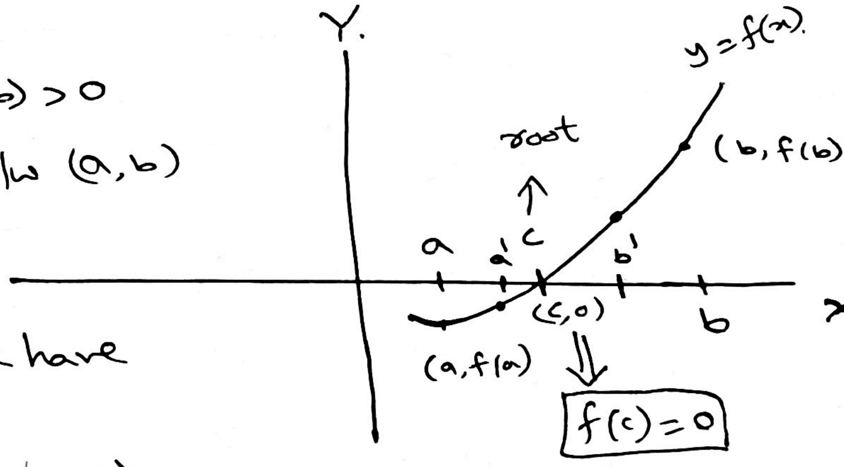
UNIT - 1

Theorem: If $f(x)$ is Continous on $[a, b]$ and $f(a) \cdot f(b) < 0$, then $\exists\ c \in (a, b)$ such that

$$f(c) = 0$$

we Call 'c' as root of $f(x) = 0$

Bisection Method



Clearly $f(a) < 0$, $f(b) > 0$
So root lies b/w (a, b)

Let $b' = \dfrac{a+b}{2}$

Then, from the graph, we have

$f(a) < 0$ $\quad f(b') > 0$

$\therefore$ root lies b/w (a, b')

Let $a' = \dfrac{a+b'}{2}$ then we Can see $f(a') < 0$, $f(b') > 0$
So root lies b/w (a', b')

The Process Continues till we reach the root.

① obtain an approximated root for $x^3-5x+1=0$ upto 2 decimals of accuracy.

Solution: $f(x) = x^3-5x+1$

$\left.\begin{array}{l} f(0) = 1 \\ f(1) = -3 \end{array}\right\} \Rightarrow$ $f(a) > 0$, $f(b) < 0$
$\therefore$ root lies b/w (a, b)

let $a=0$, $b=1$

S.NO	a	b	Sign of $f(a)$	Sign of $f(b)$	$c = \frac{a+b}{2}$	Sign of $f(c)$
1	0	1	>0	<0	0.5	<0
2	0	0.5	>0	<0	0.25	<0
3	0	0.25	>0	<0	0.125	>0
4	0.125	0.25	>0	<0	0.187	>0
5	0.187	0.25	>0	<0	0.218	<0
6	0.187	0.218	>0	<0	0.202	<0
7	0.187	0.202	>0	<0	0.194	>0
8	0.194	0.202	>0	<0	0.198	—

$\therefore$ The approximated root of $x^3-5x+1=0$ is 0.198.

(2) Find a approximate root of $xe^{x} = 1$, Perform 8 iterations.

<u>Solution</u> Let $f(x) = xe^{x} - 1 = 0$

$$\left.\begin{array}{l} f(0) = -1 < 0 \\ f(1) = 1.718 > 0 \end{array}\right\} \begin{array}{l} f(a) < 0,\ f(b) > 0 \\ \text{So root lies b/w } (a, b) \end{array}$$

S.NO	a	b	Sign of f(a)	Sign of f(b)	c $= \frac{a+b}{2}$	sign of f(c)
1	0	1	< 0	> 0	0.5	< 0
2	0.5	1	< 0	> 0	0.75	> 0
3	0.5	0.75	< 0	> 0	0.625	> 0
4	0.5	0.625	< 0	> 0	0.562	< 0
5	0.562	0.625	< 0	> 0	0.593	> 0
6	0.562	0.593	< 0	> 0	0.577	> 0
7	0.562	0.577	< 0	> 0	0.569	> 0
8	0.562	0.569	< 0	> 0	$\boxed{0.565}$	

Approximate root of $xe^{x} - 1 = 0$ is 0.565.

(3) Find an approximate root of $\sin x = \frac{1}{x}$, carry out computations upto 8^{th} stage.

Solution: Let $f(x) = x\sin x - 1$

$f(0) = -1 < 0$

$f(1) = -0.158 < 0$

$f(2) = 0.818 > 0$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $f(a) < 0$, $f(b) > 0$ So Root lies b/w (a, b)

S.NO	a	b	Sign of f(a)	Sign of f(b)	$c = \frac{a+b}{2}$	Sign of f(c)
1	1	2	<0	>0	1.5	>0
2	1	1.5	<0	>0	1.25	>0
3	1	1.25	<0	>0	1.125	>0
4	1	1.125	<0	>0	1.062	<0
5	1.062	1.125	<0	>0	1.093	<0
6	1.093	1.125	<0	>0	1.109	<0
7	1.109	1.125	<0	>0	1.117	>0
8	1.109	1.117	<0	>0	$\boxed{1.113}$	

$\therefore$ $x = 1.113$ is an approximate root of $x\sin x - 1 = 0$

$x = 1.113$ is an approximate root of $\sin x = \frac{1}{x}$.

Class work on Bisection Method

① Find a root of the equation $x^3 - 4x - 9 = 0$, using the bisection Method Correct to 2 decimal places.

② Find the root of the equation $\cos x - xe^x = 0$ using Bisection Method Correct to 2 decimal places.

③ Find a real root of the equation $x \log_{10} x = 1.2$ Correct to 2 decimal places.