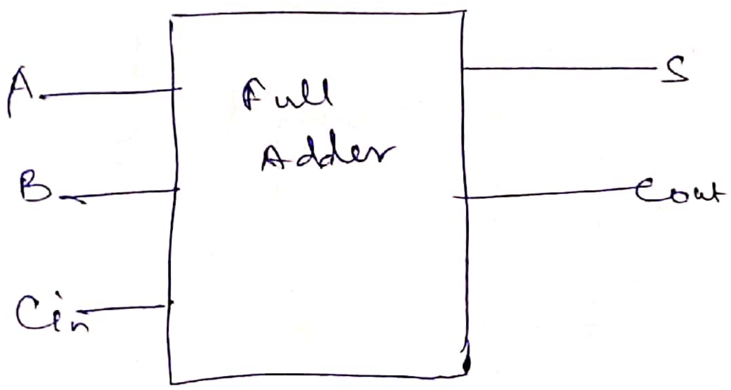
Full Adder

=> A half-adder has two 1-bit inputs and there is no provision to add any carry which could have been generated from lower bit order conditions.

=> The limitation of half-adder circuit is overcomed in Full-adder.

=> The full adder is a combinational logic circuit that has the provision to add a carry.



	Inputs			Sum (S)	Carry (Cout)
	A	B	Cin		
0	0	0	0	0	0
1	0 ($\bar{A}$)	0 ($\bar{B}$)	1 (Cin)	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1
8					

* 3 input bits, so total no. of combinations are $2^3 = 8$
* from 000 to 111

Process-1:-

To find the expression of Sum(S); we have to find the position having the value '1' for SOP.

$$\therefore S = \bar{A}\,\bar{B}\,Cin + \bar{A}\,B\,\overline{Cin} + A\,\bar{B}\,\overline{Cin} + A\,B\,Cin$$

(simplify using Boolean Algebra)

$$= (\bar{A}\,\bar{B} + A\,B)\,Cin + (\bar{A}\,B + A\,\bar{B})\,\overline{Cin}$$

$$= \overline{(A \oplus B)}\,Cin + (A \oplus B)\,\overline{Cin}$$

$[\because \overline{X\text{-}OR} = X\text{-}NOR]$

$$= A \oplus B \oplus Cin$$

$[\because$ Considering $A \oplus B = X$ & $Cin = Y$, So $\bar{X}Y + X\bar{Y} = X \oplus Y]$

Now, for $C = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$

$= \bar{A}BC_{in} + A\bar{B}C_{in} + AB(C_{in} + \overline{C_{in}})$

$= \bar{A}BC_{in} + A\bar{B}C_{in} + AB \quad [\because C_{in} + \overline{C_{in}} = 1]$

$= (\bar{A}B + A\bar{B})C_{in} + AB$

$= (A \oplus B)C_{in} + AB$

$= AB + AC_{in} + BC_{in}$

Process - 2 :- By Using K-map Technique :-

** It is suggested in earlier classes also for simplification in SOP or POS forms please use K-map **

for S :-

A \ B C_{in}	00	01	11	10
0	(0)	1 (1)	(3)	1 (2)
1	1 (4)	(5)	1 (7)	(6)

* Here, no groups are possible.
* This particular type in K-map is known as check-board configuration.

* whenever this configuration comes, we can write it is simply X-OR operation.

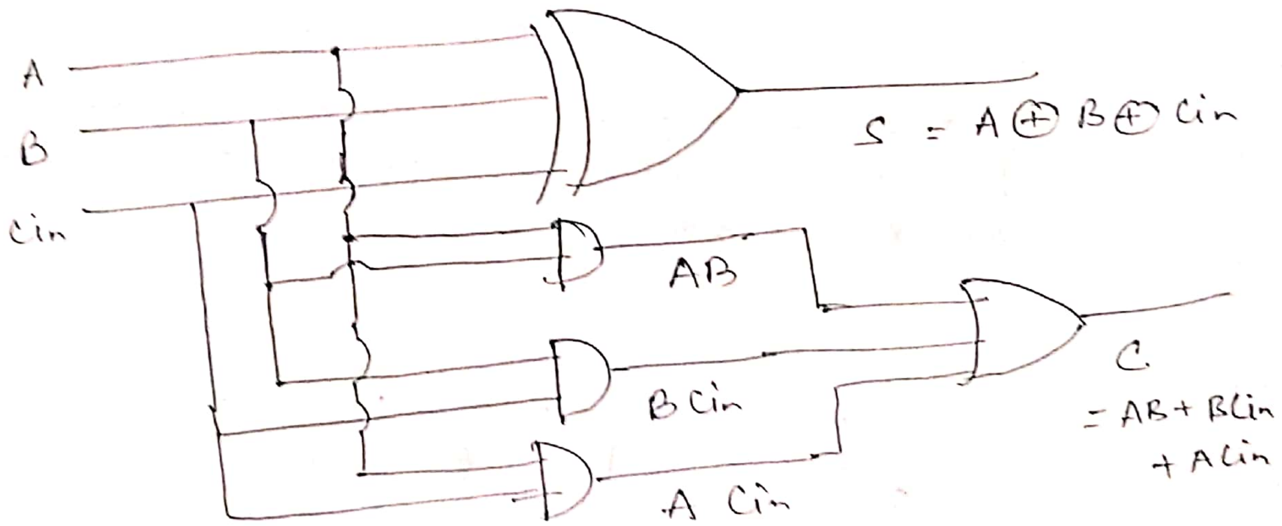
$\therefore S = A \oplus B \oplus C_{in}$

