24/9/20.

ASSIGNMENT-1

DMS

V. Greeshma
CSE-4
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1. Consider the following.

P: Today is Tuesday Q: It is raining. R: It is cold.

Write in simple sentences of the following formulas.

a) $\sim Q \rightarrow (R \wedge P)$

$\sim Q$: It is not raining.

$R \wedge P$: It is cold and today is Tuesday.

$\therefore$ $\sim Q \rightarrow (R \wedge P)$: If it is not raining then it is cold and today is Tuesday.

b) $\neg P \rightarrow (Q \vee R)$

$\neg P$: Today is not Tuesday.

$Q \vee R$: It is raining or it is cold.

$\therefore$ $\neg P \rightarrow (Q \vee R)$: If today is not Tuesday then it is raining or it is cold.

c) $(P \vee Q) \leftrightarrow R$

$P \vee Q$: Today is Tuesday or it is raining.

R : It is cold.

$\therefore$ $(P \vee Q) \leftrightarrow R$: Today is Tuesday or it is raining if and only if it is cold.

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2. a) Prove that $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ is Tautology.

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \rightarrow \neg P$	$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Since the truth values are all true, hence,
$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ is a Tautology.

b) Prove that $P \land (P \lor Q) \leftrightarrow P$ is Tautology.

P	Q	$P \lor Q$	$P \land (P \lor Q)$	$P \land (P \lor Q) \leftrightarrow P$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

Since the truth values are all true, hence $P \land (P \lor Q) \leftrightarrow P$ is a Tautology.

3. a) Show that $\neg(P \land Q) \equiv \neg P \lor \neg Q$.

P	Q	$\neg P$	$\neg Q$	$P \land Q$	$\neg(P \land Q)$	$\neg P \lor \neg Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Since the truth values of $\neg(P \land Q)$ and $\neg P \lor \neg Q$ are identical, they are said to be equivalent.

$\therefore \quad \neg(P \land Q) \equiv \neg P \lor \neg Q$.

b) Show that $P \lor (Q \land \neg Q) \equiv P$.

P	Q	$\neg Q$	$Q \land \neg Q$	$P \lor (Q \land \neg Q)$
T	T	F	F	T
T	F	T	F	T
F	T	F	F	F
F	F	T	F	F

Since the truth values of P and $P \lor (Q \land \neg Q)$ are identical, they are said to be equivalent.

$\therefore \quad P \lor (Q \land \neg Q) \equiv P$.

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4. Find PCNF of $Q \wedge (P \vee \neg Q)$

P	Q	$\neg Q$	$P \vee \neg Q$	$Q \wedge (P \vee \neg Q)$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	F
F	F	T	T	F

The max terms are $(\sim P \vee Q)$, $(P \vee \sim Q)$, $(P \vee Q)$ and their conjunction gives the PCNF.

$\therefore$ $(\sim P \vee Q) \wedge (P \vee \sim Q) \wedge (P \vee Q)$ is the required PCNF.

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5. Find PDNF of $(\neg P \vee \neg Q) \longrightarrow (\neg P \wedge R)$.

P	Q	R	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg P \wedge R$	$(\neg P \vee \neg Q) \to (\neg P \wedge R)$
T	T	T	F	F	F	F	T
T	T	F	F	F	F	F	T
T	F	T	F	T	T	F	F
T	F	F	F	T	T	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

The min terms are $(P \wedge Q \wedge R)$, $(P \wedge Q \wedge \neg R)$, $(\neg P \wedge Q \wedge R)$, $(\neg P \wedge \neg Q \wedge R)$ and their disjunction gives the PDNF.

$\therefore$ $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$ is the required PDNF.

6. If PDNF of A is $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$, then find PCNF of A.

Sol^n: Given that PDNF of A is $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$, then PDNF of $\sim A$ is the disjunction of the remaining minterms which do not appear in PDNF of A,

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i.e $\sim A \Leftrightarrow$ disjunction of remaining terms.

