**Conditional Probability**

If you know an event B has occurred, this knowledge reduces your sample space to the set of outcomes in **B**. **B** becomes the *reduced sample space,*

A More Precise Definition of Independence

Now that we have conditional probability, we can define the notion of independence more precisely. Two events A and B are independent if the probability of one event is not affected by the other event,

In other words, the occurrence of B gives us no extra knowledge about the probability of A.

Instructions: Refer to the accompanying handout for a summary of all the topics covered in this worksheet. Take out several sheets of paper. Put your name at the top of each page. Complete the following problems on these sheets. Show all of your work. Turn in everything at the front desk when you are finished.

1. **Reduced Samples Spaces.** List the *reduced* sample spaces for these conditional events:

a. Flipping three coins, given at least one of the coins is a head.

b. Rolling a die, given it has landed on an even number.

c. Selecting a card from a deck of cards, given the card selected is a Heart.

d. Selecting a card from a deck of cards, given the card selected is a King.

2. **Conditional Probability vs Classical Probability.** Using the reduced samples from #1, find the following probabilities:

a. The probability of getting two heads in three coin flips, given at least one of the coins is head.

i. If you didn't know that at least one of the coins was a head, what would the probability of getting two heads in three coin flips be? How does this answer compare to the answer in part a? What does this mean about the events of getting two heads and the event of getting at least one head?

b. Rolling a die and getting a number less than 3, given that it has landed on an even number.

i. If you didn't know the die had landed on even number, what would the probability of rolling a number less than 3 be? How does this answer compare to the answer in part b? What does this mean about the events of getting a number less than 3 and getting an even number?

c. What is the probability of selecting a Jack, given the card selected is a Heart?

i. If you didn't know the card selected was a heart, what would the probability of selecting a Jack be? How does this answer compare to the answer in part c? What does this mean about the events of selecting a Jack and selecting a Heart?

d. What is the probability of selecting a Heart, given the card selected was a King?

i. If you didn't know the card selected was a King, what would the probability of selecting a Heart be? How does this answer compare the answer in part d? What does this mean about the events of selecting a King and selecting a Heart?

3. **A Conditional Probability Table.** You conduct a very important survey on the Westminster Campus of FRCC and obtain the following results,

|  |  |  |  |
| --- | --- | --- | --- |
|  | *Believes in Ghosts* | *Doesn't Believe in Ghosts* | **Total** |
| *Believes in Aliens* | 18 | 103 | **121** |
| *Doesn't Believe in Aliens* | 7 | 405 | **412** |
| **Total** | **25** | **508** | **533** |

a. What is the probability a student believes in aliens?

b. What is the probability a student believes in ghosts?

c. What is the probability a student believes in both aliens and ghosts?

d. Given that a student believes in aliens, what is the probability he or she believes in ghosts?

e. Given that a student believes in ghosts, what is the probability he or she believes in aliens?

f. What is the probability a student believes in aliens, but doesn't believe in ghosts?

g. What is the probability a student doesn't believe in aliens, given they don't believe in ghosts?

h. What is the probability of a student believing in ghosts or not believing in aliens?

I. Of those who don't believe in aliens, what is the probability they believe in ghosts?

4. **More Conditional Tables.** The Titanic sank on April 15, 1912. The following table shows how passengers from different classes in the Titanic fared in the aftermath of its sinking,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *First Class* | *Second Class* | *Third Class* | *Crew* | **Total** |
| *Survived* | 203 | 118 | 178 | 212 | **711** |
| *Died* | 122 | 167 | 528 | 673 | **1490** |
| **Total** | **325** | **285** | **706** | **885** | **2201** |

a. What is the probability a passenger was from first class?

b. What is the probability a passenger survived?

c. What is the probability a passenger from first class survived?

d. Given that a passenger survived, what is the probability the passenger was from first class?

e. Given that a passenger was from first class, what is the probability they survived?

i. Compare this answer to part b. What does this mean about your chances of surviving on the Titanic?

f. What is the probability a passenger was a crew member?

g. What is the probability a crew member survived?

h. Given that a passenger was a crew member, what is the probability they survived?

i. Compare your answer to part b and part e. Were crew members more or less likely to survive than most abroad?

i. Given that a passenger did not survive, what is the probability they were a crew member?

j. What is the probability a passenger was from third class?

k. What is the probability a third class passenger survived?

l. Given that a passenger was from third class, what is the probability they survived?

m. Given that a passenger survived, what is the probability they were from third class?