**Combinatorics: The Counting Principle, Permutations and Combinations**

**The Counting Principle**

If an n-step process has k1, k2 , k3 … kn choices at each step of the process, then the total number of ways the process could occur is the product of the number of choices ki at each step *i = 1, 2, … n*.

Ex.

**Factorials**

Ex.

*Note:* 0! is defined to equal 1.

**Permutations:**

If there are n distinct elements, then the number of ways to select r elements *without replacement* where order matters is given by,

Ex. Consider the letters *R, G, B.* These letters are 3 distinct elements. The number of ways to select 2 of them at a time, where order matters, is given by,

When we list out all possible sequences of two letters, we get

{*R, G*}, {*G, R*}, {*R, B*}, {*B, R*},{*G, B*}, {*B, G*}

Which is exactly six, as calculated. Notice permutations distinguishes between sequences like {*R,B*} and {*B, R*}. This is what we mean when we say *ORDER MATTERS*.

You can use the counting principle to derive the number of permutation as follows: Using the previous example of the letters *R, G, B*, we have a two step process where we have 3 choices for the first step (i.e. three letters), but once we select one element that element can no longer be selected (i.e. if you pick R, you can't pick R again; this is what *without replacement* means), so we have 2 choices for the second step, i.e.,

**Combinations**

Combinations are the number of permutations left over when you do not differentiate between the order of the sequence. In other words, with combinations, *ORDER DOES NOT MATTER*.

If we select r elements from n distinct elements, then there are many sequences, with r! many overlapping sequences, i.e. sequences with the same elements in different order.

Ex. You can see this in the previous example. We selected 2 elements from 3 distinct objects. There are 2! = 2 sequences that overlap for each given combination. In other words, {R, G} & {G,R} pair together, {R, B} & {B, R} pair together and {G, B}, {G, B}. Combinations consider each pair of sequences the same since they contain the same elements. If we count the number of sequences where order does not matter, we arrive at 3. This is equal to the number of permutation divided by 2!, the number of ways to arrange 2 elements.

Therefore, we should divide the number of combinations by the number ways of arrange the r elements we are selecting, r!,

Combinations and Permutations on TI-83/TI-84

Both combinations and permutations are accessed via the PRB submenu in the MATH menu

Permutations: *MATH > PRB > 2: nPr*

Combinations: *MATH > PRB > 3: nCr*

To use one of these functions, you first type in the number of distinct elements from which you are selecting. Then, you access the program through the MATH menu. Finally type in the number of elements you are selecting and then execute. The combination or permutation should get outputted on screen.

Ex. If I have five people standing in a room and I want to pick 3 people to give 1 dollar (because I'm a generous guy), then there are ways to do this (*why?*) , so I would type in

*5 > MATH > PRB > 3 : nCr > 3 > ENTER*

And I should get the number 10 on screen! Verify that you also get 10 before proceeding!

Instructions: Take out several sheets of paper. Put your name at the top of each sheet. Clearly label each problem. Show all of your work. Turn in everything stapled together when you are finished.

**Problems**

1. Consider the word MATH.

a. How many permutations of the letters in this word can be written?

i. List each possible permutations and confirm that your answer agrees with your previous result.

b. How many possible combinations of the letters in this word can be written?

i. List each possible combination and confirm that your answer agrees with your previous result.

*Hint: part b should be deceptively obvious!*

2. Consider, once again, the word MATH.

a. If you select two letters from this word without replacement, how many possible permutations can you generate?

i. List all possible permutations and confirm that your answer agrees with your previous result.

b. If you select three letters from this word without replacement, how many possible combinations can you generate?

i. List all possible combinations and confirm that your answer agrees with your previous result.

3. A combination lock has six slots for its combination. If each slot can be a digit 0 – 9, what is the total amount of lock combinations possible?

4. Peter always makes his sandwiches with bread, cheese, lunch meat and a condiment. He has 3 types of bread, 4 types of cheese, 2 types of lunch meat and a 3 types of condiments. How many possible sandwiches can Peter create?

5. You are building a computer. For each component of the computer, you have the following choices: 5 types of motherboard, 3 types of video card, 10 choices of harddrive, 5 choices of RAM and 4 choices of power supply. How many different computers can you build?

6. Assume a license plate on an automobile must be 3 letters followed by 3 numbers. Under these assumptions, how many possible license plates can be made?

7. The chess club is choosing from its member for the position of president, secretary and treasurer. The chess club has 10 members. How many different ways can these offices be filled?

8. The local FM radio disc jockey has to fill up a slot with 4 songs. He has 8 possible songs from which to choose. How many different ways can the music for this slot be arranged?

9. A four person committee is being formed from a group of ten people. How many different ways can this committee be formed?

10. A six person committee is being formed from a group of 13 people, 5 of whom are women and 7 of whom are male. If the committee must have exactly 3 men and exactly 3 women, how many different ways can this committee be formed?

11. 30 cars are competing in a race. If only the first three cars can place in the winners circle, in how many ways can this race finish?