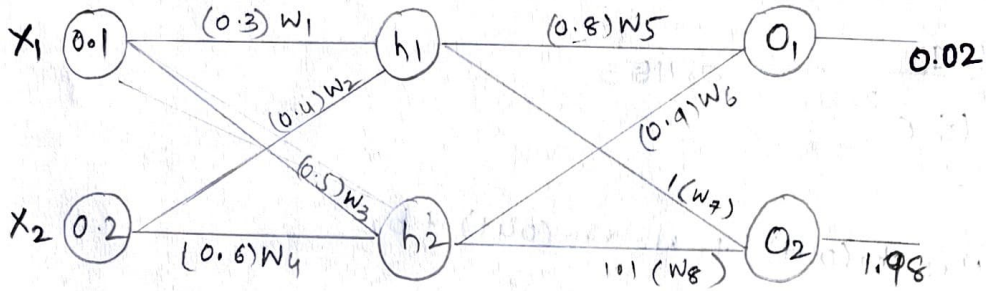
Artificial Neural Network - Back Propagation

Hand Calculation Example



$b_1 = 0.7$ $\quad b_2 = 1.2$

$$h_1(in) = w_1 x_1 + w_2 x_2 + b_1$$

$$= 0.3 \times 0.1 + 0.4 \times 0.1 + 0.7$$

$$= 0.77$$

$$h_1(out) = \frac{1}{1 + e^{-h_1(in)}} = \frac{1}{1 + e^{-0.77}} = 0.683$$

$$\therefore h_1(out) = 0.6835$$

$$h_2(in) = w_3 x_1 + w_4 x_2 + b_2$$

$$= 0.5 \times 0.1 + 0.6 \times 0.2 + 0.7$$

$$= 0.87$$

$$h_2(out) = \frac{1}{1 + e^{-h_2(out)}} = \frac{1}{1 + e^{-0.87}} = \text{0.7473} \ 0.7047$$

$$\therefore h_2(out) = \text{0.7073} \ 0.7047$$

$$O_1(In) = w_5 \times h_1(out) + w_6 \times h_2(out) + b_2$$

$$= 0.8 \times 0.6835 + 0.9 \times 0.7047 + 1.2$$

$$= 2.381$$

$$O_1(out) = \frac{1}{1+e^{-2.381}} = 0.9153$$

$$O_2(In) = w_7 \times h_1(out) + w_8 \times h_2(out) + b_2$$

$$= 1 \times 0.6835 + 1.1 \times 0.7047 + 1.2$$

$$= 2.6586$$

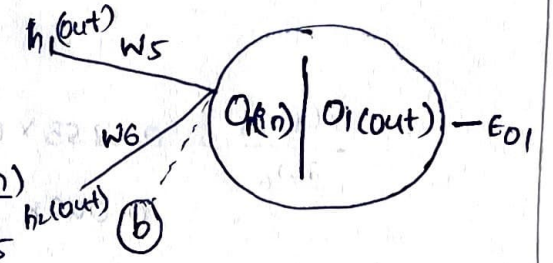
$$O_2(out) = \frac{1}{1+e^{-2.6586}} = 0.9345$$

$$E_{O1} = \frac{1}{2}(\text{target o/p} - \text{calculated o/p})^2$$

$$= \frac{1}{2}(0.02 - 0.9153)^2 = 0.40$$

$$E_{O2} = \frac{1}{2}(0.98 - 2.6586)^2 = 0.230$$

$$E_{total} = E_{O1} + E_{O2} = 0.40 + 0.230 = 0.63$$



$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial O_1(out)} * \frac{\partial O_1(out)}{\partial O_1(in)} * \frac{\partial O_1(in)}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial O_1(out)} = O_1(out) - Target_{O1}$$

$$= 0.9153 - 0.02 = 0.8953$$

$$\frac{\partial O_1(out)}{\partial O_1(in)} = O_1(out)(1 - O_1(out))$$

$$= 0.9153(1 - 0.9153) = 0.0775$$

$$\frac{\partial O_1(in)}{\partial w_5} = h_1(out) = 0.6835$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.8953 \times 0.0775 \times 0.6835$$

$$= 0.0474$$

let $\eta = 0.2$ learning rate parameter

$$w_5^+ = w_5 - \eta \times \frac{\partial E_{total}}{\partial w_5}$$

$$= 0.8 - 0.2 \times 0.0474$$

$$w_5^+ = 0.79$$

w_6:

$$\frac{\partial E_{total}}{\partial w_6} = \frac{\partial E_{total}}{\partial O_1(out)} \times \frac{\partial O_1(out)}{\partial O_1(in)} \times \frac{\partial O_1(in)}{\partial w_5}$$

