Interpolation with unequal intervals

(1) If $y = f(x)$ takes the values $y_0, y_1, \ldots, y_n$ corresponding to $x_0, x_1, \ldots, x_n$ then

$$y(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}\, y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}\, y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}\, y_n$$

This is known as Lagrange's interpolation formula.

(6) If $y(1) = -3$, $y(3) = 9$, $y(4) = 30$, $y(6) = 132$. Find the polynomial $f(x)$ using L.I.P.

$y(2)$, $y(5)$

(7) Use Lagranges formula, express $\dfrac{x^2+x-3}{x^3-2x^2-x+2}$ as sum of partial fractions.

(8)

x :	0	1	2	4	3
y :	1	3	9	81	$\boxed{}$

(1) Using Lagranges interpolation formula, find the value of y when $x=10$, Given

x:	5	6	9	11
y:	12	13	14	16

Solution:

x:	5^{x_0}	6^{x_1}	9^{x_2}	11^{x_3}
y:	12 y_0	13 y_1	14 y_2	16 y_3

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12) + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}(13)$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}(16)$$

$$y(10) = \frac{4.1.1.12}{1.4.6} + \frac{(5)(1)(-1)(13)}{1.(-3)(-5)} + \frac{5.4.(-1)(14)}{4.3.(-2)} + \frac{5.4.1(16)}{6.5.2}$$

$$= \boxed{}$$

$$= 14.7$$

② Find the Polynomial $f(x)$ and hence find $f(3)$, Given

x :	0	1	2	5
$f(x)$:	2	3	12	147

Solution

$x_0 = 0 \qquad x_1 = 1 \qquad x_2 = 2 \qquad x_3 = 5$

$y_0 = 2 \qquad y_1 = 3 \qquad y_3 = 12 \qquad y_4 = 147$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$f(x) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)$$

$$+ \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

Note: $(x-a)(x-b)(x-c) = x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc$

$$f(x) = x^3 + x^2 - x + 2$$

$$\boxed{f(3) = 35}$$

③ A curve passes through the points $(0, 18)$, $(1, 10)$, $(3, -18)$ and $(6, 90)$. Find the slope of the curve at $x = 2$.

Solution: Consider:

	x_0	x_1	x_2	x_3
x:	0	1	3	6
y:	18	10	-18	90
	y_0	y_1	y_2	y_3

Use Lagrange's formula, we get

$$\boxed{y(x) = 2x^3 - 10x^2 + 18.}$$

$$y'(x) = 6x^2 - 20x$$

Slope of the curve at $x=2$ is

$$y'(2) = 6(2)^2 - 20(2) = -16.$$

(4) Using Lagranges formula, express $\dfrac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ as the sum of partial fractions.

Solution Let us consider $y(x) = 3x^2+x+1$

	x_0	x_1	x_2
x :	1	2	3
y :	5	15	31
	y_0	y_1	y_2

$$y(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

$$y(x) = \frac{(x-2)(x-3)(5)}{(1-2)(1-3)} + \frac{(x-1)(x-3)(15)}{(2-1)(2-3)} + \frac{(x-1)(x-2)(31)}{(3-1)(3-2)}$$

$$y(x) = \frac{5}{2}(x-2)(x-3) - (x-1)(x-3)(15) + \frac{31}{2}(x-1)(x-2)$$

$$3x^2+x+1 = \frac{5}{2}(x-2)(x-3) - (x-1)(x-3)(15) + \frac{31}{2}(x-1)(x-2)$$

$$\frac{3x^2+x+1}{(x-1)(x-2)(x-3)} = \frac{5}{2}\frac{1}{x-1} - \frac{15}{x-2} + \frac{31}{2}\frac{1}{x-3}$$

(5) Find the distance moved by a Particle and its accaleration at the end of 4 Seconds if the time verses velocity is as follows:

	t_0	t_1	t_2	t_3
t :	0	1	3	4
$v(t)$:	21	15	12	10
	v_0	v_1	v_2	v_3

Solution:

$$v(t) = \frac{(t-t_1)(t-t_2)(t-t_3)}{(t_0-t_1)(t_0-t_2)(t_0-t_3)}(v_0) + \frac{(t-t_0)(t-t_2)(t-t_3)}{(t_1-t_0)(t_1-t_2)(t_1-t_3)}(v_1)$$

$$+ \frac{(t-t_0)(t-t_1)(t-t_3)}{(t_2-t_0)(t_2-t_1)(t_2-t_3)}(v_2) + \frac{(t-t_0)(t-t_1)(t-t_2)}{(t_3-t_0)(t_3-t_1)(t_3-t_2)} v_3$$

Substitute, we get

$$v(t) = \frac{-5}{12}t^3 + \frac{38}{12}t^2 - \frac{105}{12}t + \frac{252}{12}$$

$$= \frac{-5}{12}t^3 + \frac{19}{6}t^2 - \frac{35}{4}t + 21$$

$$\text{Distance moved} = s = \int_0^4 v\,dt$$

$$= \int_0^4 \left[\frac{-5}{12}t^3 + \frac{19}{6}t^2 - \frac{35}{4}t + 21\right] dt$$

$$= \boxed{\cdots} = 54.9$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{-5}{12}t^3 + \frac{19}{6}t^2 - \frac{35}{4}t + 21\right]$$

$$= -\frac{5}{12}(3t^2) + \frac{19}{6}(2t) - \frac{35}{4}$$

$$\text{Acceleration at } t=4 = -\frac{5}{12}(3)(4)^2 + \frac{19}{6}(2)(4) - \frac{35}{4} = -3.4$$