**Set Theory**

Set theory is the study of relationships that hold between collections of objects.

**Preliminaries**

1.

Implication: A symbolic representation of a *conditional* relationship between two propositions (*sentences*). This formula can be translated into English in the folllowing ways:

a. “if *p,* then *q”*

b*. “*whenever *p*, *q*”

c. “*p* implies *q*”

d. “*q* follows from *p*”

2.

Quantification: A symbolic representation of a universal proposition (*sentence*). This formula can be translated into English in the following ways:

a. “for all *p*, *q*”

b. “for every *p*, *q*”

**Definitions**

1. … ( lower-case letters )

Elements*:* The individuals, or objects, in the *domain* *of discourse;* The “*things*” being counted.

2. … ( upper-case letters )

Sets*:* Groups of elements that share a common property. *Sets* are *s*ometimes referred to as *classes* or *collections*.

3.

Universal Set:The set of all elements in the *domain of discourse.*

4.

Inclusion*:* The element *x belongs* to the set **A**.

5.

Exclusion*:* The element *x* does *not belong* to the set **A**.

6.

Cardinality*:* The number of *distinct* elements in the set **A**.

7.

Null Set*:* The unique set which contains nothing.

**Notation**

1.

List Notation: All of the elements that belong to **A** are *explicitly* written between a pair of brackets with commas separating them.

2.

Quantifer Notation: All of the elements that belong to **A** are *implicitly* written between a pair of brackets with a formula that specifies the conditions for membership.

**Immediate Consequences**

1.

The number of elements in the *null set* is 0.

2.

Nothing belongs to the *null set*.

3.

Everything belongs to the *universal set*.

**Relations**

1.

Subset*:* All of **A**'s elements are contained in set **B**. If the element *x* belongs to **A**, then the element *x* also belongs to **B**.

*Symbolically*:

2.

Proper Subset*:* The set**A** is a subsetof the set **B** and **A** is *wholly contained* in **B**; set **A** has less elements than set **B**.

*Symbolically*:

3.

Equivalence*:* The number of elements in **A** is equal to the number of elements in **B**.

*Symbolically:*

4.

Equality*:* **A** and **B** are the same set; **A** and **B** contain the same elements.

*Symbolically*:

**Operations**

1.

Complement: The set containing elements that do *not* belong to the set **A**.

*Symbolically*:

2.

Union*:* The set containing elements that belong to either the set **A** *or* theset **B**.

*Symbolically*:

3.

Intersection: The set containing elements that belong to both the set **A** *and* the set **B**.

*Symbolically*:

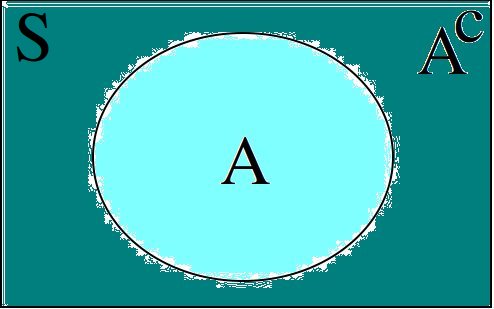
4.

Difference: The set containing the elements that belong to set **A** but not set **B**

