Diploma Programme
Programme du diplôme
Programa del Diploma

**Mathematical studies
Standard level
Paper 1**

Monday 13 May 2019 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

20 pages

20EP01

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. A sphere with diameter $3\,474\,000$ metres can model the shape of the Moon.

(a) Use this model to calculate the circumference of the Moon in **kilometres**. Give your full calculator display. [3]

(b) Give your answer to part (a) correct to three significant figures. [1]

(c) Write your answer to **part (b)** in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. [2]

Working:

Answers:

(a) ....

(b) ....

(c) ....

Turn over

2. Colorado beetles are a pest, which can cause major damage to potato crops. For a certain Colorado beetle the amount of oxygen, in millilitres (ml), consumed each day increases with temperature as shown in the following table.

Temperature, °C (*x*)	10	15	20	25	30
Oxygen consumed, ml (*y*)	90	133	230	306	391

This information has been used to plot a scatter diagram.

Temperature, °C
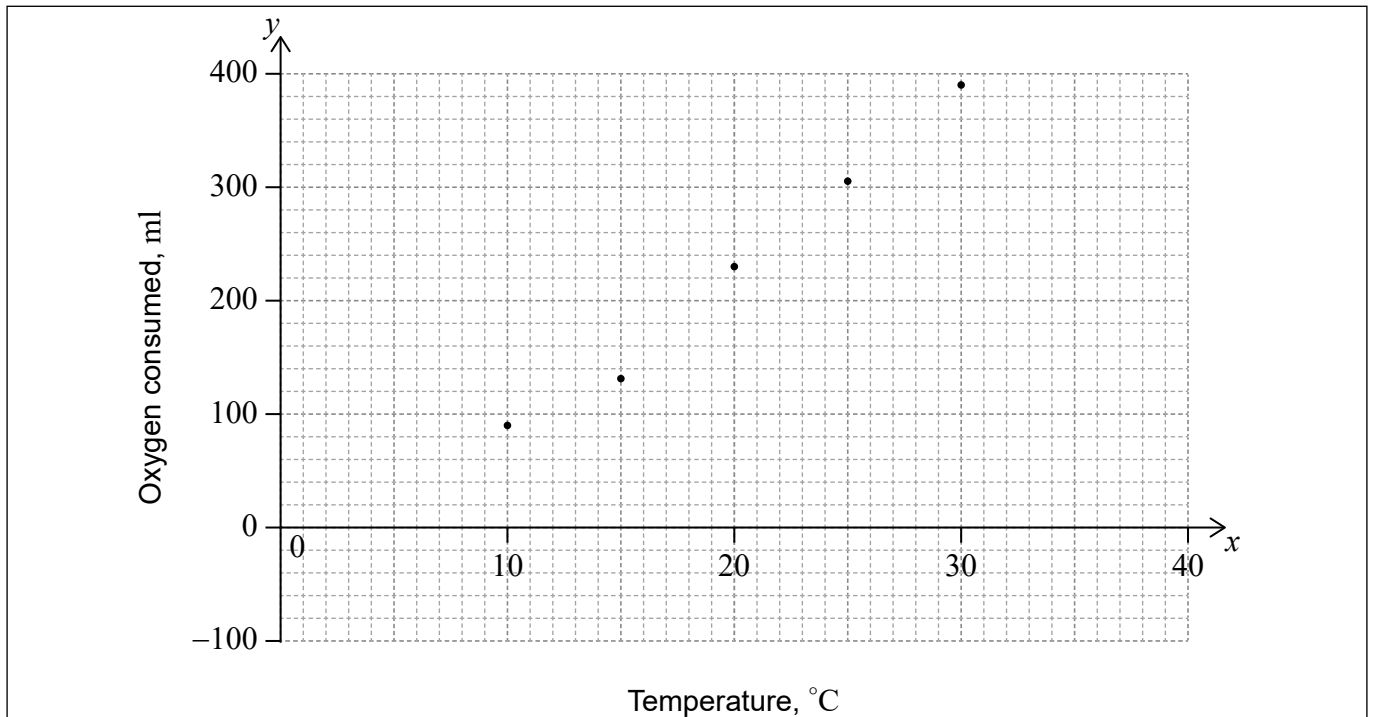
(a)   Find the equation of the regression line of y on x.   [2]

The mean point has coordinates $(20, 230)$.

(b)   Draw the regression line of y on x on the scatter diagram.   [2]

In order to estimate the amount of oxygen consumed, this regression line is considered to be reliable for a temperature x such that $a \leq x \leq b$.

(c)   Write down the value of a and of b.   [2]

Working:

Answers:

(a)   . . . .

(c)   . . . .

20EP04

3.   **In this question, give all answers to two decimal places.**

Velina travels from New York to Copenhagen with 1200 US dollars (USD). She exchanges her money to Danish kroner (DKK). The exchange rate is $1\,\mathrm{USD} = 7.0208\,\mathrm{DKK}$.

(a)   Calculate the amount that Velina receives in DKK.   [2]

At the end of her trip Velina has 3450 DKK left that she exchanges to USD. The bank charges a 5% commission. The exchange rate is still $1\,\mathrm{USD} = 7.0208\,\mathrm{DKK}$.

(b)   (i)   Calculate the amount, in DKK, that will be left to exchange after commission.

      (ii)   Hence, calculate the amount of USD she receives.   [4]

Working:

Answers:

(a)   ....

(b)   (i)   ....

      (ii)   ....

4. Consider the following statements.

p: a is divisible by 9
q: b is divisible by 9
s: The product of a and b is divisible by 9

(a) Write down $s \Leftrightarrow (p \wedge q)$ in words. [3]

(b) Complete the following truth table. [2]

p	q	s	$p \wedge q$	$s \Leftrightarrow (p \wedge q)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

(c) Justify why $s \Leftrightarrow (p \wedge q)$ is neither a tautology nor a contradiction. [1]

Working:

Answers:

(a) . . . .
. . . .
. . . .
. . . .

(c) . . . .
. . . .
. . . .

5. A school café sells three flavours of smoothies: mango (M), kiwi fruit (K) and banana (B).
85 students were surveyed about which of these three flavours they like.

> 35 students liked mango, 37 liked banana, and 26 liked kiwi fruit
> 2 liked all three flavours
> 20 liked both mango and banana
> 14 liked mango and kiwi fruit
> 3 liked banana and kiwi fruit

(a) Using the given information, complete the following Venn diagram. [2]
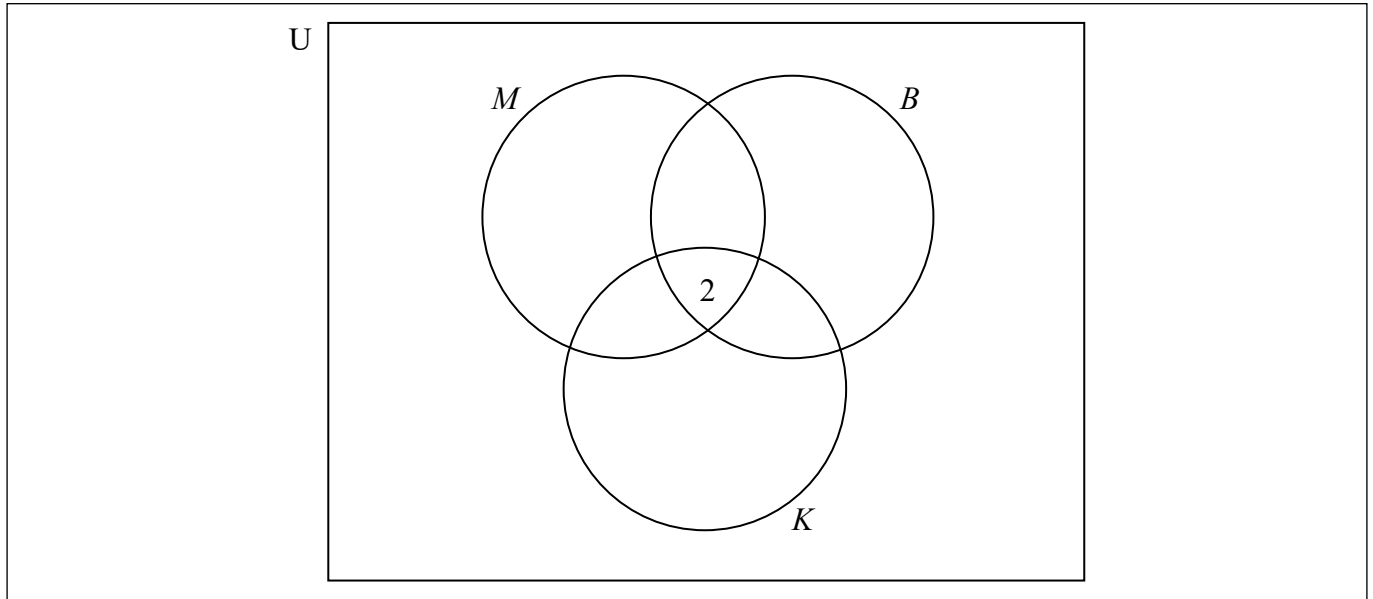
(b) Find the number of surveyed students who did not like any of the three flavours. [2]

