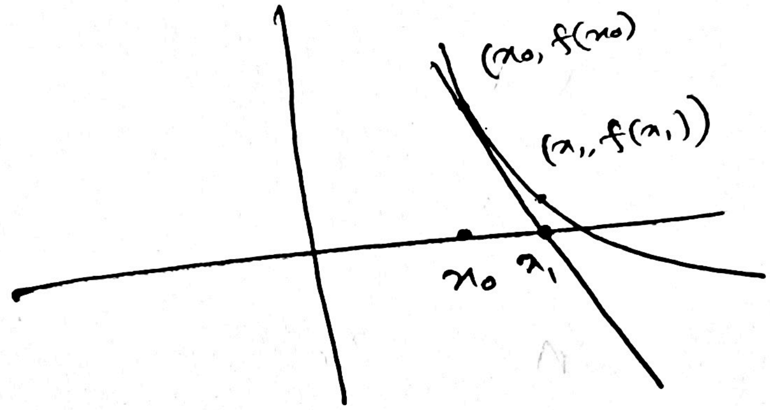
3 - Newton's Method (or) Newton - Raphson Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general, $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

→ <u>Class work</u> : (6 decimals)

1) $f(x) = x^3 - 4x - 9 = 0$

2) $\cos x - xe^x = 0$

3) $x \log x - 1.2 = 0$

① obtain approximate root of $x^3 - 5x + 1 = 0$ corrected to 6 decimals of accuracy

<u>Solution</u> $f(x) = x^3 - 5x + 1$, $f'(x) = 3x^2 - 5$

$f(0) = 1 > 0$
$f(1) = -3 < 0$ } root lies b/w 0 & 1

Let $x_0 = 0$

S.No	n	x_n	$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
1	0	$x_0 = 0$	$x_1 = 0.2$
2	1	$x_1 = 0.2$	$\boxed{x_2 = 0.2016393}$
3	2	$x_2 = 0.2016393$	$\boxed{x_3 = 0.2016396}$

The root is 0.2016396 corrected to 6 decimals of accuracy.

Note: $X - \dfrac{f(x)}{f'(x)} = X - \dfrac{x^3 - 5x + 1}{3x^2 - 5}$

(or)

$X - [(x^3 - 5x + 1) \div (3x^2 - 5)]$

(2) $xe^x - 1 = 0$

$f(x) = xe^x - 1$

$f'(x) = xe^x + e^x$

$f(0) = -1 < 0$

$f(1) = 1.718 > 0$

So root lies b/w 0, 1

S.NO	n	x_n	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1	0	$x_0 = 0$	$x_1 = 1$
2	1	$x_1 = 1$	$x_2 = 0.6839397$
3	2	$x_2 = 0.6839397$	$x_3 = 0.5774544$
4	3	$x_3 = 0.5774544$	$x_4 = 0.5672297$
5	4	$x_4 = 0.5672297$	$\boxed{x_5 = 0.5671432}$
6	5	$x_5 = 0.5671432$	$\boxed{x_6 = 0.5671432}$

The root is 0.5671432.

note: $x - \frac{f(x)}{f'(x)} = x - \frac{xe^x - 1}{xe^x + e^x}$

(or)

$x - [(xe^x - 1) \div (xe^x + e^x)]$

(3) $f(x) = x \sin x - 1$

$f'(x) = x \cos x + \sin x$

$f(1) = -0.1585 < 0$
$f(2) = 0.81859 > 0$ $\}$ root lies in $(1, 2)$

S.NO	n	x_n	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1	0	$x_0 = 1$	$x_1 = 1.1147286$
2	1	$x_1 = 1.1147286$	$x_2 = 1.1141571$
3	2	$x_2 = 1.1141571$	$x_3 = 1.1141571$

The root is 1.1141571.

note: $x - \frac{f(x)}{f'(x)} = x - \frac{x\sin(x) - 1}{x\cos(x) + \sin(x)}$

(or)

$x - [(x\sin(x) - 1) \div (x\cos(x) + \sin(x))]$