MLM Final Project Part 1

April 30 2020

Team Members and division of work:

Question 0.

Load classroom.csv and create MATH1ST (fit all models using REML, use lmerTest::lmer)

```
# Insert code to load data and create math1st variable
classroom <- read_csv("/Users/dennishilgendorf/Desktop/NYU MPH/Spring 2020 (Y2)/MLM/1/classroom.csv")
```

```
## Parsed with column specification:
## cols(
##   sex = col_double(),
##   minority = col_double(),
##   mathkind = col_double(),
##   mathgain = col_double(),
##   ses = col_double(),
##   yearstea = col_double(),
##   mathknow = col_double(),
##   housepov = col_double(),
##   mathprep = col_double(),
##   classid = col_double(),
##   schoolid = col_double(),
##   childid = col_double()
## )
```

```
classroom <- classroom %>% mutate(math1st = mathkind + mathgain)

save(classroom, file = "/Users/dennishilgendorf/Desktop/NYU MPH/Spring 2020 (Y2)/MLM/1/classroom.RData")
```

Question 1.

Estimate UMM model with random intercepts for both schools and classrooms.

```
# Insert code to fit model and print summary

M1_UMM <- lmerTest::lmer(math1st ~ (1 | schoolid/classid), data=classroom)
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.407   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a. Report the ICC for schools and classrooms:

```
# Insert code if you'd like but you can also do this inline
```

Response:

$\hat{\sigma}^2_\eta = 85.47$ $\hat{\sigma}^2_\zeta = 280.69$ $\hat{\sigma}^2_\varepsilon = 1146.79$

ICC for classrooms $= \hat{\sigma}^2_\eta/(\hat{\sigma}^2_\eta + \hat{\sigma}^2_\zeta + \hat{\sigma}^2_\varepsilon) = (85.47)/(85.47 + 280.69 + 1146.79) = 5.65\%$ is the proportion of variance explained by between classroom differences.

ICC for schools $= \hat{\sigma}^2_\zeta/(\hat{\sigma}^2_\eta + \hat{\sigma}^2_\zeta + \hat{\sigma}^2_\varepsilon) = (280.69)/(85.47 + 280.69 + 1146.79) = 18.55\%$ is the proportion of variance explained by between school differences.

b. Write out the model:

Model 1 Equation:

$MATH1ST_{ijk} = b_0 + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model with i representing students, j representing classrooms and k representing schools.

Question 2.

Add all school-level predictors:

Model 2 Equation:

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model with i representing students, j representing classrooms and k representing schools.

```
# Insert code to fit model and print summary

M2 <- lmerTest::lmer(math1st ~ housepov + (1 | schoolid/classid), data=classroom)
summary(M2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
```

```
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.95  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.809 159.024   <2e-16 ***
## housepov     -45.783     14.236 111.063  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

a. Report if the additional predictors are justified:

```
# Insert code to compare models
anova(M1_UMM, M2, refit = F)
```

```
## Data: classroom
## Models:
## M1_UMM: math1st ~ (1 | schoolid/classid)
## M2: math1st ~ housepov + (1 | schoolid/classid)
##        npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M1_UMM    4 11953 11973 -5972.3    11945
## M2        5 11937 11963 -5963.7    11927 17.186  1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
rand(M2)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ housepov + (1 | classid:schoolid) + (1 | schoolid)
##                         npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                     5 -5963.7 11937
## (1 | classid:schoolid)     4 -5966.7 11941  6.014  1     0.0142 *
## (1 | schoolid)             4 -5988.9 11986 50.359  1   1.28e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The additional school level covariate of HOUSEPOV is justified because the p-value ($p < 0.05$) of the chi square ANOVA test is significant at the 0.05 alpha significance level suggesting the benefit of adding school

level predictors showing significant difference in deviance from log likelihood of the model comparisons. The school level predictor of HOUSEPOV is also significant at the alpha 0.05 significance level (p-value = 0.0017).

b. Report the change to school variance:

`Response:`

Change in school variance ($\hat{\sigma}^2_\zeta$) changed from 280.69 in Model 1 (UMM null model) to 250.93 indicating that adding the covariate, HOUSEPOV, accounted for variance at the school level.

Change in $\sigma^2_\zeta = 250.93 - 280.69 = -29.76$

Question 3: Add all class-level predictors

`Model 3 Equation:`

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 MATHKNOW_{jk} + b_3 MATHPREP_{jk} + b_4 YEARSTEA_{jk} + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model with i representing students, j representing classrooms and k representing schools.

