MLM Final Project Part 1

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Team Members and division of work:

Question 0.

Load classroom.csv and create MATH1ST (fit all models using REML, use lmerTest::lmer)

```
# Insert code to load data and create math1st variable
classroom <- read_csv("/Users/dennishilgendorf/Desktop/NYU MPH/Spring 2020 (Y2)/MLM/1/classroom.csv")
```

```
## Parsed with column specification:
## cols(
##   sex = col_double(),
##   minority = col_double(),
##   mathkind = col_double(),
##   mathgain = col_double(),
##   ses = col_double(),
##   yearstea = col_double(),
##   mathknow = col_double(),
##   housepov = col_double(),
##   mathprep = col_double(),
##   classid = col_double(),
##   schoolid = col_double(),
##   childid = col_double()
## )
```

```
attach(classroom)
classroom$math1st <-  mathkind + mathgain

save(classroom, file = "/Users/dennishilgendorf/Desktop/NYU MPH/Spring 2020 (Y2)/MLM/1/classroom.RData")
```

Question 1.

Estimate UMM model with random intercepts for both schools and classrooms.

```
# Insert code to fit model and print summary

M1_UMM <- lmerTest::lmer(math1st ~ 1 + (1 | schoolid/classid), data=classroom)
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ 1 + (1 | schoolid/classid)
##    Data: classroom
##
```

```
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                      1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a. Report the ICC for schools and classrooms:

```
# Insert code if you'd like but you can also do this inline
```

Response:

$\hat{\sigma}^2_\eta = 85.47$ $\hat{\sigma}^2_\zeta = 280.69$ $\hat{\sigma}^2_\varepsilon = 1146.79$

ICC for classrooms $= \hat{\sigma}^2_\eta/(\hat{\sigma}^2_\eta + \hat{\sigma}^2_\zeta + \hat{\sigma}^2_\varepsilon) = (85.47)/(85.47 + 280.69 + 1146.79) = 5.65\%$ is the proportion of variance explained by between classroom differences.

ICC for schools $= \hat{\sigma}^2_\zeta/(\hat{\sigma}^2_\eta + \hat{\sigma}^2_\zeta + \hat{\sigma}^2_\varepsilon) = (280.69)/(85.47 + 280.69 + 1146.79) = 18.55\%$ is the proportion of variance explained by between school differences.

b. Write out the model:

Model 1 Equation:

$MATH1ST_{ijk} = b_0 + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model.

Question 2.

Add all school-level predictors:

Model 2 Equation:

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model.

```
# Insert code to fit model and print summary

M2 <- lmerTest::lmer(math1st ~ housepov + (1 | schoolid/classid), data=classroom)
summary(M2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: classroom
```

```
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.96  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.807 159.023   <2e-16 ***
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

a. Report if the additional predictors are justified:

```
# Insert code to compare models
anova(M1_UMM, M2, refit = F)
```

```
## Data: classroom
## Models:
## M1_UMM: math1st ~ 1 + (1 | schoolid/classid)
## M2: math1st ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M1_UMM  4 11953 11973 -5972.3    11945
## M2      5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
rand(M2)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ housepov + (1 | classid:schoolid) + (1 | schoolid)
##                         npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                     5 -5963.7 11937
## (1 | classid:schoolid)     4 -5966.7 11941  6.014  1     0.0142 *
## (1 | schoolid)             4 -5988.9 11986 50.359  1   1.28e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The additional school level covariate of HOUSEPOV is justified because the p-value ($p < 0.05$) of the chi

square ANOVE test is significant at the 0.05 alpha significance level suggesting the benefit of adding school level predictors showing significant difference in deviance from log likelihood of the model comparisons. The school level predictor of HOUSEPOV is also significant at the alpha 0.05 significance level (p-value = 0.0017).

b. Report the change to school variance:

Response:

Change in school variance ($\hat{\sigma}^2_\zeta$) changed from 280.69 in Model 1 (UMM null model) to 250.93 indicating that adding the covariate, HOUSEPOV, accounted for variance at the school level.

Question 3: Add all class-level predictors

Model 3 Equation:

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 MATHKNOW_{jk} + b_3 MATHPREP_{jk} + b_4 YEARSTEA_{jk} + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model.

```
# Insert code to fit model and print summary

M3 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid), data=
summary(M3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62117   14.08834 109.83230  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp
## housepov -0.568
```

```
## mathknow -0.052  0.082
## mathprep -0.666  0.032  0.004
## yearstea -0.264  0.077  0.030 -0.175
```

a. Report if adding the predictors is justified:

```
# Insert code to compare models
# Must compare blocks using subset as they are nested
save.options <- options()
options(na.action = "na.pass")
mm <- model.matrix(math1st ~ housepov + mathknow + mathprep + yearstea, data = classroom)
in_sample <- apply(is.na(mm), 1, sum) == 0
options(save.options)

# re-fit mlms using only fully observed observations (no listwise deletions)

M1a_UMM <- lmerTest::lmer(math1st ~ 1 + (1 | schoolid/classid), data=classroom, subset = in_sample)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00266688 (tol = 0.002, component 1)
```

```
summary(M1a_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ 1 + (1 | schoolid/classid)
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10844.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5202 -0.6151 -0.0242  0.5959  3.8621
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)  100.5   10.02
##  schoolid         (Intercept)  248.5   15.76
##  Residual                      1135.9   33.70
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.016      2.017 103.702   259.3   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## convergence code: 0
## Model failed to converge with max|grad| = 0.00266688 (tol = 0.002, component 1)
```

