

# MLM Final Project Part 1

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## Question 0.

Load classroom.csv and create MATH1ST (fit all models using REML, use lmerTest::lmer)

```
# Insert code to load data and create math1st variable
classroom <- read.csv("/Users/mbp/Documents/NYU/APSTA 2042 - Multi-level Models (Nested)/Project 1/MLM-1")
classroom <- classroom %>% mutate(math1st = mathkind + mathgain)
```

## Question 1.

Estimate UMM model with random intercepts for both schools and classrooms.

```
# Insert code to fit model and print summary

M1_UMM <- lmerTest::lmer(math1st ~ (1 | schoolid/classid), data=classroom)
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
## Groups              Name                Variance Std.Dev.
## classid:schoolid (Intercept)    85.47     9.245
## schoolid          (Intercept)   280.69    16.754
## Residual                        1146.79    33.864
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)   522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

### a. Report the ICC for schools and classrooms:

Response:

- $\hat{\sigma}_{\eta}^2 = 85.47$

- $\hat{\sigma}_{\zeta}^2 = 280.69$
- $\hat{\sigma}_{\epsilon}^2 = 1146.79$

$$ICC_{school} = \frac{280.69}{85.47 + 280.69 + 1146.79} = 18.55\%$$

$$ICC_{classroom} = \frac{85.47}{85.47 + 280.69 + 1146.79} = 5.65\%$$

**b. Write out the model:**

Model 1 Equation:

$$MATH1ST_{ijk} = b_0 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with  $\zeta_k \sim N(0, \sigma_{\zeta}^2)$ ,  $\eta_{jk} \sim N(0, \sigma_{\eta}^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$ , independent of each other  
and  $k$  = schools,  $j$  = classrooms and  $i$  = students

**Question 2.**

**Add all school-level predictors:**

Model 2 Equation:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with  $\zeta_k \sim N(0, \sigma_{\zeta}^2)$ ,  $\eta_{jk} \sim N(0, \sigma_{\eta}^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$ , independent of each other  
and  $k$  = schools,  $j$  = classrooms and  $i$  = students

```
# Insert code to fit model and print summary
M2 <- lmerTest::lmer(math1st ~ housepov + (1 | schoolid/classid), data=classroom)
summary(M2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
## Groups             Name             Variance Std.Dev.
## classid:schoolid (Intercept)    82.36    9.075
## schoolid          (Intercept)   250.93   15.841
## Residual                        1146.96   33.867
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)   531.294      3.341 102.807 159.023 <2e-16 ***
```

```
## housepov      -45.783      14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## housepov -0.810
```

a. Report if the additional predictors are justified:

```
# Insert code to compare models
anova(M1_UMM, M2, refit = F)
```

```
## Data: classroom
## Models:
## M1_UMM: math1st ~ (1 | schoolid/classid)
## M2: math1st ~ housepov + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## M1_UMM  4 11953 11973 -5972.3    11945
## M2      5 11937 11963 -5963.7    11927 17.186      1 3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the school-level predictor is justified according to the ANOVA between the unconditional means model, and the model with the school-level predictor HOUSEPOV. The chi-square test results in a statistically significant p-value (at the 0.05 alpha level) of approximately 0.

b. Report the change to school variance:

Response:

- Change in  $\sigma_{\zeta}^2 = 250.93 - 280.69 = -29.76$
- The school variance changed from 280.69 in Model 1 (UMM null model) to 250.93 indicating that adding the covariate, HOUSEPOV, accounted for variance at the school level.

### Question 3: Add all class-level predictors

Model 3 Equation:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with  $\zeta_k \sim N(0, \sigma_{\zeta}^2)$ ,  $\eta_{jk} \sim N(0, \sigma_{\eta}^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$ , independent of each other

and  $k$  = schools,  $j$  = classrooms and  $i$  = students

```
# Insert code to fit model and print summary
```

```
M3 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid), data=
summary(M3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
```

```
## Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid:schoolid (Intercept)    94.36   9.714
## schoolid         (Intercept)   223.31  14.943
## Residual                        1136.43  33.711
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  532.29852    5.20495  228.85767 102.268 < 2e-16 ***
## housepov     -41.62117   14.08834  109.83230  -2.954  0.00383 **
## mathknow       2.55143    1.44530  231.06560   1.765  0.07883 .
## mathprep      -0.75440    1.42809  203.20755  -0.528  0.59790
## yearstea       0.06193    0.14717  223.76570   0.421  0.67432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw mthprp
## housepov    -0.568
## mathknow     -0.052  0.082
## mathprep     -0.666  0.032  0.004
## yearstea     -0.264  0.077  0.030 -0.175
```

a. Report if adding the predictors is justified:

```
linearHypothesis(M3, c("mathknow", "mathprep", "yearstea"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## mathknow = 0
## mathprep = 0
## yearstea = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + mathknow + mathprep + yearstea + (1 | schoolid/classid)
##
##      Df  Chisq Pr(>Chisq)
## 1
## 2  3 3.4804    0.3233
```

Response:

The addition of classroom level covariates of MATHKNOW, MATHPREP, and YEARSTEA is not justified because the p-value ( $p = 0.3233$ ) of the WALD test is not significant at the 0.05 alpha level. This suggests the benefit of adding classroom level predictors jointly is not significant in comparison to the previous model

containing student level predictors.

**b. Report changes in class-level variance and individual variance:**

Response:

- Change in  $\sigma_{\eta}^2 = 94.36 - 82.36 = 12.00$
- Change in  $\sigma_{\epsilon}^2 = 1136.43 - 1146.96 = -10.53$
- Change in classroom level variance  $\hat{\sigma}_{\eta}^2$  changed from 85.47 in Model 1 (UMM null model) to 82.36 in Model 2 and finally to 94.36 in Model 3 indicating that adding the classroom level covariates increases classroom level variance.
- Change in individual level variance  $\hat{\sigma}_{\epsilon}^2$  changed from 1146.79 in Model 1 (UMM null model) to 1146.96 in Model 2 and finally to 1136.43 in Model 3 indicating that adding the classroom level covariates reduced individual variance.

**c. Give a potential reason to explain why individual variance but not class variance is reduced:**

Response:

Adding the classroom-level predictors shows a potential misspecification of the model. The classroom level predictors can make it difficult to estimate the individual level variance (i.e overstated individual variance) due to individual outliers in classrooms that have a very small amount of students. > GO THROUGH RECORDING

**Question 4.**

Add all student-level predictors excepting mathgain and mathkind:

```
# Insert code to fit model and print summary
```

```
M4 <- lmerTest::lmer(math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority + sex + (1 | schoolid/classid))
summary(M4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## sex + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##   Groups             Name             Variance Std.Dev.
## classid:schoolid (Intercept)    93.89    9.689
## schoolid          (Intercept)   169.45   13.017
## Residual                        1064.96   32.634
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error          df t value Pr(>|t|)
```

```
## (Intercept) 539.63041    5.31209  275.39010 101.585 < 2e-16 ***
## housepov   -17.64850   13.21755  113.87814  -1.335    0.184
## mathknow    1.35004    1.39168  234.49768   0.970    0.333
## mathprep   -0.27705    1.37583  205.27111  -0.201    0.841
## yearstea    0.01129    0.14141  226.80861   0.080    0.936
## ses         10.05076   1.54485 1066.56211   6.506 1.18e-10 ***
## minority   -16.18676   3.02605  704.47787  -5.349 1.20e-07 ***
## sex        -1.21419    2.09483 1022.42110  -0.580    0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw mthprp yearst ses      minrty
## housepov -0.451
## mathknow  -0.083  0.058
## mathprep  -0.631  0.038  0.004
## yearstea  -0.259  0.071  0.029 -0.172
## ses       -0.121  0.082 -0.007  0.053 -0.028
## minority  -0.320 -0.178  0.115  0.001  0.024  0.162
## sex       -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011
```

