Antecedent rules:

$\neg \Rightarrow$: if $\underbrace{\overset{S}{\Longrightarrow} x}$ then $\neg x \overset{S}{\Longrightarrow}$

x is Tautology the $\neg x$ becomes contradiction.

$\wedge \Rightarrow$: if $\underbrace{x, y \overset{S}{\Longrightarrow}}$ then $x \wedge y \overset{S}{\Longrightarrow}$

if x or y is false then $x \wedge y$ becomes True

$\vee \Rightarrow$: if $x \overset{S}{\Rightarrow}$ and $y \overset{S}{\Rightarrow}$ then $x \vee y \overset{S}{\Rightarrow}$

x - False y - False

$\rightarrow \Rightarrow$: if $\overset{S}{\Rightarrow} x$ and $y \overset{S}{\Rightarrow}$ then $x \rightarrow y \overset{S}{\Rightarrow}$ x - True, y - False

$\rightleftarrows \Rightarrow$: if $x, y \overset{S}{\Rightarrow}$ and $\overset{S}{\Rightarrow} x, y$ then $x \rightleftarrows y \overset{S}{\Rightarrow}$

x or y false   x or y true

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Consequent rules:

$\Rightarrow \neg$: if $x \overset{S}{\Rightarrow}$ then $\overset{S}{\Rightarrow} \neg x$ $\neg x$ = True, x = False

$\Rightarrow \wedge$: if $\overset{S}{\Rightarrow} x$ and $\overset{S}{\Rightarrow} y$ then $\overset{S}{\Rightarrow} x \wedge y$ x, y = True

$\Rightarrow \vee$: if $\overset{S}{\Rightarrow} x, y$ then $\overset{S}{\Rightarrow} x \vee y$ = x or y True

$\Rightarrow \rightarrow$: if $x \overset{S}{\Rightarrow} y$ then $\overset{S}{\Rightarrow} x \rightarrow y$ = x false y True

$\Rightarrow \rightleftarrows$: if $x \overset{S}{\Rightarrow} y$ and $y \overset{S}{\Rightarrow} x$ then $\overset{S}{\Rightarrow} x \rightleftarrows y$ identical truth value

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problem:- show that $P \vee Q$ follows from P

we need to show that (1) $\Rightarrow P \rightarrow (P \vee Q)$

(1) if (2) $P \Rightarrow P \vee Q$ ($\Rightarrow \rightarrow$)

(2) if (3) $P \Rightarrow P, Q$ ($\Rightarrow \vee$) Axiom

$\therefore$ Hence $P \vee Q$ follows from P problem:- Does p follows from $p \vee Q$?

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Sol:- (1) $\Rightarrow (p \vee Q) \rightarrow p$

(1) if (2) $p \vee Q \Rightarrow p$ $(\Rightarrow \rightarrow)$

(2) if (3) $p \Rightarrow p$ and (4) $Q \Rightarrow p$ $(\vee \Rightarrow)$

$\downarrow$ Axiom $\downarrow$ Not Axiom

$\therefore$ Hence p does not follow from $p \vee Q$.

problem: prove that $p \rightarrow \neg p$ is not a Tautology

Sol:- let us assume that $p \rightarrow \neg p$ is Tautology

$\overset{s}{\Rightarrow} p \rightarrow \neg p$ (1)

(1) if (2) $p \overset{s}{\Rightarrow} \neg p$ (2) ($\because$ By $\Rightarrow \rightarrow$)

(2) is not an axiom.

There fore our assumption is wrong.

hence $p \rightarrow \neg p$ is not a tautology.

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Relations:-

set: A well defined collection of objects is called a set. We denote the set with a capital letter where as elements with a small letters.

Example: 1) $A = \{x : X \text{ is an even number}\}$ is a set

2) $B = \{x : X \text{ is tall}\}$ is not a set is not well defined.

cartesian product: if A and B are any two non-empty finite sets, then the set of all distinct ordered 1
pairs whose first coordinate belongs to A & 9
Second Coordinate belongs to B, is called 1
3
the cartesian product of A and B (in that 1
A
order) and is denoted by $A \times B$ 0
5

$$A \times B = \{(a,b) : a \in A, b \in B\}$$ Q
6

Ex-1

$A = \{x, y\}$ $B = \{a, b, c\}$

$A \times B = \{(x,a), (x,b), (x,c), (y,a), (y,b), (y,c)\}$

Ex-2

$A = \{1, 2, 3, 4\}$ $B = \{2, 5\}$

$A \times B = \{(1,2), (1,5), (2,2), (2,5), (3,2), (3,5), (4,2), (4,5)\}$

what are the orderpairs comes under 'lessthan'

