

language is not K.L.

\*

$$L = ww \mid w \in (a,b)^*$$

given string:-  $\underline{a} \underline{b} \underline{a} \underline{a} \underline{b} \underline{a}$   
 $\downarrow \quad \downarrow \quad \downarrow$   
 $x \quad y \quad z$

if  $i=4$ ;  $x y^i z \Rightarrow x y^4 z \Rightarrow abbbbbaaba$ ; not satisfied

$|y| > 0 \Rightarrow 1 > 0 \Rightarrow$  satisfied

$|xy| \leq n \Rightarrow 5 \leq n \Rightarrow$  satisfied

1<sup>st</sup> condition is not satisfied so not a regular lang.

# Closure Properties of Regular Sets :-

## 1. Union

Union of 2 regular language is regular.  
if  $L$  and  $M$  are regular languages, then so is  $L \cup M$ .

## 2. Intersection

Intersection of 2 regular languages are regular.  
if  $L$  and  $M$  are regular languages, then so is  $L \cap M$ .

## 3. Complement of a language is regular.

if  $L$  is a regular language over alphabet  $\Sigma$ , then  $\Sigma^* - L$  is also a regular language.

## 4. Difference of two regular languages is regular.

if  $L$  and  $M$  are regular languages, then so is  $L - M$ .

## 5. Reversal of a regular language is regular.

Reversing a string means writing the string backwards. for Eg., reverse of the string  $abcde$  is the string  $edcba$ .

The reversal of a language  $L$  is the language consisting of reversal of all its strings, reversed.

for Eg., if  $L = \{001, 110\}$ , then  $L(R) = \{100, 011\}$ .

## 6. Closure of a regular language is regular.

if  $L$  is a regular language, then so is  $L^*$ .

## 7. Concatenation of regular languages is regular

if  $L$  and  $M$  are regular languages, then so is  $LM$ .

## 8. Homomorphism of a regular language is regular

A homomorphism is the substitution of strings for symbols.

for Eg:- let the function  $h$  be defined by  $h(0) = a$  &  $h(1) = b$ . Then  $h$  applied to  $0011$  is simply  $aabb$ .  
if  $h$  is a homomorphism on alphabet  $\Sigma$  and if  $w$

is a string of symbols  $abcd \dots z$ , then

$$h(w) = h(a)h(b)h(c)h(d) \dots h(z)$$

The mathematical definition for homomorphism is

$$h: \Sigma^* \rightarrow \Gamma^* \text{ Such that } \forall x, y \in \Sigma^* \text{ and } h(x), h(y) \in \Gamma^*.$$

A homomorphism can also be applied to a language by applying it to each string in the language. Let  $L$  be a language over alphabet  $\Sigma$ , and  $h$  be a homomorphism on  $\Sigma$  then,

$$h(L) = \{h(w) \mid w \text{ is in } L\}$$

Then theorem can be stated as follows. 'if  $L$  is a regular language over alphabet  $\Sigma$ , and if  $h$  is a homomorphism on  $\Sigma$ , then  $h(L)$  is also regular'.

9. Inverse homomorphism of two regular languages is regular.

Suppose  $h$  be a homomorphism from some alphabet  $\Sigma$  to strings in another alphabet  $T$  and suppose  $L$  be a language over  $T$ , then  $h$  inverse of  $L$ .

$h^{-1}(L)$ , is the set of strings  $w$  in  $\Sigma^*$  such that  $h(w)$  is in  $L$ .

The theorem states that: 'if  $h$  is a homomorphism from alphabet  $\Sigma$  to alphabet  $T$ , and  $L$  is a regular language on  $T$ , then  $h^{-1}(L)$  is also a regular language.'

Eg:-

$$h(0) = a$$

$$h(1) = b$$

$$h(2) = ab$$

$$h(L) = \{abab\}$$

$$h^{-1}(L) = \{0101, 22, 012, 201\}$$

$$\begin{aligned} h(h^{-1}(L)) &= \{abab, abab, abab, abab\} \\ &= \{abab\} \end{aligned}$$



# Applications of Regular Expression

## \* Lexical Analysis

The compiler has a lexical analysis phase which forms the most important initial processing where the source program is scanned for recognizing the tokens. The tokens are defined by a regular expression for each pattern.

Eg:- An identifier is a token which has a pattern: 'an alphabet followed by any no. of alphanumeric characters'. The corresponding regular expression is