a. Report if the block of predictors is justified:

```
# Insert code to compare models
```

```
linearHypothesis(M4, c("sex", "minority", "ses"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## sex = 0
## minority = 0
## ses = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##          sex + (1 | schoolid/classid)
##
##    Df  Chisq Pr(>Chisq)
##  1
##  2  3 85.055 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of student level covariates of SES, MINORITY, and SEX is justified because the p-value ( $p = 0$ ) of the WALD test is significant at the 0.05 alpha level. This suggests the benefit of adding student level predictors jointly is significant in comparison to the previous model containing school and classroom-level predictors.

b. Report change in all variance components

Response:

- Change in  $\sigma_{\zeta}^2 = 169.45 - 223.31 = -53.86$

- Change in  $\sigma_\eta^2 = 93.89 - 94.36 = -0.47$
- Change in  $\sigma_\epsilon^2 = 1064.96 - 1136.43 = -71.47$
- Change in classroom level variance ( $\hat{\sigma}_\eta^2$ ) changed from 85.47 in Model 1 (UMM null model), to 82.36 in Model 2, to 94.36 in Model 3, and then to 93.89 in Model 4 with a marginal decrease in classroom level variance.
- Change in school variance ( $\hat{\sigma}_\zeta^2$ ) changed from 280.69 in Model 1 (UMM null model), to 250.93 in Model 2, to 223.31 in Model 3, and finally to 169.45 in Model 4 with a decrease in school level variance observed after adding student level covariates in Model 4.
- Change in individual student level variance ( $\hat{\sigma}_\epsilon^2$ ) changed from 1146.79 in Model 1 (UMM null model), to 1146.96 in Model 2, to 1136.43 in Model 3, and finally to 1064.96 in Model 4 indicating that student level variance decreased after adding student level covariates in Model 4.

c. Give a potential reason as to why the school variance drops from the last model:

Response:

The student-level predictors explain some variance at the school level. SES, MINORITY, and SEX of children vary between different schools and may impact math scores. For example, some schools located in poorer areas with a different demographic of students will have much different individual math scores than those in more affluent areas.

d. Write this model out:

Model 4 Equation:

$$MATH1ST_{ijk} = b_0 + b_1HOUSEPOV_k + b_2YEARSTEA_{jk} + b_3MATHKNOW_{jk} + b_4MATHPREP_{jk} + b_5SEX_{ijk} + b_6MINORITY_{ijk} + b_7SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with  $\zeta_k \sim N(0, \sigma_\zeta^2)$ ,  $\eta_{jk} \sim N(0, \sigma_\eta^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$ , independent of each other  
and  $k$  = schools,  $j$  = classrooms and  $i$  = students

## Question 5.

a. Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately - not all together)

b. Report the models and their fit.

```
M5 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + yearstea | schoolid) + (1|schoolid/classid), data = classroom)

## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0080545
## (tol = 0.002, component 1)

summary(M5)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
## Data: classroom
##
```

```

## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
##      Groups             Name             Variance  Std.Dev.
## classid.schoolid (Intercept) 9.247e+01  9.6159
## schoolid         (Intercept) 1.684e+02 12.9758
## schoolid.1       yearstea      1.008e-02  0.1004
## Residual                        1.065e+03 32.6361
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780  266.47953 101.662 < 2e-16 ***
## housepov     -17.72082   13.21686  113.58577  -1.341  0.183
## yearstea       0.01128    0.14192  122.87741   0.079  0.937
## mathknow       1.33106    1.39155  234.33195   0.957  0.340
## mathprep      -0.26584    1.37588  204.90504  -0.193  0.847
## sex           -1.21060    2.09480 1022.21558  -0.578  0.563
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## ses           10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.450
## yearstea    -0.258  0.070
## mathknow    -0.082  0.057  0.028
## mathprep    -0.632  0.037 -0.172  0.003
## sex         -0.191 -0.007  0.015  0.006 -0.006
## minority    -0.320 -0.179  0.023  0.115  0.001 -0.010
## ses         -0.121  0.082 -0.027 -0.007  0.053  0.020  0.162
## convergence code: 0
## Model failed to converge with max|grad| = 0.0080545 (tol = 0.002, component 1)

```

```
anova(M4,M5, refit = F)
```

```

## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M5: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M5:      ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M5 12 10754 10813 -5364.8    10730 0.007      1      0.9336

```

Response:

The addition of random slope on the *YEARSTEA* variable is not significant (p-value = 0.9336) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.



```

M6 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + mathknow | schoolid) + (1|schoolid/classid), data = classroom)

## boundary (singular) fit: see ?isSingular

summary(M6)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups           Name             Variance Std.Dev.
## classid.schoolid (Intercept) 9.389e+01  9.689914
## schoolid         (Intercept) 1.694e+02 13.016328
## schoolid.1       mathknow      1.700e-06  0.001304
## Residual                             1.065e+03 32.633705
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63047    5.31204   275.40357 101.586 < 2e-16 ***
## housepov     -17.64821   13.21718   113.88792  -1.335  0.184
## yearstea       0.01129    0.14141   226.81110   0.080  0.936
## mathknow       1.34993    1.39168   234.50067   0.970  0.333
## mathprep      -0.27708    1.37583   205.27196  -0.201  0.841
## sex           -1.21417    2.09483  1022.42010  -0.580  0.562
## minority     -16.18681    3.02603   704.47306  -5.349 1.20e-07 ***
## ses            10.05075    1.54485  1066.56262   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.451
## yearstea    -0.259  0.071
## mathknow    -0.083  0.058  0.029
## mathprep    -0.631  0.038 -0.172  0.004
## sex         -0.190 -0.007  0.016  0.007 -0.006
## minority    -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses         -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular

```

```
anova(M4, M6, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M6: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M6:      ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M6 12 10754 10813 -5364.8    10730      0      1      1
```

Response:

The addition of random slope on the *MATHKNOW* variable is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M7 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + mathprep | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M7)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups             Name             Variance Std.Dev.
## classid.schoolid (Intercept) 9.388e+01 9.689e+00
## schoolid          (Intercept) 1.694e+02 1.302e+01
## schoolid.1        mathprep      2.171e-07 4.659e-04
## Residual                          1.065e+03 3.263e+01
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39223 101.586 < 2e-16 ***
## housepov     -17.64851   13.21749  113.87941  -1.335    0.184
## yearstea       0.01129    0.14141  226.80838   0.080    0.936
## mathknow       1.35003    1.39167  234.49786   0.970    0.333
## mathprep     -0.27705    1.37582  205.27063  -0.201    0.841
## sex          -1.21419    2.09483 1022.42070  -0.580    0.562
## minority    -16.18676    3.02605  704.47629 -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56201  6.506 1.18e-10 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv yearst mthknw mthprp sex    minrty
## houspov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
anova(M4, M7, refit = F)

## Data: classroom
## Models:
## M4: math1st ~ houspov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M7: math1st ~ houspov + yearstea + mathknow + mathprep + sex + minority +
## M7:      ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8    10730
## M7 12 10754 10813 -5364.8    10730      0      1          1
```

Response:

The addition of random slope on the *MATHPREP* variable is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