(c) A student is chosen at random from the surveyed students.

Find the probability that this student likes kiwi fruit smoothies given that they like mango smoothies. [2]

Working:

Answers:

(b) ....

(c) ....

Turn over

6. A health inspector analysed the amount of sugar in 500 different **snacks** prepared in various school cafeterias. The collected data are shown in the following box-and-whisker diagram.

Amount of sugar per snack in grams
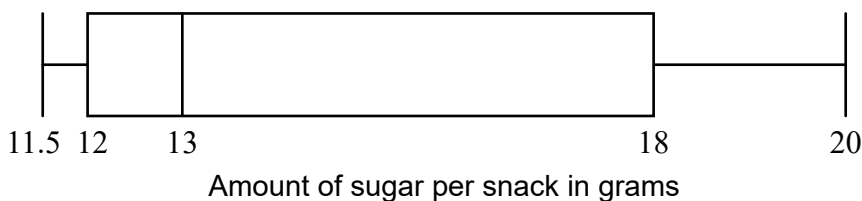
(a) State what 13 represents in the given diagram.   [1]

(b) (i) Write down the interquartile range for this data.

   (ii) Write down the approximate number of snacks whose amount of sugar ranges from 18 to 20 grams.   [3]

The health inspector visits two school cafeterias. She inspects the same number of **meals** at each cafeteria. The data is shown in the following box-and-whisker diagrams.

School Cafeteria 1

Amount of sugar per meal in grams

School Cafeteria 2

Amount of sugar per meal in grams
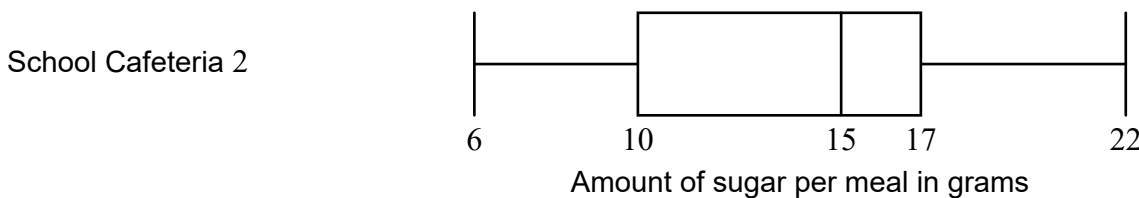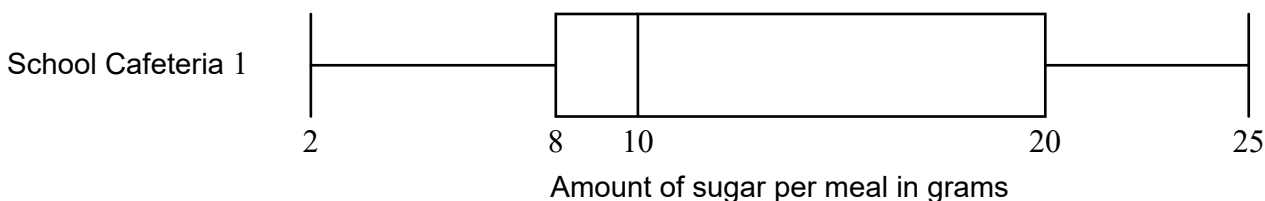
Meals prepared in the school cafeterias are required to have less than 10 grams of sugar.