$$\frac{\partial O_1(in)}{\partial w_6} = h_2(out) = 0.7047$$

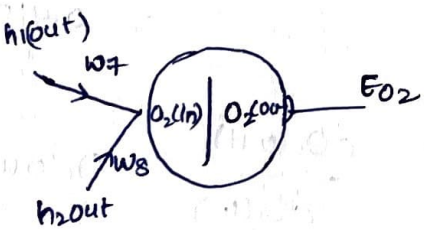
$$\therefore \frac{\partial E_{total}}{\partial w_6} = 0.8953 \times 0.0775 \times 0.7047$$

$$= 0.0488$$

$$w_6^* = w_6 - \eta \times \frac{\partial E_{total}}{\partial w_6} = 0.9 - 0.2 \times 0.0488$$

$$= 0.89$$

w_7:



$$\frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial O_2(out)} \times \frac{\partial O_2(out)}{\partial O_2(in)} \times \frac{\partial O_2(in)}{\partial w_7}$$

$$\frac{\partial E_{total}}{\partial O_2(out)} = O_2(out) - \text{Target } O_2$$

$$= 0.9345 - 1.98 = -1.0455$$

$$\frac{\partial O_2(out)}{\partial O_2(in)} = O_2(out)(1 - O_2(out))$$

$$= 0.9345(1 - 0.9345) = 0.06$$

$$\frac{\partial O_2(in)}{\partial w_7} = h_1(out) = 0.6835$$

$$\frac{\partial E_{total}}{\partial w_7} = (-1.0455)(0.06)(0.6835) = -0.0428$$

$$w_7^* = w_7 - \eta \times \frac{\partial E_{total}}{\partial w_7}$$

$$w_7^* = 1 + 0.2 \times 0.0428 = 0.9914$$

$$= 1.0085$$

w_8 :

$$\frac{\partial E_{total}}{\partial w_8} = \frac{\partial E_{total}}{\partial O_2(out)} \times \frac{\partial O_2(out)}{\partial O_2(in)} \times \frac{\partial O_2(in)}{\partial w_8}$$

$$\frac{\partial O_2(in)}{\partial w_8} = h_2(out) = 0.7047$$

$$\frac{\partial E_{total}}{\partial w_8} = (-1.0455)(0.06)(0.7047)$$

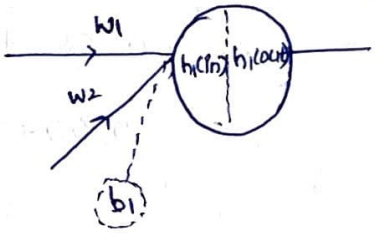
$$= -0.044$$

$$w_8^* = w_8 - \eta \frac{\partial E_{total}}{\partial w_8} = 1.1 + 0.2 \times (0.044)$$

$$= 1.1088$$

Hidden Layer:

w_1 :



$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial h_1(out)} \times \frac{\partial h_1(out)}{\partial h_1(in)} \times \frac{\partial h_1(in)}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial h_1(out)} = \frac{\partial E_{O1}}{\partial h_1(out)} + \frac{\partial E_{O2}}{\partial h_1(out)}$$

$$\frac{\partial E_{O1}}{\partial O_1(in)} \times \underbrace{\frac{\partial O_1(in)}{\partial h_1(out)}}_{w_5} + \frac{\partial E_{O2}}{\partial O_2(in)} + \underbrace{\frac{\partial O_2(in)}{\partial h_1(out)}}_{w_7}$$

$$\frac{\partial E_{O1}}{\partial O_1(out)} \times \frac{\partial O_1(out)}{\partial O_1(in)} \times w_5 \qquad \frac{\partial E_{O2}}{\partial O_2(out)} \times \frac{\partial O_2(out)}{\partial O_2(in)} \times w_7$$

$$\frac{\partial E_{o1}}{\partial o_1(out)} = (o_1(out) - \text{target } o_1) = 0.8953$$

$$\frac{\partial E_{o2}}{\partial o_2(out)} = (o_2(out) - \text{target } o_2) = -1.0455$$

$$\frac{\partial o_2(out)}{\partial o_1(in)} = o_1(out)(1 - o_1(out)) = 0.0775$$

$$\frac{\partial o_2(out)}{\partial o_2(in)} = o_2(out)(o_2\ 1 - o_2(out)) = 0.06$$

$$\frac{\partial E_{total}}{\partial w_1} = 0.8953 \times 0.0775 \times 0.8 + (-1.0455)(0.06) \times 1$$

$$= 0.0555 + (-0.0627)$$

$$= -0.0072$$

$$w_1^* = w_1 - \eta \times \frac{\partial E_{total}}{\partial w_1}$$

$$= 0.3 - 0.2 \times (-0.0072) = 0.3014$$

$$\frac{\partial h_1(out)}{\partial h_1(in)} = h_1(out)(1 - h_1(out)) = 0.2163$$

$$\frac{\partial h_1(in)}{\partial w_1} = \frac{\partial (w_1x_1 + w_2x_2 + b_1)}{\partial w_1} = x_1 = 0.1$$

$$\frac{\partial E_{total}}{\partial w_1} = (-0.0072) \times 0.2163 \times 0.1 = -0.0001557$$