for c :-

A \ B Cin	00	01	11	10
0	(0)	(1)	1 (3)	(2)
1	(4)	1 (5)	1 (7)	1 (6)

$$C = AB + ACin + BCin$$

Representation using ~~Half~~ Logic Gates :-



Full-Adder using Universal Logic:-

① NAND Logic:-

As, we have seen in half-adder logic, $S = \bar{x}y + x\bar{y} = \overline{\overline{x \cdot \overline{xy}} \cdot \overline{y \cdot \overline{xy}}}$

Here, we represented; input as A & B so,

for half-adder $S = \bar{A}B + A\bar{B} = \overline{\overline{A \cdot \overline{AB}} \cdot \overline{B \cdot \overline{AB}}}$

Now, for full-adder,

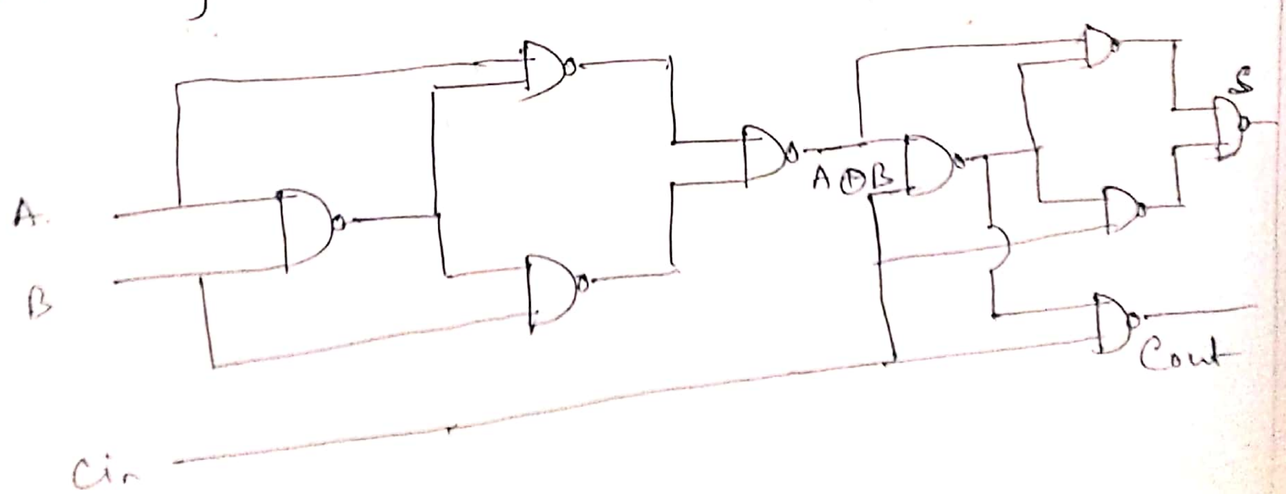
$$S = \underbrace{A \oplus B}_{x} \oplus \underbrace{Cin}_{y}$$

$$= \overline{\overline{(A \oplus B) \cdot \overline{(A \oplus B) Cin}} \cdot \overline{Cin \cdot \overline{(A \oplus B) Cin}}}$$

$$Cin = Cin(A \oplus B) + AB = \overline{\overline{Cin(A \oplus B)} \cdot \overline{AB}}$$

[from De-Morgan's]

Similarly like half adder →



② **NOR-Logic:-** from half-adder →

$$A \oplus B = \overline{\overline{A+B} + \overline{\bar{A}+\bar{B}}}$$

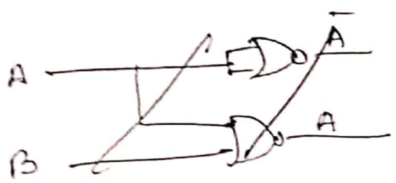
For full-adder; $S = \underline{A \oplus B} \oplus \underline{C_{in}}$

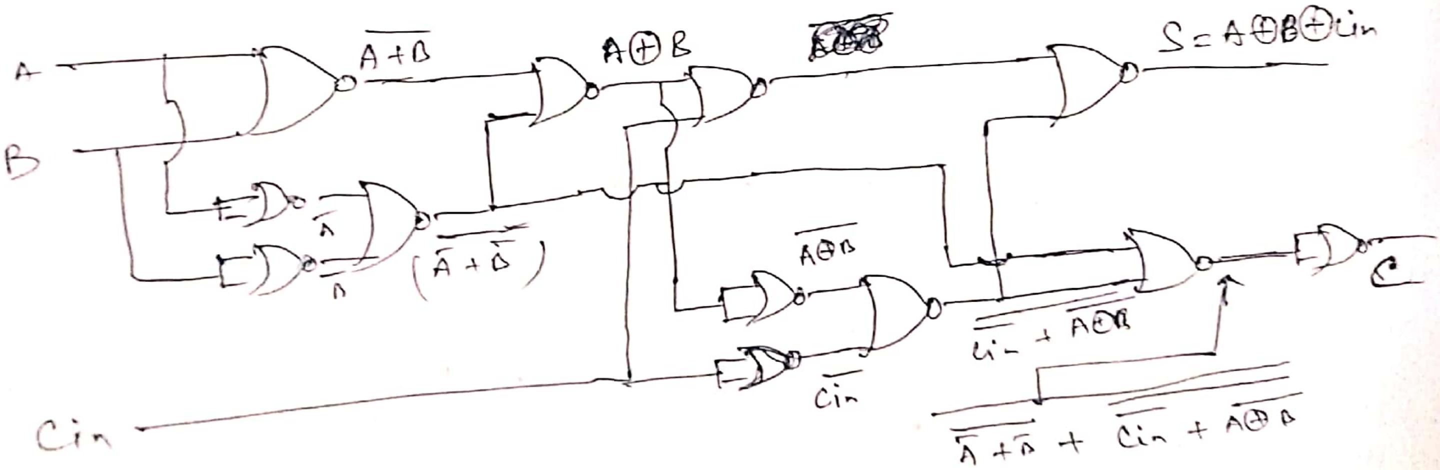
$$= \overline{\overline{(A \oplus B) + C_{in}} + \overline{\overline{(A \oplus B)} + \overline{C_{in}}}}$$

$$C_{out} = \underline{AB} + C_{in}(A \oplus B)$$

$$= \overline{\bar{A}+\bar{B}} + \overline{\overline{C_{in}} + \overline{(A \oplus B)}}$$

[using De-morgan's Theorem]





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Full-adder using Half-adders:-

The boolean expression of full Adder are →

$$\left.\begin{aligned} S_f &= C_{in} \oplus A \oplus B \\ \text{and } C_{out} &= AB + AC_{in} + BC_{in} \end{aligned}\right\} \quad (1)$$

The boolean Expression for half-adder is

$$\left.\begin{aligned} S_h &= \cancel{C_{in}\oplus}\ A \oplus B \\ \text{and } C_h &= AB \end{aligned}\right\} \quad (2)$$

(1) =>

$$S_f = C_{in} \oplus \underline{A \oplus B}$$
$$= \underline{C_{in} \oplus S_h}$$

and
$$C_{out} = AB + AC_{in} + BC_{in}$$
$$= AB + AC_{in} + BC_{in}(A + \bar{A}) \quad [\because A + \bar{A} = 1]$$
$$= AB + AC_{in} + ABC_{in} + \bar{A}BC_{in}$$
$$= AB(1 + C_{in}) + AC_{in} + \bar{A}BC_{in}$$
$$= AB + AC_{in} + \bar{A}BC_{in}$$
$$= AB + AC_{in}(B + \bar{B}) + \bar{A}BC_{in} \quad [\because B + \bar{B} = 1]$$
$$= AB + ABC_{in} + A\bar{B}C_{in} + \bar{A}BC_{in}$$
$$= AB(1 + C_{in}) + C_{in}(A\bar{B} + \bar{A}B)$$
$$= AB + C_{in}(A \oplus B) \quad [\because 1 + C_{in} = 1]$$
$$= \underline{AB} + C_{in} \times S_h \quad [\because A \oplus B = S_h]$$

o/p of half adder

$$= C_h + C_{in} \times S_h$$

Logic Diagram of full adder using half-adders :-