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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 13 May 2019 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2219 – 7305

11 pages

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following table shows the probability distribution of a discrete random variable X.

X	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{3}{13}$	$\frac{1}{13}$	$\frac{4}{13}$	k

(a) Find the value of k. [3]

(b) Find $\mathrm{E}(X)$. [3]

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2. [Maximum mark: 6]

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Find the value of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are

(a) parallel; [2]

(b) perpendicular. [4]

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3. [Maximum mark: 6]

Consider the function $f(x) = \frac{3x+1}{x-2}$, $x \neq 2$.

(a) For the graph of f,

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

Let $g(x) = x^2 + 4$, $x \in \mathbb{R}$.

(b) Find $(f \circ g)(1)$. [3]

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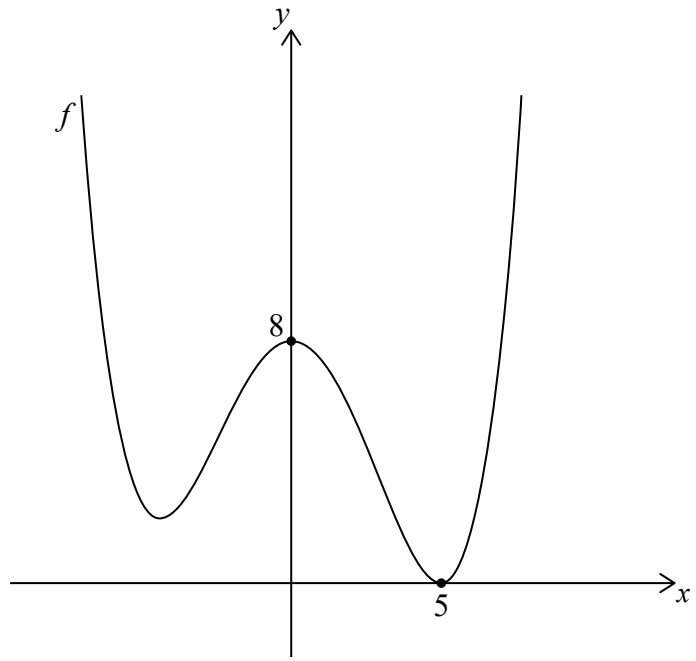
4. [Maximum mark: 7]

The following diagram shows part of the graph of f with x-intercept $(5, 0)$ and y-intercept $(0, 8)$.



(a) Find the y-intercept of the graph of

(i) $f(x) + 3$;

(ii) $f(4x)$. [3]

(b) Find the x-intercept of the graph of $f(2x)$. [2]

(c) Describe the transformation $f(x + 1)$. [2]

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5. [Maximum mark: 6]

Consider the function $f(x) = (1-k)x^2 + x + k$, $x \in \mathbb{R}$. Find the value of k for which $f(x)$ has two equal real roots.

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6. [Maximum mark: 7]

Solve $\log_4(2-x) = \log_{16}(13-4x)$.

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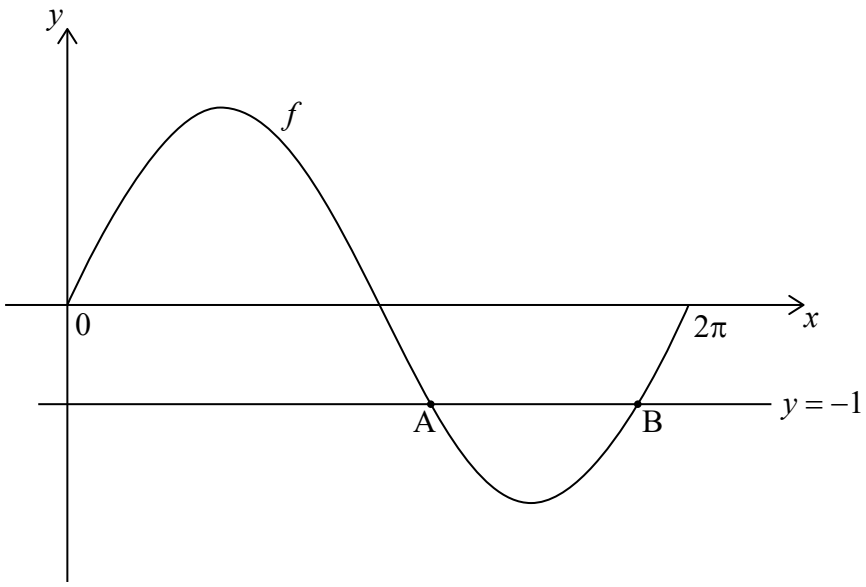
7. [Maximum mark: 7]