```
M2a <- lmerTest::lmer(math1st ~ housepov + (1 | schoolid/classid), data=classroom, subset = in_sample)
summary(M2a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
```

```
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10827.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5574 -0.6102 -0.0397  0.5727  3.8408
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.6    9.726
##  schoolid         (Intercept)  221.1   14.871
##  Residual                      1137.5   33.727
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.481      3.307 101.710 160.691  < 2e-16 ***
## housepov     -43.738     13.952 108.705  -3.135  0.00221 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.809
```

```
M3a <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid), data
summary(M3a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62117   14.08834 109.83230  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
```

```
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp
## housepov -0.568
## mathknow -0.052  0.082
## mathprep -0.666  0.032  0.004
## yearstea -0.264  0.077  0.030 -0.175
```

```
#anova of subsets using FML to compare fixed effects as a block
anova(M1a_UMM, M3a, refit = T)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: classroom
## Subset: in_sample
## Models:
## M1a_UMM: math1st ~ 1 + (1 | schoolid/classid)
## M3a: math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M1a_UMM  4 10855 10875 -5423.7    10847
## M3a      8 10850 10890 -5417.1    10834 13.114      4    0.01073 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(M2a, M3a, refit = T)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: classroom
## Subset: in_sample
## Models:
## M2a: math1st ~ housepov + (1 | schoolid/classid)
## M3a: math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M2a  5 10848 10873 -5418.9    10838
## M3a  8 10850 10890 -5417.1    10834 3.5061      3       0.32
```

Response:

The addition of classroom level covariates of MATHKNOW MATHPREP and YEARSTEA is not justified because the p-value ($p > 0.32$) of the chi square ANOVA test is non-significant at the 0.05 alpha significance level suggesting the benefit of adding classroom level predictors jointly is non-significant in comparison to the previous model. This means that there is not a statistically significant difference in deviance from log likelihood of the model comparisons with the previous model. The addition of classroom level covariates is justified in comparison to the null model (pvalue = 0.01073) at the 0.05 alpha signifiance level.

b. Report changes in class-level variance and individual variance:

Response:

Change in classroom level variance ($\hat{\sigma}^2_\eta$) changed from 85.47 in Model 1 (UMM null model) to 82.36 in Model 2 and finally to 94.36 in Model 3 indicating that adding the classroom level covariates increases classroom level variance.

Change in individual level variance ($\hat{\sigma}^2_\varepsilon$) changed from 1146.79 in Model 1 (UMM null model) to 1146.96 in Model 2 and finally to 1136.43 in Model 3 indicating that adding the classroom level covariates reduced individual variance.

c. Give a potential reason to explain why individual variance but not class variance is reduced:

Response:

Individual variance could be reduced by the addition of these classroom level covariates while classroom level variance is not reduced because classroom level covariates could explain more individual level variance (individuals varying within a classroom) and thus more of this variance is partitioned into the classroom level variance. This makes sense as individuals vary between classrooms (accounting for variance of the classroom) while also varying within a classroom.

Question 4.

Add all student-level predictors excepting mathgain and mathkind:

```
# Insert code to fit model and print summary
M4 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority + sex + (1 | s
summary(M4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                     1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) housepv mthknw mthprp yearst ses    minrty
## housepov -0.451
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

a. Report if the block of predictors is justified:

```
# Insert code to compare models

# Must compare blocks using subset as they are nested
save.options <- options()
options(na.action = "na.pass")
mm2 <- model.matrix(math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority + sex, data = c
in_sample <- apply(is.na(mm2), 1, sum) == 0
options(save.options)

# re-fit mlms using only fully observed observations (no listwise deletions)

M1b_UMM <- lmerTest::lmer(math1st ~ 1 + (1 | schoolid/classid), data=classroom, subset = in_sample)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00266688 (tol = 0.002, component 1)
```

```
summary(M1b_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ 1 + (1 | schoolid/classid)
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10844.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5202 -0.6151 -0.0242  0.5959  3.8621
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)  100.5   10.02
##  schoolid         (Intercept)  248.5   15.76
##  Residual                      1135.9   33.70
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.016      2.017 103.702   259.3   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## convergence code: 0
```

```
## Model failed to converge with max|grad| = 0.00266688 (tol = 0.002, component 1)
```

```
M2b <- lmerTest::lmer(math1st ~ housepov + (1 | schoolid/classid), data=classroom, subset = in_sample)
summary(M2b)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10827.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5574 -0.6102 -0.0397  0.5727  3.8408
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.6    9.726
##  schoolid         (Intercept)  221.1   14.871
##  Residual                      1137.5   33.727
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.481      3.307 101.710 160.691  < 2e-16 ***
## housepov     -43.738     13.952 108.705  -3.135  0.00221 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.809
```

```
M3b <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid), data
summary(M3b)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
```

```
##  Residual                     1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62117   14.08834 109.83230  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp
## housepov -0.568
## mathknow -0.052  0.082
## mathprep -0.666  0.032  0.004
## yearstea -0.264  0.077  0.030 -0.175
```

```
M4b <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority + sex + (1 |
summary(M4b)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##     sex + (1 | schoolid/classid)
##    Data: classroom
##  Subset: in_sample
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp yearst ses    minrty
## housepov -0.451
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