```
# Insert code to fit model and print summary

M3 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid), data=
summary(M3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62117   14.08834 109.83230  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) housepv mthknw mthprp
```

```
## housepov -0.568
## mathknow -0.052  0.082
## mathprep -0.666  0.032  0.004
## yearstea -0.264  0.077  0.030 -0.175
```

a. Report if adding the predictors is justified:

```
linearHypothesis(M3, c("mathknow", "mathprep", "yearstea"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## mathknow = 0
## mathprep = 0
## yearstea = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 3.4804     0.3233
```

Response:

The addition of classroom level covariates of MATHKNOW MATHPREP and YEARSTEA is not justified because the p-value ($p > 0.3233$) of the WALD test is non-significant at the 0.05 alpha significance level suggesting the benefit of adding classroom level predictors jointly is non-significant in comparison to the previous model containing student level predictors.

b. Report changes in class-level variance and individual variance:

Response:

Change in classroom level variance ($\hat{\sigma}^2_\eta$) changed from 85.47 in Model 1 (UMM null model) to 82.36 in Model 2 and finally to 94.36 in Model 3 indicating that adding the classroom level covariates increases classroom level variance.

Change in individual level variance ($\hat{\sigma}^2_\varepsilon$) changed from 1146.79 in Model 1 (UMM null model) to 1146.96 in Model 2 and finally to 1136.43 in Model 3 indicating that adding the classroom level covariates reduced individual variance.

Change in $\sigma^2_\eta = 94.36 - 82.36 = 12.00$ Change in $\sigma^2_\epsilon = 1136.43 - 1146.96 = -10.53$

c. Give a potential reason to explain why individual variance but not class variance is reduced:

Response:

Adding the classroom-level predictors shows a potential misspecification of the model. The classroom level predictors can make it difficult to estimate the individual level variance (i.e overstated) due to individual outliers in classrooms that have a very small amount of students. > GO THROUGH RECORDING

Question 4.

Add all student-level predictors excepting mathgain and mathkind:

```r
# Insert code to fit model and print summary
M4 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority + sex + (1 |
summary(M4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp yearst ses    minrty
## housepov -0.451
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

a. Report if the block of predictors is justified:

```r
# Insert code to compare models

linearHypothesis(M4, c("sex","minority", "ses"))
```

```
## Linear hypothesis test
```

```
##
## Hypothesis:
## sex = 0
## minority = 0
## ses = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 85.055  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of student level covariates of SES MINORITY and SEX is justified because the p-value ($p < 0.05$) of the WALD test is significant at the 0.05 alpha significance level suggesting the benefit of adding student level predictors jointly is significant in comparison to the previous model containing school and classroom-level predictors.

b. Report change in all variance components

Response:

Change in classroom level variance ($\hat{\sigma}^2_\eta$) changed from 85.47 in Model 1 (UMM null model) to 82.36 in Model 2 and finally to 94.36 in Model 3 to 93.89 in Model 4 with a marginal decrease in classroom level variance.

Change in school variance ($\hat{\sigma}^2_\zeta$) changed from 280.69 in Model 1 (UMM null model) to 250.93 in Model 2 to 223.31 in Model 3 and finally to 169.45 in Model 4 with a decrease in school level variance observed after adding student level covariates in Model 4

Change in individual student level variance ($\hat{\sigma}^2_\varepsilon$) changed from 1146.79 in Model 1 (UMM null model) to 1146.96 in Model 2 to 1136.43 in Model 3 and finally to 1064.96 in Model 4 indicating that student level variance decreased after adding student level covariates in Model 4.

Change in $\sigma^2_\zeta = 169.45 - 223.31 = -53.86$ Change in $\sigma^2_\eta = 93.89 - 94.36 = -0.47$ Change in $\sigma^2_\epsilon = 1064.96 - 1136.43 = -71.47$

c. Give a potential reason as to why the school variance drops from the last model:

Response:

The student-level predictors explain some variance at the school level. SES & Minority Status, and SEX composition of children vary between different schools and may impact math scores. For example, some schools located in poorer areas with a different demographic of students will have much different individual math scores than those in more affluent areas.

d. Write this model out:

Model 4 Equation:

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 MATHKNOW_{jk} + b_3 MATHPREP_{jk} + b_4 YEARSTEA_{jk} + b_5 SES_{ijk} + b_6 MINORITY_{ijk} + b_7 SEX_{ijk} + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model with i representing students, j representing classrooms and k representing schools.

Question 5.

a. Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately - not all together)

b. Report the models and their fit.

```
M5 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + yearstea | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0080545 (tol = 0.002, component 1)
```

```
summary(M5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.247e+01  9.6159
##  schoolid         (Intercept) 1.684e+02 12.9758
##  schoolid.1       yearstea    1.008e-02  0.1004
##  Residual                     1.065e+03 32.6361
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780  266.47953 101.662  < 2e-16 ***
## housepov     -17.72082   13.21686  113.58577  -1.341    0.183
## yearstea       0.01128    0.14192  122.87741   0.079    0.937
## mathknow       1.33106    1.39155  234.33195   0.957    0.340
## mathprep      -0.26584    1.37588  204.90504  -0.193    0.847
## sex           -1.21060    2.09480 1022.21558  -0.578    0.563
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## ses           10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.450
## yearstea -0.258  0.070
## mathknow -0.082  0.057  0.028
## mathprep -0.632  0.037 -0.172  0.003
## sex      -0.191 -0.007  0.015  0.006 -0.006
```

```
## minority -0.320 -0.179  0.023  0.115  0.001 -0.010
## ses      -0.121  0.082 -0.027 -0.007  0.053  0.020  0.162
## convergence code: 0
## Model failed to converge with max|grad| = 0.0080545 (tol = 0.002, component 1)
```