$R_1(<) = \{(1,2), (1,5), (2,5), (3,5), (4,5)\}$

Domain $(R_1) = \{1,2,3,4\}$ co-domain $(R_1) = \{2,5\}$

what are the orderpairs comes under 'greater than'

$R_2(>) = \{(3,\underline{2}), (4,\underline{2})\}$ → Range

Domain$(R_2) = \{1,2,3,4\}$

co-domain$(R_2) = \{2,5\}$

Range $(R_2) = \{2\}$

what are the ordered pairs comes under 'equal'

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$R_3(=) = \{(2,2)\}$ → Range

Domain$(R_3) = \{1,2,3,4\}$
Co-domain$(R_3) = \{2,5\}$
Range$(R_3) = \{2\}$

clearly R_1, R_2, R_3 are subsets of $A \times B$

Binary relation: Any subset of $A \times B$ is defined a relation or binary relation. The word "binary" in this definition refers to a relation b/w two things. We denote the relation with "R". So the relation R is defined to the set of all such ordered pairs (x,y), where xRy (i.e, x is related to y).

Thus $(x,y) \in R \Leftrightarrow xRy$

Note: (1) for any sets A and B, there are 2 extreme relations, 1st is example of empty relation ϕ, 2nd is total relation $A \times B$

(2) A relation R is said to be universal if $R = A \times A$

(3) A relation R is said to be void relation if $R = \phi$

Ex-1: The set $\{(x,y) / x, y \in R$ and $x > y\}$ is a relation, and it relates a real number to every real number less than it.

Ex-2: The set $\{(a,b) / a$ is the mother of $b\}$ is a relation on the set of human beings which relates the female to every human being

Domain and Range of a Relation:

If R is a relation from a set A to another set B, then the set of elements in A that are related to some element or elements in B is called the domain of R and the set B is called codomain of R. Symbolically

$D = \{x : (x,y) \in R, x \in A$ for some $y \in B\}$

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The range, say E of a relation R is the set of elements in B that appear as second element in the ordered pair which belong to R. Symbolically

$$E = \{y : (x,y) \in R,\ x \in A\}$$

Note: i) Domain of a relation R is denoted by D(R) and range of R by R(R)

ii) if R is a relation from a set 'A' to itself, that is, if R is a subset of $A \times A$, then we say R is a relation on set A instead of saying that R is a relation from A to A.

problem: let $A = \{1,2,3,4,6\}$ let R be the relation on A defined by $\{(x,y) \in R,\ x,y \in A,\ y$ is excatly divisible by $x\}$

1
9
1
3
1
A
0
5
9
6

i) write R in roster form

ii) find the domain of R.

iii) find the range of R

Sol:- i) $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$

ii) $D(R) = \{1,2,3,4,6\}$

iii) $R(R) = \{1,2,3,4,6\}$

Set operations on Relations:

Union of R and S: $R \cup S$ defines a relation such that $x(R \cup S)y \Leftrightarrow xRy$ or xSy (i.e $xRy \vee xSy$)

intersection of R and S: $R \cap S$ defines a relation such that $x(R \cap S)y \Leftrightarrow xRy$ and xSy (i.e, $xRy \wedge xSy$)

Difference of R & S: $R-S$ defines a relation such that $x(R-S)y \Leftrightarrow xRy$ and $x \not{S} y$ (i.e., $xRy \wedge x \not{S} y$)

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Complement of R: $\sim R$ (or R') defines a relation such that $x(\sim R)y \iff x \not R y$

Ex-1 if $A = \{2,3,5\}$, $B = \{6,8,10\}$, $C = \{2,3\}$ & $D = \{8,1$

Suppose R from A to B is defined as $R = \{(2,6)(2,8)(3,1$

and the relation $S = \{(2,8)(3,10)\}$. find $R \cup S$,

$R \cap S$, $R - S$ and R'

Sol:- $R \cup S = \{(2,6),(2,8),(3,10)\}$

$R \cap S = \{(2,8),(3,10)\}$

$R - S = \{(2,6)\}$

$R' = \{(2,10),(3,6),(3,8),(5,6),(5,8),(5,10)\}$

Ex-2 let $X = \{1,2,3,4\}$

if $S = \{(x,y) \in X \times X \,/\, (x-y)$ is an integral nonzero multiple of $3)\}$