### c. Why is it a bad idea to include a random slope on the houspov effect?

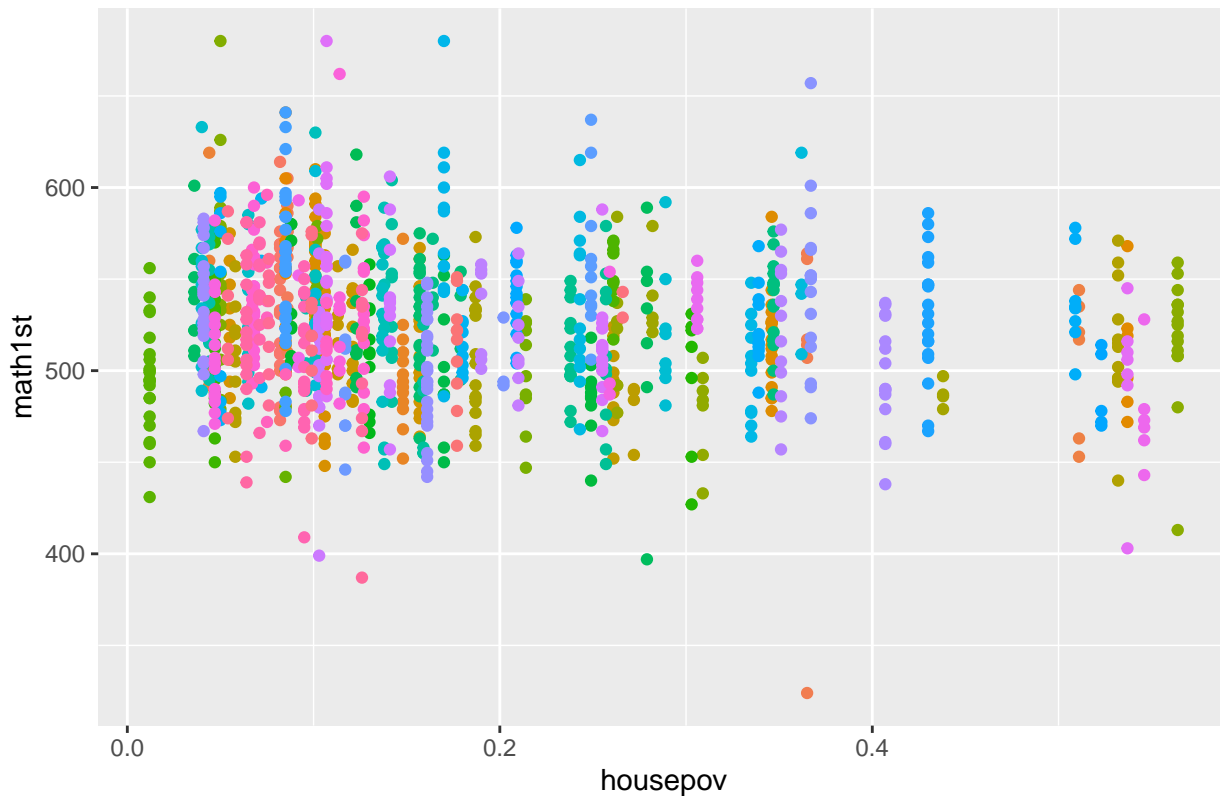
Response:

It is a bad idea to include a random slope on the *HOUSEPOV* effect because its variance is already being accounted for in the null model by including the differential effects of schoolid on the outcome *MATH1ST*. *HOUSEPOV* is a school-level predictor and cannot vary at the classroom or individual level - every individual would have the same *HOUSEPOV* level within a school.

The plot below illustrates that each individual in a school have the same *HOUSEPOV*:

```
ggplot(classroom, aes(x=houspov, y=math1st, color = factor(schoolid))) +
  geom_point() +
  ggtitle("MATH1ST Scores v. HOUSEPOV Level") +
  theme(legend.position = "none")
```

## MATH1ST Scores v. HOUSEPOV Level



d. Retry the above models, allowing the slopes to be correlated with the random intercepts (still one by one):

```
M8 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (yearstea | schoolid) + (1 | classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0287242
## (tol = 0.002, component 1)
```

```
summary(M8)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##   ses + (yearstea | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##   Min       1Q   Median       3Q      Max
## -3.7469 -0.6029 -0.0282  0.6034  3.8430
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##
```

```
## classid (Intercept) 38.7450 6.2245
## schoolid (Intercept) 365.8546 19.1273
## yearstea 0.5508 0.7422 -0.78
## Residual 1066.0342 32.6502
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 538.95831 5.49083 222.68166 98.156 < 2e-16 ***
## housepov -17.12865 13.46221 119.58919 -1.272 0.206
## yearstea 0.02197 0.15769 75.74039 0.139 0.890
## mathknow 1.04763 1.34485 210.60696 0.779 0.437
## mathprep 0.04798 1.34661 191.68881 0.036 0.972
## sex -1.33487 2.08766 1024.47814 -0.639 0.523
## minority -16.44853 2.99666 669.72921 -5.489 5.74e-08 ***
## ses 10.14964 1.53869 1062.66735 6.596 6.64e-11 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) housepv yearst mthknw mthprp sex minrty
## housepov -0.455
## yearstea -0.370 0.084
## mathknow -0.085 0.049 0.012
## mathprep -0.606 0.050 -0.139 0.014
## sex -0.184 -0.004 0.009 0.008 -0.004
## minority -0.305 -0.169 0.032 0.122 -0.007 -0.012
## ses -0.119 0.079 -0.019 -0.001 0.049 0.022 0.168
## convergence code: 0
## Model failed to converge with max|grad| = 0.0287242 (tol = 0.002, component 1)
```

```
anova(M4, M8, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4: sex + (1 | schoolid/classid)
## M8: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M8: ses + (yearstea | schoolid) + (1 | classid)
## Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8 10730
## M8 13 10750 10814 -5361.8 10724 5.825 2 0.05434 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *YEARSTEA* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value = 0.05434) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M9 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (mathknow | schoolid) + (1 | classid), data = classroom)
summary(M9)
```

```

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## mathlst ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (mathknow | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 9.393e+01 9.6915
## schoolid (Intercept) 1.693e+02 13.0118
## mathknow 9.182e-04 0.0303 0.97
## Residual 1.065e+03 32.6341
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 539.64041 5.31203 275.38950 101.588 < 2e-16 ***
## housepov -17.64141 13.21242 103.98187 -1.335 0.185
## yearstea 0.01114 0.14141 226.85275 0.079 0.937
## mathknow 1.35458 1.39201 214.62548 0.973 0.332
## mathprep -0.27754 1.37599 201.27747 -0.202 0.840
## sex -1.21328 2.09485 1021.79809 -0.579 0.563
## minority -16.19378 3.02608 703.80364 -5.351 1.18e-07 ***
## ses 10.04788 1.54488 1062.12259 6.504 1.20e-10 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv yearst mthknw mthprp sex minrty
## housepov -0.451
## yearstea -0.259 0.071
## mathknow -0.082 0.057 0.029
## mathprep -0.631 0.038 -0.173 0.004
## sex -0.190 -0.007 0.016 0.007 -0.006
## minority -0.320 -0.178 0.024 0.115 0.001 -0.011
## ses -0.121 0.082 -0.028 -0.007 0.053 0.020 0.162
anova(M4, M9, refit = F)

## Data: classroom
## Models:
## M4: mathlst ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4: sex + (1 | schoolid/classid)
## M9: mathlst ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M9: ses + (mathknow | schoolid) + (1 | classid)
## Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4 11 10752 10806 -5364.8 10730
## M9 13 10756 10820 -5364.8 10730 3e-04 2 0.9998

```

Response:

The addition of random slope on the *MATHKNOW* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value = 0.9998) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M10 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +  
  sex + minority + ses + (mathprep | schoolid) + (1 | classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [  
## lmerModLmerTest]  
## Formula:  
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +  
##      ses + (mathprep | schoolid) + (1 | classid)  
## Data: classroom  
##  
## REML criterion at convergence: 10724.7  
##  
## Scaled residuals:  
##      Min       1Q   Median       3Q      Max   
## -3.8542 -0.6034 -0.0221  0.5915  3.6475   
##  
## Random effects:  
## Groups Name Variance Std.Dev. Corr  
## classid (Intercept) 78.46 8.858  
## schoolid (Intercept) 552.68 23.509  
## mathprep 15.88 3.985 -1.00  
## Residual 1064.27 32.623  
## Number of obs: 1081, groups: classid, 285; schoolid, 105  
##  
## Fixed effects:  
## Estimate Std. Error df t value Pr(>|t|)  
## (Intercept) 538.60872 5.60800 159.90129 96.043 < 2e-16 ***  
## housepov -14.01327 12.88651 116.07030 -1.087 0.279  
## yearstea -0.02586 0.13948 223.50199 -0.185 0.853  
## mathknow 1.29875 1.37192 229.68405 0.947 0.345  
## mathprep 0.04067 1.34844 139.04813 0.030 0.976  
## sex -1.16756 2.08698 1023.14893 -0.559 0.576  
## minority -16.46421 2.99522 663.67387 -5.497 5.52e-08 ***  
## ses 10.14167 1.53961 1060.93434 6.587 7.04e-11 ***  
## ---  
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Correlation of Fixed Effects:  
## (Intr) houspv yearst mthknw mthprp sex minrty  
## housepov -0.461  
## yearstea -0.260 0.089  
## mathknow -0.071 0.027 0.048  
## mathprep -0.692 0.107 -0.155 0.012  
## sex -0.183 0.003 0.023 0.002 -0.008  
## minority -0.275 -0.187 0.025 0.107 -0.035 -0.013  
## ses -0.121 0.095 -0.033 -0.001 0.061 0.024 0.161
```

```
## convergence code: 0
## boundary (singular) fit: see ?isSingular
anova(M4, M10, refit = F)

## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M10: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M10:      ses + (mathprep | schoolid) + (1 | classid)
##      Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M10  13 10751 10816 -5362.3    10725  4.8144     2    0.09007 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *MATHPREP* variable, allowing the random slope to be correlated with the random intercept, is not significant (p-value=0.09007) at the 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

**e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occurred (hint: what did you add to the model?).**

Response:

- After adding uncorrelated random slopes varying by schools on *YEARSTEA*, *MATHKNOW*, and *MATHPREP*, the variances of the random slopes are all close to 0. All other variance components remained almost the same as the random-intercept model.
- After adding correlated random slopes varying by schools on *YEARSTEA*, and *MATHKNOW*, the variances due to classrooms decreased for both. For the *YEARSTEA* model, the variances due to classrooms increased. After adding a correlated random slope for *MATHPREP*, the variance due to schools increased significantly, and had a -1 correlation with its random slope (i.e. singularity).