```
#anova of subsets using FML to compare fixed effects as a block
anova(M1b_UMM, M4b, refit = T)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: classroom
## Subset: in_sample
## Models:
## M1b_UMM: math1st ~ 1 + (1 | schoolid/classid)
## M4b: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4b:     sex + (1 | schoolid/classid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M1b_UMM  4 10855 10875 -5423.7    10847
## M4b     11 10774 10829 -5376.1    10752 95.131      7  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(M3b, M4b, refit = T)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: classroom
## Subset: in_sample
## Models:
## M3b: math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
## M4b: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4b:     sex + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M3b  8 10850 10890 -5417.1    10834
## M4b 11 10774 10829 -5376.1    10752 82.017      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of student level covariates of SES MINORITY and SEX is justified because the p-value ($p < 0.05$) of the chi square ANOVA test is significant at the 0.05 alpha significance level suggesting the benefit of adding classroom level predictors jointly is significant in comparison to the previous model. This means that there is a statistically significant difference in deviance from log likelihood of the model comparisons with the previous model. The addition of classroom level covariates is also justified in comparison to the null model (pvalue < 0.05) at the 0.05 alpha signifiance level.

b. Report change in all variance components

Response:

Change in classroom level variance ($\hat{\sigma}^2_\eta$) changed from 85.47 in Model 1 (UMM null model) to 82.36 in Model 2 and finally to 94.36 in Model 3 to 93.89 in Model 4 with a marginal decrease in classroom level variance.

Change in school variance ($\hat{\sigma}^2_\zeta$) changed from 280.69 in Model 1 (UMM null model) to 250.93 in Model 2 to 223.31 in Model 3 and finally to 169.45 in Model 4 with a decrease in school level variance observed after adding student level covariates in Model 4

Change in individual student level variance ($\hat{\sigma}^2_\varepsilon$) changed from 1146.79 in Model 1 (UMM null model) to 1146.96 in Model 2 to 1136.43 in Model 3 and finally to 1064.96 in Model 4 indicating that student level variance decreased after adding student level covariates in Model 4.

c. Give a potential reason as to why the school variance drops from the last model:

`Response:`

One potential reason why the school variance drops from model 3 to model 4 after the addition of student level covariates is that more of the variance is accounted for by the student level covariates, which as a level 1 predictor also changes the partitioning of variance explained in the classroom and school levels. Put another way, within individual variance is explained by SES, MINORITY, and SEX which may be relatively similar between the schools in this sample resulting in a lower variance at the school level.

d. Write this model out:

`Model 4 Equation:`

$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 MATHKNOW_{jk} + b_3 MATHPREP_{jk} + b_4 YEARSTEA_{jk} + b_5 SES_{ijk} + b_6 MINORITY_{ijk} + b_7 SEX_{ijk} + \eta_{jk} + \zeta_k + \varepsilon_{ijk}$ with normality assumptions of $\zeta_k \sim N(0, \sigma^2_\zeta)$,$\varepsilon_{ijk} \sim N(0, \sigma^2_\varepsilon)$ and $\eta_{jk} \sim N(0, \sigma^2_\eta)$ $\varepsilon_{ijk}, \zeta_{0k}, \eta_{jk}$ assumed independent for random terms in the model.

Question 5.

a. Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately - not all together)

b. Report the models and their fit.

```
# Insert code to fit first model in Q5b model and print summary
M5 <- lmerTest::lmer(math1st~mathknow + (0+mathknow|schoolid)+(1|schoolid)+(1|classid),data=classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ mathknow + (0 + mathknow | schoolid) + (1 | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10837.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.4990 -0.6102 -0.0182  0.5947  3.8510
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   96.03   9.799
```

```
##  schoolid   (Intercept)  250.33  15.822
##  schoolid.1 mathknow       0.00   0.000
##  Residual               1134.50  33.682
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.064      2.016 103.740 259.402   <2e-16 ***
## mathknow       2.908      1.457 232.556   1.996   0.0471 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## mathknow 0.012
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Insert code to compare models

#First without random slopes

M5a <- lmerTest::lmer(math1st~mathknow + (1|schoolid/classid),data=classroom)
summary(M5a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ mathknow + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10837.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.4990 -0.6102 -0.0182  0.5947  3.8510
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   96.03   9.799
##  schoolid         (Intercept)  250.33  15.822
##  Residual                      1134.50  33.682
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.064      2.016 103.740 259.402   <2e-16 ***
## mathknow       2.908      1.457 232.556   1.996   0.0471 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## mathknow 0.012
```

```
#Now with random slopes - M5

anova(M5a, M5, refit = F)
```

```
## Data: classroom
## Models:
## M5a: math1st ~ mathknow + (1 | schoolid/classid)
## M5: math1st ~ mathknow + (0 + mathknow | schoolid) + (1 | schoolid) +
## M5:     (1 | classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M5a  5 10848 10872 -5418.8    10838
## M5   6 10850 10880 -5418.8    10838     0      1          1
```

```
rand(M5)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ mathknow + (0 + mathknow | schoolid) + (1 | schoolid) +
##     (1 | classid)
##                                        npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                    6 -5418.8 10850
## mathknow in (0 + mathknow | schoolid)    5 -5418.8 10848  0.000  1    0.99998
## (1 | schoolid)                           5 -5439.0 10888 40.330  1  2.145e-10
## (1 | classid)                            5 -5422.0 10854  6.437  1    0.01117
##
## <none>
## mathknow in (0 + mathknow | schoolid)
## (1 | schoolid)                         ***
## (1 | classid)                          *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of mathknow across schools as p-value is not statistically significant (pvalue = 1.00), fail to reject the null hypothesis of the ANOVA test between the two models. The extremely small value for the estimate of $\sigma^2_{\zeta_1}$ random slope estimate of mathknow also indicates little systematic variation across schools for mathknow.