(c) State, giving a reason, which school cafeteria has more meals that **do not** meet the requirement.   [2]

(This question continues on the following page)

(Question 6 continued)

Working:

Answers:

(a) . . . .

. . . .

(b) (i) . . . .

(ii) . . . .

(c) . . . .

. . . .

. . . .

7.   The width of a rectangular garden is 4.5 metres shorter than its length, which is x metres.

(a)   Write down an expression for the width of the garden in terms of x.   [1]

The perimeter of the garden is $111\,\text{m}$.

(b)   Write down an equation for the perimeter of the garden in terms of x.   [1]

(c)   Find the value of x.   [2]

A gardener measured the length of the garden as $25\,\text{m}$.

(d)   Find the percentage error in his measurement.   [2]

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

(d)   . . . .

8. Line L has a y-intercept at $(0, 3)$ and an x-intercept at $(4, 0)$, as shown on the following diagram.
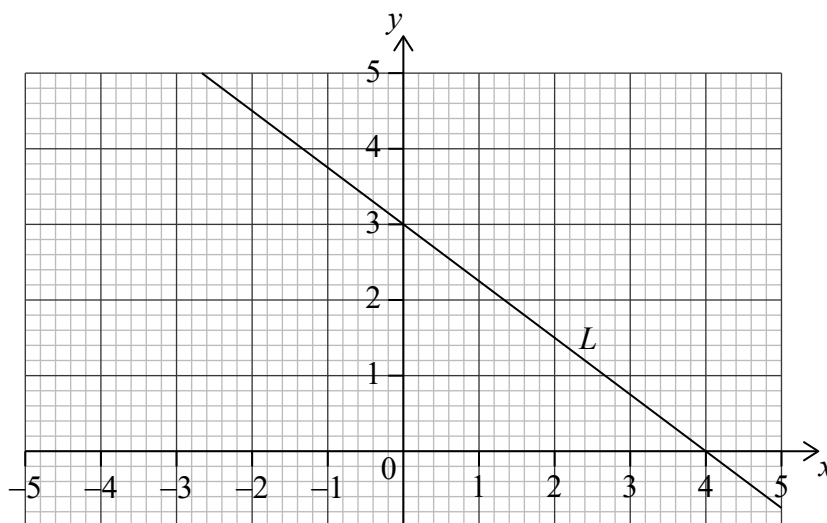
(a) (i) Find the gradient of L.

　　(ii) Write down the equation of L in the form $y = mx + c$. [3]

Line N is perpendicular to L, and passes through point $P(2, 1)$.

(b) (i) Write down the gradient of N.

　　(ii) Find the equation of N in the form $y = mx + c$. [3]

Working:

Answers:

(a) (i) . . . .

　　(ii) . . . .

(b) (i) . . . .

　　(ii) . . . .

Turn over

9. A ladder on a fire truck has its base at point B which is 4 metres above the ground. The ladder is extended and its other end rests on a vertical wall at point C, 16 metres above the ground. The horizontal distance between B and C is 9 metres.