$[A-Za-z][A-Za-z|0-9]^*$

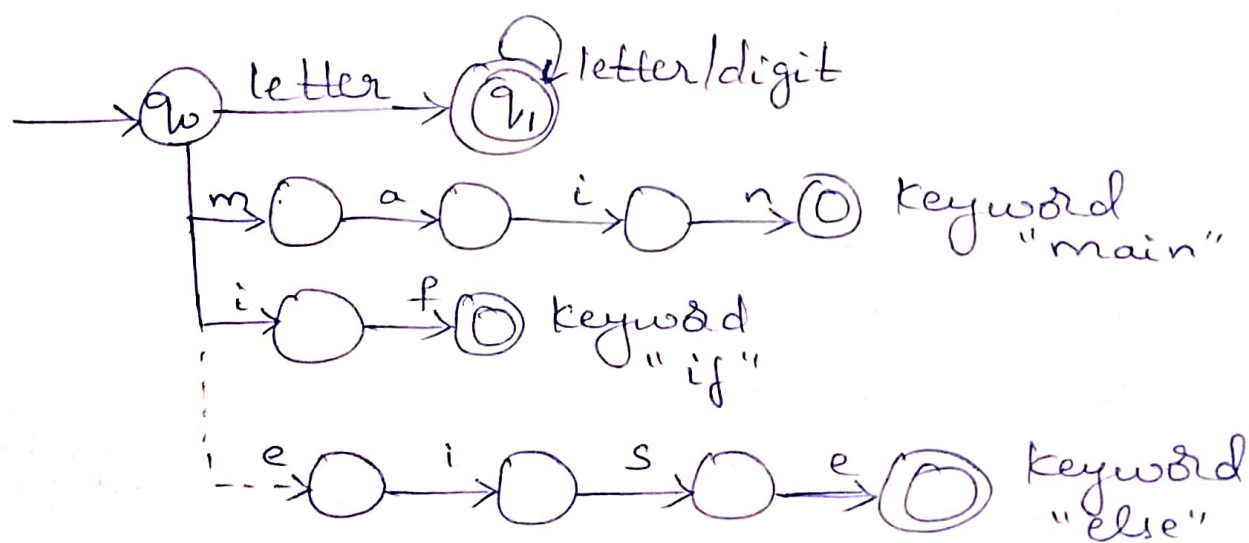
when the expression is specified using regular Expression, the corresponding finite automata generated is shown.



letter -  $[A-Za-z]$

digit -  $[0-9]$

⇒ DFA for Identifiers



⇒ DFA for Keywords

## \* Finding Patterns

The regular Expression technology is found to be useful in finding words corresponding to some patterns to be searched which are defined Vaguely.

- \* To Scan a Very large no. of webpages and detect addresses.
- \* To Create mailing list.
- \* To classify the business by their location and to answer the queries.
- \* To Search files in the system based on a pattern

Regular Grammars

Grammar contains 4 tuples

$\{V, T, P, S\}$

V - Variables

T - terminals

P - Production rule

S - Start symbol

$\boxed{s \rightarrow aSb \mid \epsilon}$   $\rightarrow$  Entire it is a production  
 $\downarrow \quad \downarrow$   
 $a \ \& \ b \rightarrow$  terminals  $S \rightarrow$  Variable.

Eg:- \*  $s \rightarrow aSb \mid \epsilon$

[There are 2 diff. productions  
 one is  $aSb$   
 other is  $\epsilon$ ]

Considering one production.

$s \rightarrow aSb$  [in place of  $s$  substitute  $aSb$ ]

$S \rightarrow \underline{aasbb}$   
 $\downarrow$

from here we got  $aabbb$

now again substitute  $aSb$  in place of  $s$

$s \rightarrow \underline{aaasbbb}$

Now if  $s$  is  $\epsilon$  then we would  
 will get  $aaabbb$

$\Rightarrow$  if  $\epsilon$  is placed instead of  $s$ , it is known as terminating.

$\Rightarrow$  if  $aSb$  is placed instead of  $s$  then the no. of  $a$ 's and  $b$ 's will be increasing and also no. of  $a$ 's = no. of  $b$ 's.

So, the language generated by this grammar is  $a^n b^n$ . [i.e., no. of a's = no. of b's]

$$\begin{array}{l} * \quad \left. \begin{array}{l} S \rightarrow SS \\ S \rightarrow aSb \\ S \rightarrow bSa \\ S \rightarrow \epsilon \end{array} \right\} \text{Given Grammar} \end{array}$$

The 4 productions can <sup>also</sup> be written as  
 $S \rightarrow SS | aSb | bSa | \epsilon$

cond. consider  $S \rightarrow SS$  production.

$$S \rightarrow SS \quad [\text{substitute } S = aSb]$$

$$S \rightarrow \underline{a}S\underline{b}aS\underline{b} \quad [\text{again sub. } S = aSb]$$

$$S \rightarrow a\underline{a}S\underline{b}b\underline{a}aS\underline{b}b \quad [\text{sub. } S = bSa]$$

$$S \rightarrow a\underline{a}b\underline{S}a\underline{b}b\underline{a}a\underline{b}S\underline{a}b \quad [\text{sub. } S = \epsilon] \\ \text{i.e. terminating}$$

$$S \rightarrow \underline{aabaabbaababb}$$

6 a's, 6 b's [no. of a's = no. of b's]

So, the language generated by this grammar is  $(ab + ba)^*$

diff. b/w the above 2 eg's :-

1<sup>st</sup> eg:- no. of a's = no. of b's, & a should follow b and b should follow a. (condition)

2<sup>nd</sup> eg:-

no. of a's = no. of b's but there is no condition that a should follow b and b should follow a.