**Probability**

Probability is the study of the properties of random events.

**Preliminaries**

1.

Extended Union: A symbol that represents the union of a sequence of sets .

2.

Summation: A symbol that represents the sum of the elements.

**Definitions**

0. Terms

i. Experiment: An *uncertain* event.

ii. Mutual Exclusivity: Two sets, **A** and **B**, are *mutually exclusive* if they are disjoint.

“**A** and **B** are mutually exclusive”

1. ( lower-case letters )

Outcome*: A* possible way an experiment might occur.

2.

Sample Space:The set of all possible outcomes to an experiment.

3. ( upper-case letters )

( upper-case letters with subscripts )

Event*:* A subset of the sample space; A set of outcomes.

4.

Probability: The “*likelihood*” of event **A**; the “*chance*” that event **A** occurs.

**Axioms of Probability**

1.

All probabilities are positive; No probabilities are negative.

2.

The probability of *some* outcome from the sample space **S** occurring is equal to 1.

3.

If each event **Ai** in the sample space **S** is *mutually exclusive* with all of the other events , then the probability of the union of all of these events is equal to the sum of the probabilities of each individual event.

**Immediate Consequences**

1. **The Law of Complements**

Proof:

i.   
 … by **Complement Theorem 12 ( Set Theory )**

ii.   
 … by **Complement Theorem 13 ( Set Theory )**

iii.

… by *ii.* and **Axiom 3 ( Probability )**

**=>**    
 … by *i.,* **Axiom 2 ( Probability )** and *iii.*

2.

Proof:

i.   
 … by *1*

ii.   
 … by **Axiom 1 ( Probability )**

**=>**

… by *ii.*

3.

Proof: Follows immediately from *2* and **Axiom 1** **( Probability )**

4.

Proof:

i.

… by **Identity Theorem 1 ( Set Theory )**

ii.

… by **Axiom 3 ( Probability )**

iii.

… by **Axiom 1 ( Probability )**

iv.

… by *2*

v.

… by *iv.,*  *iii.* and *ii.*

**=>**

… by *v.* and **Axiom 1 ( Probability )**

**Partitions**

1.

Partition: A *partition* of a sample space **S** is a set of events such that the union of all of the events **Ai** ( *i = 1 , 2 , … , n – 1 , n* ) is equal to the sample space **S** *and* all of the events **Ai** are *mutually exclusive.*

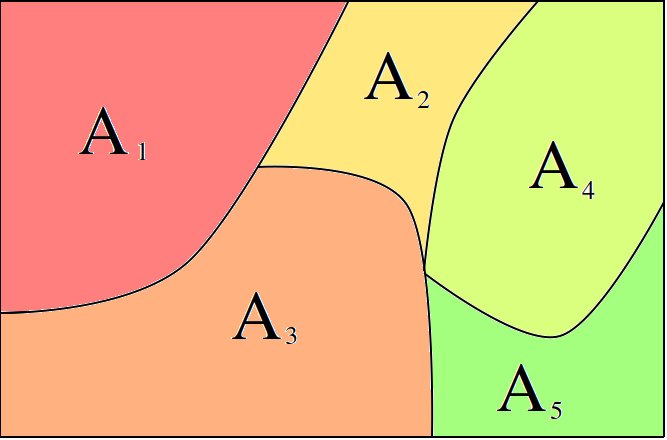


Figure: The sets **A**i ( i = 1, 2, 3, 4, 5) form a *partition* of the sample space **S**. Recall the sample space **S** is represented as the *entire* rectangle

2.

Even Partition: If all of the events **Ai** ( *i = 1 , 2 , … , n – 1 , n* ) are *equally likely*, then the is called an *even partition,* denoted *.*

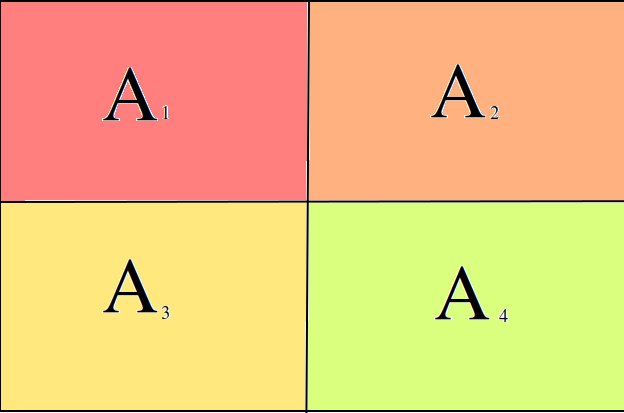


Figure: The sets **A**i ( i = 1, 2, 3, 4) form a *even partition* of the sample space **S**. This is represented graphically by four squares of equal area.

**Classical Theorems of Probability \***

\* Sometimes known as the *Frequentist Interpretations of Probability*

1.

If the eventsare an *even partition* of the sample space **S**, then for all *i* = *1, 2, … , n – 1, n.*

Proof:

i.

… by the assumption:

and **Axiom 3**

ii.

… by the assumption:

iii.

… by **Preliminary 2**, *i.* and *ii.*

iv.

… by the assumption: , *i.* and **Axiom 2**

v.

… by *iii.* and *iv.*

**=>**

2.

Proof:

i.

ii.

**=>**

3. **The Law of Unions**

Proof:

i.

ii.

**=>**