```
# Insert code to fit second model in Q5b model and print summary

M6 <- lmerTest::lmer(math1st~ mathprep + (0+mathprep|schoolid)+(1|schoolid)+(1|classid),data=classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ mathprep + (0 + mathprep | schoolid) + (1 | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 11941.8
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -5.1890 -0.6159 -0.0237  0.5837  3.8445
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   86.96   9.325
##  schoolid   (Intercept)  280.58  16.751
##  schoolid.1 mathprep       0.00   0.000
##  Residual               1146.65  33.862
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
## (Intercept) 524.4407     4.0456 264.9523 129.631   <2e-16 ***
## mathprep     -0.7349     1.3497 222.2216  -0.544    0.587
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## mathprep -0.864
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Insert code to compare models

#First without random slopes

M6a <- lmerTest::lmer(math1st~mathprep + (1|schoolid/classid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00325823 (tol = 0.002, component 1)
```

```
summary(M6a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11941.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1890 -0.6159 -0.0238  0.5838  3.8445
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   86.96   9.325
##  schoolid         (Intercept)  280.45  16.747
##  Residual                      1146.69  33.863
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
```

```
## (Intercept) 524.4410     4.0454 264.9837 129.638   <2e-16 ***
## mathprep     -0.7349     1.3497 222.2209  -0.544    0.587
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## mathprep -0.864
## convergence code: 0
## Model failed to converge with max|grad| = 0.00325823 (tol = 0.002, component 1)
```

```
#Now with random slopes - M6

anova(M6a, M6, refit = F)
```

```
## Data: classroom
## Models:
## M6a: math1st ~ mathprep + (1 | schoolid/classid)
## M6: math1st ~ mathprep + (0 + mathprep | schoolid) + (1 | schoolid) +
## M6:     (1 | classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M6a  5 11952 11977 -5970.9    11942
## M6   6 11954 11984 -5970.9    11942     0      1     0.9982
```

```
rand(M6)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00325823 (tol = 0.002, component 1)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ mathprep + (0 + mathprep | schoolid) + (1 | schoolid) +
##     (1 | classid)
##                                        npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                                    6 -5970.9 11954
## mathprep in (0 + mathprep | schoolid)     5 -5970.9 11952  0.0000  1     0.9982
## (1 | schoolid)                            5 -5981.2 11972 20.6727  1  5.449e-06
## (1 | classid)                             5 -5974.2 11958  6.4814  1     0.0109
##
## <none>
## mathprep in (0 + mathprep | schoolid)
## (1 | schoolid)                         ***
## (1 | classid)                          *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of mathprep across schools as p-value is not statistically significant (pvalue = 0.9982), fail to reject the null hypothesis of the ANOVA test between the two models. The extremely small value for the estimate of $\sigma^2_{\zeta_1}$ random slope estimate of mathprep also indicates little systematic variation across schools for mathprep

```
# Insert code to fit third model in Q5b model and print summary
M7 <- lmerTest::lmer(math1st~ yearstea + (0+yearstea|schoolid)+(1|schoolid)+(1|classid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00312491 (tol = 0.002, component 1)
```

```
summary(M7)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ yearstea + (0 + yearstea | schoolid) + (1 | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2354 -0.6095 -0.0289  0.5825  3.8322
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept)   65.5667  8.0973
##  schoolid   (Intercept)  260.7883 16.1489
##  schoolid.1 yearstea       0.1662  0.4077
##  Residual               1148.9710 33.8965
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 521.67248    2.61441 172.39951 199.537   <2e-16 ***
## yearstea      0.07081    0.14808 119.35087   0.478    0.633
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## yearstea -0.627
## convergence code: 0
## Model failed to converge with max|grad| = 0.00312491 (tol = 0.002, component 1)
```

```
# Insert code to compare models

#First without random slopes

M7a <- lmerTest::lmer(math1st~yearstea + (1|schoolid/classid),data=classroom)
summary(M7a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ yearstea + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11946.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1743 -0.6190 -0.0209  0.5818  3.8316
```

```
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.92   9.269
##  schoolid         (Intercept)  278.19  16.679
##  Residual                     1147.85  33.880
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 521.52011    2.66634 198.09948 195.594   <2e-16 ***
## yearstea      0.08311    0.14021 236.52588   0.593    0.554
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## yearstea -0.648
```

```
#Now with random slopes - M6

anova(M7a, M7, refit = F)
```

```
## Data: classroom
## Models:
## M7a: math1st ~ yearstea + (1 | schoolid/classid)
## M7: math1st ~ yearstea + (0 + yearstea | schoolid) + (1 | schoolid) +
## M7:     (1 | classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M7a  5 11956 11982 -5973.2    11946
## M7   6 11957 11987 -5972.3    11945 1.6488      1     0.1991
```

```
rand(M7)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ yearstea + (0 + yearstea | schoolid) + (1 | schoolid) +
##     (1 | classid)
##                                        npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                    6 -5972.3 11957
## yearstea in (0 + yearstea | schoolid)    5 -5973.2 11956  1.649  1    0.19913
## (1 | schoolid)                           5 -5990.5 11991 36.416  1  1.594e-09
## (1 | classid)                            5 -5973.8 11958  3.020  1    0.08224
##
## <none>
## yearstea in (0 + yearstea | schoolid)
## (1 | schoolid)                        ***
## (1 | classid)                         .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of yearstea across schools as p-value is not statistically significant (pvalue = 0.1991), fail to reject the null hypothesis of the ANOVA test between the two models. The extremely small value for the estimate of $\sigma^2_{\zeta_1}$ random slope estimate of yearstea also indicates little systematic variation

across schools for yearstea.

c. Why is it a bad idea to include a random slope on the housepov effect?

Response:

It is a bad idea to include a random slope on the housepov effect because its variance is already being accounted for in the null model by including the differential effects of schoolid on the outcome math1st.

d. Retry the above models, allowing the slopes to be correlated with the random intercepts (still one by one):

```
# Insert code to fit first model in Q5d model and print summary

M5_corr <- lmerTest::lmer(math1st~mathknow + (mathknow|schoolid) + (1|classid),data=classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M5_corr)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ mathknow + (mathknow | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10837.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5001 -0.6127 -0.0196  0.5950  3.8513
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.606e+01  9.8009
##  schoolid (Intercept) 2.494e+02 15.7933
##           mathknow    1.946e-02  0.1395  1.00
##  Residual             1.135e+03 33.6861
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.073      2.014 102.425 259.658   <2e-16 ***
## mathknow       2.923      1.458 207.666   2.004   0.0464 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## mathknow 0.022
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Insert code to compare models

anova(M5a, M5_corr, refit = F)
```

```
## Data: classroom
```

```
## Models:
## M5a: math1st ~ mathknow + (1 | schoolid/classid)
## M5_corr: math1st ~ mathknow + (mathknow | schoolid) + (1 | classid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M5a      5 10848 10872 -5418.8    10838
## M5_corr  7 10852 10886 -5418.8    10838 0.0052      2     0.9974
```

```
rand(M5_corr)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ mathknow + (mathknow | schoolid) + (1 | classid)
##                                    npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                7 -5418.8 10852
## mathknow in (mathknow | schoolid)     5 -5418.8 10848 0.0052  2    0.99740
## (1 | classid)                         6 -5422.0 10856 6.4361  1    0.01118 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of mathknow across schools as p-value is not statistically significant (pvalue = 0.9974), fail to reject the null hypothesis of the ANOVA test between the two models indicating no need for additional 2 parameters for correlated random slopes compared to a random intercept model.

```
# Insert code to fit second model in Q5d model and print summary

M6_corr <- lmerTest::lmer(math1st~ mathprep + (mathprep|schoolid)+(1|classid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0171966 (tol = 0.002, component 1)
```

```
summary(M6_corr)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ mathprep + (mathprep | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 11938.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1691 -0.6023 -0.0245  0.5887  3.9100
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   81.187  9.01
##  schoolid (Intercept)  596.479 24.42
##           mathprep       8.881  2.98    -1.00
##  Residual             1146.068 33.85
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  524.030      4.294 122.612 122.046   <2e-16 ***
```

```
## mathprep      -0.596      1.300 205.249  -0.458    0.647
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## mathprep -0.887
## convergence code: 0
## Model failed to converge with max|grad| = 0.0171966 (tol = 0.002, component 1)
```

```
# Insert code to compare models

anova(M6a, M6_corr, refit = F)
```

```
## Data: classroom
## Models:
## M6a: math1st ~ mathprep + (1 | schoolid/classid)
## M6_corr: math1st ~ mathprep + (mathprep | schoolid) + (1 | classid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M6a      5 11952 11977 -5970.9    11942
## M6_corr  7 11953 11988 -5969.4    11939 3.0338      2     0.2194
```

```
rand(M6_corr)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00325823 (tol = 0.002, component 1)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0117108 (tol = 0.002, component 1)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ mathprep + (mathprep | schoolid) + (1 | classid)
##                                    npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                7 -5969.4 11953
## mathprep in (mathprep | schoolid)     5 -5970.9 11952 3.0338  2    0.21939
## (1 | classid)                         6 -5972.2 11956 5.5754  1    0.01821 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of mathprep across schools as p-value is not statistically significant (pvalue = 0.2194), fail to reject the null hypothesis of the ANOVA test between the two models indicating no need for additional 2 parameters for correlated random slopes compared to a random intercept model.

```
# Insert code to fit third model in Q5d model and print summary
M7_corr <- lmerTest::lmer(math1st~ yearstea + (yearstea|schoolid)+(1|classid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0115986 (tol = 0.002, component 1)
```

```
summary(M7_corr)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ yearstea + (yearstea | schoolid) + (1 | classid)
##    Data: classroom
```

```
##
## REML criterion at convergence: 11940.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2213 -0.6073 -0.0314  0.5948  3.8358
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   25.845  5.0838
##  schoolid (Intercept)  448.054 21.1673
##           yearstea       0.636  0.7975  -0.62
##  Residual             1148.954 33.8962
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 521.73595    2.94932  74.20756 176.900   <2e-16 ***
## yearstea      0.09675    0.16237  75.67340   0.596    0.553
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## yearstea -0.723
## convergence code: 0
## Model failed to converge with max|grad| = 0.0115986 (tol = 0.002, component 1)
```

```
# Insert code to compare models

anova(M7a, M7_corr, refit = F)
```

```
## Data: classroom
## Models:
## M7a: math1st ~ yearstea + (1 | schoolid/classid)
## M7_corr: math1st ~ yearstea + (yearstea | schoolid) + (1 | classid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M7a      5 11956 11982 -5973.2    11946
## M7_corr  7 11954 11990 -5970.2    11940 5.8748      2      0.053 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
rand(M7_corr)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.0115311 (tol = 0.002, component 1)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ yearstea + (yearstea | schoolid) + (1 | classid)
##                                    npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                7 -5970.2 11954
## yearstea in (yearstea | schoolid)     5 -5973.2 11956 5.8748  2     0.0530 .
## (1 | classid)                         6 -5970.4 11953 0.4498  1     0.5024
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

Observations show some significant changes of yearstea across schools as p-value is marginally statistically significant (pvalue = 0.053) indicating a potential need for additional 2 parameters for correlated random slopes compared to a random intercept model for yearstea.

e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occured (hint: what did you add to the model?).

