

Composed Difference

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A composed difference is a function 'h' that equals the difference in value of a function 'g' after transforming the argument with a function 'f'. We write down the 3 functions as a tuple with 3 values.

$$\begin{aligned} h(x) &= g(f(x)) - g(x) \\ (f(x), g(x), h(x)) \end{aligned}$$

It is useful to think of the tuple as a set. The most general set is:

$$(f(x), g(x), g(f(x)) - g(x))$$

If 'g' is a linear function, then we have:

$$(f(x), g(x), g(f(x) - x))$$

If 'f' is adding an infinitesimal, we get information about the derivative in 'h':

$$\begin{aligned} \epsilon^2 &= 0 \\ (x + \epsilon, g(x), g'(x)\epsilon) \end{aligned}$$

The square function is symmetric because there is does not matter wether we change the sign:

$$(-x, x^2, 0)$$

The general invariant set is given by:

$$(f(x), g(x), 0)$$

When 'f' is identity, we have:

$$(x, g(x), 0)$$

Any constant 'c' added to 'g' remains invariant:

$$(f(x), g(x) + c, 0)$$

For a quadratic function we have additional invariant:

$$\left(-\left(\frac{a_1}{a_2} + x\right), a_0 + a_1 x + a_2 x^2, 0\right)$$

For degree 3 we have two additional invariants:

$$\left(\frac{\pm\sqrt{-4a_1a_3 + a_2^2 - 2a_2a_3x - 3a_3^2x^2} - a_2 - a_3x}{2a_3}, a_0 + a_1x + a_2x^2 + a_3x^3, 0\right)$$