diagram not to scale
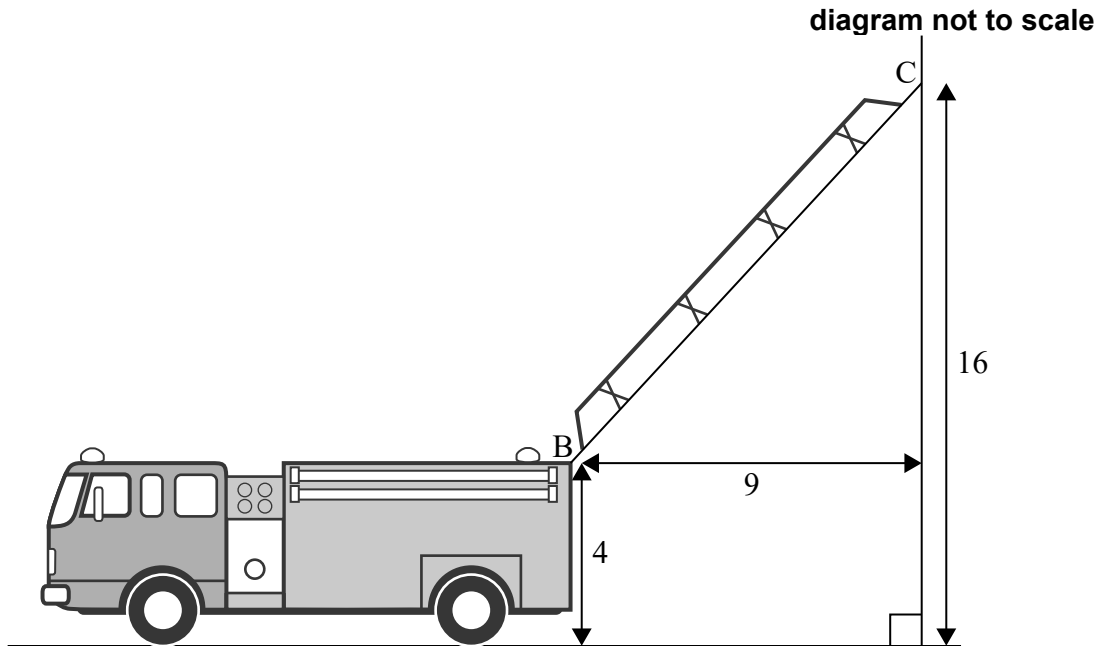
(a) Find the angle of elevation from B to C. [3]

A second truck arrives whose ladder, when fully extended, is 30 metres long. The base of this ladder is also 4 metres above the ground. For safety reasons, the maximum angle of elevation that the ladder can make is $70°$.

(b) Find the maximum height on the wall that can be reached by the ladder on the second truck. [3]

(This question continues on the following page)

(Question 9 continued)

Working:

Answers:

(a) ....

(b) ....

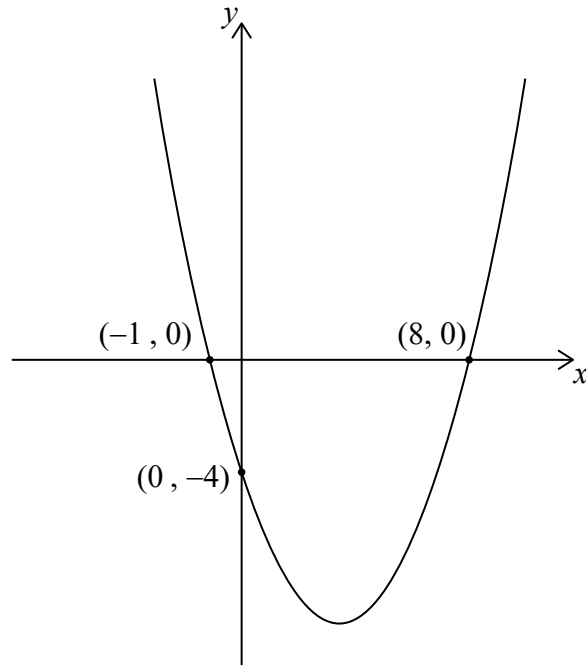**Turn over**

10. Consider the function $f(x) = a(x - p)(x - q)$, shown on the following graph.

(a) Find the equation of the axis of symmetry. [2]

(b) Find the value of p and of q. [2]

(c) Find the value of a. [2]

Working:

Answers:

(a) ....

(b) ....

(c) ....

11. Consider the following sets:

The universal set U consists of all positive integers less than 15;
A is the set of all numbers which are multiples of 3;
B is the set of all even numbers.

