is the expression for intrinsic carrier concentration.

Carrier concentration in n-type semiconductor:

Let N_D be the concentration of donors in the material. With rise in temperature, more atoms get ionized and e^- concentration in conduction band increases. e^- require an energy E_D for their transition in the conduction band from donor levels. Hence we assume e^- concentration n in the conduction band is,
$$n = N_D^+$$
$$n = N_D - N_D^0$$

$N_D^+ \rightarrow$ no. of donor atoms ionized, $N_D^0 \longrightarrow$ no. of atoms left to be ionized.

$$N_D^+ = (N_D - N_D^0) = N_D[1-f(E_D)] = \frac{N_D}{1+e^{-(E_D-E_F)/kT}}$$

$$n = \frac{N_D}{1+e^{-(E_D-E_F)/kT}}$$

On simplifying, $n = N_D e^{(E_D-E_F)/kT}$

In conduction band, $n = N_c e^{-(E_c-E_F)/kT}$

$$N_D e^{(E_D-E_F)/kT} = N_c e^{-(E_c-E_F)/kT}$$

Taking logarithm and rearranging the terms we get,

$$\left(\frac{E_D-E_F}{kT}\right)+\left(\frac{E_c-E_F}{kT}\right) = \ln\frac{N_c}{N_D}$$

$$(E_D+E_c)-2E_F = (kT)\ln\frac{N_c}{N_D}$$

$$\Rightarrow \quad E_F = \frac{E_D+E_c}{2} - \left(\frac{kT}{2}\right)\ln\frac{N_c}{N_D}$$

$$\Rightarrow \quad E_F = \frac{E_D+E_c}{2} + \left(\frac{kT}{2}\right)\ln\frac{N_D}{N_c}$$

$$\Rightarrow \quad E_F = \frac{E_D+E_c}{2} + \left(\frac{kT}{2}\right)\ln\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}$$

At $T=0K$, $E_F = \frac{E_D+E_c}{2}$.

Now, $$\exp\left[\frac{E_F-E_c}{kT}\right] = \exp\left[\frac{E_D+E_c}{2kT} + \left(\frac{1}{2}\right)\ln\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}} - \frac{E_c}{kT}\right]$$

$$= \exp\left[\frac{E_D-E_c}{2kT} + \ln\sqrt{\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}}\right].$$

$$= \exp\left[\left(\frac{E_D-E_c}{2kT}\right)\right]\cdot\left[\sqrt{\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}}\right].$$

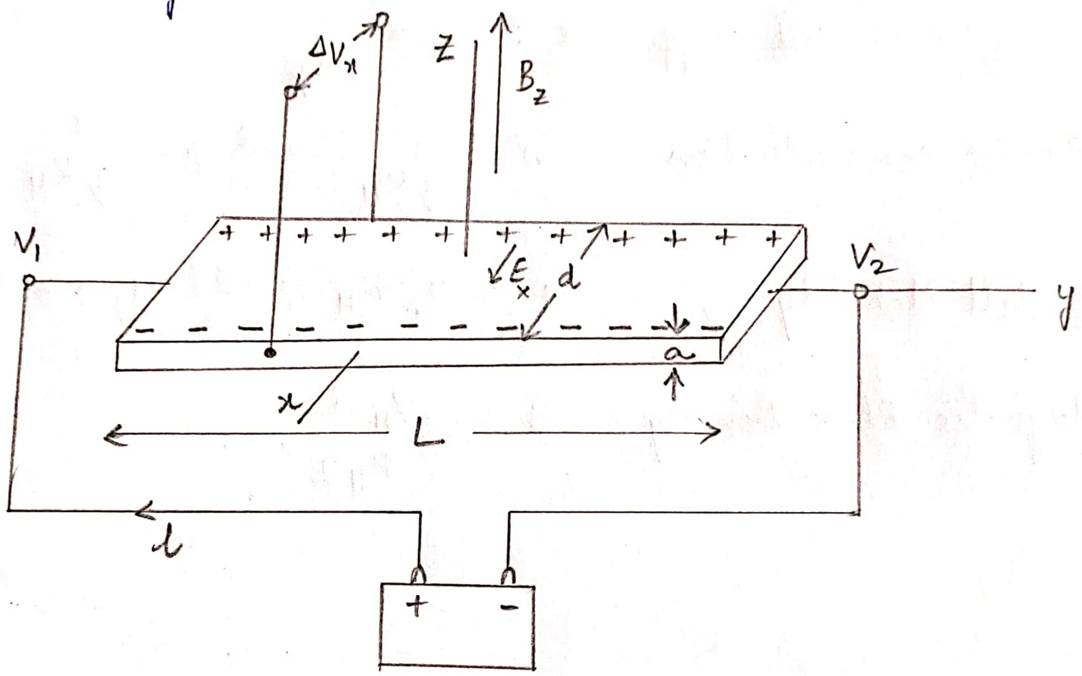
$$n = N_c\exp\left[\frac{E_F-E_c}{kT}\right] = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}\exp\left[\frac{E_D-E_c}{2kT}\right]\cdot\left[\sqrt{\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}}\right].$$

$$\therefore \quad \boxed{n = (2N_D)^{1/2}\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/4}\exp\left[\left(\frac{E_D-E_c}{2kT}\right)\right].}$$

2) Hall Effect.

In 1879, E. H. Hall observed that when an electrical current passes through a sample placed in a magnetic field, a potential proportional to the current and to the magnetic field is developed across the material in a direction ⊥'r to both the current and to the magnetic field. This effect is known as Hall effect.

Consider a conducting slab with length L in x direction, width w in y direction and thickness t in the z direction.



Assume the conductor to have charge carrier of charge q, charge carrier number density n, charge carrier drift velocity V_x, current I_x. If J_x is current density and crossectional area of conductor is 'da'. Then,

$$I_x = J_x da = nqv_x da .$$

In the case where current is directly proportional to field, we say material obeys Ohm's law and,

$$J_x = \sigma E_x .$$

$\sigma \rightarrow$ conductivity of material in conductor.

Now, if conductor is placed in magnetic field ⊥ to plane of slab, then charge carriers experience a Lorentz force $qV \times B$ that will deflect them toward one side of slab which creates a transverse electric field E_y.

$$E_y = v_x B_z$$

We measure the potential difference across the sample – the Hall voltage V_H – which is related to Hall field by,

$$V_H = -\int_0^z E_y \cdot dy = -E_y w.$$

$$\boxed{V_H = \frac{1}{nq} \frac{I_x \cdot B_z}{t}}$$

where Hall coefficient : $R_H = \frac{1}{nq}$

Carrier concentration, $n = \frac{1}{q R_H}$ and $p = \frac{1}{q R_H}$

Hall Mobility, $\mu_n = \sigma_n R_H$ and $\mu_p = \sigma_p R_H$.

Magnetic Flux Density, $B = \frac{V_H d}{R_H I}$.