Theory Assignment – I

Design and Analysis of Algorithms **B.Tech (CSE) SEM IV**

Date of Issue: May 13, 2021 **Submission Date June 03, 2021**

Instructions:

1. No copy paste from Internet or friends otherwise strict disciplinary action will be taken.
2. Help your friend in understanding the concept instead of simply passing your "done assignment".

Q.1 What is an algorithm? Explain various characteristics of an algorithm.

Q.2 List and explain various way of expressing an algorithm.

Q.3 What is amortized analysis? Explain in detail.

Q.4 what is Probabilistic analysis of algorithms?

Q.5 what is a recurrence relation? How to solve recurrence relation?

Q.6 Explain all the cases of master theorem for following recurrence relations:
T (n) = aT(n/b) + f (n) (Dividing Function) & T(n)=aT(n-b)+f(n) (Decreasing function)

Q7. Solve the following recurrence relation using substitution, recurrence tree and master method (if applicable).

1) T(n) = 7T(n/2) + 3n^2 + 2
2) $T(n) = 2T(n/2) + n^2$.
3) $T(n) = T(n/3) + T(2n/3) + n$.
4) $T(n) = 4T(n/2) + n^3$.
5) $T(n) = cn + 3T(2n/3)$
6) $T(n)=T(n-1)+T(n/2)+n$.
7) $T(n)=T(\alpha n)+T((1-\alpha)n)+cn$, where α is a constant in the range $0 < \alpha < 1$, and $c > 0$ is also a constant.
8) $T(n)=T(n-a)+T(a)+cn$, where $a \geq 1$ and $c > 0$ are constants.
9) T(n) = 2T(n / 4) + 1
10) $T(n)=2T(n/4)+\sqrt{n}$.
11) T(n) = 2T(n / 4) + n.
12) T(n) = 2T(n / 4) + n^2
13) $T(n)=4T(n/2)+n$^2lg$n$
14) $T(n)=T(n-1)+n$
15) $T(n)=T(\lceil n/2\rceil)+1$
16) $T(n)=2T(\lfloor n/2\rfloor+17)+n$
17) $T(n)=4T(n/2)+n$^2
18) $T(n)=3T(\sqrt{n})+\log n$
19) $T(n)=2T(n/2)+n$^4.
20) T(n) = T(7n / 10) + n.
21) T(n) = 16T(n / 4) + n^2.
22) T(n) = 7T(n / 3) + n^2.
23) T(n) = 7T(n / 2) + n^2.
24) T(n) = 2T(n / 4) + $\sqrt{n}$.
25) T(n) = T(n - 2) + n^2.

Q.8 Group the functions so that f(n) and g(n) are in same group iff f(n)=O(g(n)) and g(n)=O(f(n)). List the group in increasing order.

$\sqrt{n}$	n	2^n
$nlogn$	$n - n^3 + 7n^5$	$n^2 + logn$
n^2	n^3	$logn$
$n^{\frac{1}{3}} + logn$	$(logn)^2$	$n!$
lnn	$\frac{n}{\log n}$	$loglogn$
$(\frac{1}{3})^n$	$(\frac{3}{2})^n$	6

Q9

[redacted] ***Relative asymptotic growths***

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

Q10.

i. Consider the following fourteen functions for the question that follows:

(a) $\log_3(2n)$
(b) $\sqrt{n}$
(c) $n \log_3(n/2)$
(d) $\log_2(3n^2)$
(e) 2^n
(f) 2^{n+2}
(g) 2^{2n}
(h) $3n + 5\log_2(n)$
(i) $5n + \sqrt{n}$
(j) $\sum_{k=1}^{n} k = 1 + 2 + 3 + 4 + \ldots + n$
(k) $\sum_{k=1}^{2n} k = 1 + 2 + 3 + 4 + \ldots + 2n$
(l) $\sum_{k=1}^{n^2} k = 1 + 2 + 3 + 4 + \ldots + n^2$
(m) $\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3$
(n) $\sum_{k=1}^{n} 2^k = 1 + 2 + 4 + 8 + \ldots + 2^n$

Make a table in which each function is in a column dictated by its Θ growth rate. Functions with the same asymptotic growth rate should be in the same column. Columns should be ordered left to right by the rate of growth of their functions: columns with slower growing functions should be to the left of columns with faster growing functions.

Q.11

For each of the following pairs of functions, either f(n) is O(g(n)), f(n) is Ω(g(n)), or f(n) Θ(g(n)). Determine which relationship is correct and explain.

a. $f(n) = n^{0.25}$; $g(n) = n^{0.5}$

b. $f(n) = n$; $g(n) = \log^2 n$

c. $f(n) = \log n$; $g(n) = \ln n$

d. $f(n) = 1000n^2$; $g(n) = 0.0002n^2 - 1000n$

e. $f(n) = n\log n$; $g(n) = n\sqrt{n}$

f. $f(n) = e^n$; $g(n) = 3^n$

g. $f(n) = 2^n$; $g(n) = 2^{n+1}$

h. $f(n) = 2^n$; $g(n) = 2^{2n}$

i. $f(n) = 2^n$; $g(n) = n!$

j. $f(n) = \lg n$; $g(n) = \sqrt{n}$

$f(n) = \log n^2$; $g(n) = \log n + 5$

$f(n) = n$; $g(n) = \log n^2$

$f(n) = \log \log n$; $g(n) = \log n$

$f(n) = n$; $g(n) = \log^2 n$

$f(n) = n \log n + n$; $g(n) = \log n$

$f(n) = 10$; $g(n) = \log 10$

$f(n) = 2^n$; $g(n) = 10n^2$

$f(n) = 2^n$; $g(n) = 3^n$