**Exam 1**

*MAT 120*

June 12th 2018

**Exam Start Time**: 2: 00 pm

**Exam End Time**: 3: 30 pm

Some Useful Information

∨ → Union

∧ → Intersection

~ → Complement

**A** – **B →** Subtraction

*n(* **A** *)* → Cardinality of A

**P (** A **)** → Power Set of A

*Buddha's First Noble Trut*h: All Life is Suffering

Bolivia's top export is tin.

Instructions: Take out several sheets of paper. Put your name at the top of each sheet. Solve the following problems on these sheets. Clearly label each problem and circle the answer. Partial credit will be awarded if you show your work and your logic is correct, so make sure to write everything down and clearly explain your steps. Turn in everything stapled together when you are done.

**Chapter 2**

**Problems**

Instructions: For the first nine problems, you are given the universal set **U,**

**U** = { *Anakin, Luke, Kylo, Han, Leia, Bail, C-3P0, R2-D2, BB-8, Obi-wan, Rey* }

And the subsets **Seq**, **Ori**, **Preq**, **Sky**, **Sol**, **Drd**, **Jed** defined on **U**,

**Seq** = { *Rey, BB-8, Kylo* }

**Ori** = { *Luke, Leia, Han, R2-D2, C-3P0* }

**Pre** = {*Anakin, Obi-wan, Bail, General Grievous* }

**Sky** = {*Anakin, Luke, Leia, Kylo* }

**Sol** = { *Han, Kylo*}

**Drd** = { *C-3P0, R2D2, BB-8, General Grievous* }

**Jed** = { *Anakin, Luke, Obi-wan, Plo Koon*}

**Sit** = { *Anakin, Kylo* }

1. **Venn Diagrams with Two Sets**. Draw a Venn Diagram with Two Sets for the following pairs of sets. Fill in the members in the appropriate region. Be sure to include the members in the universal set that are not in either set somewhere in each diagram. *State the type of set relation shown in the Venn Diagram* (i.e., subset, overlapping or disjoint):

a. **Jed**, **Sky**

b. **Sit**, **Sky**

c. **Pre**, **Seq**

d. **Drd**, **Ori**

2. **Set Operations**. Perform the following set operations and list the result in Roster Notation.

a. **Jed** ∧ **Sky**

b. ( ~**Sit** ) ∧ **Sky**

c. **Sit** ∨ **Sky**

d. **Drd** - **Ori**

*Hint*: *Use your Venn Diagrams from #1 for these problems!*

3. **Complements.**

a. Find *n*(**Sol**).

b. Find *n(***U**).

c. Without performing the set operation, determine *n*(~**Sol**).

i. What fundamental counting law did you use to solve this?

4. **Union-Intersection**. Find the set **Jed** V **Sit** . List the set in Roster Notation.

a. What is *n(* **Jed** V **Sit** *)?*

b. Why doesn't *n(* **Jed** V **Sit** *) = n(* **Jed**  *) + n(*  **Sit** *) ?*

5. **Symmetric Difference.** The Symmetric Difference **A** [ - ] **B** is a type of set operation we did not cover in class. It can be defined in terms of the other operations we studied, though. Find the Symmetric Difference of **Drd** and **Ori** by applying the identity,

**Drd** [ - ] **Ori** = (**Drd** – **Ori**) ∨ (**Ori** – **Drd**)

*(Symmetric Difference) = (Set Operation Definition)*

List the members of the set **Drd** [ - ] **Ori** in Roster Notation and then shade in the region described by the Symmetric Difference **Drd** [ - ] **Ori** in a Venn Diagram of the two sets.

*Hint: You should already have the Venn Diagram of the two sets from #1.*

6. **The First Absorption Law**. Show the *First Absorption Law* *of Set Theory* is valid,

**A** ∨ ( **A** ∧ **B** ) = **A**

(First Absorption Law: *the union of a set with its intersection of another set results in identity*)

Where **A** and **B** are any two sets. Let **A** = **Ori** and **B** = **Drd**. Show the operations on the left-hand side of the equation result in the set on the right-hand side of the equation.

7. **The Second Absorption Law.** Show the *Second Absorption Law of Set Theory* is valid,

**A** ∧ ( **A** ∨ **B** ) = **A**

(Second Absorption Law: *the intersection of a set with its union with another set results in identity)*

Where **A** and **B** are any two sets. Let **A** = **Ori** and **B** = **Drd**. Show the operations on the left-hand side of the equation result in the set on the right-hand side of the equation.

8. **Venn Diagrams with Three Sets.** Make a Venn diagram of the three sets **Sky**, **Jed** and **Ori** and then answer the following questions**.** Fill in the members in the appropriate region. Be sure to include the members in the universal set that are not in either of the three set somewhere in the diagram.

a. n( **Ori** – **Jed** – **Sky**) = ?

b. n (**Sky** – **Jed** – **Ori** ) = ?

c. n (**Jed**– **Sky** – **Ori** ) = ?

b. n( **Ori** ∧ **Jed** – **Sky** ) = ?

d. Are the sets **Sky** – **Jed** – **Ori** and **Jed**– **Sky** – **Ori** equivalent, equal or neither?

9. **Power Sets**. Find **P**( **Sky** ) and list its members in Roster Notation. State the cardinality of this Power Set, *n (***P ( Sky)** *).*

**Chapter 1**

**Problems**

Instructions: The last problem on the exam is an example of an inductive argument made by the genius mathematician Friedrich Gauss (seriously, this guy was smarter than Einstein, but gets no credit) at the ripe old age of 7 years old when his elementary school teacher gave him busy work while he took a nap at the front desk after a late night (*don't look up*).

10. **Inductive Reasoning**. Calculate the sum of the first 1000 natural numbers starting at 1 by following the steps in this problem.

a. Write out the first few terms of the sum and the last few terms of the sum, i.e.,

1 + 2 + 3 + 4 + …. + 997 + 998 + 999 + 1000

(*I swear this is going somewher*e)

b. Now suppose you knew the sum of these numbers and call it***S***. Let***S***= the expression written in *part a*.

c. Now write the sum in reverse order, starting at 1000 and going down to 1. Line up each term with the equation from *part a* like so,

S = 1 + 2 + 3 + 4 + ... + 997 + 998 + 999 + 1000

**S** = 1000 + 999 + 998 + 997 + …+ 4 + 3 + 2 + 1

(*How many terms are on the right-hand side of each equation?*)

d. Now add together both equations term by term.

i. Notice anything strange about each term that results from adding the vertically paired terms on the right-hand side?

e. Find the sum of the first 1000 numbers, ***S***.

11. **Bonus. (1 pt each)**

a. Show how the First and Second Absorption Laws from # 6 and #7 can be proven with Venn Diagrams.

*Hint: Create the left-hand side diagram in two or three separate shading steps and show the result is equal to the shaded area of the right-hand side diagram.*

b. Derive a general formula for the sum of the first *n* numbers from the argument presented in #10 by replacing 1000 with an arbitrary number *n.*