Response:

Yearsteaching is statististically significantly varies across schools (alpha = 0.05) as it decreases variance at the classroom level and increases variance at the school level.

Question 6.

a. Try to add a random slope for each student level predictor (varying at the classroom level; one by one - not all together)

```
# Insert code to fit first model in Q6a model and print summary
M8 <- lmerTest::lmer(math1st~ses + (0+ses|classid)+(1|schoolid)+(1|classid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00422542 (tol = 0.002, component 1)
```

```
summary(M8)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + (0 + ses | classid) + (1 | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 11879.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1953 -0.5778 -0.0474  0.5892  3.6219
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   69.18   8.318
##  classid.1 ses           50.05   7.074
##  schoolid  (Intercept)  238.83  15.454
##  Residual              1080.11  32.865
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.102      1.912 100.940 273.551  < 2e-16 ***
## ses           11.856      1.579 167.948   7.509 3.35e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##     (Intr)
## ses 0.048
## convergence code: 0
## Model failed to converge with max|grad| = 0.00422542 (tol = 0.002, component 1)
```

```
# Insert code to compare models

#First without random slopes

M8a <- lmerTest::lmer(math1st~ses + (1|schoolid/classid),data=classroom)
summary(M8a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11880.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2117 -0.5768 -0.0533  0.5851  3.6162
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   74.67   8.641
##  schoolid         (Intercept)  237.20  15.401
##  Residual                     1102.00  33.196
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
## (Intercept)  523.169      1.909  101.251  273.99  < 2e-16 ***
## ses           11.811      1.484 1180.367    7.96 4.01e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##     (Intr)
## ses 0.037
```

```
#Now with random slopes - M5

anova(M8a, M8, refit = F)
```

```
## Data: classroom
## Models:
## M8a: math1st ~ ses + (1 | schoolid/classid)
## M8: math1st ~ ses + (0 + ses | classid) + (1 | schoolid) + (1 | classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M8a  5 11890 11916 -5940.3    11880
## M8   6 11891 11922 -5939.5    11879 1.4568      1     0.2274
```

```
rand(M8)
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## math1st ~ ses + (0 + ses | classid) + (1 | schoolid) + (1 | classid)
##                            npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                        6 -5939.5 11891
## ses in (0 + ses | classid)    5 -5940.3 11890  1.457  1     0.2274
## (1 | schoolid)                5 -5962.5 11935 45.847  1  1.278e-11 ***
## (1 | classid)                 5 -5941.6 11893  4.184  1     0.0408 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of ses across schools as p-value is not statistically significant (pvalue = 0.2274), fail to reject the null hypothesis of the ANOVA test between the two models. The relatively small value for the estimate of $\sigma^2_{\zeta_1}$ random slope estimate of ses also indicates some systematic variation across schools for ses but not enough for statistical significance justidying model preference in the ANOVA test.

```
# Insert code to fit second model in Q6a model and print summary

M9 <- lmerTest::lmer(math1st~ minority + (0+minority|classid)+(1|schoolid)+(1|classid),data=classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M9)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ minority + (0 + minority | classid) + (1 | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 11893.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2046 -0.6278 -0.0233  0.5859  3.8416
##
## Random effects:
##  Groups    Name        Variance  Std.Dev.
##  classid   (Intercept) 9.614e+01  9.805262
##  classid.1 minority    1.652e-06  0.001285
##  schoolid  (Intercept) 1.921e+02 13.859446
##  Residual              1.117e+03 33.418246
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  537.364      2.755 229.886 195.040  < 2e-16 ***
## minority     -20.576      2.904 643.070  -7.086 3.64e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## minority -0.752
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Insert code to compare models

#First without random slopes

M9a <- lmerTest::lmer(math1st~minority + (1|schoolid/classid),data=classroom)
summary(M9a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ minority + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11893.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2046 -0.6278 -0.0233  0.5859  3.8416
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   96.12   9.804
##  schoolid         (Intercept)  192.10  13.860
##  Residual                      1116.79  33.418
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  537.364      2.755 229.880 195.038  < 2e-16 ***
## minority     -20.576      2.904 643.075  -7.086 3.64e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## minority -0.752
```

```
#Now with random slopes - M6

anova(M9a, M9, refit = F)
```

```
## Data: classroom
## Models:
## M9a: math1st ~ minority + (1 | schoolid/classid)
## M9: math1st ~ minority + (0 + minority | classid) + (1 | schoolid) +
## M9:     (1 | classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M9a  5 11904 11929 -5946.9    11894
## M9   6 11906 11936 -5946.9    11894     0      1          1
```