```
anova(M4,M5, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M5: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M5:     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##    npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M5   12 10754 10813 -5364.8    10730 0.007  1     0.9336
```

Response:

The addition of random slope on the *yearstea* variable is not significant (p-value = 0.9336) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M6 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathknow | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.389e+01  9.689654
##  schoolid         (Intercept) 1.694e+02 13.017245
##  schoolid.1       mathknow    2.323e-07  0.000482
##  Residual                     1.065e+03 32.633630
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38873 101.585  < 2e-16 ***
## housepov     -17.64848   13.21759  113.87742  -1.335    0.184
## yearstea       0.01129    0.14141  226.80898   0.080    0.936
## mathknow       1.35004    1.39169  234.49763   0.970    0.333
```

```
## mathprep      -0.27705    1.37583  205.27161  -0.201    0.841
## sex           -1.21419    2.09483 1022.42143  -0.580    0.562
## minority     -16.18678    3.02605  704.47917  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M6, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M6: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M6:     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##    npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M6   12 10754 10813 -5364.8    10730     0  1          1
```

Response:

The addition of random slope on the *mathknow* variable is not significant (p-value = 1.00) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M7 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathprep | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M7)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
```

```
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.388e+01 9.689e+00
##  schoolid         (Intercept) 1.694e+02 1.302e+01
##  schoolid.1       mathprep    2.171e-07 4.659e-04
##  Residual                     1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39223 101.586  < 2e-16 ***
## housepov     -17.64851   13.21749  113.87941  -1.335    0.184
## yearstea       0.01129    0.14141  226.80838   0.080    0.936
## mathknow       1.35003    1.39167  234.49786   0.970    0.333
## mathprep      -0.27705    1.37582  205.27063  -0.201    0.841
## sex           -1.21419    2.09483 1022.42070  -0.580    0.562
## minority     -16.18676    3.02605  704.47629  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56201   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M7, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M7: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M7:     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##    npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M7   12 10754 10813 -5364.8    10730     0  1          1
```

Response:

The addition of random slope on the *mathprep* variable is not significant (p-value = 1.00) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

c. Why is it a bad idea to include a random slope on the housepov effect?

Response:

It is a bad idea to include a random slope on the housepov effect because its variance is already being accounted for in the null model by including the differential effects of schoolid on the outcome math1st.

INSERT JEFFREY PLOT

d. Retry the above models, allowing the slopes to be correlated with the random intercepts (still one by one):

```
M8 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (yearstea | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge: degenerate Hessian with 1 negative eigenvalues
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -7.9e-02
```

```
summary(M8)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (yearstea | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7464 -0.6037 -0.0291  0.6038  3.8452
##
## Random effects:
##  Groups           Name        Variance  Std.Dev. Corr
##  classid.schoolid (Intercept)   37.8871  6.1553
##  schoolid         (Intercept)  114.0528 10.6796
##  schoolid.1       (Intercept)  252.9156 15.9033
##                   yearstea       0.5531  0.7437  -0.94
##  Residual                      1066.3788 32.6555
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95153    5.48972  222.41069  98.175  < 2e-16 ***
## housepov     -17.13430   13.46400  119.54537  -1.273    0.206
## yearstea       0.02204    0.15769   75.69579   0.140    0.889
## mathknow       1.04750    1.34382  209.71207   0.779    0.437
## mathprep       0.05100    1.34550  190.82638   0.038    0.970
## sex           -1.33602    2.08769 1024.47431  -0.640    0.522
## minority     -16.44710    2.99670  669.29320  -5.488 5.76e-08 ***
## ses           10.15068    1.53871 1062.64465   6.597 6.61e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.455
```

```
## yearstea -0.370  0.084
## mathknow -0.085  0.049  0.012
## mathprep -0.606  0.050 -0.139  0.014
## sex      -0.184 -0.004  0.009  0.008 -0.004
## minority -0.305 -0.169  0.032  0.122 -0.007 -0.012
## ses      -0.119  0.079 -0.019 -0.001  0.049  0.022  0.168
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 1 negative eigenvalues
```

```
anova(M4, M8, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M8: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M8:     ses + (yearstea | schoolid) + (1 | schoolid/classid)
##    npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M8   14 10752 10822 -5361.8    10724 5.8254  3     0.1204
```

Response:

The addition of random slope on the *yearstea* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value = 0.1204) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M9 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (mathknow| schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M9)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (mathknow | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5970  3.6602
##
## Random effects:
##  Groups           Name        Variance  Std.Dev. Corr
##  classid.schoolid (Intercept) 9.393e+01  9.69189
##  schoolid         (Intercept) 1.100e+02 10.48988
##  schoolid.1       (Intercept) 5.930e+01  7.70080
##                   mathknow    1.738e-03  0.04169 1.00
##  Residual                     1.065e+03 32.63382
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63882    5.31214  275.37813 101.586  < 2e-16 ***
## housepov     -17.64251   13.21357  104.31444  -1.335    0.185
## yearstea       0.01116    0.14142  226.84643   0.079    0.937
## mathknow       1.35388    1.39199  214.47020   0.973    0.332
## mathprep      -0.27746    1.37599  201.40762  -0.202    0.840
## sex           -1.21344    2.09484 1021.81598  -0.579    0.563
## minority     -16.19268    3.02609  703.85005  -5.351 1.18e-07 ***
## ses           10.04833    1.54488 1062.25217   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.082  0.057  0.029
## mathprep -0.631  0.038 -0.173  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M9, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M9: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M9:     ses + (mathknow | schoolid) + (1 | schoolid/classid)
##    npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M9   14 10758 10827 -5364.8    10730 3e-04  3          1
```