## Question 6.

**a. Try to add a random slope for each student level predictor (varying at the classroom level; one by one - not all together)**

```
M11 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + sex | classid) + (1|schoolid/classid), data = classroom)
summary(M11)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + sex | classid) + (1 | schoolid/classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
```



```
##      Min      1Q  Median      3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups          Name          Variance Std.Dev.
## classid         sex          3.310e-05  0.005753
## classid:schoolid (Intercept) 9.387e+01  9.688824
## schoolid        (Intercept) 1.695e+02 13.017987
## Residual                    1.065e+03 32.633681
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63033    5.31211   275.37965 101.585 < 2e-16 ***
## housepov     -17.64878   13.21784   113.87028  -1.335  0.184
## yearstea      0.01129    0.14141   226.80606   0.080  0.936
## mathknow      1.35013    1.39167   234.49478   0.970  0.333
## mathprep     -0.27702    1.37582   205.26985  -0.201  0.841
## sex          -1.21421    2.09483  1022.41563  -0.580  0.562
## minority     -16.18672    3.02607   704.48078  -5.349 1.20e-07 ***
## ses           10.05076    1.54485  1066.56153   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) housepv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

```
anova(M4, M11, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M11: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M11:      ses + (0 + sex | classid) + (1 | schoolid/classid)
##      Df    AIC    BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M11  12 10754 10813 -5364.8    10730      0      1      1
```

Response:

The addition of random slope on the *SEX* variable, varying by classrooms, is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M12 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(M12)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + minority | classid) + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups             Name                Variance Std.Dev.
## classid            minority             0.00   0.00
## classid:schoolid (Intercept)    93.89   9.69
## schoolid           (Intercept)  169.45  13.02
## Residual                        1064.95  32.63
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38908 101.585 < 2e-16 ***
## housepov     -17.64848   13.21758  113.87764  -1.335  0.184
## yearstea       0.01129    0.14141  226.80896   0.080  0.936
## mathknow       1.35004    1.39168  234.49773   0.970  0.333
## mathprep      -0.27705    1.37583  205.27155  -0.201  0.841
## sex           -1.21419    2.09483 1022.42137  -0.580  0.562
## minority     -16.18678    3.02605  704.47894  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56222   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex       -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses       -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(M4, M12, refit = F)
```

```
## Data: classroom
## Models:
```

```
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M12: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M12:      ses + (0 + minority | classid) + (1 | schoolid/classid)
##      Df    AIC    BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M12 12 10754 10813 -5364.8    10730      0      1      0.9999
```

Response:

The addition of random slope on the *MINORITY* variable, varying by classrooms, is not significant (p-value = 1) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M13 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
      sex + minority + ses + (0 + ses | classid) + (1|schoolid/classid), data = classroom)
summary(M13)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + ses | classid) + (1 | schoolid/classid)
##      Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##      Groups             Name             Variance Std.Dev.
##      classid             ses              49.60    7.043
##      classid:schoolid (Intercept)      87.11    9.333
##      schoolid             (Intercept)  171.02   13.077
##      Residual                        1043.44   32.302
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710 < 2e-16 ***
## housepov     -17.50879   13.21775  113.44882  -1.325   0.188
## yearstea       0.01103    0.14117  226.97682   0.078   0.938
## mathknow       1.36796    1.38563  229.40643   0.987   0.325
## mathprep     -0.27938    1.37171  204.89332  -0.204   0.839
## sex          -1.37733    2.09334 1022.81814  -0.658   0.511
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.451
```

```
## yearstea -0.259 0.070
## mathknow -0.082 0.058 0.029
## mathprep -0.631 0.040 -0.172 0.005
## sex -0.190 -0.007 0.014 0.006 -0.005
## minority -0.321 -0.180 0.025 0.111 0.002 -0.011
## ses -0.108 0.081 -0.026 0.002 0.050 0.020 0.145
```

```
anova(M4, M13, refit = F)
```

```
## Data: classroom
```

```
## Models:
```

```
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
```

```
## M4:      sex + (1 | schoolid/classid)
```

```
## M13: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
```

```
## M13:      ses + (0 + ses | classid) + (1 | schoolid/classid)
```

```
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
```

```
## M4  11 10752 10806 -5364.8    10730
```

```
## M13 12 10752 10812 -5364.0    10728 1.5969      1    0.2063
```

Response:

The addition of random slope on the *SES* variable, varying by classrooms, is not significant (p-value = 0.2063) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

**b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?**

Response:

It is a bad idea to include a classroom level variable with random slopes at the classroom level because its variance is already being accounted for in the null model by including the differential effects of *classid* clustering on the outcome *MATH1ST*.

The plots below illustrate that each classroom level predictors (*MATHKNOW*, *MATHPREP*, *YEARSTEA*) are the same across each classroom:

```
p2 <- ggplot(classroom, aes(x=yearstea, y=math1st, color = factor(classid))) +
  geom_point() +
  #geom_smooth(method=lm, color="red", fill="#69b3a2", se=TRUE)+
  theme(legend.position = "none")+
  ggtitle("MATH1ST Scores v. Classroom-Level Predictors Across Classrooms")
```

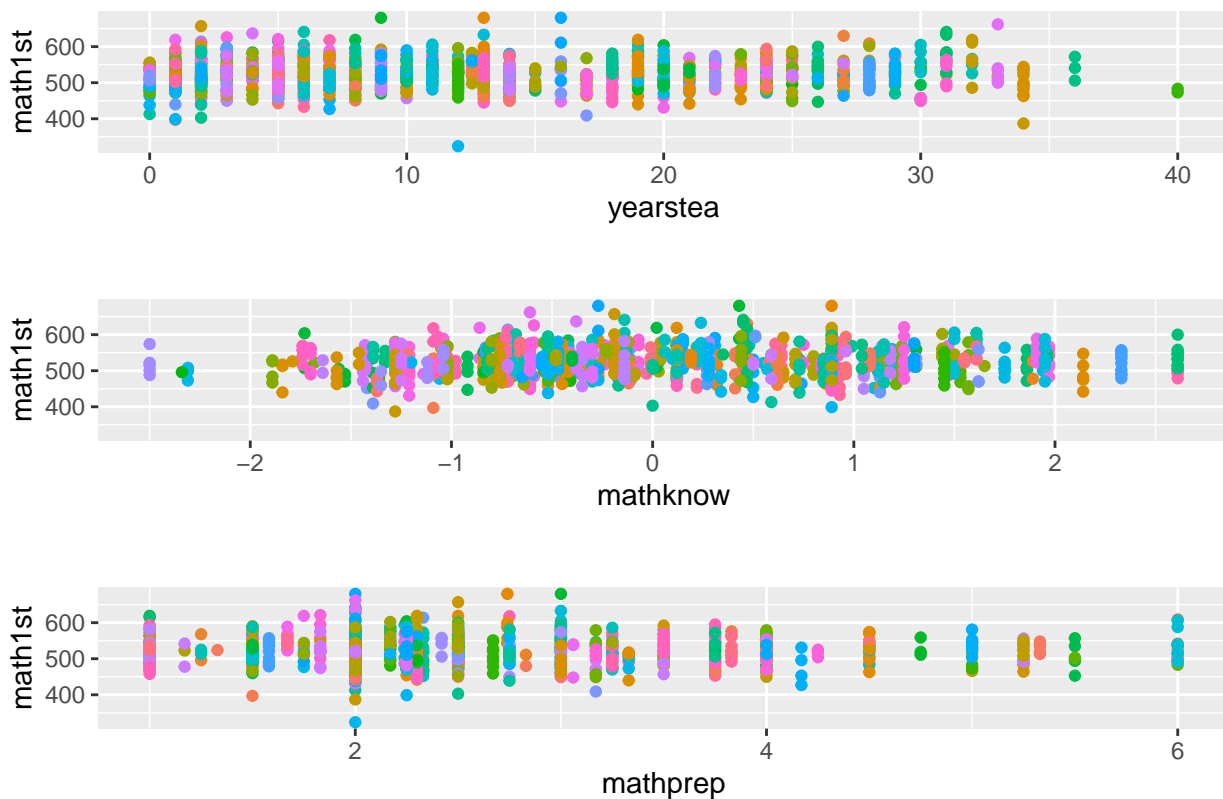
```
p3 <- ggplot(classroom, aes(x=mathknow, y=math1st, color = factor(classid))) +
  geom_point() +
  #geom_smooth(method=lm, color="red", fill="#69b3a2", se=TRUE)+
  theme(legend.position = "none")+
  ggtitle("")
```

```
p4 <- ggplot(classroom, aes(x=mathprep, y=math1st, color = factor(classid))) +
  geom_point() +
  #geom_smooth(method=lm, color = "red", fill="#69b3a2", se=TRUE)+
  theme(legend.position = "none")+
  ggtitle("")
```

```
gridExtra::grid.arrange(p2, p3, p4)
```

```
## Warning: Removed 109 rows containing missing values (geom_point).
```

## MATH1ST Scores v. Classroom-Level Predictors Across Classrooms



c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.

```
M14 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (sex | classid) + (1 | schoolid), data = classroom)
summary(M14)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##   ses + (sex | classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##   Min       1Q   Median       3Q      Max
## -3.7562 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   classid  (Intercept)         130.2    11.411
##           sex                 31.5     5.612  -0.67
##   schoolid (Intercept)         169.9    13.035
##   Residual                    1056.3    32.502
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
```

```
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.279 < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382   0.170
## yearstea     3.053e-03  1.416e-01  2.270e+02   0.022   0.983
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939   0.349
## mathprep    -3.460e-01  1.374e+00  2.014e+02  -0.252   0.801
## sex         -1.197e+00  2.123e+00  2.158e+02  -0.564   0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses         1.010e+01  1.544e+00  1.065e+03   6.539 9.61e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.452
## yearstea -0.258  0.072
## mathknow -0.085  0.060  0.029
## mathprep -0.628  0.040 -0.174  0.005
## sex      -0.203 -0.005  0.015  0.003 -0.008
## minority -0.321 -0.178  0.024  0.116  0.003 -0.009
## ses      -0.123  0.083 -0.027 -0.005  0.054  0.020  0.164
anova(M4, M14, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M14: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M14:      ses + (sex | classid) + (1 | schoolid)
##      Df   AIC   BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8   10730
## M14  13 10755 10820 -5364.5   10729 0.5003     2    0.7787
```

Response:

The addition of random slope on the *SEX* variable, varying by classrooms and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.7787) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M15 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (minority | classid) + (1 | schoolid), data = classroom)
summary(M15)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (minority | classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10726.3
##
```

```
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.9036 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##   Groups   Name      Variance Std.Dev. Corr
##   classid (Intercept) 225.4    15.01
##           minority    171.3    13.09  -0.82
##   schoolid (Intercept) 157.4    12.55
##   Residual          1045.3    32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.73593    5.38021  270.70839 100.319 < 2e-16 ***
## housepov    -17.34685    12.91273   103.34823  -1.343  0.182
## yearstea    -0.01636     0.14285   234.25604  -0.115  0.909
## mathknow     1.45697     1.39354   234.05425   1.046  0.297
## mathprep    -0.13522     1.37018   203.97781  -0.099  0.921
## sex         -1.01014     2.08966  1015.73459  -0.483  0.629
## minority    -16.48615     3.21756   183.24221  -5.124 7.55e-07 ***
## ses          9.89350     1.54595  1062.82952   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.435
## yearstea -0.265  0.080
## mathknow -0.079  0.061  0.038
## mathprep -0.618  0.037 -0.171 -0.006
## sex      -0.188 -0.009  0.015  0.009 -0.005
## minority -0.368 -0.171  0.025  0.108 -0.004 -0.009
## ses      -0.117  0.085 -0.023  0.001  0.051  0.021  0.149
anova(M4, M15, refit = F)

## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M15: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M15:      ses + (minority | classid) + (1 | schoolid)
##      Df   AIC   BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M15  13 10752 10817 -5363.2    10726 3.1967     2    0.2022
```

Response:

The addition of random slope on the *MINORITY* variable, varying by classrooms and allowing for correlation between random slope and random intercept, is not significant (p-value = 0.2022) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M16 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  ses + minority + ses + (ses | classid) + (1 | schoolid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
## Warning: Model failed to converge with 1 negative eigenvalue: -3.5e+02
summary(M16)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (ses | classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10733.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7815 -0.6172 -0.0348  0.6046  3.8587
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 0.00 0.00
##      ses 66.59 8.16 NaN
## schoolid (Intercept) 198.66 14.09
## Residual 1091.72 33.04
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 539.20226 5.04059 366.90683 106.972 < 2e-16 ***
## housepov -18.33722 13.12647 110.81073 -1.397 0.165
## yearstea 0.03344 0.12846 827.45931 0.260 0.795
## mathknow 1.48810 1.26459 754.59810 1.177 0.240
## mathprep -0.16719 1.23860 867.95858 -0.135 0.893
## sex -1.44037 2.10650 1035.47989 -0.684 0.494
## minority -15.99717 3.03333 689.35568 -5.274 1.79e-07 ***
## ses 10.32651 1.67733 184.92901 6.157 4.52e-09 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv yearst mthknw mthprp sex minrty
## housepov -0.464
## yearstea -0.249 0.068
## mathknow -0.085 0.060 0.035
## mathprep -0.600 0.040 -0.177 0.003
## sex -0.198 -0.007 0.015 0.001 -0.011
## minority -0.340 -0.182 0.023 0.103 0.004 -0.009
## ses -0.107 0.079 -0.032 0.000 0.051 0.027 0.138
## convergence code: 0
## boundary (singular) fit: see ?isSingular
anova(M4, M16, refit = F)

## Data: classroom
## Models:
```



```
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M16: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M16:      ses + (ses | classid) + (1 | schoolid)
##      Df    AIC    BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8    10730
## M16 13 10760 10825 -5366.9    10734      0      2      1
```

Response:

The addition of random slope on the *SES* variable, varying by classrooms and allowing for correlation between the random slope and random intercept, is not significant (p-value = 1) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

## Question 7.

a. Try to add a random slope for each student level predictor varying at the school level:

```
M17 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
                      sex + minority + ses + (0 + sex | schoolid) + (1 | schoolid/classid), data = classroom)
summary(M17)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + sex | schoolid) + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5556
##
## Random effects:
## Groups          Name          Variance Std.Dev.
## classid.schoolid (Intercept)   96.08   9.802
## schoolid         (Intercept)  161.63  12.713
## schoolid.1       sex           35.85   5.987
## Residual                    1054.36  32.471
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.43513    5.30741  272.54817 101.638 < 2e-16 ***
## housepov     -16.77631   13.22883  112.39531  -1.268   0.207
## yearstea       0.01448    0.14163  226.44545   0.102   0.919
## mathknow       1.40068    1.39464  234.45910   1.004   0.316
## mathprep      -0.27193    1.38011  205.78600  -0.197   0.844
## sex          -1.33538    2.18749  138.10018  -0.610   0.543
## minority     -16.16537    3.02862  704.25875 -5.338 1.27e-07 ***
## ses           9.98475    1.54243 1058.28030  6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
##      (Intr) houspv yearst mthknw mthprp sex    minrty
## housepv -0.449
## yearstea -0.259  0.070
## mathknow -0.081  0.055  0.028
## mathprep -0.633  0.036 -0.172  0.004
## sex      -0.179 -0.010  0.013  0.007 -0.004
## minority -0.320 -0.178  0.024  0.114  0.001 -0.015
## ses      -0.120  0.081 -0.029 -0.007  0.052  0.020  0.161
```