```
rand(M9)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## math1st ~ minority + (0 + minority | classid) + (1 | schoolid) +
##     (1 | classid)
##                                     npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                 6 -5946.9 11906
## minority in (0 + minority | classid)    5 -5946.9 11904  0.000  1    1.00000
## (1 | schoolid)                         5 -5962.7 11935 31.682  1  1.816e-08
## (1 | classid)                          5 -5948.9 11908  4.157  1    0.04147
##
## <none>
## minority in (0 + minority | classid)
## (1 | schoolid)                      ***
## (1 | classid)                       *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of minority across schools as p-value is not statistically significant (pvalue = 1.00), fail to reject the null hypothesis of the ANOVA test between the two models. The extremely small value for the estimate of $\sigma^2_{\zeta_1}$ random slope estimate of minority also indicates little systematic variation across schools for minority.

```
# Insert code to fit third model in Q6a model and print summary
M10 <- lmerTest::lmer(math1st~ sex + (0+sex|classid)+(1|schoolid)+(1|classid),data=classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + (0 + sex | classid) + (1 | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 11941.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1922 -0.6131 -0.0238  0.5774  3.8410
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   85.37   9.24
##  classid.1 sex            0.00   0.00
##  schoolid  (Intercept)  280.66  16.75
##  Residual              1147.87  33.88
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  522.7741     2.2888  163.7302 228.405   <2e-16 ***
## sex           -0.4633     2.0727 1123.8032  -0.224    0.823
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
##     (Intr)
## sex -0.456
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
# Insert code to compare models

#First without random slopes

M10a <- lmerTest::lmer(math1st~sex + (1|schoolid/classid),data=classroom)
summary(M10a)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11941.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1922 -0.6131 -0.0238  0.5774  3.8410
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.38   9.24
##  schoolid         (Intercept)  280.66  16.75
##  Residual                     1147.87  33.88
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  522.7741     2.2888  163.7311 228.405   <2e-16 ***
## sex           -0.4633     2.0727 1123.8032  -0.224    0.823
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##     (Intr)
## sex -0.456
```

```
#Now with random slopes - M6

anova(M10a, M10, refit = F)
```

```
## Data: classroom
## Models:
## M10a: math1st ~ sex + (1 | schoolid/classid)
## M10: math1st ~ sex + (0 + sex | classid) + (1 | schoolid) + (1 | classid)
##      Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M10a  5 11951 11977 -5970.6    11941
## M10   6 11953 11984 -5970.6    11941     0      1          1
```

```
rand(M10)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ sex + (0 + sex | classid) + (1 | schoolid) + (1 | classid)
##                            npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                        6 -5970.6 11953
## sex in (0 + sex | classid)    5 -5970.6 11951  0.000  1    0.99998
## (1 | schoolid)                5 -5997.7 12006 54.273  1  1.745e-13 ***
## (1 | classid)                 5 -5973.4 11957  5.581  1    0.01816 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of sex across schools as p-value is not statistically significant (pvalue = 1.00), fail to reject the null hypothesis of the ANOVA test between the two models. The extremely small value for the estimate of $\sigma^2_{\zeta_1}$ random slope estimate of sex also indicates little systematic variation across schools for sex.

b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

Response:

It is a bad idea to include a classroom level variable with random slopes at the classroom level because its variance is already being accounted for in the null model by including the differential effects of classid clustering on the outcome math1st.

c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.

```
# Insert code to fit first model in Q5d model and print summary

M8_corr <- lmerTest::lmer(math1st~ ses + (ses|classid) + (1|schoolid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00213079 (tol = 0.002, component 1)
```

```
summary(M8_corr)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ ses + (ses | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 11878.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.3245 -0.5682 -0.0489  0.5807  3.5812
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   69.35   8.328
```

```
##           ses           45.38   6.736   0.51
##  schoolid (Intercept)  234.59  15.316
##  Residual              1084.72  32.935
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  523.111      1.903 101.084 274.908  < 2e-16 ***
## ses           11.794      1.580 166.635   7.466 4.39e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##     (Intr)
## ses 0.090
## convergence code: 0
## Model failed to converge with max|grad| = 0.00213079 (tol = 0.002, component 1)
```

```
# Insert code to compare models

anova(M8a, M8_corr, refit = F)
```

```
## Data: classroom
## Models:
## M8a: math1st ~ ses + (1 | schoolid/classid)
## M8_corr: math1st ~ ses + (ses | classid) + (1 | schoolid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M8a      5 11890 11916 -5940.3    11880
## M8_corr  7 11892 11928 -5939.1    11878 2.4026      2     0.3008
```

```
rand(M8_corr)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ ses + (ses | classid) + (1 | schoolid)
##                         npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                     7 -5939.1 11892
## ses in (ses | classid)     5 -5940.3 11890  2.403  2     0.3008
## (1 | schoolid)             6 -5961.2 11934 44.244  1  2.899e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of ses across schools as p-value is not statistically significant (pvalue = 0.3008), fail to reject the null hypothesis of the ANOVA test between the two models indicating no need for additional 2 parameters for correlated random slopes compared to a random intercept model.

```
# Insert code to fit second model in Q5d model and print summary

M9_corr <- lmerTest::lmer(math1st~ minority + (minority|classid)+(1|schoolid),data=classroom)
summary(M9_corr)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ minority + (minority | classid) + (1 | schoolid)
```

```
##    Data: classroom
##
## REML criterion at convergence: 11890.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2691 -0.6214 -0.0238  0.5924  3.8787
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  208.6   14.44
##           minority     254.7   15.96    -0.80
##  schoolid (Intercept)  176.5   13.29
##  Residual              1092.0   33.05
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  537.693      2.918 155.982 184.287  < 2e-16 ***
## minority     -21.020      3.128 194.325  -6.719 1.97e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## minority -0.793
```

```
# Insert code to compare models

anova(M9a, M9_corr, refit = F)
```

```
## Data: classroom
## Models:
## M9a: math1st ~ minority + (1 | schoolid/classid)
## M9_corr: math1st ~ minority + (minority | classid) + (1 | schoolid)
##         Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M9a      5 11904 11929 -5946.9    11894
## M9_corr  7 11904 11940 -5945.1    11890 3.5944      2     0.1658
```

```
rand(M9_corr)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## math1st ~ minority + (minority | classid) + (1 | schoolid)
##                                  npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                              7 -5945.1 11904
## minority in (minority | classid)    5 -5946.9 11904  3.5944  2     0.1658
## (1 | schoolid)                      6 -5957.3 11927 24.5383  1  7.285e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of minority across schools as p-value is not statistically significant (pvalue = 0.1658), fail to reject the null hypothesis of the ANOVA test between the two models indicating no need for additional 2 parameters for correlated random slopes compared to a random intercept model.