Solve PDNF of $A \Leftrightarrow (P\wedge Q)\vee(\neg P\wedge R)\vee(Q\wedge R)$

$$\equiv [(P\wedge Q)\wedge T]\vee[(\neg P\wedge R)\wedge T]\vee[(Q\wedge R)\wedge T]$$

$[\because P\wedge T\equiv P]$.

$$\equiv [(P\wedge Q)\wedge(R\vee\neg R)]\vee[(\neg P\wedge R)\wedge(Q\vee\neg Q)]$$
$$\vee[(Q\wedge R)\wedge(P\vee\neg P)]$$

$[\because P\vee\neg P\equiv T]$.

$$\equiv (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R)$$
$$\vee(P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R)$$

$[\because$ By distributive property$]$.

$$\equiv (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(\neg P\wedge Q\wedge R)\vee(\neg P\wedge\neg Q\wedge R).$$

Hence this is the PDNF of A.

Now for $\sim A$,

i.e $\sim A \Leftrightarrow (P\wedge\sim Q\wedge R)\vee(P\wedge\sim Q\wedge\sim R)$
$\vee(\sim P\wedge Q\wedge\sim R)\vee(\sim P\wedge\sim Q\wedge\sim R)$.

Applying negation on both sides, we get,

$$\sim\sim A \Leftrightarrow \sim[(P\wedge\sim Q\wedge R)\vee(P\wedge\sim Q\wedge\sim R)\vee(\sim P\wedge Q\wedge\sim R)$$
$$\vee(\sim P\wedge\sim Q\wedge\sim R)].$$

Therefore $A \Leftrightarrow \sim(P\wedge\sim Q\wedge R)\wedge\sim(P\wedge\sim Q\wedge\sim R)\wedge\sim(\sim P\wedge Q\wedge\sim R)$
$\wedge\sim(\sim P\wedge\sim Q\wedge\sim R)$

$$\Leftrightarrow (\sim P\vee Q\vee\sim R)\wedge(\sim P\vee Q\vee R)\wedge(P\vee\sim Q\vee R)$$
$$\wedge(P\vee Q\vee R)$$

$$\boxed{\begin{array}{l}\text{Hence PCNF of } A \text{ is}\\ \Leftrightarrow (\sim P\vee Q\vee\sim R)\wedge(\sim P\vee Q\vee R)\wedge(P\vee\sim Q\vee R)\wedge(P\vee Q\vee R)\end{array}}.$$

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7. Write Antecedent and Consequent Rules.

Antecedent rules :-

$\neg \Rightarrow$: if $\overset{s}{\Rightarrow} X$ then $\neg X \overset{s}{\Rightarrow}$

$\wedge \Rightarrow$: if $X, Y \overset{s}{\Rightarrow}$ then $X \wedge Y \overset{s}{\Rightarrow}$

$\vee \Rightarrow$: if $X \overset{s}{\Rightarrow}$ and $Y \overset{s}{\Rightarrow}$ then $X \vee Y \overset{s}{\Rightarrow}$

$\rightarrow \Rightarrow$: if $\overset{s}{\Rightarrow} X$ and $Y \overset{s}{\Rightarrow}$ then $X \rightarrow Y \overset{s}{\Rightarrow}$

$\rightleftarrows \Rightarrow$: if $X, Y \overset{s}{\Rightarrow}$ and $\overset{s}{\Rightarrow} X, Y$ then $X \rightleftarrows Y \overset{s}{\Rightarrow}$

Consequent rules :-

$\Rightarrow \neg$: if $X \overset{s}{\Rightarrow}$ then $\overset{s}{\Rightarrow} \neg X$

$\Rightarrow \wedge$: if $\overset{s}{\Rightarrow} X$ and $\overset{s}{\Rightarrow} Y$ then $\overset{s}{\Rightarrow} X \wedge Y$

$\Rightarrow \vee$: if $\overset{s}{\Rightarrow} X, Y$ then $\overset{s}{\Rightarrow} X \vee Y$.

$\Rightarrow \rightarrow$: if $X \overset{s}{\Rightarrow} Y$ then $\overset{s}{\Rightarrow} X \rightarrow Y$

$\Rightarrow \rightleftarrows$: if $X \overset{s}{\Rightarrow} Y$ and $Y \overset{s}{\Rightarrow} X$ then $\overset{s}{\Rightarrow} X \rightleftarrows Y$.

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8. Prove that $(P \wedge Q) \rightarrow P$ is ~~not~~ a Tautology.

P	Q	$P \wedge Q$	$(P \wedge Q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since the truth values are all true, hence, $(P \wedge Q) \rightarrow P$ is a Tautology.