1. Given $\sin 45° = 0.7071$, $\sin 50° = 0.7660$, $\sin 55° = 0.8192$, $\sin 60° = 0.8660$ find $\sin 52°$ using newtons forward formula.

Sol:-

x_i	y_i	Δy_i	Δy_i^2	Δy_i^3
45	0.7071			
		0.0589 Δy_0		
50	0.7660		-0.0057 $\Delta^2 y_0$	
		0.0532		-0.0007 $\Delta^3 y_0$
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

given $x_0 = 45$, $h = 5$, $p = \frac{x - x_0}{h} = \frac{x-45}{5}$

We need $\sin 52° \Rightarrow y(52)$

so $p = \frac{52-45}{5} = 7/5 = 1.4$

By newton's forward interpolation formula we have

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 0.7071 + (1.4)(0.0589) + \frac{(1.4)(0.4)(-0.0057)}{2} + \frac{(1.4)(0.4)(-0.6)(-0.0007)}{6}$$

$y(52) = 0.7880032$

$\therefore \sin 52° = 0.7880032$

② Using Newtons forward formula find $f(1.6)$.

If

x	1	1.4	1.8	2.2	2.6	3.0
y	3.49	4.82	5.96	6.5	7.2	8.4

Sol:-

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$
1	3.49					
		1.33 Δy_0				
1.4	4.82		-0.19 $\Delta^2 y_0$			
		1.14		~~0.09~~ -0.41 $\Delta^3 y_0$		
1.8	5.96		-0.6		1.17 $\Delta^4 y_0$	
		0.54		0.76		-1.59 $\Delta^5 y_0$
2.2	6.5		0.16		-0.42	
		0.7		0.34		
2.6	7.2		0.5			
		1.2				
3.0	8.4					

we need $y(1.6)$ so $1.6 = x_0 + ph$

$h = 0.4, x_0 = 1$

$1.6 = 1 + p(0.4)$

$0.6 = p(0.4) \Rightarrow p = \frac{0.6}{0.4} = 1.5$

By newtons forward interpolation formula

$$y(x_0+ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 y_0$$

$$= 3.49 + 1.5(1.33) + \frac{(1.5)(0.5)}{2}(-0.19) + \frac{(1.5)(0.5)(-0.5)}{6}(-0.41)$$

$$+ \frac{(1.5)(0.5)(-0.5)(-1.5)}{24}(1.17) + \frac{(1.5)(0.5)(-0.5)(-1.5)(-2.5)}{120}(-1.59)$$

$= 3.49 + 1.995 + (-0.07125) + 0.025625 + 0.027421$
$+ 0.018632$

$= \underline{5.485428}$

③ the population of a town in the decimal census was given below estimate the population for the year 1895

Sol:-

year	1891	1901	1911	1921	1931
POP in thousands	46	66	81	93	101

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
1891	46				
		Δy_0 20			
1901	66		$\Delta^2 y_0$ -5		
		15		$\Delta^3 y_0$ 2	
1911	81		-3		$\Delta^4 y_0$ -3
		12		-1	
1921	93		-4		
		8			
1931	101				

we need $y(1895)$ so $1895 = 1891 + p(10)$

$$p = \frac{1895 - 1891}{10} = 0.4$$

By newtons forward interpolation formula

$$y(x_0+ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$= 46 + (0.4)(20) + \frac{(0.4)(-0.6)}{2}(-5) + \frac{(0.4)(-0.6)(-1.6)}{6}(2) + \frac{(0.4)(-0.6)(-1.6)(-2.6)}{24}(-3)$$

$= 46 + 8 + 0.6 + 0.128 + 0.1248$

$= 54.8528$

$\therefore f(1895) = 54.8528.$

④ A second degree polynomial passes through the points $(1,-1)$, $(2,-1)$, $(3,1)$ and $(4,5)$ find the polynomial

Sol:

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
1	-1			
		0		
2	-1		2	
		2		0
3	1		2	
		4		
4	5			

Given $x_0 = 1$ $h = 1$ $p = \frac{x - x_0}{h} = \frac{x-1}{1} = x-1$

By Newtons forward intapolation formula, we have

$$f(x) = f(0) + P\Delta f(0) + \frac{P(P-1)}{2!}\Delta^2 f(0) + \frac{P(P-1)(P-2)}{3!}\Delta^3 f(0)$$

$$= -1 + (x-1)(0) + \frac{(x-1)(x-2)}{\cancel{2}}(\cancel{2}) + \frac{(x-1)(x-2)(x-3)}{6}(0)$$

$$= -1 + (x-1)(x-2)$$

$$= -1 + x^2 - 2x - x + 2$$

$$= x^2 - 3x + 1$$

⑤ find $f(1.75)$, if $f(1.7) = 5.474$, $f(1.8) = 6.050$, $f(1.9) = 6.686$, $f(2) = 7.389$

Sol:-

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
1.7	5.474			
		0.576		
1.8	6.050		0.06	
		0.636		0.007
1.9	6.686		0.067	
		0.703		
2	7.389			

Given $x_0 = 1.7$, $h = 0.1$, $P = \frac{x - x_0}{h} = \frac{1.75 - 1.7}{0.1}$

$= 0.5$

By Newton's forward interpolation formula, we have

$$f(x) = f(0) + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 .$$

$$= 5.474 + (0.5)(0.576) + \frac{(0.5)(-0.5)}{2}(0.06) + \frac{(0.5)(-0.5)(-1.5)}{6}(0.007)$$

$$= 5.474 + 0.288 + (-0.0075) + 0.0004375$$

$$= 5.75493$$

(6) Using newtons forward formula Compute $f(142)$ from following table

x	140	150	160	170	180
y	3.685	4.854	6.302	8.046	10.225

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
140	3.685				
		1.169 Δy_0			
150	4.854		0.279 $\Delta^2 y_0$		
		1.448		0.017 $\Delta^3 y_0$	
160	6.302		0.296		0.122 $\Delta^4 y_0$
		1.744		0.139	
170	8.046		0.435		

~~180~~ 2.179

180 10.225

Given $x_0 = 140$, $h = 10$ $\quad p = \frac{x - x_0}{h} = \frac{142-140}{10}$

$= \frac{2}{10} = \frac{1}{5} = 0.2$

By newton's forward interpolation formula, we have

$$f(x) = f(0) + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0$$

$$= 3.685 + (0.2)(1.169) + \frac{(0.2)(-0.8)}{2}(0.279) + \frac{(0.2)(-0.8)(-1.8)}{6}(0.017)$$

$$+ \frac{(0.2)(-0.8)(-1.8)(-2.8)}{24}(0.122)$$

$$= 3.685 + 0.2338 + (-0.02232) + 0.000816 + (-0.0040992)$$

$$= 3.8931968$$

7. Calculate the value $f(7.5)$ for the table

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

x_i	y_i	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		0
		91		6	
6	216		36		$0_{\nabla^4 y_n}$
		127		$6_{\nabla^3 y_n}$	
7	343		$42_{\nabla^2 y_n}$		
		$169_{\nabla y_n}$			

$\nabla^5 y_n = 0$

$\nabla^6 y_n = 0$

$\nabla^7 y_n = 0$

~~so the~~

so the equation will be

$$y = f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$p = \frac{x - x_n}{h} = \frac{x-8}{1} \Rightarrow p = \frac{7.5-8}{1} = -\frac{0.5}{1}$$

here $x_n = 8$ $h = 1$. given $x = 7.5$

$$p = -0.5$$

$$y = 512 + (-0.5)(169) + \frac{(0.5)(0.5)}{2}(42) + \frac{(0.5)(0.5)(1.5)}{6}(6)$$

$$= 512 + (-84.5) - 5.25 - 0.375$$

$$= 512 - 90.125$$

$$= 421.875$$

(8) Estimate value of $f(22)$ & $f(42)$ from the following table:

x	20	25	30	35	40	45
$f(x)$	354	332	291	260	231	204

sol:-

x_i	$f(x)=y_i$	$\nabla y, \Delta y$	$\nabla^2 y, \Delta^2 y$	$\nabla^3 y, \Delta^3 y$	$\Delta^4 y, \nabla^4 y$	$\Delta^5 y, \nabla^5 y$
20	354 y_0					
		-22 Δy_0				
25	332		-19 $\Delta^2 y_0$			
		-41		29 $\Delta^3 y_0$		
30	291		10		-37 $\Delta^4 y_0$	
		-31		-8		45 $\Delta^5 y_0$ $\nabla^5 y_n$
35	260		2		8 $\nabla^4 y_n$	
		-29		0 $\nabla^3 y_n$		
40	231		2 $\nabla^2 y_n$			
		-27 ∇y_n				
45	204 y_n					

For calculating $f(22)$

$$x_0 = 20, h = 5, \quad p = \frac{x - x_0}{h} = \frac{22-20}{5} = \frac{2}{5} = 0.4$$

By Newton's forward interpolation formula

$$f(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0$$

$$+ \frac{P(P-1)(P-2)(P-3)(P-4)}{5!}\Delta^5 y_0$$

$$f(x) = 354 + (0.4)(-22) + \frac{(0.4)(-0.6)}{2}(-19) + \frac{(0.4)(-0.6)(-1.6)}{6}(29)$$

$$+ \frac{(0.4)(0.6)(-1.6)(-2.6)}{24}(-37) + \frac{(0.4)(-0.6)(-1.6)(-2.6)(-3.6)}{120}(45)$$

$$= 354 - 8.8 + 2.28 + 1.856 + 1.5392 + 1.34784$$

$$= 352.22 \approx 352$$

for $f(42)$ by newton's backward interpolation

$$P = \frac{x - x_n}{h} = \frac{42 - 45}{5} = -0.6$$

$$y = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_n$$

$$+ \frac{P(P+1)(P+2)(P+3)(P+4)}{9!}\nabla^5 y_n$$

$$= 204 + (-0.6)(-27) + \frac{(-0.6)(0.4)\cancel{(1.4)}}{\cancel{6}\,2}(2) + \frac{(-0.6)(0.4)(1.4)}{6}(0)$$

$$+ \frac{(-0.6)(0.4)(1.4)(2.4)}{\cancel{24}\,3}(8) + \frac{(-0.6)(0.4)(1.4)(2.4)(3.4)}{120}(45)$$

$$= 204 + 16.2 - 0.24 - 0.2688 - 1.02816$$

$$= 218.66304 \approx 219$$

9. From following data ~~to~~ find θ at $x = 43$ & $x = 84$

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$
40	184			
		20		
50	204		2	
		22		0
60	226		2	
		24		0
70	250		2	
		26		0
80	276		2	
		28		
90	304			

$\Delta^4 y_0 = 0$

$\Delta^5 y_0 = 0$

$lly\ \nabla^4 y_n = 0$

$\nabla^5 y_n = 0$

For calculating $f(43)$

$x_0 = 40,\ h = 10,\ p = \frac{x - x_0}{h} = \frac{43-40}{10} = \frac{3}{10}$

$= 0.3$

By newtons forward interpolation formula

$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$

$= 184 + (0.3)(20) + \frac{(0.3)(-0.7)}{\cancel{2}}(\cancel{2})$

$= 184 + 6 + (-0.21)$

$= 190 - 0.21 = 189.79$

for calculating $f(84)$

$x_n = 90\quad \cancel{x}\ h = 10\quad p = \frac{x - x_n}{h} = \frac{84-90}{10} = -0.6$

By newtons backward interpolation formula

$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n$

$= 304 + (-0.6)(28) + \frac{(-0.6)(0.4)}{\cancel{2}} \times \cancel{2}$

$= 304 - 16.8 - 0.24$

$= 304 - 17.04 = 286.96$

⑩ Evaluate $\sqrt{8.5}$ given that $\sqrt{5} = 2.236$, $\sqrt{6} = 2.449$, $\sqrt{7} = 2.646$, $\sqrt{8} = 2.828$ by newton backward interpolation formula.

Sol:-

x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$
5	2.236			
		0.213		
6	2.449		-0.016	
		0.197		0.001
7	2.646		-0.015	
		0.182		
8	2.828			

For calculating $f(8.5)$

$$x_n = 8 \quad h = 1 \quad p = \frac{x - x_n}{h} = \cancel{\frac{8.5 - 8.5}{1}}$$

$$= \frac{8.5 - 8}{1} = 0.5$$

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$= 2.828 + (0.5)(0.182) + \frac{(0.5)(1.5)}{2} \times (-0.015) + \frac{(0.5)(1.5)(2.5)}{6}(0.001)$$

$$= 2.828 + 0.091 + (-0.005625) + 0.0003125$$

$$= 2.9136875$$