MLM Final Project Part 1

May 06 2020

Team Members and division of work:

- Bilal Waheed, Dennis Hilgendorf, Trey Dellucci, Joe Marlo, Yi-Hung Wang

Question 0.

Load classroom.csv and create MATH1ST (fit all models using REML, use lmerTest::lmer)

```
# Insert code to load data and create math1st variable
classroom <- read.csv("/Users/mbp/Documents/NYU/APSTA 2042 - Multi-level Models (Nested)/Project 1/MLM-
classroom <- classroom %>% mutate(math1st = mathkind + mathgain)
```

Question 1.

Estimate UMM model with random intercepts for both schools and classrooms.

```
# Insert code to fit model and print summary

M1_UMM <- lmerTest::lmer(math1st ~ (1 | schoolid/classid), data=classroom)
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                      1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a. Report the ICC for schools and classrooms:

Response:

- $\hat{\sigma}^2_\eta = 85.47$

- $\hat{\sigma}^2_\zeta = 280.69$
- $\hat{\sigma}^2_\varepsilon = 1146.79$

$$ICC_{school} = \frac{280.69}{85.47 + 280.69 + 1146.79} = 18.55\%$$

$$ICC_{classroom} = \frac{85.47}{85.47 + 280.69 + 1146.79} = 5.65\%$$

b. Write out the model:

`Model 1 Equation:`

$$MATH1ST_{ijk} = b_0 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

Question 2.

Add all school-level predictors:

`Model 2 Equation:`

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

```
# Insert code to fit model and print summary
M2 <- lmerTest::lmer(math1st ~ housepov + (1 | schoolid/classid), data=classroom)
summary(M2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.96  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.807 159.023   <2e-16 ***
```

```
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

a. Report if the additional predictors are justified:

```
# Insert code to compare models
anova(M1_UMM, M2, refit = F)
```

```
## Data: classroom
## Models:
## M1_UMM: math1st ~ (1 | schoolid/classid)
## M2: math1st ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M1_UMM  4 11953 11973 -5972.3    11945
## M2      5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the school-level predictor is justified according to the ANOVA between the unconditional means model, and the model with the school-level predictor HOUSEPOV. The chi-square test results in a statistically significant p-value (at the 0.05 alpha level) of approximately 0.

b. Report the change to school variance:

Response:

- Change in $\sigma^2_\zeta = 250.93 - 280.69 = -29.76$
- The school variance changed from 280.69 in Model 1 (UMM null model) to 250.93 indicating that adding the covariate, HOUSEPOV, accounted for variance at the school level.

Question 3: Add all class-level predictors

Model 3 Equation:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

```
# Insert code to fit model and print summary
M3 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid), data=
summary(M3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
```

```
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62117   14.08834 109.83230  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp
## housepov -0.568
## mathknow -0.052  0.082
## mathprep -0.666  0.032  0.004
## yearstea -0.264  0.077  0.030 -0.175
```

a. Report if adding the predictors is justified:

```
linearHypothesis(M3, c("mathknow", "mathprep", "yearstea"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## mathknow = 0
## mathprep = 0
## yearstea = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 3.4804     0.3233
```

Response:

The addition of classroom level covariates of MATHKNOW, MATHPREP, and YEARSTEA is not justified because the p-value ($p = 0.3233$) of the WALD test is not significant at the 0.05 alpha level. This suggests the benefit of adding classroom level predictors jointly is not significant in comparison to the previous model

containing student level predictors.

b. Report changes in class-level variance and individual variance:

`Response:`

- Change in $\sigma^2_\eta = 94.36 - 82.36 = 12.00$
- Change in $\sigma^2_\epsilon = 1136.43 - 1146.96 = -10.53$
- Change in classroom level variance $\hat{\sigma}^2_\eta$ changed from 85.47 in Model 1 (UMM null model) to 82.36 in Model 2 and finally to 94.36 in Model 3 indicating that adding the classroom level covariates increases classroom level variance.
- Change in individual level variance $\hat{\sigma}^2_\varepsilon$ changed from 1146.79 in Model 1 (UMM null model) to 1146.96 in Model 2 and finally to 1136.43 in Model 3 indicating that adding the classroom level covariates reduced individual variance.

c. Give a potential reason to explain why individual variance but not class variance is reduced:

`Response:`

Adding the classroom-level predictors shows a potential misspecification of the model. The classroom level predictors can make it difficult to estimate the individual level variance (i.e overstated individual variance) due to individual outliers in classrooms that have a very small amount of students. > GO THROUGH RECORDING

Question 4.

Add all student-level predictors excepting mathgain and mathkind:

```
# Insert code to fit model and print summary
M4 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority + sex + (1 | s
summary(M4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
```

```
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp yearst ses    minrty
## housepov -0.451
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

a. Report if the block of predictors is justified:

```
# Insert code to compare models

linearHypothesis(M4, c("sex","minority", "ses"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## sex = 0
## minority = 0
## ses = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 85.055  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of student level covariates of SES, MINORITY, and SEX is justified because the p-value (p = 0) of the WALD test is significant at the 0.05 alpha level. This suggests the benefit of adding student level predictors jointly is significant in comparison to the previous model containing school and classroom-level predictors.

b. Report change in all variance components

Response:

- Change in $\sigma^2_\zeta = 169.45 - 223.31 = -53.86$

- Change in $\sigma^2_\eta = 93.89 - 94.36 = -0.47$
- Change in $\sigma^2_\epsilon = 1064.96 - 1136.43 = -71.47$
- Change in classroom level variance ($\hat{\sigma}^2_\eta$) changed from 85.47 in Model 1 (UMM null model), to 82.36 in Model 2, to 94.36 in Model 3, and then to 93.89 in Model 4 with a marginal decrease in classroom level variance.
- Change in school variance ($\hat{\sigma}^2_\zeta$) changed from 280.69 in Model 1 (UMM null model), to 250.93 in Model 2, to 223.31 in Model 3, and finally to 169.45 in Model 4 with a decrease in school level variance observed after adding student level covariates in Model 4.
- Change in individual student level variance ($\hat{\sigma}^2_\varepsilon$) changed from 1146.79 in Model 1 (UMM null model), to 1146.96 in Model 2, to 1136.43 in Model 3, and finally to 1064.96 in Model 4 indicating that student level variance decreased after adding student level covariates in Model 4.

c. Give a potential reason as to why the school variance drops from the last model:

`Response:`

The student-level predictors explain some variance at the school level. SES, MINORITY, and SEX of children vary between different schools and may impact math scores. For example, some schools located in poorer areas with a different demographic of students will have much different individual math scores than those in more affluent areas.

d. Write this model out:

`Model 4 Equation:`

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + b_5 SEX_{ijk} + b_6 MINORITY_{ijk} + b_7 SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

Question 5.

a. Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately - not all together)

b. Report the models and their fit.