```r
# Insert code to fit third model in Q5d model and print summary
M10_corr <- lmerTest::lmer(math1st~ sex + (sex|classid)+(1|schoolid),data=classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl = control$checkConv, :
## Model failed to converge with max|grad| = 0.00253493 (tol = 0.002, component 1)
```

```r
summary(M10_corr)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ sex + (sex | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 11941
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1092 -0.6121 -0.0203  0.5889  3.8714
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  109.800 10.479
##           sex            9.413  3.068   -0.79
##  schoolid (Intercept)  281.758 16.786
##  Residual              1144.594 33.832
## Number of obs: 1190, groups:  classid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.727      2.313 143.097   226.0   <2e-16 ***
## sex           -0.416      2.081 228.694    -0.2    0.842
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##     (Intr)
## sex -0.472
## convergence code: 0
## Model failed to converge with max|grad| = 0.00253493 (tol = 0.002, component 1)
```

```r
# Insert code to compare models

anova(M10a, M10_corr, refit = F)
```

```
## Data: classroom
## Models:
## M10a: math1st ~ sex + (1 | schoolid/classid)
## M10_corr: math1st ~ sex + (sex | classid) + (1 | schoolid)
##          Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M10a      5 11951 11977 -5970.6    11941
## M10_corr  7 11955 11991 -5970.5    11941 0.2221      2     0.8949
```

```r
rand(M10_corr)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
```

```
## Model:
## math1st ~ sex + (sex | classid) + (1 | schoolid)
##                         npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                     7 -5970.5 11955
## sex in (sex | classid)     5 -5970.6 11951  0.222  2     0.8949
## (1 | schoolid)             6 -5997.7 12007 54.431  1   1.61e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

No observation of significant changes of sex across schools as p-value is not statistically significant (pvalue = 0.8949), fail to reject the null hypothesis of the ANOVA test between the two models indicating no need for additional 2 parameters for correlated random slopes compared to a random intercept model.

Question 7.

a. Try to add a random slope for each student level predictor varying at the school level:

```
# Insert code to fit first model in Q7a model and print summary
```

```
# Insert code to compare models
```

Response:

```
# Insert code to fit second model in Q7a model and print summary
```

```
# Insert code to compare models
```

Response:

```
# Insert code to fit third model in Q7a model and print summary
```

```
# Insert code to compare models
```

Response:

b. Retry the above, allowing the slopes to be correlated with the random intercepts.

```
# Insert code to fit first model in Q7b model and print summary
```

```
# Insert code to compare models
```

Response:

```
# Insert code to fit second model in Q7b model and print summary
```

```
# Insert code to compare models
```

Response:

```
# Insert code to fit third model in Q7b model and print summary
```

```
# Insert code to compare models
```

Response:

c. Report anything unusual about the variance components (changes that are unexpected)

Response:

Question 8.

a. Take the two predictors that had significant random slopes, in the forms in which they worked (indep. or correlated) and add both to the model, and test for need of one conditional on needing the other.

```
# Fit models and run LRT tests
```

Response:

b. Is the more complex model (with both random slopes in it) justified?

```
# Insert code to compare models
```

Reponse:

c. WRITE OUT THIS MODEL in your preferred notation

The model is:

Question 9.

a. For UMM, write down: V_S, V_C, V_E for the three variance components (simply the estimates)

```
# If you want to look at your UMM insert code here or you can just do this in line
```

V_S =

V_C =

V_E =

b. For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

```
# If you want to look at your model insert code here or you can just do this in line
```

V_S =

V_C =

V_E =

c. By what fraction did these each decrease with the new predictors in the model?

V_S:

V_C:

V_E:

Question 10. Now consider the model with a random slope in ses.

a. What are: V_C, V_S(ses=0), V_E ?

```
# If you want to look at your model insert code here or you can just do this in line
```

V_S(ses=0) =

V_C =

V_E =

b. What are: V_S(ses=-0.50), V_S(ses=+0.5) ?

V_S(ses=0.5) =

V_S(ses=-0.5) =

Question 11.

Now consider the model with a random slope in minority.

a. What are: V_C, V_S(minority=0), V_E ?

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
```

V_S(minority=0) =

V_C =

V_E =

b. What are: V_S(minority=0.25), V_S(minority=+0.50), V_S(minority=+0.75) ?

V_S(minority=0.25) =

```
# V_S(minority = 0.25)
# Insert code if you want to do the calculations in R
```

V_S(minority=0.5) =

```
# V_S(minority = 0.50)
# Insert code if you want to do the calculations in R
```

V_S(minority=0.75) =

```
# V_S(minority = 0.75)
# Insert code if you want to do the calculations in R
```

Question 12.

Now consider the model with a random slope in ses & minority.

a. What are: V_C, V_S(minority=0,ses=0), V_E ? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
```

V_S(sex=0, minority=0) =

V_C =

V_E =

b. In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Response:

c. Can we make a similar statement about ζ_{0k}?

Response:

d. If you had a large value for η_{0jk}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Response:

e. If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Response: