**Confidence Interval for the True Mean of the Population**

*CONFIDENCE INTERVALS WILL ALWAYS LOOK LIKE*

Where MOE is the *margin of the error.*

The m*argin of error* depends on whether or not you know the standard deviation beforehand.

**STEPS FOR FINDING THE CONFIDENCE INTERVAL**

→ A significance level is chosen so we can be the value of the true mean will be within the boundaries of the interval we construct.

→ is estimated from a sample with n observations.

**Case 1:** Standard Deviation Known

→ is known before the sample is made, perhaps due to historical data (*such as corn or soy yields*) or due to current data (*such as stock market volatility, which is fairly accurately predicted by the Chicago Board of Exchange's VIX ticker*).

→ Use the standard normal distribution to find the Z Score where of the distribution is to the right.

**Case 2:** Standard Deviation Known

→ If is unknown, replace sigma with our best estimate of it, *s*, the sample standard deviation. This introduces another element of uncertainty into the sample; in order to account for this uncertainty, we modify the distribution of the sample mean slightly, by making it into a *studentized* t-distribution.

→ The t-distribution is essentially a Normal Distribution with an additional parameter: the degrees of freedom (*df*). Not knowing the standard deviation removes a single degree of freedom from the sample, therefore *df = n -1*.

→ Use the student's t-distribution to find the t-score where of the distribution is to the right.

→ Compute Margin Of Error (*MOE*).

→ Add and subtract the MOE to the sample mean to find a

Problems

1. Suppose you are making samples from a population that you know has a standard deviation of *5*. Your summary of the sample is given in the table below:

|  |  |
| --- | --- |
|  | 25.3 |
| s | 4.2 |
| n | 14 |

a. Find the 90% Confidence Interval for the true mean of this population.

b. Find the 95% Confidence Interval for the true mean of this population.

c. Find the 99% Confidence Interval for the true mean of this population.

i. How does changing the confidence level affect the width of the interval?

d. Suppose the number sampled was n = 50, instead of 34. Let's say you wound up with the exact same mean. What would be the 95% Confidence Interval for the true mean of this population?

ii. How does changing the number sampled affect the Confidence Interval?

2. Suppose you are making samples from a population with an unknown standard deviation. Your summary of the sample is given in the table below:

|  |  |
| --- | --- |
|  | 25.3 |
| s | 4.2 |
| n | 14 |

a. Find the 90% Confidence Interval for the true mean of this population.

b. Find the 95% Confidence Interval for the true mean of this population.

c. Find the 99% Confidence Interval for the true mean of this population.

i. Compare your results against the results in #1. How does the additional uncertainty of the not knowing the standard deviation affect the width of each interval?

3. Suppose you are making samples from a population with a standard deviation of 3. If you want your 95% Confidence Interval to be within 3 units of the sample mean, how many samples should you make to ensure your interval is this big?

4. Open the accompanying spreadsheet. The spreadsheet should contain two samples of data. In the first you will be constructing a confidence interval for the true mean when the standard deviation is known. In the second you will be constructing a confidence interval for the true mean when the standard deviation is unknown. There are additional instructions on the spreadsheet.

Replace your name in the filename. Save all of your work. Upload it to the appropriate drop-box on D2L when you are finished.