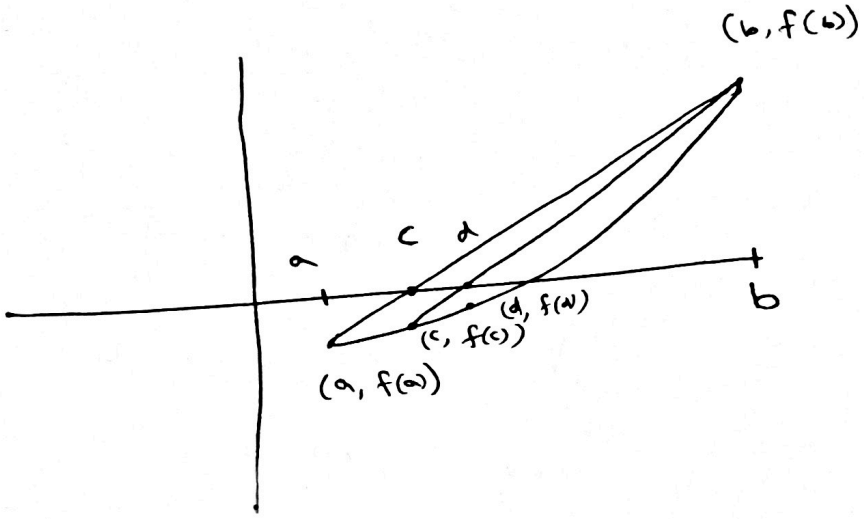
2 - Regula Falsi Method



Class notes work: on Regula falsi Method:

① $f(x) = x^3 - 4x - 9 = 0$, (4 decimals)

② $\cos x - xe^x = 0$ (")

③ $x \log_{10} x - 1.2 = 0$ (").

① obtain a approximated root for $x^3 - 5x + 1 = 0$ upto 4 decimals of accuracy

Solution:

$f(x) = x^3 - 5x + 1$

$f(0) = 1 > 0$

$f(1) = -3 < 0$

So root lies b/w 0 & 1 i.e (0, 1).

S NO	a	b	Sign of f(a)	Sign of f(b)	$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$	Sign of f(c)
1	0	1	>0	<0	0.25	<0
2	0	0.25	>0	<0	0.20253	<0
3	0	0.20253	>0	<0	0.20165	<0
4	0	0.20165	>0	<0	0.20163	

∴ The root is 0.20163 Grected to 4 decimals of accuracy.

Note:

$$\frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{a[b^3 - 5b + 1] - b[a^3 - 5a + 1]}{(b^3 - 5b + 1) - (a^3 - 5a + 1)}$$

$$[a(b^3 - 5b + 1) - b(a^3 - 5a + 1)] \div [(b^3 - 5b + 1) - (a^3 - 5a + 1)]$$

(2) Find a approximate root of $xe^x - 1 = 0$ upto 4 decimals of accuracy.

Solution

$f(x) = xe^x - 1 = 0$

$f(0) = -1 < 0$

$f(1) = 1.718 > 0$

} root lies b/w (0,1)

S.NO	a	b	Sign f(a)	Sign f(b)	$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$	Sign f(c)
1	0	1	<0	>0	0.36787	<0
2	0.36787	1	<0	>0	0.50331	<0
3	0.50331	1	<0	>0	0.54741	<0
4	0.54741	1	<0	>0	0.56111	<0
5	0.56111	1	<0	>0	0.56530	<0
6	0.56530	1	<0	>0	0.56658	<0
7	0.56658	1	<0	>0	0.56697	<0
8	0.56697	1	<0	>0	0.56709	<0
9	0.56709	1	<0	>0	0.56712	<0
10	0.56712	1	<0	>0	0.56713	—

∴ The root is 0.56713 Corrected to 4 decimals of accuracy.

(3) $f(x) = x\sin x - 1$

$f(0) = -1 < 0$

$f(1) = -0.158 < 0$ } root lies b/w (1, 2)

$f(2) = 0.818 > 0$ }

S.No	a	b	Sign of f(a)	Sign of f(b)	$c = \frac{af(b) - bf(a)}{f(b) - f(a)}$	Sign of f(c)
1	1	2	<0	>0	1.16224	>0
2	1	1.16224	<0	>0	1.11425	>0
3	1	1.11425	<0	>0	1.11415	<0
4	1.11415	1.11425	<0	>0	1.11415	—

The root is 1.11415 Corrected to 4 decimals of accuracy.

note: $\frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{a[b\sin b - 1] - b(a\sin a - 1)}{(b\sin b - 1) - (a\sin a - 1)}$

$= \left(a(b\sin b - 1) - b(a\sin a - 1)\right) \div \left(b\sin b - a\sin a\right)$