```
anova(M4, M17, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepv + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M17: math1st ~ housepv + yearstea + mathknow + mathprep + sex + minority +
## M17:      ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M17  12 10753 10813 -5364.4    10729 0.6137      1    0.4334
```

Response:

The addition of random slope on the *SEX* variable, varying by schools, is not significant (p-value = 0.4334) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M18 <- lmerTest::lmer(math1st ~ housepv + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + minority | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -3.9e+00
```

```
summary(M18)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepv + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + minority | schoolid) + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups           Name              Variance Std.Dev.
## classid.schoolid (Intercept) 9.388e+01  9.689369
## schoolid         (Intercept) 1.694e+02 13.017176
## schoolid.1       minority      1.777e-06  0.001333
## Residual                    1.065e+03 32.633690
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
```

```
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63040    5.31208  275.39128 101.586 < 2e-16 ***
## housepov     -17.64850   13.21752  113.87888  -1.335  0.184
## yearstea       0.01129    0.14141  226.80855   0.080  0.936
## mathknow       1.35003    1.39168  234.49782   0.970  0.333
## mathprep      -0.27705    1.37582  205.27091  -0.201  0.841
## sex           -1.21419    2.09483 1022.42090  -0.580  0.562
## minority      -16.18676    3.02605  704.47696  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56207   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) housepv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex       -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses       -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
anova(M4, M18, refit = F)

## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M18: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M18:      ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8   10730
## M18  12 10754 10813 -5364.8   10730      0    1      1
```

Response:

The addition of random slope on the *MINORITY* variable, varying by schools, is not significant (p-value = 1.00) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M19 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
summary(M19)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##      Data: classroom
##
## REML criterion at convergence: 10724.8
```

```
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##      Groups             Name             Variance Std.Dev.
## classid.schoolid (Intercept)    88.56    9.411
## schoolid          (Intercept)  168.00   12.961
## schoolid.1        ses           72.50    8.515
## Residual                                1035.11  32.173
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926   270.52802 102.124 < 2e-16 ***
## housepov     -16.94575   13.21161   112.81447  -1.283   0.202
## yearstea      0.03079    0.14052   223.94252   0.219   0.827
## mathknow      1.35586    1.38461   232.19737   0.979   0.328
## mathprep     -0.19799    1.35995   198.59551  -0.146   0.884
## sex          -1.40187    2.08169  1011.29089  -0.673   0.501
## minority     -16.52526    3.02191   700.07600  -5.468 6.32e-08 ***
## ses           9.78982    1.82216    79.01650   5.373 7.61e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.451
## yearstea    -0.260  0.070
## mathknow    -0.079  0.056  0.028
## mathprep    -0.628  0.041 -0.172  0.002
## sex         -0.190 -0.007  0.018  0.006 -0.007
## minority    -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses         -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
anova(M4, M19, refit = F)

## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M19: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M19:      ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8   10730
## M19  12 10749 10809 -5362.4   10725 4.6972     1    0.03021 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *SES* variable, varying by schools, is **significant** (p-value = 0.03021) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Retry the above, allowing the slopes to be correlated with the random intercepts.

```
M20 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (sex | schoolid) + (1 | schoolid:classid), data = classroom)

## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00526732
## (tol = 0.002, component 1)

summary(M20)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | schoolid) + (1 | schoolid:classid)
## Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8050 -0.6094 -0.0223  0.5970  3.5528
##
## Random effects:
## Groups              Name                Variance Std.Dev. Corr
## schoolid:classid (Intercept)    97.29   9.863
## schoolid          (Intercept)  206.02  14.353
## sex                sex           83.73   9.151  -0.43
## Residual                        1041.89  32.278
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.675 < 2e-16 ***
## housepov     -1.742e+01  1.325e+01  1.136e+02  -1.314   0.191
## yearstea      6.921e-03  1.418e-01  2.277e+02   0.049   0.961
## mathknow      1.379e+00  1.396e+00  2.364e+02   0.988   0.324
## mathprep     -2.796e-01  1.378e+00  2.061e+02  -0.203   0.839
## sex          -1.340e+00  2.300e+00  8.740e+01  -0.583   0.562
## minority     -1.642e+01  3.027e+00  7.076e+02  -5.425 7.97e-08 ***
## ses           9.929e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.449
## yearstea    -0.258  0.072
## mathknow    -0.082  0.060  0.027
## mathprep    -0.627  0.038 -0.172  0.004
## sex         -0.222 -0.003  0.014  0.006 -0.005
## minority    -0.319 -0.178  0.024  0.114  0.004 -0.011
## ses         -0.121  0.083 -0.028 -0.006  0.053  0.018  0.163
## convergence code: 0
```

```
## Model failed to converge with max|grad| = 0.00526732 (tol = 0.002, component 1)
```

```
anova(M4, M20, refit = F)
```

```
## Data: classroom
```

```
## Models:
```

```
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
```

```
## M4:      sex + (1 | schoolid/classid)
```

```
## M20: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
```

```
## M20:      ses + (sex | schoolid) + (1 | schoolid:classid)
```

```
##      Df    AIC    BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
```

```
## M4   11 10752 10806 -5364.8    10730
```

```
## M20  13 10754 10818 -5363.8    10728 1.863      2      0.394
```

```
Response:
```

The addition of random slope on the *SEX* variable, varying by schools and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.394) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M21 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +  
      sex + minority + ses + (minority | schoolid) + (1 | schoolid:classid), data = classroom)  
summary(M21)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
```

```
## lmerModLmerTest]
```

```
## Formula:
```

```
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
```

```
##      ses + (minority | schoolid) + (1 | schoolid:classid)
```

```
##      Data: classroom
```

```
##
```

```
## REML criterion at convergence: 10717.5
```

```
##
```

```
## Scaled residuals:
```

```
##      Min      1Q  Median      3Q      Max
```

```
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
```

```
##
```

```
## Random effects:
```

```
##      Groups      Name      Variance Std.Dev. Corr
```

```
## schoolid:classid (Intercept)  86.7    9.311
```

```
## schoolid      (Intercept) 381.2   19.524
```

```
##      minority      343.2   18.525  -0.83
```

```
## Residual      1039.4   32.240
```

```
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
```

```
##
```

```
## Fixed effects:
```

```
##      Estimate Std. Error      df t value Pr(>|t|)
```

```
## (Intercept) 539.49369    5.65513 173.09179  95.399 < 2e-16 ***
```

```
## housepov    -16.06251   12.57477  99.99134  -1.277  0.204
```

```
## yearstea     -0.00437    0.13765 217.17884  -0.032  0.975
```

```
## mathknow     1.63216    1.35929 224.78144   1.201  0.231
```

```
## mathprep    -0.29178    1.33537 198.06922  -0.218  0.827
```

```
## sex         -0.86278    2.08382 1021.81437  -0.414  0.679
```

```
## minority    -16.37547    3.89604  58.24604  -4.203 9.17e-05 ***
```

```
## ses          9.43095    1.54335 1063.13485   6.111 1.39e-09 ***
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv yearst mthknw mthprp sex    minrty
## housepv -0.394
## yearstea -0.253  0.091
## mathknow -0.078  0.061  0.024
## mathprep -0.576  0.037 -0.167 -0.002
## sex      -0.172 -0.013  0.014  0.010 -0.005
## minority -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses      -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
```