$R = \{(x,y) \in X \times X \,/\, (x-y)$ is an integral nonzero multiple of $2\}$

then find $R \cup S$, $R \cap S$; $R - S$, $\sim R$

$S = \{(1,4),(4,1)\}$ $R = \{(1,3),(3,1),(2,4),(4,2)\}$

$R \cup S = \{(1,3),(3,1),(2,4),(4,2),(1,4),(4,1)\}$

$R \cap S = \{\phi\}$

$R - S = \{(1,3),(3,1),(2,4),(4,2)\}$

$\sim R = \{(1,2)$ $\longrightarrow$ All elements in $X \times X$ except those in R

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2. Properties of relations

Reflexive relation: A relation R on a set X is said to be reflexive iff each element in X is related to itself, i.e, for each $x \in X$ we have xRx or

R is reflexive in $X \iff ((x,x) \in R, \forall x \in X)$

<u>Ex-1</u> let $A = \{1,2,3,4\}$. Then the relation $R_1 = \{(1,1)(2,4)(3,3)(4,1)(4,4)\}$ on A is not reflexive because $2 \in A$ but $(2,2) \notin A$ where as the relation $R_2 = \{(1,1)(2,2)(3,3)(4,4)(2,3)\}$ is a reflexive relation on A

↳ Similar pairs must be there along with them any order pairs can be written no problem

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<u>Ex-2</u> The relation $\leq$ on natural numbers is reflexive, because any $x \in N$ is less than or equal to x.

<u>EX-6</u> (Divisibility)

Consider the relation | of divisibility on the natural numbers, defined as $n|m$ if and only if there exists a $K \in N$ such that $nK = m$.

pronounce $n|m$ "n is a divisor of m" or "n divides m". This relation is reflexive because $n \cdot 1 = n$.

<u>Ex-4</u> Consider the relation $R = \{(x,y) \in Z \times Z / x+y \text{ is odd}\}$.

This relation is not reflexive because $x+x=2x$ may not odd always.

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problem: for real numbers x and y we write xRy iff $x-y+\sqrt{2}$ is an irrational number. Then Verify the relation is reflexive or not?

Sol:- let $x \in R$ arbitarily

$xRy \Leftrightarrow x-y+\sqrt{2}$

x relates to y

$xRx \Leftrightarrow x-x+\sqrt{2} = \sqrt{2}$ is an irrational number

x relates to x

$\therefore$ the given relation is reflexive

Symmetric relation: A relation R on a set X is said to be symmetric iff for every $x, y \in X$, whenever xRy, then yRx or R is symmetric in $X \Leftrightarrow (xRy \rightarrow yRx, \forall x, y \in X)$

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Ex:- let $A = \{1,2,3,4\}$ and relation $R = \{(1,2),(2,2),(2,1),(1,3),(3,1)\}$ is symmetric relation.

R_1 defined on A such that $R_1 = \{(1,1),(2,2),(3,3),(4,4),(1,4)\}$ is reflexive but not symmetric.

Transitive Relation: A relation R on X is said to be transitive iff for each $a, b, c \in X$ we have that if aRb and bRc then aRc.

Ex-1 for any sets A, B and C ARB, BRC $\Leftrightarrow A \subset B$ and $B \subseteq C \Rightarrow A \subseteq C \Rightarrow ARC$. Hence the relation $\subseteq$ is transitive

Ex-2 let $A = \{1,2,3,4\}$ and relation R defined on A such that $R = \{(1,2),(2,2),(2,1),(1,3),(3,1),(1,1),(2,3)\}$

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$(1,3),(3,1) \in R$
but $(3,3) \notin R$

Since $(3,1),(1,3)$ belongs to R but $(3,3)$ doesn't belong to R.

$\therefore$ It is not transitive.

Irreflexive relation: A relation R on X is said to be irreflexive iff for each $x \in X$ we have $x \not R x$ or R is irreflexive in $X \Leftrightarrow (x)(x \in X \rightarrow (x,x) \notin R)$

Ex-1 let $A = \{1,2,3,4\}$ then the relation $R_1 = \{(1,3),(2,4),(3,4),(4,1),(4,2)\}$ on A is irreflexive.

The relation $R_2 = \{(1,1),(1,3),(2,4),(3,4),(4,1),(4,2)\}$ on A is not irreflexive since $(1,1) \in R$

Anti-Symmetric relation: A relation R on X is said to be anti-symmetric when for each $a,b \in X$ we have bRa and aRb then a=b or R is anti-symmetric in $X \Leftrightarrow (x)(x \in X \wedge y \in X \wedge xRy \wedge yRx \rightarrow x=y)$

Note: that in this definition of anti-symmetry aRb, bRa can not both hold simultaneously for different a, b

Ex-1 The relation inclusion in X is anti-symmetric because $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$