**Probability and Combinatorics**

1. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag without replacement.

a. What is the probability all of them are red?

b. What is the probability two of them are blue?

c. What is the probability two of them are blue and one is red?

2. What is the probability of getting a full house in a game of five card poker? Recall a full house is a three of a kind and a pair simultaneously. Example: Kings full of 8s is given by KKK88.

(*Hint:* A full house has the pattern AAABB. Consider the number of choices available for A. Then consider how many different ways A can be picked from the deck. Likewise for B.)

3. A PIN is selected at random from 4 digits. What is the probability of no digits being repeated?   
  
4. In a certain state’s lottery, 48 balls numbered 1 through 48 are placed in a machine and six of them are drawn at random. If the six numbers drawn match the numbers that a player had chosen, the player wins $1,000,000. In this lottery, the order the numbers are drawn in doesn’t matter. Compute the probability that you win the million-dollar prize if you purchase a single lottery ticket.

5. In the state lottery from the previous example, if five of the six numbers drawn match the numbers that a player has chosen, the player wins a second prize of $1,000. Compute the probability that you win the second prize if you purchase a single lottery ticket.

6. **The Birthday Problem**

a. Suppose three people are in a room. What is the probability that there is at least one shared birthday among these three people?

b. Suppose ten people are in a room. What is the probability that there is at least one shared birthday among these ten people?

c. Suppose thirty people are in a room. What is the probability that there is at least one shared birthday among these thirty people?

*Hint*: First find the probability of *no* matching birthdays in each case!