(a)   Write down the elements that belong to $A \cap B$.   [3]

(b)   Write down

(i)   the elements that belong to $A \cap B'$;

(ii) $n(A \cap B')$.   [3]

Working:

Answers:

(a)   . . . .

(b) (i)   . . . .

(ii)   . . . .

12. University students were surveyed and asked how many hours, h, they worked each month. The results are shown in the following table.

Hours per month, h	Frequency	Cumulative frequency
$0 < h \leq 10$	3	3
$10 < h \leq 20$	7	10
$20 < h \leq 30$	10	20
$30 < h \leq 40$	14	34
$40 < h \leq 50$	p	44
$50 < h \leq 60$	6	50
$60 < h \leq 70$	4	54
$70 < h \leq 80$	2	q

(a) Use the table to find the following values.
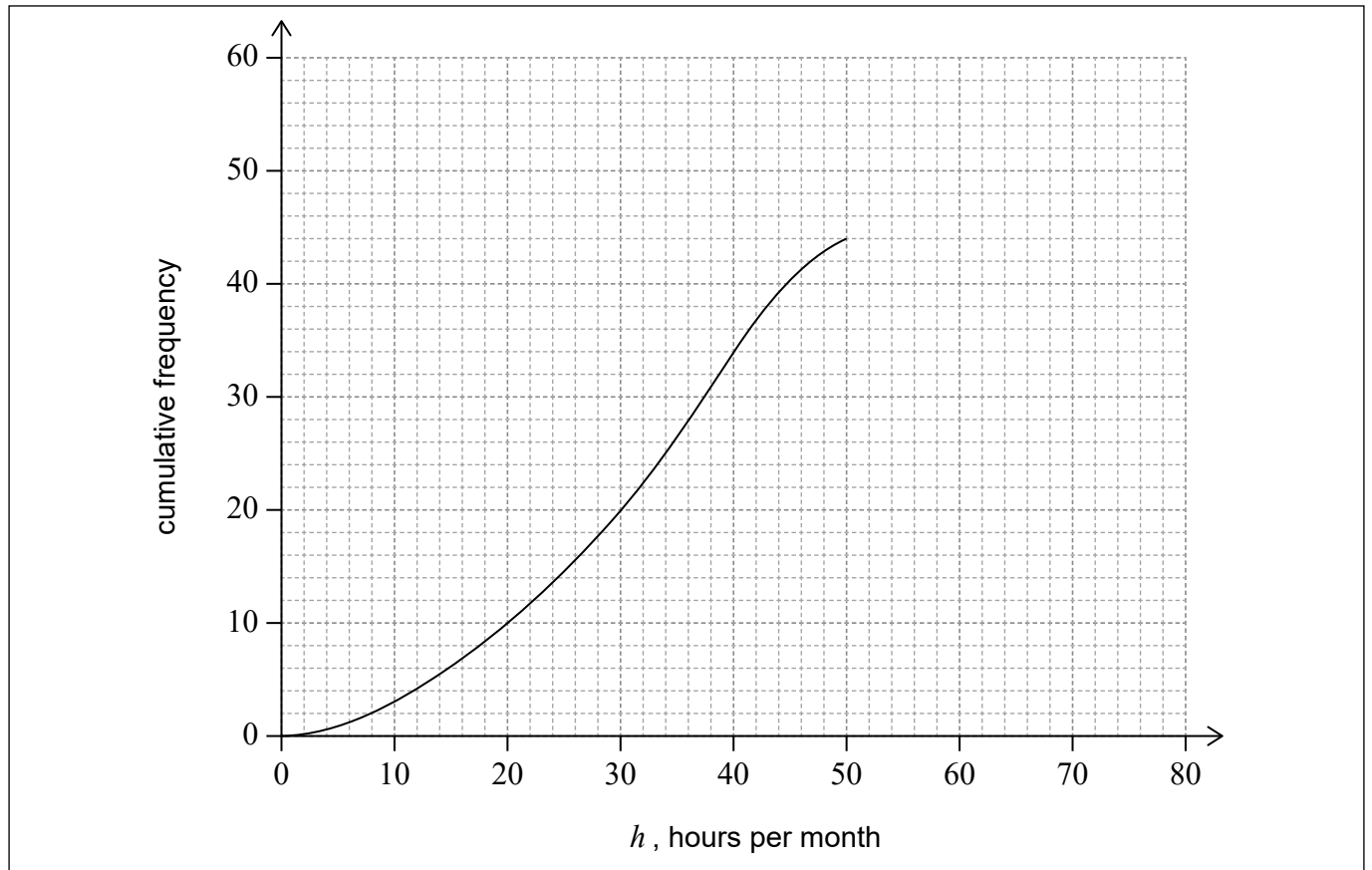
(i) p

(ii) q [2]

(This question continues on the following page)

(Question 12 continued)

The first five class intervals, indicated in the table, have been used to draw part of a cumulative frequency curve as shown.

(b)   On the same grid, complete the cumulative frequency curve for these data.   [2]

(c)   Use the cumulative frequency curve to find an estimate for the number of students who worked at most 35 hours per month.   [2]

Working:

Answers:

(a) (i)   . . . .

       (ii)   . . . .

(c)   . . . .

13. Little Green island originally had no turtles. After 55 turtles were introduced to the island, their population is modelled by

$$N(t) = a \times 2^{-t} + 10, \ t \geq 0,$$

where a is a constant and t is the time in years since the turtles were introduced.

(a) Find the value of a. [2]

(b) Find the time, in years, for the population to decrease to 20 turtles. [2]

There is a number m beyond which the turtle population will not decrease.

(c) Find the value of m. Justify your answer. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

. . . .

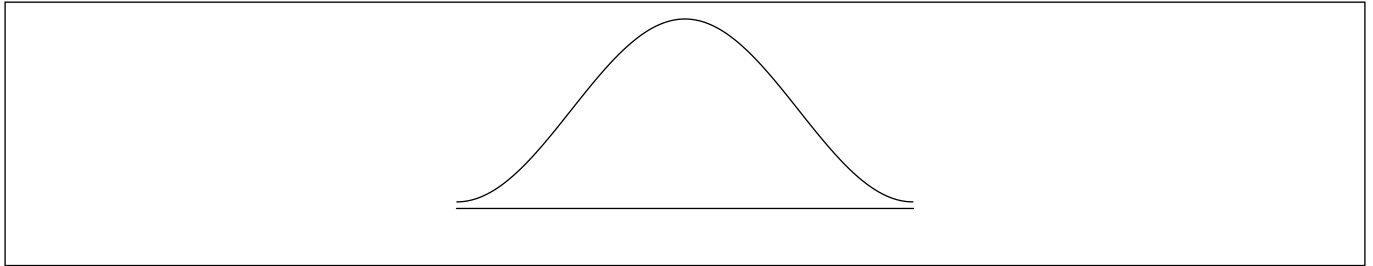. . . . .

14. The price per kilogram of tomatoes, in euro, sold in various markets in a city is found to be normally distributed with a mean of 3.22 and a standard deviation of 0.84.

(a) (i) On the following diagram, shade the region representing the probability that the price of a kilogram of tomatoes, chosen at random, will be higher than 3.22 euro.

(ii) Find the price that is two standard deviations above the mean price. [2]

(b) Find the probability that the price of a kilogram of tomatoes, chosen at random, will be between 2.00 and 3.00 euro. [2]

To stimulate reasonable pricing, the city offers a free permit to the sellers whose price of a kilogram of tomatoes is in the lowest 20%.

(c) Find the highest price that a seller can charge and still receive a free permit. [2]

Working:

Answers:

(a) (ii) . . . .

(b) . . . .

(c) . . . .

15. A potter sells x vases per month.

His monthly profit in Australian dollars (AUD) can be modelled by

$$P(x) = -\frac{1}{5}x^3 + 7x^2 - 120, \, x \geq 0.$$

(a) Find the value of P if no vases are sold. [1]

(b) Differentiate $P(x)$. [2]

(c) **Hence**, find the number of vases that will maximize the profit. [3]

Working:

Answers:

(a) ....

(b) ....

(c) ....