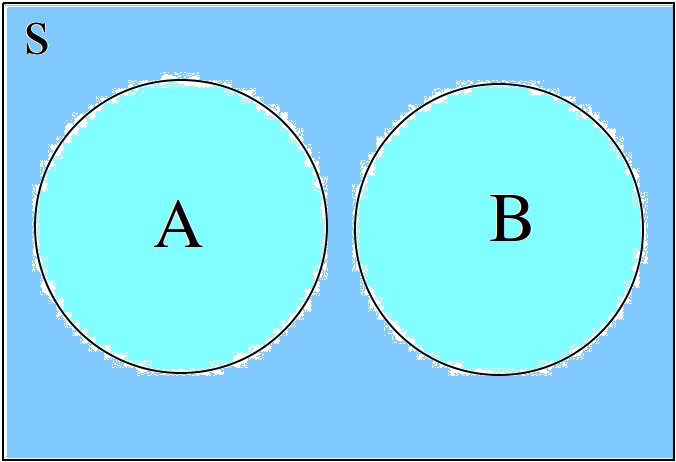
*Symbollically:*

**Venn Diagrams**

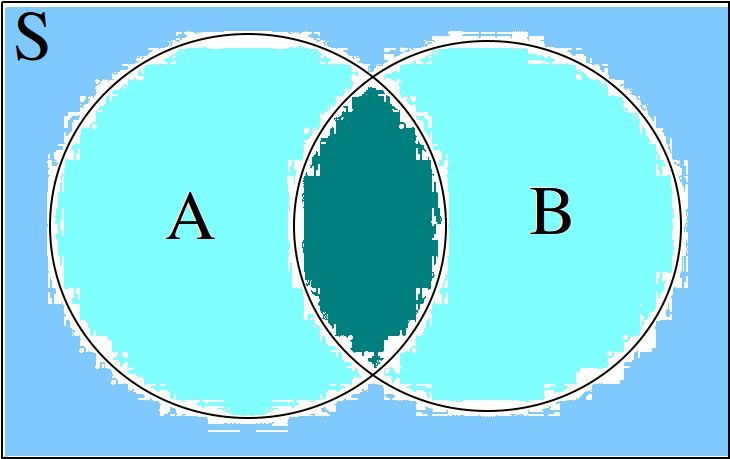
Note: In the following diagrams, **S** refers to the rectangle around the border containing everything.

1. Complement

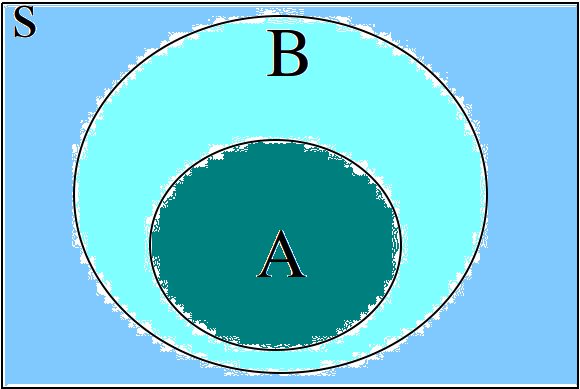
3. Disjoint



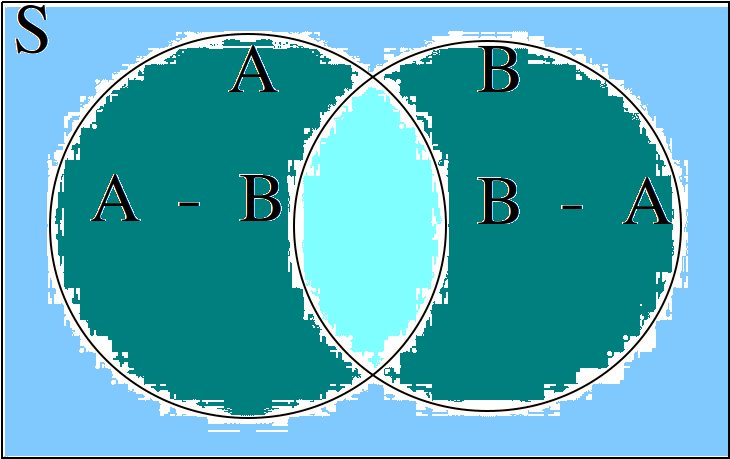
3. Overlapping



4. Subset



5. Difference

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**Special Sets**

1. Natural Numbers

2. Real Numbers

**Theorems**

**Identity Theorems**

1.

The intersection of **A** and the *null set* is equal to the *null set*.

2.

The union of **A** and the *null set* is equal to **A**.

3.

The union of **A** and **A** is equal to **A**.

4.

The intersection of **A** and **A** is equal to **A**.

**Subset Theorems**

5.

The intersection of **A** and **B** is a subset of **A**.

6.

**A** is a subset of the union of **A** and **B**.

7.

The intersection of **A** and **B** is a subset of the union of **A** and **B**.

8.

If **A** is a subset of **B**, then the intersection of **A** and **B** is equal to **A**

9.

If **A** is a subset of **B**, then the union of **A** and **B** is equal to **B**

10.

If **A** is a subset of **B**, then the difference of **A** and **B** is the *null set*

11.

If **A** is a subset of **B** and **B** is a subset of **C**, then **A** is a subset of **C**.

**Complement Theorems**

12.

The complement of the complement of **A** is equal to **A**.

13.

The union of **A** and its complement is the universal set.

14.

The intersection of **A** and its complement is the empty set.

15.

The difference of **A** and **B** is equal to the intersection of **A** and the complement of **B**.

**Counting Theorems**

16.

The number of elements in **A** and the number of elements in is equal total number of elements in the universal set.

17.

The number of elements in the union of **A** with **B** (**A** or **B**)is equal to the sum of number of elements in **A** and the number of elements in **B** , minus the number of elements in the intersection of **A** ***with* B** (**A** and **B**).