Consider the graph of the function $f(x) = 2\sin x$, $0 \le x < 2\pi$. The graph of f intersects the line $y = -1$ exactly twice, at point A and point B. This is shown in the following diagram.



(a) Find the x-coordinate of A and of B. [4]

Consider the graph of $g(x) = 2\sin px$, $0 \le x < 2\pi$, where $p > 0$.

(b) Find the greatest value of p such that the graph of g does not intersect the line $y = -1$. [3]

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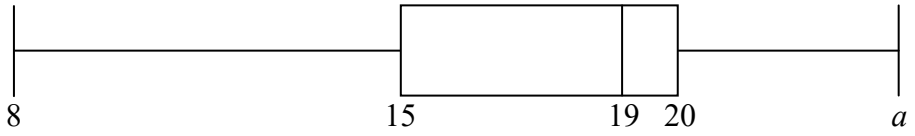
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

A group of 10 girls recorded the number of hours they spent watching television during a particular week. Their results are summarized in the box-and-whisker plot below.



(a) The range of the data is 16. Find the value of a. [2]

(b) Find the value of the interquartile range. [2]

The group of girls watched a total of 180 hours of television.

(c) Find the mean number of hours that the girls in this group spent watching television that week. [2]

A group of 20 boys also recorded the number of hours they spent watching television that same week. Their results are summarized in the table below.

$\overline{x} = 21$	$\sigma = 3$

(d) (i) Find the total number of hours the group of boys spent watching television that week.

(ii) Find the mean number of hours that **all 30** girls and boys spent watching television that week. [5]

The following week, the group of boys had exams. During this exam week, the boys spent half as much time watching television compared to the previous week.

(e) For this exam week, find

(i) the mean number of hours that the group of boys spent watching television;

(ii) the variance in the number of hours the group of boys spent watching television. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Let θ be an **obtuse** angle such that $\sin\theta = \frac{3}{5}$.

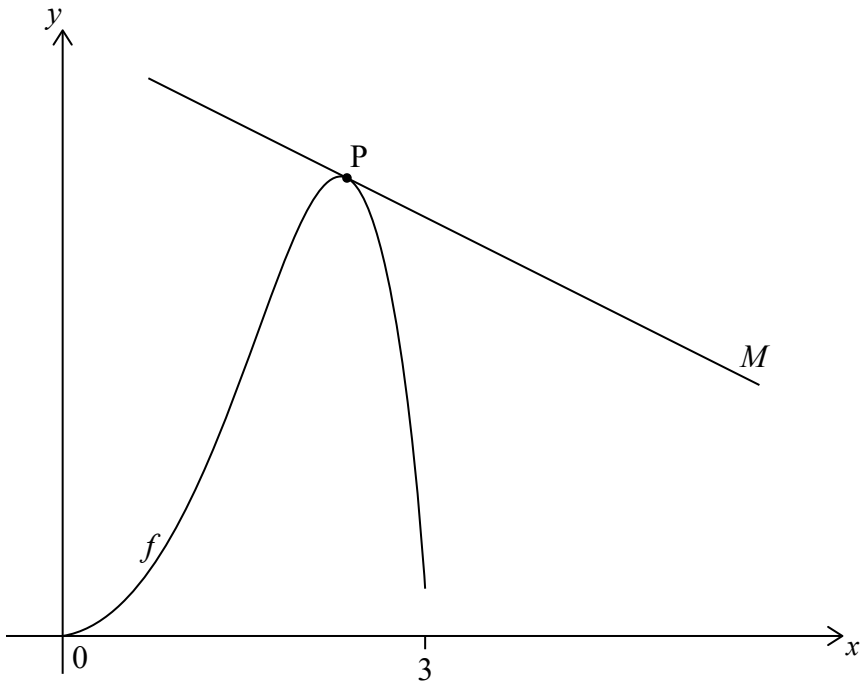
(a) Find the value of $\tan\theta$. [4]

(b) Line L passes through the origin and has a gradient of $\tan\theta$. Find the equation of L. [2]

Let $f(x) = e^x \sin x - \frac{3x}{4}$.

(c) Find the derivative of f. [5]

The following diagram shows the graph of f for $0 \leq x \leq 3$. Line M is a tangent to the graph of f at point P.



(d) Given that M is parallel to L, find the x-coordinate of P. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

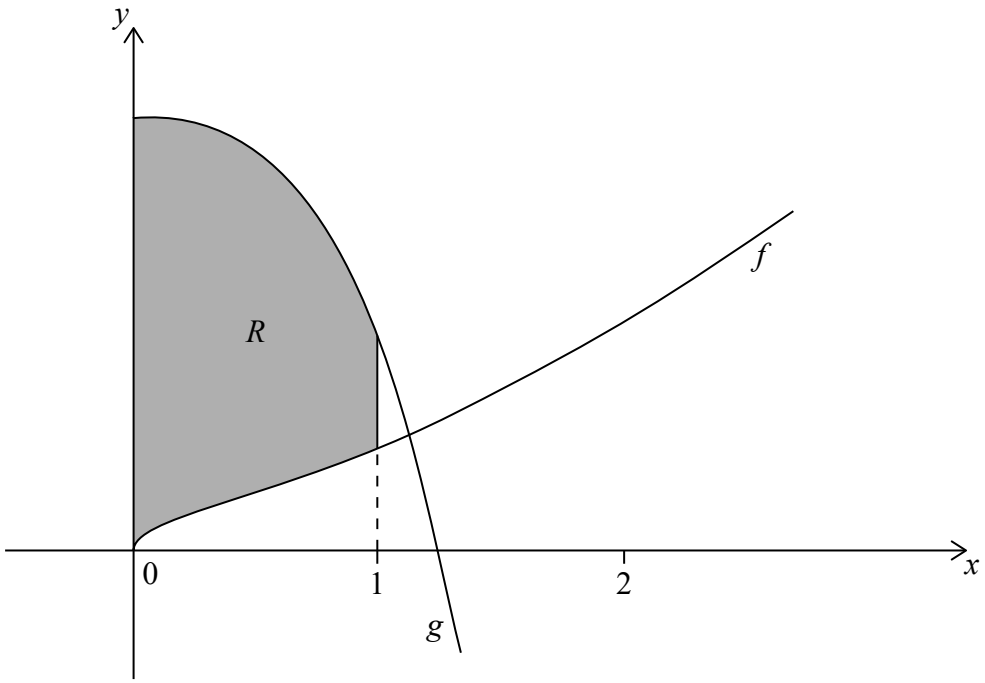
Let $y = \left(x^3 + x\right)^{\frac{3}{2}}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Hence find $\int \left(3x^2 + 1\right)\sqrt{x^3 + x}\ \mathrm{d}x$. [3]

Consider the functions $f(x) = \sqrt{x^3 + x}$ and $g(x) = 6 - 3x^2\sqrt{x^3 + x}$, for $x \geq 0$.

The graphs of f and g are shown in the following diagram.



The shaded region R is enclosed by the graphs of f, g, the y-axis and $x = 1$.

(c) Write down an expression for the area of R. [2]

(d) Hence find the exact area of R. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.