Response:

The addition of random slope on the *mathknow* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value = 1.00) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M10 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (mathprep| schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (mathprep | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
```

```
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
##  Groups           Name        Variance  Std.Dev. Corr
##  classid.schoolid (Intercept) 7.846e+01  8.85755
##  schoolid         (Intercept) 7.465e-05  0.00864
##  schoolid.1       (Intercept) 5.528e+02 23.51221
##                   mathprep    1.589e+01  3.98630 -1.00
##  Residual                     1.064e+03 32.62311
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60842    5.60816  159.87872  96.040  < 2e-16 ***
## housepov     -14.01234   12.88673  116.06322  -1.087    0.279
## yearstea      -0.02587    0.13949  223.50025  -0.185    0.853
## mathknow       1.29879    1.37193  229.68130   0.947    0.345
## mathprep       0.04075    1.34846  139.01480   0.030    0.976
## sex           -1.16758    2.08697 1023.15099  -0.559    0.576
## minority     -16.46429    2.99523  663.67010  -5.497 5.52e-08 ***
## ses           10.14167    1.53961 1060.93393   6.587 7.04e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.461
## yearstea -0.260  0.089
## mathknow -0.071  0.027  0.049
## mathprep -0.692  0.107 -0.155  0.012
## sex      -0.183  0.003  0.023  0.002 -0.008
## minority -0.275 -0.187  0.025  0.107 -0.035 -0.013
## ses      -0.121  0.095 -0.033 -0.001  0.061  0.024  0.161
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M10, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M10: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M10:     ses + (mathprep | schoolid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M10   14 10753 10822 -5362.3    10725 4.8144  3     0.1859
```

Response:

The addition of random slope on the *mathprep* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value=0.1859) (at the 0.05 level of significance), according to the

ANOVA LRT comparing the model with and without the random slope addition.

e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occured (hint: what did you add to the model?).

Response:

> Yearsteaching is statististically significantly varies across schools (alpha = 0.05) as it decreases variance at the classroom level and increases variance at the school level.

Question 6.

a. Try to add a random slope for each student level predictor (varying at the classroom level; one by one - not all together)

```
M11 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | classid) + (1|schoolid/classid), data = classroom)
summary(M11)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid          sex         2.698e-05  0.005195
##  classid:schoolid (Intercept) 9.388e+01  9.689264
##  schoolid         (Intercept) 1.695e+02 13.017397
##  Residual                     1.065e+03 32.633680
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63039    5.31209  275.38791 101.585  < 2e-16 ***
## housepov     -17.64857   13.21761  113.87644  -1.335    0.184
## yearstea       0.01129    0.14141  226.80795   0.080    0.936
## mathknow       1.35006    1.39168  234.49700   0.970    0.333
## mathprep      -0.27704    1.37582  205.27072  -0.201    0.841
## sex           -1.21420    2.09483 1022.41650  -0.580    0.562
## minority     -16.18675    3.02605  704.47832  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56195   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

```
anova(M4, M11, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M11: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M11:     ses + (0 + sex | classid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M11   12 10754 10813 -5364.8    10730     0  1          1
```

Response:

The addition of random slope on the *sex* variable, varying by classrooms, is not significant (p-value = 1.00) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M12 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M12)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          minority       0.00   0.00
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38908 101.585  < 2e-16 ***
## housepov     -17.64848   13.21758  113.87764  -1.335    0.184
## yearstea       0.01129    0.14141  226.80896   0.080    0.936
## mathknow       1.35004    1.39168  234.49773   0.970    0.333
## mathprep      -0.27705    1.37583  205.27155  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47894  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56222   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M12, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M12: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M12:     ses + (0 + minority | classid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M12   12 10754 10813 -5364.8    10730     0  1     0.9999
```

Response:

The addition of random slope on the *minority* variable, varying by classrooms, is not significant (p-value = 0.9999) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M13 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | classid) + (1|schoolid/classid), data = classroom)
summary(M13)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          ses           49.60   7.043
##  classid:schoolid (Intercept)   87.11   9.333
##  schoolid         (Intercept)  171.02  13.077
##  Residual                     1043.44  32.302
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.44882  -1.325    0.188
## yearstea       0.01103    0.14117  226.97682   0.078    0.938
## mathknow       1.36796    1.38563  229.40643   0.987    0.325
## mathprep      -0.27938    1.37171  204.89332  -0.204    0.839
## sex           -1.37733    2.09334 1022.81814  -0.658    0.511
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.070
## mathknow -0.082  0.058  0.029
## mathprep -0.631  0.040 -0.172  0.005
## sex      -0.190 -0.007  0.014  0.006 -0.005
## minority -0.321 -0.180  0.025  0.111  0.002 -0.011
## ses      -0.108  0.081 -0.026  0.002  0.050  0.020  0.145
```

```
anova(M4, M13, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M13: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M13:     ses + (0 + ses | classid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M13   12 10752 10812 -5364.0    10728 1.5969  1     0.2063
```

Response:

The addition of random slope on the *ses* variable, varying by classrooms, is not significant (p-value = 0.2063) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

Response:

It is a bad idea to include a classroom level variable with random slopes at the classroom level because its variance is already being accounted for in the null model by including the differential effects of classid clustering on the outcome math1st.