```
M5 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + yearstea | schoolid) + (1|schoolid/classid), data = classroom
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0080545
## (tol = 0.002, component 1)
```

```
summary(M5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
```

```
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.247e+01  9.6159
##  schoolid         (Intercept) 1.684e+02 12.9758
##  schoolid.1       yearstea    1.008e-02  0.1004
##  Residual                     1.065e+03 32.6361
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780  266.47953 101.662  < 2e-16 ***
## housepov     -17.72082   13.21686  113.58577  -1.341    0.183
## yearstea       0.01128    0.14192  122.87741   0.079    0.937
## mathknow       1.33106    1.39155  234.33195   0.957    0.340
## mathprep      -0.26584    1.37588  204.90504  -0.193    0.847
## sex           -1.21060    2.09480 1022.21558  -0.578    0.563
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## ses           10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.450
## yearstea -0.258  0.070
## mathknow -0.082  0.057  0.028
## mathprep -0.632  0.037 -0.172  0.003
## sex      -0.191 -0.007  0.015  0.006 -0.006
## minority -0.320 -0.179  0.023  0.115  0.001 -0.010
## ses      -0.121  0.082 -0.027 -0.007  0.053  0.020  0.162
## convergence code: 0
## Model failed to converge with max|grad| = 0.0080545 (tol = 0.002, component 1)
```

```
anova(M4,M5, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M5: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M5:     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M5 12 10754 10813 -5364.8    10730 0.007      1     0.9336
```

Response:

The addition of random slope on the *YEARSTEA* variable is not significant (p-value = 0.9336) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M6 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathknow | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.389e+01  9.689914
##  schoolid         (Intercept) 1.694e+02 13.016328
##  schoolid.1       mathknow    1.700e-06  0.001304
##  Residual                     1.065e+03 32.633705
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63047    5.31204  275.40357 101.586  < 2e-16 ***
## housepov     -17.64821   13.21718  113.88792  -1.335    0.184
## yearstea       0.01129    0.14141  226.81110   0.080    0.936
## mathknow       1.34993    1.39168  234.50067   0.970    0.333
## mathprep      -0.27708    1.37583  205.27196  -0.201    0.841
## sex           -1.21417    2.09483 1022.42010  -0.580    0.562
## minority     -16.18681    3.02603  704.47306  -5.349 1.20e-07 ***
## ses           10.05075    1.54485 1066.56262   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M6, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M6: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M6:     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M6 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *MATHKNOW* variable is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M7 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathprep | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M7)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.388e+01 9.689e+00
##  schoolid         (Intercept) 1.694e+02 1.302e+01
##  schoolid.1       mathprep    2.171e-07 4.659e-04
##  Residual                     1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39223 101.586  < 2e-16 ***
## housepov     -17.64851   13.21749  113.87941  -1.335    0.184
## yearstea       0.01129    0.14141  226.80838   0.080    0.936
## mathknow       1.35003    1.39167  234.49786   0.970    0.333
## mathprep      -0.27705    1.37582  205.27063  -0.201    0.841
## sex           -1.21419    2.09483 1022.42070  -0.580    0.562
## minority     -16.18676    3.02605  704.47629  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56201   6.506 1.18e-10 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M7, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M7: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M7:     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M7 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *MATHPREP* variable is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

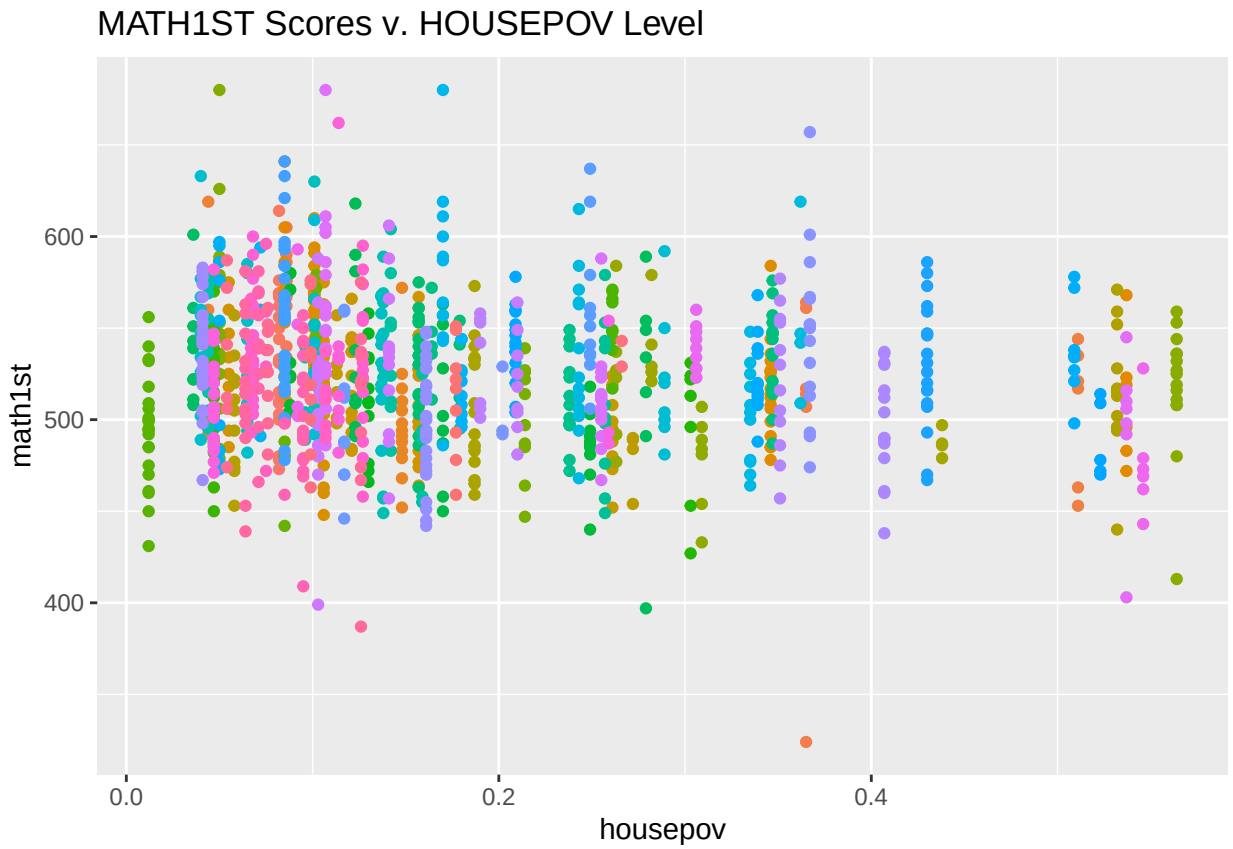
c. Why is it a bad idea to include a random slope on the housepov effect?

Response:

It is a bad idea to include a random slope on the *HOUSEPOV* effect because its variance is already being accounted for in the null model by including the differential effects of schoolid on the outcome *MATH1ST*. *HOUSEPOV* is a school-level predictor and cannot vary at the classroom or individual level - every individual would have the same *HOUSEPOV* level within a school.

The plot below illustrates that each individual in a school have the same *HOUSEPOV*:

```
ggplot(classroom, aes(x=housepov, y=math1st, color = factor(schoolid))) +
  geom_point() +
  ggtitle("MATH1ST Scores v. HOUSEPOV Level") +
  theme(legend.position = "none")
```



d. Retry the above models, allowing the slopes to be correlated with the random intercepts (still one by one):

```
M8 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (yearstea | schoolid) + (1 | classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0287242
## (tol = 0.002, component 1)
```

```
summary(M8)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (yearstea | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7469 -0.6029 -0.0282  0.6034  3.8430
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
```

```
##  classid  (Intercept)   38.7450  6.2245
##  schoolid (Intercept)  365.8546 19.1273
##           yearstea       0.5508  0.7422  -0.78
##  Residual             1066.0342 32.6502
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95831    5.49083  222.68166  98.156  < 2e-16 ***
## housepov     -17.12865   13.46221  119.58919  -1.272    0.206
## yearstea       0.02197    0.15769   75.74039   0.139    0.890
## mathknow       1.04763    1.34485  210.60696   0.779    0.437
## mathprep       0.04798    1.34661  191.68881   0.036    0.972
## sex           -1.33487    2.08766 1024.47814  -0.639    0.523
## minority     -16.44853    2.99666  669.72921  -5.489 5.74e-08 ***
## ses           10.14964    1.53869 1062.66735   6.596 6.64e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.455
## yearstea -0.370  0.084
## mathknow -0.085  0.049  0.012
## mathprep -0.606  0.050 -0.139  0.014
## sex      -0.184 -0.004  0.009  0.008 -0.004
## minority -0.305 -0.169  0.032  0.122 -0.007 -0.012
## ses      -0.119  0.079 -0.019 -0.001  0.049  0.022  0.168
## convergence code: 0
## Model failed to converge with max|grad| = 0.0287242 (tol = 0.002, component 1)
```

```
anova(M4, M8, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M8: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M8:     ses + (yearstea | schoolid) + (1 | classid)
##    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M8 13 10750 10814 -5361.8    10724 5.825      2    0.05434 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *YEARSTEA* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value = 0.05434) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M9 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (mathknow | schoolid) + (1 | classid), data = classroom)

summary(M9)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (mathknow | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.393e+01  9.6915
##  schoolid (Intercept) 1.693e+02 13.0118
##           mathknow    9.182e-04  0.0303  0.97
##  Residual             1.065e+03 32.6341
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.64041    5.31203  275.38950 101.588  < 2e-16 ***
## housepov     -17.64141   13.21242  103.98187  -1.335    0.185
## yearstea       0.01114    0.14141  226.85275   0.079    0.937
## mathknow       1.35458    1.39201  214.62548   0.973    0.332
## mathprep      -0.27754    1.37599  201.27747  -0.202    0.840
## sex           -1.21328    2.09485 1021.79809  -0.579    0.563
## minority     -16.19378    3.02608  703.80364  -5.351 1.18e-07 ***
## ses           10.04788    1.54488 1062.12259   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.082  0.057  0.029
## mathprep -0.631  0.038 -0.173  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

```
anova(M4, M9, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M9: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M9:     ses + (mathknow | schoolid) + (1 | classid)
##    Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M9 13 10756 10820 -5364.8    10730 3e-04      2     0.9998
```

Response:

The addition of random slope on the *MATHKNOW* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value = 0.9998) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M10 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (mathprep | schoolid) + (1 | classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (mathprep | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   78.46   8.858
##  schoolid (Intercept)  552.68  23.509
##           mathprep      15.88   3.985   -1.00
##  Residual             1064.27  32.623
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60872    5.60800  159.90129  96.043  < 2e-16 ***
## housepov     -14.01327   12.88651  116.07030  -1.087    0.279
## yearstea      -0.02586    0.13948  223.50199  -0.185    0.853
## mathknow       1.29875    1.37192  229.68405   0.947    0.345
## mathprep       0.04067    1.34844  139.04813   0.030    0.976
## sex           -1.16756    2.08698 1023.14893  -0.559    0.576
## minority     -16.46421    2.99522  663.67387  -5.497 5.52e-08 ***
## ses           10.14167    1.53961 1060.93434   6.587 7.04e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.461
## yearstea -0.260  0.089
## mathknow -0.071  0.027  0.048
## mathprep -0.692  0.107 -0.155  0.012
## sex      -0.183  0.003  0.023  0.002 -0.008
## minority -0.275 -0.187  0.025  0.107 -0.035 -0.013
## ses      -0.121  0.095 -0.033 -0.001  0.061  0.024  0.161
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M10, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M10: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M10:     ses + (mathprep | schoolid) + (1 | classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M10 13 10751 10816 -5362.3    10725 4.8144      2    0.09007 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *MATHPREP* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value=0.09007) at the 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occured (hint: what did you add to the model?).

Response:

- After adding an uncorrelated random slopes varying by schools on *YEARSTEA*, *MATHKNOW*, and *MATHPREP*, the variances of the random slopes are all close to 0. All other variance components remained almost the same as the random-intercept model.
- After adding correlated random slopes varying by schools on *YEARSTEA*, and *MATHKNOW*, the variances due to classrooms decreased for both. For the *YEARSTEA* model, the variances due to classrooms increased. After adding a correlated random slope for *MATHPREP*, the variance due to schools increased significantly, and had a -1 correlation with its random slope (i.e. singularity).

Question 6.

a. Try to add a random slope for each student level predictor (varying at the classroom level; one by one - not all together)

```
M11 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | classid) + (1|schoolid/classid), data = classroom)
summary(M11)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid          sex         3.310e-05  0.005753
##  classid:schoolid (Intercept) 9.387e+01  9.688824
##  schoolid         (Intercept) 1.695e+02 13.017987
##  Residual                     1.065e+03 32.633681
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63033    5.31211  275.37965 101.585  < 2e-16 ***
## housepov     -17.64878   13.21784  113.87028  -1.335    0.184
## yearstea       0.01129    0.14141  226.80606   0.080    0.936
## mathknow       1.35013    1.39167  234.49478   0.970    0.333
## mathprep      -0.27702    1.37582  205.26985  -0.201    0.841
## sex           -1.21421    2.09483 1022.41563  -0.580    0.562
## minority     -16.18672    3.02607  704.48078  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56153   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

```
anova(M4, M11, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M11: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M11:     ses + (0 + sex | classid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M11 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *SEX* variable, varying by classrooms, is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M12 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M12)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          minority       0.00   0.00
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38908 101.585  < 2e-16 ***
## housepov     -17.64848   13.21758  113.87764  -1.335    0.184
## yearstea       0.01129    0.14141  226.80896   0.080    0.936
## mathknow       1.35004    1.39168  234.49773   0.970    0.333
## mathprep      -0.27705    1.37583  205.27155  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47894  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56222   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M12, refit = F)
```

```
## Data: classroom
## Models:
```

```
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M12: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M12:     ses + (0 + minority | classid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M12 12 10754 10813 -5364.8    10730     0      1     0.9999
```

Response:

The addition of random slope on the *MINORITY* variable, varying by classrooms, is not significant (p-value = 1) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M13 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | classid) + (1|schoolid/classid), data = classroom)
summary(M13)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          ses           49.60   7.043
##  classid:schoolid (Intercept)   87.11   9.333
##  schoolid         (Intercept)  171.02  13.077
##  Residual                     1043.44  32.302
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.44882  -1.325    0.188
## yearstea       0.01103    0.14117  226.97682   0.078    0.938
## mathknow       1.36796    1.38563  229.40643   0.987    0.325
## mathprep      -0.27938    1.37171  204.89332  -0.204    0.839
## sex           -1.37733    2.09334 1022.81814  -0.658    0.511
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
```

```
## yearstea -0.259  0.070
## mathknow -0.082  0.058  0.029
## mathprep -0.631  0.040 -0.172  0.005
## sex      -0.190 -0.007  0.014  0.006 -0.005
## minority -0.321 -0.180  0.025  0.111  0.002 -0.011
## ses      -0.108  0.081 -0.026  0.002  0.050  0.020  0.145
```

```
anova(M4, M13, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M13: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M13:     ses + (0 + ses | classid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M13 12 10752 10812 -5364.0    10728 1.5969      1     0.2063
```

Response:

The addition of random slope on the *SES* variable, varying by classrooms, is not significant (p-value = 0.2063) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

Response:

It is a bad idea to include a classroom level variable with random slopes at the classroom level because its variance is already being accounted for in the null model by including the differential effects of *classid* clustering on the outcome *MATH1ST*.

> The plots below illustrate that each classroom level predictors (*MATHKNOW*, *MATHPREP*, *YEARSTEA*) are the same across each classroom:

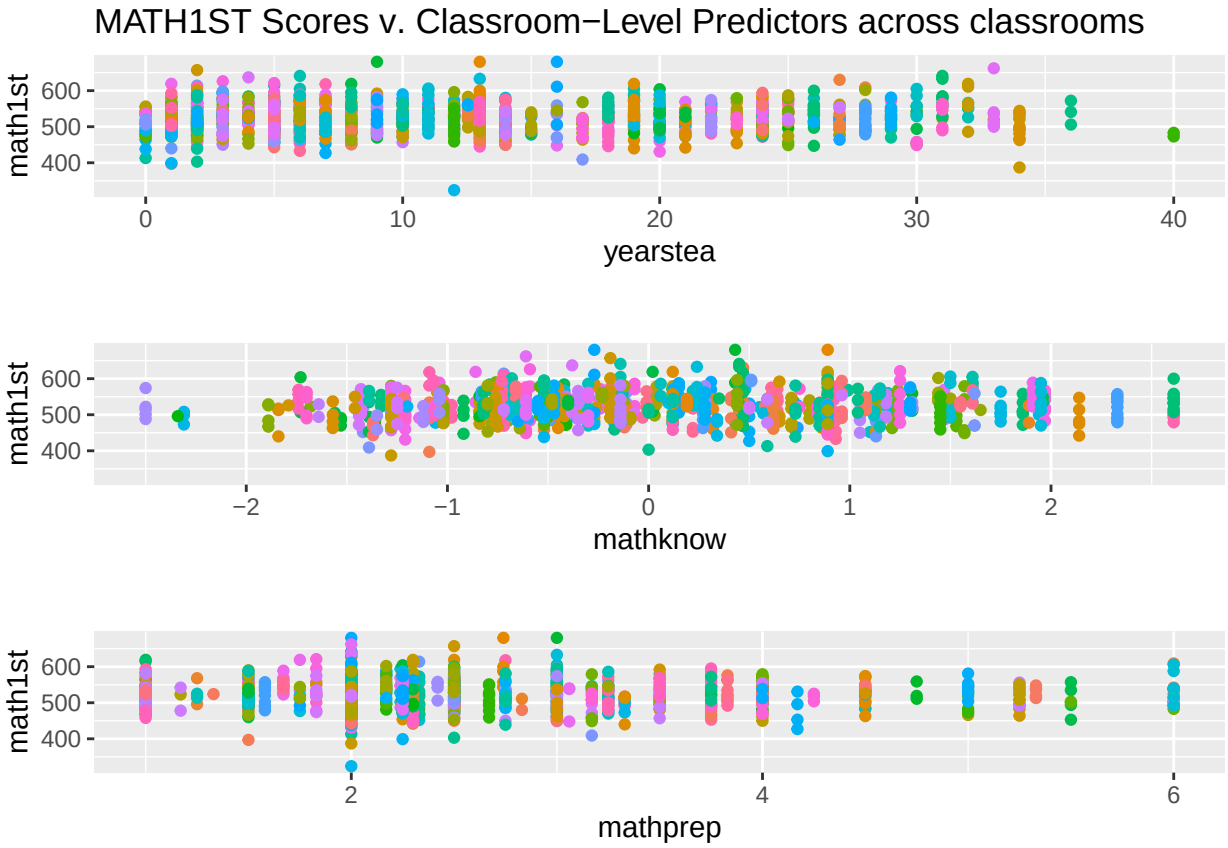
```
p2 <- ggplot(classroom, aes(x=yearstea, y=math1st, color = factor(classid))) +
  geom_point() +
  #geom_smooth(method=lm , color="red", fill="#69b3a2", se=TRUE)+
  theme(legend.position = "none")+
  ggtitle("MATH1ST Scores v. Classroom-Level Predictors across classrooms")

p3 <- ggplot(classroom, aes(x=mathknow, y=math1st, color = factor(classid))) +
  geom_point() +
  #geom_smooth(method=lm , color="red", fill="#69b3a2", se=TRUE)+
  theme(legend.position = "none")+
  ggtitle("")

p4 <- ggplot(classroom, aes(x=mathprep, y=math1st, color = factor(classid))) +
  geom_point( ) +
  #geom_smooth(method=lm ,color = "red" ,fill="#69b3a2", se=TRUE)+
  theme(legend.position = "none")+
  ggtitle("")

gridExtra::grid.arrange(p2, p3, p4)
```

```
## Warning: Removed 109 rows containing missing values (geom_point).
```



c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.

```
M14 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (sex | classid) + (1 | schoolid), data = classroom)
summary(M14)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7562 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  130.2   11.411
##           sex           31.5    5.612   -0.67
##  schoolid (Intercept)  169.9   13.035
##  Residual              1056.3   32.502
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
```

```
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.279  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
## yearstea     3.053e-03  1.416e-01  2.270e+02   0.022    0.983
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## mathprep    -3.460e-01  1.374e+00  2.014e+02  -0.252    0.801
## sex         -1.197e+00  2.123e+00  2.158e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.61e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.452
## yearstea -0.258  0.072
## mathknow -0.085  0.060  0.029
## mathprep -0.628  0.040 -0.174  0.005
## sex      -0.203 -0.005  0.015  0.003 -0.008
## minority -0.321 -0.178  0.024  0.116  0.003 -0.009
## ses      -0.123  0.083 -0.027 -0.005  0.054  0.020  0.164
```

```
anova(M4, M14, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M14: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M14:     ses + (sex | classid) + (1 | schoolid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M14 13 10755 10820 -5364.5    10729 0.5003      2     0.7787
```

Response:

The addition of random slope on the *SEX* variable, varying by classrooms and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.7787) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M15 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | classid) + (1 | schoolid), data = classroom)
summary(M15)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9036 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  225.4   15.01
##           minority     171.3   13.09    -0.82
##  schoolid (Intercept)  157.4   12.55
##  Residual             1045.3   32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73593    5.38021  270.70839 100.319  < 2e-16 ***
## housepov     -17.34685   12.91273  103.34823  -1.343    0.182
## yearstea      -0.01636    0.14285  234.25604  -0.115    0.909
## mathknow       1.45697    1.39354  234.05425   1.046    0.297
## mathprep      -0.13522    1.37018  203.97781  -0.099    0.921
## sex           -1.01014    2.08966 1015.73459  -0.483    0.629
## minority     -16.48615    3.21756  183.24221  -5.124 7.55e-07 ***
## ses            9.89350    1.54595 1062.82952   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.435
## yearstea -0.265  0.080
## mathknow -0.079  0.061  0.038
## mathprep -0.618  0.037 -0.171 -0.006
## sex      -0.188 -0.009  0.015  0.009 -0.005
## minority -0.368 -0.171  0.025  0.108 -0.004 -0.009
## ses      -0.117  0.085 -0.023  0.001  0.051  0.021  0.149
```

```
anova(M4, M15, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M15: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M15:     ses + (minority | classid) + (1 | schoolid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M15 13 10752 10817 -5363.2    10726 3.1967      2     0.2022
```

Response:

The addition of random slope on the *MINORITY* variable, varying by classrooms and allowing for correlation between random slope and random intercept, is not significant (p-value = 0.2022) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M16 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (ses | classid) + (1 | schoolid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -3.5e+02
```

```
summary(M16)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (ses | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10733.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7815 -0.6172 -0.0348  0.6046  3.8587
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)    0.00   0.00
##           ses           66.59   8.16     NaN
##  schoolid (Intercept)  198.66  14.09
##  Residual             1091.72  33.04
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.20226    5.04059  366.90683 106.972  < 2e-16 ***
## housepov     -18.33722   13.12647  110.81073  -1.397    0.165
## yearstea       0.03344    0.12846  827.45931   0.260    0.795
## mathknow       1.48810    1.26459  754.59810   1.177    0.240
## mathprep      -0.16719    1.23860  867.95858  -0.135    0.893
## sex           -1.44037    2.10650 1035.47989  -0.684    0.494
## minority     -15.99717    3.03333  689.35568  -5.274 1.79e-07 ***
## ses           10.32651    1.67733  184.92901   6.157 4.52e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.464
## yearstea -0.249  0.068
## mathknow -0.085  0.060  0.035
## mathprep -0.600  0.040 -0.177  0.003
## sex      -0.198 -0.007  0.015  0.001 -0.011
## minority -0.340 -0.182  0.023  0.103  0.004 -0.009
## ses      -0.107  0.079 -0.032  0.000  0.051  0.027  0.138
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M16, refit = F)
```

```
## Data: classroom
## Models:
```

```
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M16: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M16:     ses + (ses | classid) + (1 | schoolid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M16 13 10760 10825 -5366.9    10734     0      2          1
```

Response:

The addition of random slope on the *SES* variable, varying by classrooms and allowing for correlation between the random slope and random intercept, is not significant (p-value = 1) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

Question 7.

a. Try to add a random slope for each student level predictor varying at the school level:

```
M17 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | schoolid) + (1 | schoolid/classid), data = classroom)
summary(M17)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5556
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   96.08   9.802
##  schoolid         (Intercept)  161.63  12.713
##  schoolid.1       sex           35.85   5.987
##  Residual                      1054.36  32.471
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43513    5.30741  272.54817 101.638  < 2e-16 ***
## housepov     -16.77631   13.22883  112.39531  -1.268    0.207
## yearstea       0.01448    0.14163  226.44545   0.102    0.919
## mathknow       1.40068    1.39464  234.45910   1.004    0.316
## mathprep      -0.27193    1.38011  205.78600  -0.197    0.844
## sex           -1.33538    2.18749  138.10018  -0.610    0.543
## minority     -16.16537    3.02862  704.25875  -5.338 1.27e-07 ***
## ses            9.98475    1.54243 1058.28030   6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.259  0.070
## mathknow -0.081  0.055  0.028
## mathprep -0.633  0.036 -0.172  0.004
## sex      -0.179 -0.010  0.013  0.007 -0.004
## minority -0.320 -0.178  0.024  0.114  0.001 -0.015
## ses      -0.120  0.081 -0.029 -0.007  0.052  0.020  0.161
```

```
anova(M4, M17, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M17: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M17:     ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M17 12 10753 10813 -5364.4    10729 0.6137      1     0.4334
```

Response:

The addition of random slope on the *SEX* variable, varying by schools, is not significant (p-value = 0.4334) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M18 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -3.9e+00
```

```
summary(M18)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.388e+01  9.689369
##  schoolid         (Intercept) 1.694e+02 13.017176
##  schoolid.1       minority    1.777e-06  0.001333
##  Residual                     1.065e+03 32.633690
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
```

```
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63040    5.31208  275.39128 101.586  < 2e-16 ***
## housepov     -17.64850   13.21752  113.87888  -1.335    0.184
## yearstea       0.01129    0.14141  226.80855   0.080    0.936
## mathknow       1.35003    1.39168  234.49782   0.970    0.333
## mathprep      -0.27705    1.37582  205.27091  -0.201    0.841
## sex           -1.21419    2.09483 1022.42090  -0.580    0.562
## minority     -16.18676    3.02605  704.47696  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56207   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M18, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M18: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M18:     ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M18 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *MINORITY* variable, varying by schools, is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M19 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
summary(M19)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
```

```
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   88.56   9.411
##  schoolid         (Intercept)  168.00  12.961
##  schoolid.1       ses           72.50   8.515
##  Residual                     1035.11  32.173
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926  270.52802 102.124  < 2e-16 ***
## housepov     -16.94575   13.21161  112.81447  -1.283    0.202
## yearstea       0.03079    0.14052  223.94252   0.219    0.827
## mathknow       1.35586    1.38461  232.19737   0.979    0.328
## mathprep      -0.19799    1.35995  198.59551  -0.146    0.884
## sex           -1.40187    2.08169 1011.29089  -0.673    0.501
## minority     -16.52526    3.02191  700.07600  -5.468 6.32e-08 ***
## ses            9.78982    1.82216   79.01650   5.373 7.61e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.260  0.070
## mathknow -0.079  0.056  0.028
## mathprep -0.628  0.041 -0.172  0.002
## sex      -0.190 -0.007  0.018  0.006 -0.007
## minority -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses      -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
```

```
anova(M4, M19, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M19: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M19:     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M19 12 10749 10809 -5362.4    10725 4.6972      1    0.03021 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *SES* variable, varying by schools, **is significant** (p-value = 0.03021) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Retry the above, allowing the slopes to be correlated with the random intercepts.

```
M20 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (sex | schoolid) + (1 | schoolid:classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00526732
## (tol = 0.002, component 1)
```

```
summary(M20)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8050 -0.6094 -0.0223  0.5970  3.5528
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   97.29   9.863
##  schoolid         (Intercept)  206.02  14.353
##                   sex           83.73   9.151   -0.43
##  Residual                      1041.89  32.278
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.675  < 2e-16 ***
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314    0.191
## yearstea     6.921e-03  1.418e-01  2.277e+02   0.049    0.961
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203    0.839
## sex         -1.340e+00  2.300e+00  8.740e+01  -0.583    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.97e-08 ***
## ses          9.929e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.258  0.072
## mathknow -0.082  0.060  0.027
## mathprep -0.627  0.038 -0.172  0.004
## sex      -0.222 -0.003  0.014  0.006 -0.005
## minority -0.319 -0.178  0.024  0.114  0.004 -0.011
## ses      -0.121  0.083 -0.028 -0.006  0.053  0.018  0.163
## convergence code: 0
```

```
## Model failed to converge with max|grad| = 0.00526732 (tol = 0.002, component 1)
```

```
anova(M4, M20, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M20: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M20:     ses + (sex | schoolid) + (1 | schoolid:classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M20 13 10754 10818 -5363.8    10728 1.863      2      0.394
```

Response:

The addition of random slope on the *SEX* variable, varying by schools and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.394) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M21 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | schoolid) + (1 | schoolid:classid), data = classroom)
summary(M21)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.7    9.311
##  schoolid         (Intercept)  381.2   19.524
##                   minority     343.2   18.525   -0.83
##  Residual                      1039.4   32.240
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.49369    5.65513  173.09179  95.399  < 2e-16 ***
## housepov     -16.06251   12.57477   99.99134  -1.277    0.204
## yearstea      -0.00437    0.13765  217.17884  -0.032    0.975
## mathknow       1.63216    1.35929  224.78144   1.201    0.231
## mathprep      -0.29178    1.33537  198.06922  -0.218    0.827
## sex           -0.86278    2.08382 1021.81437  -0.414    0.679
## minority     -16.37547    3.89604   58.24604  -4.203 9.17e-05 ***
## ses            9.43095    1.54335 1063.13485   6.111 1.39e-09 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.394
## yearstea -0.253  0.091
## mathknow -0.078  0.061  0.024
## mathprep -0.576  0.037 -0.167 -0.002
## sex      -0.172 -0.013  0.014  0.010 -0.005
## minority -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses      -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
```

```
anova(M4, M21, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M21: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M21:     ses + (minority | schoolid) + (1 | schoolid:classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M21 13 10744 10808 -5358.8    10718 11.967      2    0.00252 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *MINORITY* variable, varying by schools and allowing for correlation between the random slope and random intercept, **is significant** (p-value= 0.00252) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M22 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (ses | schoolid) + (1 | schoolid:classid), data = classroom)
summary(M22)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (ses | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5647 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.62   9.307
##  schoolid         (Intercept)  171.12  13.081
##                   ses           73.35   8.565   0.19
##  Residual                      1035.89  32.185
```

```
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.72268    5.27645  271.16175 102.099  < 2e-16 ***
## housepov     -15.89853   13.15319  111.73535  -1.209    0.229
## yearstea       0.03616    0.14002  220.43198   0.258    0.796
## mathknow       1.26005    1.38204  230.90960   0.912    0.363
## mathprep      -0.21707    1.35647  197.11400  -0.160    0.873
## sex           -1.40428    2.08074 1011.40155  -0.675    0.500
## minority     -16.26714    3.03575  668.89455  -5.359 1.16e-07 ***
## ses            9.72644    1.82981   78.36254   5.316 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.259  0.073
## mathknow -0.077  0.057  0.028
## mathprep -0.627  0.039 -0.172  0.001
## sex      -0.188 -0.009  0.017  0.005 -0.008
## minority -0.325 -0.183  0.021  0.108  0.002 -0.011
## ses      -0.062  0.070 -0.021  0.007  0.045  0.018  0.117
```

```
anova(M4, M22, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M22: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M22:     ses + (ses | schoolid) + (1 | schoolid:classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M22 13 10750 10815 -5362.2    10724 5.1385      2    0.07659 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *SES* variable, varying by schools and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.07659) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

Response:

c. Report anything unusual about the variance components (changes that are unexpected)

Response:

- After adding an uncorrelated random slope on *MINORITY* varying by schools, the variance due to the random slope is approximately 0. For the other variables, *SEX* and *SES*, the variances for the slopes are positive, indiciating that these random slopes explain some of the variation between schools.
- After adding a correlated random slope on *MINORITY* varying by schools, the variances for the random slope and intercept on schools increased significantly (343.2 and 381.2, respectively), and the correlation

is -0.83. For the other variables, *SEX* and *SES*, the variances remained similar to the uncorrelated version.

Question 8.

a. Take the two predictors that had significant random slopes, in the forms in which they worked (indep. or correlated) and add both to the model, and test for need of one conditional on needing the other.

```
# Fit models and run LRT tests
Mslope1 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
Mslope2 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | schoolid) + (1|schoolid:classid), data = classroom)
Mslope3 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid)+ (minority | schoolid) + (1|schoolid:classid)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00333241
## (tol = 0.002, component 1)
```

```
anova(Mslope3, Mslope1, refit = F)
```

```
## Data: classroom
## Models:
## Mslope1: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope1:     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:     schoolid:classid)
##         Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## Mslope1 12 10749 10809 -5362.4    10725
## Mslope3 14 10740 10810 -5356.2    10712 12.39      2    0.00204 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(Mslope3, Mslope2, refit = F)
```

```
## Data: classroom
## Models:
## Mslope2: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope2:     ses + (minority | schoolid) + (1 | schoolid:classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:     schoolid:classid)
##         Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## Mslope2 13 10744 10808 -5358.8    10718
## Mslope3 14 10740 10810 -5356.2    10712  5.12      1    0.02365 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the random slopes on the two predictors, *SES* and *MINORITY*, with *SES* random slope not correlated with *SES* random intercept and *MINORITY* random slope correlated with *MINORITY* random intercept, is significant (p-value = 0.00204, p-value = 0.02365, respectively) at the alpha 0.05 significance

level, and are needed based on the LRT comparing the need of one random slope, conditional on needing the other.

b. Is the more complex model (with both random slopes in it) justified?

```
# Insert code to compare models
anova(M4, Mslope3, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:     schoolid:classid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4      11 10752 10806 -5364.8    10730
## Mslope3 14 10740 10810 -5356.2    10712 17.087      3  0.0006782 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Reponse:

The addition of the random slopes on the two predictors, *SES* and *MINORITY*, with *SES* random slope not correlated with *SES* random intercept and *MINORITY* random slope correlated with *MINORITY* random intercept, is significant (p-value = 0.0006782) at the alpha 0.05 significance level, meaning that the more complex model with both random slopes is justified based on the LRT comparing the complex model with the random intercepts only model.

c. WRITE OUT THIS MODEL in your preferred notation

The model is:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + b_5 SEX_{ijk} + (b_6 + \zeta_{6k}) MINORITY_{ijk} + (b_7 + \zeta_{7k}) SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with $\zeta_k \sim N(0, \sigma^2_\zeta)$, $\eta_{jk} \sim N(0, \sigma^2_\eta)$, $\zeta_{6k} \sim N(0, \sigma^2_{\zeta_6})$, $\zeta_{7k} \sim N(0, \sigma^2_{\zeta_7})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, independent of each other,

$$corr(\zeta_{0k}, \zeta_{6k}) = \rho_{\zeta_0, \zeta_6},$$

and $k =$ schools, $j =$ classrooms and $i =$ students

Question 9.

a. For UMM, write down: V_S, V_C, V_E for the three variance components (simply the estimates)

```
# If you want to look at your UMM insert code here or you can just do this in line
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
##    Data: classroom
##
```

```
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                      1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
V_S = 280.69

V_C = 85.47

V_E = 1146.79
```

b. For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

```
# If you want to look at your model insert code here or you can just do this in line
summary(M4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
```

```
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp yearst ses    minrty
## housepov -0.451
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

V_S = 169.45

V_C = 93.89

V_E = 1064.96

c. By what fraction did these each decrease with the new predictors in the model?

V_S: 39.63% decrease

V_C: 9.85% increase

V_E: 7.14% decrease

Question 10. Now consider the model with a random slope in ses.

a. What are: V_C, V_S(ses=0), V_E ?

```
# If you want to look at your model insert code here or you can just do this in line
summary(Mslope1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
```

```
##  classid.schoolid (Intercept)   88.56   9.411
##  schoolid         (Intercept)  168.00  12.961
##  schoolid.1       ses           72.50   8.515
##  Residual                     1035.11  32.173
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926  270.52802 102.124  < 2e-16 ***
## housepov     -16.94575   13.21161  112.81447  -1.283    0.202
## yearstea       0.03079    0.14052  223.94252   0.219    0.827
## mathknow       1.35586    1.38461  232.19737   0.979    0.328
## mathprep      -0.19799    1.35995  198.59551  -0.146    0.884
## sex           -1.40187    2.08169 1011.29089  -0.673    0.501
## minority     -16.52526    3.02191  700.07600  -5.468 6.32e-08 ***
## ses            9.78982    1.82216   79.01650   5.373 7.61e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.260  0.070
## mathknow -0.079  0.056  0.028
## mathprep -0.628  0.041 -0.172  0.002
## sex      -0.190 -0.007  0.018  0.006 -0.007
## minority -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses      -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
```

```
V_S(ses=0) = 168.00

V_C = 88.56

V_E = 1035.11
```

b. What are: V_S(ses=-0.50), V_S(ses=+0.5) ?

```
V_S(ses=0.5) = 168 + (0.5^2)*(72.50) = 186.125

V_S(ses=-0.5) = 168 + (-0.5^2)*(72.50) = 186.125
```

Question 11.

Now consider the model with a random slope in minority.

a. What are: V_C, V_S(minority=0), V_E ?

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
summary(Mslope2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | schoolid) + (1 | schoolid:classid)
##    Data: classroom
```

```
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.7    9.311
##  schoolid         (Intercept)  381.2   19.524
##                   minority     343.2   18.525   -0.83
##  Residual                      1039.4   32.240
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.49369    5.65513  173.09179  95.399  < 2e-16 ***
## housepov     -16.06251   12.57477   99.99134  -1.277    0.204
## yearstea      -0.00437    0.13765  217.17884  -0.032    0.975
## mathknow       1.63216    1.35929  224.78144   1.201    0.231
## mathprep      -0.29178    1.33537  198.06922  -0.218    0.827
## sex           -0.86278    2.08382 1021.81437  -0.414    0.679
## minority     -16.37547    3.89604   58.24604  -4.203 9.17e-05 ***
## ses            9.43095    1.54335 1063.13485   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.394
## yearstea -0.253  0.091
## mathknow -0.078  0.061  0.024
## mathprep -0.576  0.037 -0.167 -0.002
## sex      -0.172 -0.013  0.014  0.010 -0.005
## minority -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses      -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
```

V_S(minority=0) = 381.2

V_C = 86.70

V_E = 1039.40

b. What are: V_S(minority=0.25), V_S(minority=+0.50), V_S(minority=+0.75) ?

V_S(minority=0.25) = 252.55

```
# V_S(minority = 0.25)
# Insert code if you want to do the calculations in R

  # Calculate variance using the formula:
  # V_S(minority = 0.25) = Var(Zeta_0) + Minority^2*Var(Zeta_1) + 2*Minority*CoV(Zeta_0,Zeta_1)

(var_minority.25 = 381.2 + (0.25^2)*(343.2) + 2*0.25*(18.525*19.524*-0.83))
```

```
## [1] 252.5519
```

V_S(minority=0.5) = 166.80

```
# V_S(minority = 0.50)
# Insert code if you want to do the calculations in R

    # Calculate variance using the formula:
    # V_S(minority = 0.50) = Var(Zeta_0) + Minority^2*Var(Zeta_1) + 2*Minority*CoV(Zeta_0,Zeta_1)

(var_minority.50 = 381.2 + (0.50^2)*(343.2) + 2*0.50*(18.525*19.524*-0.83))
```

```
## [1] 166.8039
```

V_S(minority=0.75) = 123.96

```
# V_S(minority = 0.75)
# Insert code if you want to do the calculations in R

# Calculate variance using the formula:
# V_S(minority = 0.75) = Var(Zeta_0) + Minority^2*Var(Zeta_1) + 2*Minority*CoV(Zeta_0,Zeta_1)

(var_minority.75 = 381.2 + (0.75^2)*(343.2) + 2*0.75*(18.525*19.524*-0.83))
```

```
## [1] 123.9558
```

Question 12.

Now consider the model with a random slope in ses & minority.

a. What are: V_C, V_S(minority=0,ses=0), V_E ? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
summary(Mslope3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
##     schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6525 -0.6251 -0.0339  0.6048  3.6961
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid.classid (Intercept)   80.62   8.979
##  schoolid         (Intercept)  404.52  20.113
##                   minority     335.67  18.321   -0.84
##  schoolid.1       ses           74.98   8.659
##  Residual                      1009.73  31.776
```

```
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05254    5.66462  165.75707  95.161  < 2e-16 ***
## housepov     -15.32101   12.49397   99.25400  -1.226    0.223
## yearstea       0.02105    0.13657  213.65286   0.154    0.878
## mathknow       1.67490    1.35000  221.33073   1.241    0.216
## mathprep      -0.23545    1.31729  191.22021  -0.179    0.858
## sex           -1.03898    2.06951 1010.41104  -0.502    0.616
## minority     -16.72828    3.90662   55.39944  -4.282 7.42e-05 ***
## ses            9.19642    1.82287   82.48705   5.045 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.395
## yearstea -0.254  0.093
## mathknow -0.072  0.060  0.024
## mathprep -0.568  0.040 -0.166 -0.004
## sex      -0.170 -0.014  0.017  0.010 -0.005
## minority -0.509 -0.149  0.027  0.092 -0.003 -0.013
## ses      -0.080  0.083 -0.011  0.006  0.041  0.020  0.087
## convergence code: 0
## Model failed to converge with max|grad| = 0.00333241 (tol = 0.002, component 1)
```

```
V_S(sex=0, minority=0) = 404.52
```

```
V_C = 80.62
```

```
V_E = 1009.73
```

b. In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

```
Response: [-8.979, 8.979]
```

c. Can we make a similar statement about ζ_{0k}?

```
Response:
```

Yes, because we assume that $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$. Therefore the range would be [-20.113, 20.113]

d. If you had a large value for η_{0jk}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

```
Response:
```

The two random slopes can take on any value because they are not related to classrooms (i.e. independent and not correlated).

e. If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

```
Response:
```

We would expect any value for the SES random slope term since there is no correlation between the random slope and the random intercept, and a small value for the MINORITY random slope due to a negative correlation with the intercept (-0.84).