```
anova(M4, M21, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepv + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M21: math1st ~ housepv + yearstea + mathknow + mathprep + sex + minority +
## M21:      ses + (minority | schoolid) + (1 | schoolid:classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4   11 10752 10806 -5364.8    10730
## M21  13 10744 10808 -5358.8    10718 11.967      2    0.00252 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *MINORITY* variable, varying by schools and allowing for correlation between the random slope and random intercept, **is significant** (p-value= 0.00252) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

```
M22 <- lmerTest::lmer(math1st ~ housepv + yearstea + mathknow + mathprep +
                      sex + minority + ses + (ses | schoolid) + (1 | schoolid:classid), data = classroom)
summary(M22)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepv + yearstea + mathknow + mathprep + sex + minority +
##      ses + (ses | schoolid) + (1 | schoolid:classid)
##      Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5647 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##      Groups              Name              Variance Std.Dev. Corr
## schoolid:classid (Intercept)      86.62     9.307
## schoolid         (Intercept)    171.12    13.081
##                  ses             73.35     8.565    0.19
## Residual                  1035.89    32.185
```

```
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  538.72268    5.27645   271.16175 102.099 < 2e-16 ***
## housepov     -15.89853   13.15319   111.73535  -1.209   0.229
## yearstea      0.03616    0.14002   220.43198   0.258   0.796
## mathknow      1.26005    1.38204   230.90960   0.912   0.363
## mathprep     -0.21707    1.35647   197.11400  -0.160   0.873
## sex          -1.40428    2.08074  1011.40155  -0.675   0.500
## minority     -16.26714    3.03575   668.89455  -5.359 1.16e-07 ***
## ses           9.72644    1.82981    78.36254   5.316 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) housepv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.259  0.073
## mathknow -0.077  0.057  0.028
## mathprep -0.627  0.039 -0.172  0.001
## sex      -0.188 -0.009  0.017  0.005 -0.008
## minority -0.325 -0.183  0.021  0.108  0.002 -0.011
## ses      -0.062  0.070 -0.021  0.007  0.045  0.018  0.117
anova(M4, M22, refit = F)
```

```
## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## M22: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## M22:      ses + (ses | schoolid) + (1 | schoolid:classid)
##      Df   AIC   BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## M4  11 10752 10806 -5364.8   10730
## M22 13 10750 10815 -5362.2   10724 5.1385     2    0.07659 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *SES* variable, varying by schools and allowing for correlation between the random slope and random intercept, is not significant (p-value = 0.07659) at the alpha 0.05 level of significance, according to the ANOVA LRT comparing the model with and without the random slope addition.

Response:

### c. Report anything unusual about the variance components (changes that are unexpected)

Response:

- After adding an uncorrelated random slope on *MINORITY* varying by schools, the variance due to the random slope is approximately 0. For the other variables, *SEX* and *SES*, the variances for the slopes are positive, indicating that these random slopes explain some of the variation between schools.
- After adding a correlated random slope on *MINORITY* varying by schools, the variances for the random slope and intercept on schools increased significantly (343.2 and 381.2, respectively), and the correlation



is -0.83. For the other variables, *SEX* and *SES*, the variances remained similar to the uncorrelated version.

## Question 8.

a. Take the two predictors that had significant random slopes, in the forms in which they worked (indep. or correlated) and add both to the model, and test for need of one conditional on needing the other.

```
# Fit models and run LRT tests
Mslope1 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
Mslope2 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (minority | schoolid) + (1|schoolid:classid), data = classroom)
Mslope3 <- lmerTest::lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
  sex + minority + ses + (0 + ses | schoolid) + (minority | schoolid) + (1|schoolid:classid)

## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00333241
## (tol = 0.002, component 1)

anova(Mslope3, Mslope1, refit = F)

## Data: classroom
## Models:
## Mslope1: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope1:      ses + (0 + ses | schoolid) + (1 | schoolid/classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:      ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:      schoolid:classid)
##          Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## Mslope1 12 10749 10809 -5362.4    10725
## Mslope3 14 10740 10810 -5356.2    10712 12.39      2    0.00204 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

anova(Mslope3, Mslope2, refit = F)

## Data: classroom
## Models:
## Mslope2: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope2:      ses + (minority | schoolid) + (1 | schoolid:classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:      ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:      schoolid:classid)
##          Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## Mslope2 13 10744 10808 -5358.8    10718
## Mslope3 14 10740 10810 -5356.2    10712  5.12      1    0.02365 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the random slopes on the two predictors, *SES* and *MINORITY*, with *SES* random slope not correlated with *SES* random intercept and *MINORITY* random slope correlated with *MINORITY* random intercept, is significant (p-value = 0.00204, p-value = 0.02365, respectively) at the alpha 0.05 significance

level, and are needed based on the LRT comparing the need of one random slope, conditional on needing the other.

**b. Is the more complex model (with both random slopes in it) justified?**

```
# Insert code to compare models
anova(M4, Mslope3, refit = F)

## Data: classroom
## Models:
## M4: math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
## M4:      sex + (1 | schoolid/classid)
## Mslope3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## Mslope3:      ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## Mslope3:      schoolid:classid)
##      Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## M4      11 10752 10806 -5364.8    10730
## Mslope3 14 10740 10810 -5356.2    10712 17.087      3 0.0006782 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Reponse:

The addition of the random slopes on the two predictors, *SES* and *MINORITY*, with *SES* random slope not correlated with *SES* random intercept and *MINORITY* random slope correlated with *MINORITY* random intercept, is significant (p-value = 0.0006782) at the alpha 0.05 significance level, meaning that the more complex model with both random slopes is justified based on the LRT comparing the complex model with the random intercepts only model.

**c. WRITE OUT THIS MODEL in your preferred notation**

The model is:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + b_5 SEX_{ijk} + (b_6 + \zeta_{6k}) MINORITY_{ijk} + (b_7 + \zeta_{7k}) SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with  $\zeta_k \sim N(0, \sigma_\zeta^2)$ ,  $\eta_{jk} \sim N(0, \sigma_\eta^2)$ ,  $\zeta_{6k} \sim N(0, \sigma_{\zeta_6}^2)$ ,  $\zeta_{7k} \sim N(0, \sigma_{\zeta_7}^2)$  and  $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$ , independent of each other,

$$corr(\zeta_{0k}, \zeta_{6k}) = \rho_{\zeta_0, \zeta_6},$$

and  $k$  = schools,  $j$  = classrooms and  $i$  = students

**Question 9.**

**a. For UMM, write down: V\_S, V\_C, V\_E for the three variance components (simply the estimates)**

```
# If you want to look at your UMM insert code here or you can just do this in line
summary(M1_UMM)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ (1 | schoolid/classid)
## Data: classroom
##
```

```
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##   Groups             Name             Variance Std.Dev.
##   classid:schoolid (Intercept)    85.47    9.245
##   schoolid          (Intercept)  280.69   16.754
##   Residual                        1146.79   33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

V_S = 280.69

V_C = 85.47

V_E = 1146.79
```

b. For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V\_C, V\_S, V\_E?

*# If you want to look at your model insert code here or you can just do this in line*  
[`summary\(M4\)`](#)

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + mathknow + mathprep + yearstea + ses + minority +
##      sex + (1 | schoolid/classid)
##   Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##   Groups             Name             Variance Std.Dev.
##   classid:schoolid (Intercept)    93.89    9.689
##   schoolid          (Intercept)  169.45   13.017
##   Residual                        1064.96   32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209 275.39010 101.585 < 2e-16 ***
## housepov     -17.64850   13.21755 113.87814  -1.335   0.184
```

```
## mathknow      1.35004      1.39168  234.49768   0.970   0.333
## mathprep      -0.27705      1.37583  205.27111  -0.201   0.841
## yearstea       0.01129      0.14141  226.80861   0.080   0.936
## ses           10.05076      1.54485 1066.56211   6.506 1.18e-10 ***
## minority      -16.18676      3.02605  704.47787  -5.349 1.20e-07 ***
## sex           -1.21419      2.09483 1022.42110  -0.580   0.562
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) housepv mthknw mthprp yearst ses      minrty
## housepv -0.451
## mathknow -0.083  0.058
## mathprep -0.631  0.038  0.004
## yearstea -0.259  0.071  0.029 -0.172
## ses      -0.121  0.082 -0.007  0.053 -0.028
## minority -0.320 -0.178  0.115  0.001  0.024  0.162
## sex      -0.190 -0.007  0.007 -0.006  0.016  0.020 -0.011

V_S = 169.45

V_C = 93.89

V_E = 1064.96
```

c. By what fraction did these each decrease with the new predictors in the model?

V\_S: 39.63% decrease

V\_C: 9.85% increase

V\_E: 7.14% decrease

Question 10. Now consider the model with a random slope in ses.

a. What are: V\_C, V\_S(ses=0), V\_E ?

```
# If you want to look at your model insert code here or you can just do this in line
summary(Mslope1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepv + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
## Groups              Name              Variance Std.Dev.
```

```
## classid.schoolid (Intercept) 88.56 9.411
## schoolid (Intercept) 168.00 12.961
## schoolid.1 ses 72.50 8.515
## Residual 1035.11 32.173
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 539.13752 5.27926 270.52802 102.124 < 2e-16 ***
## housepov -16.94575 13.21161 112.81447 -1.283 0.202
## yearstea 0.03079 0.14052 223.94252 0.219 0.827
## mathknow 1.35586 1.38461 232.19737 0.979 0.328
## mathprep -0.19799 1.35995 198.59551 -0.146 0.884
## sex -1.40187 2.08169 1011.29089 -0.673 0.501
## minority -16.52526 3.02191 700.07600 -5.468 6.32e-08 ***
## ses 9.78982 1.82216 79.01650 5.373 7.61e-07 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) housepv yearst mthknw mthprp sex minrty
## housepov -0.451
## yearstea -0.260 0.070
## mathknow -0.079 0.056 0.028
## mathprep -0.628 0.041 -0.172 0.002
## sex -0.190 -0.007 0.018 0.006 -0.007
## minority -0.323 -0.180 0.024 0.110 0.001 -0.010
## ses -0.091 0.076 -0.019 0.006 0.042 0.017 0.124
```

$V_S(\text{ses}=0) = 168.00$

$V_C = 88.56$

$V_E = 1035.11$

b. What are:  $V_S(\text{ses}=-0.50)$ ,  $V_S(\text{ses}=+0.5)$  ?

$V_S(\text{ses}=0.5) = 168 + (0.5^2) \cdot (72.50) = 186.125$

$V_S(\text{ses}=-0.5) = 168 + (-0.5^2) \cdot (72.50) = 186.125$

## Question 11.

Now consider the model with a random slope in minority.

a. What are:  $V_C$ ,  $V_S(\text{minority}=0)$ ,  $V_E$  ?

*# If you want to look at your model/variance components insert code here or you can just do this in line*  
`summary(Mslope2)`

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## ses + (minority | schoolid) + (1 | schoolid:classid)
## Data: classroom
```

```
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##      Groups             Name             Variance Std.Dev. Corr
## schoolid:classid (Intercept)      86.7      9.311
## schoolid          (Intercept)    381.2     19.524
##                minority          343.2     18.525    -0.83
## Residual                        1039.4     32.240
## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.49369    5.65513    173.09179  95.399 < 2e-16 ***
## housepov     -16.06251   12.57477     99.99134  -1.277   0.204
## yearstea      -0.00437    0.13765    217.17884  -0.032   0.975
## mathknow       1.63216    1.35929    224.78144   1.201   0.231
## mathprep      -0.29178    1.33537    198.06922  -0.218   0.827
## sex           -0.86278    2.08382   1021.81437  -0.414   0.679
## minority     -16.37547    3.89604     58.24604  -4.203 9.17e-05 ***
## ses           9.43095    1.54335   1063.13485   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.394
## yearstea    -0.253  0.091
## mathknow    -0.078  0.061  0.024
## mathprep    -0.576  0.037 -0.167 -0.002
## sex         -0.172 -0.013  0.014  0.010 -0.005
## minority    -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses         -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
```

$V\_S(\text{minority}=0) = 381.2$

$V\_C = 86.70$

$V\_E = 1039.40$

b. What are:  $V\_S(\text{minority}=0.25)$ ,  $V\_S(\text{minority}=+0.50)$ ,  $V\_S(\text{minority}=+0.75)$  ?

$V\_S(\text{minority}=0.25) = 252.55$

```
# V_S(minority = 0.25)
# Insert code if you want to do the calculations in R

# Calculate variance using the formula:
# V_S(minority = 0.25) = Var(Zeta_0) + Minority^2*Var(Zeta_1) + 2*Minority*CoV(Zeta_0,Zeta_1)

(var_minority.25 = 381.2 + (0.25^2)*(343.2) + 2*0.25*(18.525*19.524*-0.83))
```

```
## [1] 252.5519
V_S(minority=0.5) = 166.80
# V_S(minority = 0.50)
# Insert code if you want to do the calculations in R

# Calculate variance using the formula:
#  $V_S(\text{minority} = 0.50) = \text{Var}(\text{Zeta}_0) + \text{Minority}^2 \text{Var}(\text{Zeta}_1) + 2 \text{Minority} \text{CoV}(\text{Zeta}_0, \text{Zeta}_1)$ 
(var_minority.50 = 381.2 + (0.50^2)*(343.2) + 2*0.50*(18.525*19.524*-0.83))

## [1] 166.8039
V_S(minority=0.75) = 123.96
# V_S(minority = 0.75)
# Insert code if you want to do the calculations in R

# Calculate variance using the formula:
#  $V_S(\text{minority} = 0.75) = \text{Var}(\text{Zeta}_0) + \text{Minority}^2 \text{Var}(\text{Zeta}_1) + 2 \text{Minority} \text{CoV}(\text{Zeta}_0, \text{Zeta}_1)$ 
(var_minority.75 = 381.2 + (0.75^2)*(343.2) + 2*0.75*(18.525*19.524*-0.83))

## [1] 123.9558
```

## Question 12.

Now consider the model with a random slope in ses & minority.

a. What are:  $V_C$ ,  $V_S(\text{minority}=0, \text{ses}=0)$ ,  $V_E$  ? We need to list ‘ses=0, minority=0’ here, or we don’t know how to use the slope variance

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
summary(Mslope3)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
##      schoolid:classid)
## Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6525 -0.6251 -0.0339  0.6048  3.6961
##
## Random effects:
## Groups          Name              Variance Std.Dev. Corr
## schoolid.classid (Intercept)    80.62    8.979
## schoolid         (Intercept)   404.52   20.113
##                  minority      335.67   18.321  -0.84
## schoolid.1       ses           74.98    8.659
## Residual                    1009.73   31.776
```

```

## Number of obs: 1081, groups: schoolid:classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.05254    5.66462   165.75707  95.161 < 2e-16 ***
## housepov     -15.32101   12.49397    99.25400  -1.226  0.223
## yearstea      0.02105    0.13657   213.65286   0.154  0.878
## mathknow      1.67490    1.35000   221.33073   1.241  0.216
## mathprep     -0.23545    1.31729   191.22021  -0.179  0.858
## sex          -1.03898    2.06951  1010.41104  -0.502  0.616
## minority     -16.72828    3.90662    55.39944  -4.282 7.42e-05 ***
## ses           9.19642    1.82287    82.48705   5.045 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov    -0.395
## yearstea    -0.254  0.093
## mathknow    -0.072  0.060  0.024
## mathprep    -0.568  0.040 -0.166 -0.004
## sex         -0.170 -0.014  0.017  0.010 -0.005
## minority    -0.509 -0.149  0.027  0.092 -0.003 -0.013
## ses         -0.080  0.083 -0.011  0.006  0.041  0.020  0.087
## convergence code: 0
## Model failed to converge with max|grad| = 0.00333241 (tol = 0.002, component 1)
V_S(sex=0, minority=0) = 404.52
V_C = 80.62
V_E = 1009.73

```

**b. In the last model, what is a “likely” (+/- 1 sd) range for  $\eta_{0jk}$**

Response: [-8.979, 8.979]

**c. Can we make a similar statement about  $\zeta_{0k}$ ?**

Response:

Yes, because we assume that  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ . Therefore the range would be [-20.113, 20.113]

**d. If you had a large value for  $\eta_{0jk}$ , would you expect a large or small or “any” value for the two random slope terms,  $\zeta_{1k}$  and  $\zeta_{2k}$  for ses and minority?**

Response:

The two random slopes can take on any value because they are not related to classrooms (i.e. independent and not correlated).

**e. If you had a large value for  $\zeta_{0k}$ , would you expect a large or small or “any” value for the two random slope terms,  $\zeta_{1k}$  and  $\zeta_{2k}$  for ses and minority (discuss each separately)?**

Response:



We would expect any value for the SES random slope term since there is no correlation between the random slope and the random intercept, and a small value for the MINORITY random slope due to a negative correlation with the intercept (-0.84).