Include plot JEFFREY

c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.

```
M14 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (sex | classid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge: degenerate Hessian with 1 negative eigenvalues
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -5.6e-03
```

```
summary(M14)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid          (Intercept)  104.32  10.214
##                   sex           31.35   5.599   -0.75
##  classid:schoolid (Intercept)   25.75   5.075
##  schoolid         (Intercept)  169.85  13.033
##  Residual                     1056.42  32.503
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
## yearstea     3.088e-03  1.416e-01  2.270e+02   0.022    0.983
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## mathprep    -3.459e-01  1.374e+00  2.014e+02  -0.252    0.801
```

```
## sex         -1.197e+00  2.122e+00  2.160e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.452
## yearstea -0.258  0.072
## mathknow -0.085  0.060  0.029
## mathprep -0.628  0.040 -0.174  0.005
## sex      -0.203 -0.005  0.015  0.003 -0.008
## minority -0.321 -0.178  0.024  0.116  0.003 -0.009
## ses      -0.123  0.083 -0.027 -0.005  0.054  0.020  0.164
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 1 negative eigenvalues
```

```
anova(M4, M14, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M14: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M14:     ses + (sex | classid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M14   14 10757 10827 -5364.5    10729 0.5003  3     0.9188
```

Response:

The addition of random slope on the *sex* variable, varying by classrooms and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.9188) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M15 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00448545 (tol = 0.002, component 1)
```

```
summary(M15)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid          (Intercept)  172.16  13.121
##                   minority     171.31  13.089   -0.94
##  classid:schoolid (Intercept)   53.27   7.299
##  schoolid         (Intercept)  157.39  12.546
##  Residual                     1045.29  32.331
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73596    5.38028  270.70048 100.317  < 2e-16 ***
## housepov     -17.34694   12.91285  103.34461  -1.343    0.182
## yearstea      -0.01637    0.14285  234.25241  -0.115    0.909
## mathknow       1.45704    1.39356  234.04817   1.046    0.297
## mathprep      -0.13520    1.37019  203.97272  -0.099    0.921
## sex           -1.01013    2.08966 1015.73557  -0.483    0.629
## minority     -16.48617    3.21758  183.21789  -5.124 7.55e-07 ***
## ses            9.89349    1.54595 1062.82964   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.435
## yearstea -0.265  0.080
## mathknow -0.079  0.061  0.038
## mathprep -0.618  0.037 -0.171 -0.006
## sex      -0.188 -0.009  0.015  0.009 -0.005
## minority -0.368 -0.171  0.025  0.108 -0.004 -0.009
## ses      -0.117  0.085 -0.023  0.001  0.051  0.021  0.149
## convergence code: 0
## Model failed to converge with max|grad| = 0.00448545 (tol = 0.002, component 1)
```

```
anova(M4, M15, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M15: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M15:     ses + (minority | classid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M15   14 10754 10824 -5363.2    10726 3.1967  3     0.3623
```

Response:

The addition of random slope on the *minority* variable, varying by classrooms and allowing for correlation between random slope and random intercept, is not significant (p-value=0.3623) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```r
M16 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (ses | classid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -8.9e+01
```

```r
summary(M16)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (ses | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid          (Intercept)    0.00   0.000
##                   ses           49.60   7.043    NaN
##  classid:schoolid (Intercept)   87.11   9.334
##  schoolid         (Intercept)  171.01  13.077
##  Residual                     1043.44  32.302
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71227    5.30641  273.93159 101.710  < 2e-16 ***
## housepov     -17.50877   13.21774  113.36345  -1.325    0.188
## yearstea       0.01103    0.14117  226.81927   0.078    0.938
## mathknow       1.36796    1.38563  228.69182   0.987    0.325
## mathprep      -0.27938    1.37171  203.63906  -0.204    0.839
## sex           -1.37733    2.09334 1022.76631  -0.658    0.511
## minority     -16.29363    3.02464  703.08523  -5.387 9.78e-08 ***
## ses           10.14363    1.64249  174.77088   6.176 4.48e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.070
## mathknow -0.082  0.058  0.029
## mathprep -0.631  0.040 -0.172  0.005
## sex      -0.190 -0.007  0.014  0.006 -0.005
## minority -0.321 -0.180  0.025  0.111  0.002 -0.011
## ses      -0.108  0.081 -0.026  0.002  0.050  0.020  0.145
## convergence code: 0
```

```
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M16, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M16: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M16:     ses + (ses | classid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M16   14 10756 10826 -5364.0    10728 1.5969  3     0.6601
```

Response:

The addition of random slope on the *ses* variable, varying by classrooms and allowing for correlation between the random slope and random intercept, is not significant (p-value=0.6601) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

Question 7.

a. Try to add a random slope for each student level predictor varying at the school level:

```
M17 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | schoolid) + (1|schoolid/classid), data = classroom)
summary(M17)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   96.08   9.802
##  schoolid         (Intercept)  161.63  12.713
##  schoolid.1       sex           35.84   5.986
##  Residual                      1054.36  32.471
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43517    5.30740  272.54993 101.638  < 2e-16 ***
## housepov     -16.77652   13.22879  112.39634  -1.268    0.207
## yearstea       0.01448    0.14163  226.44539   0.102    0.919
## mathknow       1.40067    1.39464  234.45909   1.004    0.316
```

```
## mathprep      -0.27193    1.38011  205.78530  -0.197    0.844
## sex           -1.33535    2.18747  138.09087  -0.610    0.543
## minority     -16.16537    3.02861  704.25756  -5.338 1.27e-07 ***
## ses            9.98477    1.54243 1058.27916   6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.259  0.070
## mathknow -0.081  0.055  0.028
## mathprep -0.633  0.036 -0.172  0.004
## sex      -0.179 -0.010  0.013  0.007 -0.004
## minority -0.320 -0.178  0.024  0.114  0.001 -0.015
## ses      -0.120  0.081 -0.029 -0.007  0.052  0.020  0.161
```

```
anova(M4, M17, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M17: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M17:     ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M17   12 10753 10813 -5364.4    10729 0.6137  1     0.4334
```

Response:

The addition of random slope on the *sex* variable, varying by schools, is not significant (p-value=0.4334) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M18 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M18)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
```

```
##  classid.schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  schoolid.1       minority       0.00   0.00
##  Residual                     1064.96  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39107 101.585  < 2e-16 ***
## housepov     -17.64847   13.21752  113.87889  -1.335    0.184
## yearstea       0.01129    0.14141  226.80889   0.080    0.936
## mathknow       1.35003    1.39168  234.49798   0.970    0.333
## mathprep      -0.27705    1.37583  205.27126  -0.201    0.841
## sex           -1.21419    2.09483 1022.42106  -0.580    0.562
## minority     -16.18677    3.02605  704.47765  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56217   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M18, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M18: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M18:     ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M18   12 10754 10813 -5364.8    10730     0  1          1
```

Response:

The addition of random slope on the *minority* variable, varying by schools, is not significant (p-value=1.00) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M19 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
summary(M19)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
```

```
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   88.56   9.411
##  schoolid         (Intercept)  167.98  12.961
##  schoolid.1       ses           72.50   8.515
##  Residual                      1035.12  32.173
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13754    5.27918  270.54292 102.125  < 2e-16 ***
## housepov     -16.94561   13.21117  112.82498  -1.283    0.202
## yearstea       0.03079    0.14052  223.94368   0.219    0.827
## mathknow       1.35576    1.38459  232.20020   0.979    0.329
## mathprep      -0.19801    1.35994  198.59490  -0.146    0.884
## sex           -1.40185    2.08170 1011.28952  -0.673    0.501
## minority     -16.52526    3.02189  700.06722  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.01642   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.260  0.070
## mathknow -0.079  0.056  0.028
## mathprep -0.628  0.041 -0.172  0.002
## sex      -0.190 -0.007  0.018  0.006 -0.007
## minority -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses      -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
```

```
anova(M4, M19, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M19: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M19:     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M19   12 10749 10809 -5362.4    10725 4.6972  1    0.03021 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *ses* variable, varying by schools, is significant (p-value = 0.03021) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Retry the above, allowing the slopes to be correlated with the random intercepts.

```
M20 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (sex | schoolid) + (1|schoolid:classid), data = classroom)
summary(M20)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   97.33   9.866
##  schoolid         (Intercept)  206.34  14.364
##                   sex           84.08   9.169   -0.43
##  Residual                      1041.76  32.276
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661  < 2e-16 ***
## housepov    -1.742e+01  1.326e+01  1.136e+02  -1.314    0.191
## yearstea     6.877e-03  1.418e-01  2.277e+02   0.048    0.961
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## mathprep    -2.795e-01  1.378e+00  2.061e+02  -0.203    0.839
## sex         -1.340e+00  2.301e+00  8.742e+01  -0.582    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.258  0.072
## mathknow -0.082  0.060  0.027
## mathprep -0.627  0.038 -0.172  0.004
## sex      -0.222 -0.003  0.014  0.006 -0.005
## minority -0.319 -0.178  0.024  0.114  0.004 -0.011
```

```
## ses      -0.121  0.083 -0.028 -0.006  0.053  0.018  0.163
```

```
anova(M4, M20, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M20: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M20:     ses + (sex | schoolid) + (1 | schoolid:classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M20   13 10754 10818 -5363.8    10728 1.8631  2      0.394
```

Response:

The addition of random slope on the *sex* variable, varying by schools and allowing for correlation between the random slope and random intercept, is not significant (p-value=0.6013) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M21 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | schoolid) + (1|schoolid:classid), data = classroom)
summary(M21)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.69   9.311
##  schoolid         (Intercept)  381.20  19.524
##                   minority     343.13  18.524   -0.83
##  Residual                     1039.39  32.240
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.394
## yearstea -0.253  0.091
## mathknow -0.078  0.061  0.024
## mathprep -0.576  0.037 -0.167 -0.002
## sex      -0.172 -0.013  0.014  0.010 -0.005
## minority -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses      -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
```

```
anova(M4, M21, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M21: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M21:     ses + (minority | schoolid) + (1 | schoolid:classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M21   13 10744 10808 -5358.8    10718 11.967  2    0.00252 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *minority* variable, varying by schools and allowing for correlation between the random slope and random intercept, is significant (p-value= 0.007497) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M22 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (ses | schoolid) + (1|schoolid:classid), data = classroom)
summary(M22)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (ses | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.57   9.305
##  schoolid         (Intercept)  171.18  13.083
##                   ses           73.36   8.565   0.19
##  Residual                      1035.90  32.185
```

```
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.72222    5.27648  271.13305 102.099  < 2e-16 ***
## housepov     -15.89873   13.15396  111.71336  -1.209    0.229
## yearstea       0.03617    0.14002  220.42240   0.258    0.796
## mathknow       1.26025    1.38201  230.89913   0.912    0.363
## mathprep      -0.21697    1.35642  197.10758  -0.160    0.873
## sex           -1.40436    2.08074 1011.40322  -0.675    0.500
## minority     -16.26698    3.03580  668.91588  -5.358 1.16e-07 ***
## ses            9.72646    1.82985   78.36212   5.315 9.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.259  0.073
## mathknow -0.077  0.057  0.028
## mathprep -0.627  0.039 -0.172  0.001
## sex      -0.188 -0.009  0.017  0.005 -0.008
## minority -0.325 -0.182  0.021  0.108  0.002 -0.011
## ses      -0.062  0.070 -0.021  0.007  0.045  0.018  0.117
```

```
anova(M4, M22, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## M22: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M22:     ses + (ses | schoolid) + (1 | schoolid:classid)
##     npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4    11 10752 10806 -5364.8    10730
## M22   13 10750 10815 -5362.2    10724 5.1385  2    0.07659 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *ses* variable, varying by schools and allowing for correlation between the random slope and random intercept, is not significant (p-value=0.1954) (at the alpha 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

Response:

c. Report anything unusual about the variance components (changes that are unexpected)

> Minority random slope when allowing for correlation between random slope and random intercept is statistically significant but 0.00 when not allowing for correlation

Response:

Question 8.

a. Take the two predictors that had significant random slopes, in the forms in which they worked (indep. or correlated) and add both to the model, and test for need of one conditional on needing the other.

```
# Fit models and run LRT tests
Mslope1 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
Mslope2 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | schoolid) + (1|schoolid:classid), data = classroom)
Mslope3 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid)+ (minority | schoolid) + (1|schoolid:classid)

anova(Mslope3, Mslope1, refit = F)
```

```
## Data: classroom
## Models:
## Mslope1: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope1:     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:     schoolid:classid)
##         npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## Mslope1   12 10749 10809 -5362.4    10725
## Mslope3   14 10740 10810 -5356.2    10712 12.39  2    0.00204 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(Mslope3, Mslope2, refit = F)
```

```
## Data: classroom
## Models:
## Mslope2: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope2:     ses + (minority | schoolid) + (1 | schoolid:classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:     schoolid:classid)
##         npar   AIC   BIC  logLik deviance Chisq Df Pr(>Chisq)
## Mslope2   13 10744 10808 -5358.8    10718
## Mslope3   14 10740 10810 -5356.2    10712  5.12  1    0.02365 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the random slopes on the two predictors *ses* and *minority* with *ses* random slope not correlated with *ses* random intercept and *minority* random slope correlated with *minority* random intercept is significant (p-value = 0.00616, p-value= 0.02365) (at the alpha 0.05 significance level) and are needed based on the LRT comparing the need of one random slope, conditional on needing the other.

b. Is the more complex model (with both random slopes in it) justified?

```
# Insert code to compare models
anova(Mslope3, M4)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:     sex + (1 | schoolid/classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:     schoolid:classid)
##         npar   AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
## M4        11 10774 10829 -5376.1    10752
## Mslope3   14 10764 10833 -5367.8    10736 16.652  3  0.0008332 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Reponse:

The addition of the random slopes on the two predictors *ses* and *minority* with *ses* random slope not correlated with *ses* random intercept and *minority* random slope correlated with *minority* random intercept is significant (p-value = 0.002258) (at the alpha 0.05 significance level) meaning that the more complex model with both random slopes is justified based on the LRT comparing the complex model with the random intercepts only model.

c. WRITE OUT THIS MODEL in your preferred notation

The model is:

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 MATHKNOW_{jk} + b_3 MATHPREP_{jk} + b_4 YEARSTEA_{jk} + b_5 SES_{ijk} + b_6 MINORITY_{ijk} + b_7 SEX_{ijk} + \eta_0 jk + \zeta_0 k + \zeta_{5k} SES + \zeta_{6k} MINORITY + \varepsilon_{ijk}$ with normality assumptions of $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$,$\varepsilon_{ijk} \sim N(0, \sigma^2_{\varepsilon})$, $\eta_{0jk} \sim N(0, \sigma^2_{\eta 0})$, $\zeta_{5k} \sim N(0, \sigma^2_{\zeta 5})$, $\zeta_{6k} \sim N(0, \sigma^2_{\zeta 6})$ and all assumed independent for random terms in the model with i representing students, j representing classrooms and k representing schools.

Question 9.

a. For UMM, write down: V_S, V_C, V_E for the three variance components (simply the estimates)

```
# If you want to look at your UMM insert code here or you can just do this in line
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
```

```
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.407   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
V_S = 280.68
```

```
V_C = 85.46
```

```
V_E = 1146.80
```

b. For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

```
# If you want to look at your model insert code here or you can just do this in line
summary(M4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                     1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp yearst ses    minrty
## housepov -0.451
```

```
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

V_S = 169.45

V_C = 93.89

V_E = 1064.96

c. By what fraction did these each decrease with the new predictors in the model?

V_S: 39.63% decrease

V_C: 9.85% increase

V_E: 7.14% decrease

Question 10. Now consider the model with a random slope in ses.

a. What are: V_C, V_S(ses=0), V_E ?

```
# If you want to look at your model insert code here or you can just do this in line
summary(Mslope1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   88.56   9.411
##  schoolid         (Intercept)  167.98  12.961
##  schoolid.1       ses           72.50   8.515
##  Residual                      1035.12  32.173
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13754    5.27918  270.54292 102.125  < 2e-16 ***
## housepov     -16.94561   13.21117  112.82498  -1.283    0.202
## yearstea       0.03079    0.14052  223.94368   0.219    0.827
## mathknow       1.35576    1.38459  232.20020   0.979    0.329
```

```
## mathprep      -0.19801    1.35994  198.59490  -0.146    0.884
## sex           -1.40185    2.08170 1011.28952  -0.673    0.501
## minority     -16.52526    3.02189  700.06722  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.01642   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.260  0.070
## mathknow -0.079  0.056  0.028
## mathprep -0.628  0.041 -0.172  0.002
## sex      -0.190 -0.007  0.018  0.006 -0.007
## minority -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses      -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
```

```
V_S(ses=0) = 167.98

V_C = 88.56

V_E = 1035.12
```

b. What are: V_S(ses=-0.50), V_S(ses=+0.5) ?

```
V_S(ses=0.5) =

V_S(ses=-0.5) =
```

Question 11.

Now consider the model with a random slope in minority.

a. What are: V_C, V_S(minority=0), V_E ?

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
summary(Mslope2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | schoolid) + (1 | schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid:classid (Intercept)   86.69   9.311
##  schoolid         (Intercept)  381.20  19.524
##                   minority     343.13  18.524   -0.83
```

```
##  Residual                     1039.39  32.240
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.394
## yearstea -0.253  0.091
## mathknow -0.078  0.061  0.024
## mathprep -0.576  0.037 -0.167 -0.002
## sex      -0.172 -0.013  0.014  0.010 -0.005
## minority -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses      -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
```

V_S(minority=0) = 308.34+72.86 = 381.2 (correlated)

V_C = 86.70

V_E = 1039.39

b. What are: V_S(minority=0.25), V_S(minority=+0.50), V_S(minority=+0.75) ?

V_S(minority=0.25) =

```
# V_S(minority = 0.25)
# Insert code if you want to do the calculations in R
```

V_S(minority=0.5) =

```
# V_S(minority = 0.50)
# Insert code if you want to do the calculations in R
```

V_S(minority=0.75) =

```
# V_S(minority = 0.75)
# Insert code if you want to do the calculations in R
```

Question 12.

Now consider the model with a random slope in ses & minority.

a. What are: V_C, V_S(minority=0,ses=0), V_E ? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
summary(Mslope3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
##     schoolid:classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  schoolid.classid (Intercept)   80.63   8.979
##  schoolid         (Intercept)  404.54  20.113
##                   minority     336.01  18.331   -0.84
##  schoolid.1       ses           74.94   8.657
##  Residual                      1009.73  31.776
## Number of obs: 1081, groups:  schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05329    5.66468  165.74699  95.160  < 2e-16 ***
## housepov     -15.32096   12.49428   99.25985  -1.226    0.223
## yearstea       0.02102    0.13657  213.65721   0.154    0.878
## mathknow       1.67475    1.35000  221.33611   1.241    0.216
## mathprep      -0.23547    1.31730  191.22079  -0.179    0.858
## sex           -1.03872    2.06951 1010.41122  -0.502    0.616
## minority     -16.72881    3.90715   55.40755  -4.282 7.43e-05 ***
## ses            9.19651    1.82273   82.48807   5.045 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.395
## yearstea -0.254  0.093
## mathknow -0.072  0.060  0.024
## mathprep -0.568  0.040 -0.166 -0.004
## sex      -0.170 -0.014  0.017  0.010 -0.005
## minority -0.509 -0.149  0.027  0.092 -0.003 -0.013
## ses      -0.080  0.083 -0.011  0.006  0.041  0.020  0.087
```

V_S(sex=0, minority=0) = 346.60+57.93 = 404.53

V_C = 80.62

V_E = 1009.73

b. In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Response:

c. Can we make a similar statement about ζ_{0k}?

Response:

d. If you had a large value for η_{0jk}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Response:

e. If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Response: