Multilevel Models - Project 1 Part 1

4/21/2020

Question 1

Estimate an Unconditional Means Model (UMM) with random intercepts for both schools and classrooms (nested in schools).

```
# Fit unconditional means model (UMM):
lm_umm <- lmer(math1st ~ 1 + (1|schoolid/classid), data = classroom)
summary(lm_umm)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ 1 + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                     1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a. Report the ICC for schools and the ICC for classrooms

Based on the summary output of the UMM model, the ICC for schools and classrooms are:

$$ICC_{school} = \frac{280.69}{85.47 + 280.69 + 1146.79} = 0.185525$$

$$ICC_{classroom} = \frac{85.47}{85.47 + 280.69 + 1146.79} = 0.05649228$$

b. WRITE OUT THIS MODEL using your preferred notation, but use the same choice of notation for the remainder of your project. Be mindful and explicit about any assumptions made.

$$MATH1ST_{ijk} = b_0 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with $\zeta_k \sim N(0, \sigma^2_\zeta)$, $\eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, independent of each other

Question 2

ADD ALL School level predictors:

```
# Add HOUSEPOV as a school level predictor to the UMM Model:
lm2 <- lmer(math1st ~ housepov + (1|schoolid/classid), data = classroom)
summary(lm2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.96  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.807 159.023   <2e-16 ***
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

a. Report if adding the predictors as a block is justifed:

```
# Run ANOVA to test if adding school-level predictor is justified
# (i.e. statistically significant change in variance components).
anova(lm_umm, lm2, refit = F)
```

```
## Data: classroom
## Models:
## lm_umm: math1st ~ 1 + (1 | schoolid/classid)
## lm2: math1st ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm_umm  4 11953 11973 -5972.3    11945
## lm2     5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The addition of the school-level predictor is justified according to the ANOVA between the unconditional means model, and the model with the school-level predictor HOUSEPOV. The chi-square test results in a p-value of approximately 0.

b. Report change in σ_ζ^2:

Change in $\sigma_\zeta^2 = 250.93 - 280.69 = -29.76$

Question 3

ADD ALL Classroom level predictors

```
# Add classroom-level predictors YEARSTEA, MATHKNOW, MATHPREP to prior model:
lm3 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep + (1|schoolid/classid), data = classroom)
summary(lm3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62116   14.08834 109.83230  -2.954  0.00383 **
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw
## housepov -0.568
## yearstea -0.264  0.077
## mathknow -0.052  0.082  0.030
## mathprep -0.666  0.032 -0.175  0.004
```

a. Report if adding the predictors as a block is justifed.

```
# Run ANOVA comparing previous model with only school-level predictors added:
  # Remove cases that have missing values:
  classroom_cc <- classroom[complete.cases(classroom),]

  # Re-fit models lm2 and lm3 using complete clases (fully observed):
  lm3_refit <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep + (1|schoolid/classid), data = c
  lm2_refit <- lmer(math1st ~ housepov + (1|schoolid/classid), data = classroom_cc)

  # Run ANOVA comparing lm2 and lm3 using complete clases (fully observed):
  anova(lm2_refit, lm3_refit, refit = F)
```

```
## Data: classroom_cc
## Models:
## lm2_refit: math1st ~ housepov + (1 | schoolid/classid)
## lm3_refit: math1st ~ housepov + yearstea + mathknow + mathprep + (1 | schoolid/classid)
##           Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm2_refit  5 10838 10862 -5413.8    10828
## lm3_refit  8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The addition of the classroom-level predictors as a block is not justified according to the ANOVA comparing the refit models using fully observed cases. The chi-square test results in a p-value of 0.08667.

b. Report change in σ^2_η and change in σ^2_ϵ

- Change in $\sigma^2_\eta = 94.36 - 82.36 = 12.00$
- Change in $\sigma^2_\epsilon = 1136.43 - 1146.96 = -10.53$

c. Give a potential reason as to why σ^2_ϵ is reduced, but not σ^2_η?

The decrease in the student-level variation (σ^2_ϵ) could be due to the classroom-level predictors impacting individual students' scores. Although classrooms may differ based on teacher, the impact on a change in a classroom-level predictor would be much greater on the student-level.

Question 4

ADD (nearly) ALL student level predictors (but not mathgain or mathkind, as these are outcomes in this context).

```
# Add student-level predictors SEX, MINORITY, SES to prior model:
lm4 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (1|schoolid/classid), data = classroom)
summary(lm4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                     1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

a. Report if justifed statistically as a block of predictors

```
# Run ANOVA comparing previous model that has only school & classroom-level predictors:
anova(lm3, lm4, refit = F)
```

```
## Data: classroom
## Models:
## lm3: math1st ~ housepov + yearstea + mathknow + mathprep + (1 | schoolid/classid)
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm3  8 10837 10877 -5410.5    10821
## lm4 11 10752 10806 -5364.8    10730 91.446      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The addition of the student-level predictors as a block is justified according to the ANOVA comparing the previous model containing school & classroom-level predictors, to the current model including school, classroom, and student-level predictors. The chi-square test results in a p-value of approximately 0.

b. Report change in variance components for all levels

- Change in $\sigma^2_\zeta = 169.45 - 223.31 = -53.86$
- Change in $\sigma^2_\eta = 93.89 - 94.36 = -0.47$
- Change in $\sigma^2_\epsilon = 1064.96 - 1136.43 = -71.47$

c. Give a potential reason as to why the school level variance component drops from prior model

The student-level predictors explain some variance at the school level. SES & Minority Status, and SEX composition of children vary between different schools and may impact math scores. For example, some schools located in poorer areas with a different demographic of students will have much different individual math scores than those in more affluent areas.

d. WRITE OUT THIS MODEL using your chosen notation.

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + b_5 SEX_{ijk} + b_6 MINORITY_{ijk} + b_7 SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with $\zeta_k \sim N(0, \sigma^2_\zeta)$, $\eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, independent of each other

Question 5

a. Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately - not all together)

```
# Add random slope effects varying at the school level, for each teacher-level predictor:

lm5 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + yearstea | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00805439
## (tol = 0.002, component 1)
```

```
lm6 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathknow | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
lm7 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathprep | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.247e+01  9.6159
##  schoolid         (Intercept) 1.684e+02 12.9758
##  schoolid.1       yearstea    1.008e-02  0.1004
##  Residual                     1.065e+03 32.6361
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780  266.47952 101.662  < 2e-16 ***
## housepov     -17.72082   13.21686  113.58577  -1.341    0.183
## yearstea       0.01128    0.14192  122.87733   0.079    0.937
## mathknow       1.33106    1.39155  234.33195   0.957    0.340
## mathprep      -0.26584    1.37588  204.90504  -0.193    0.847
## sex           -1.21060    2.09480 1022.21558  -0.578    0.563
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## ses           10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.450
## yearstea -0.258  0.070
## mathknow -0.082  0.057  0.028
## mathprep -0.632  0.037 -0.172  0.003
## sex      -0.191 -0.007  0.015  0.006 -0.006
## minority -0.320 -0.179  0.023  0.115  0.001 -0.010
## ses      -0.121  0.082 -0.027 -0.007  0.053  0.020  0.162
## convergence code: 0
## Model failed to converge with max|grad| = 0.00805439 (tol = 0.002, component 1)
```

```
summary(lm6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.389e+01  9.689914
##  schoolid         (Intercept) 1.694e+02 13.016328
##  schoolid.1       mathknow    1.700e-06  0.001304
##  Residual                     1.065e+03 32.633705
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63047    5.31204  275.40357 101.586  < 2e-16 ***
## housepov     -17.64821   13.21718  113.88792  -1.335    0.184
## yearstea       0.01129    0.14141  226.81110   0.080    0.936
## mathknow       1.34993    1.39168  234.50059   0.970    0.333
## mathprep      -0.27708    1.37583  205.27196  -0.201    0.841
## sex           -1.21417    2.09483 1022.42010  -0.580    0.562
## minority     -16.18681    3.02603  704.47306  -5.349 1.20e-07 ***
## ses           10.05075    1.54485 1066.56262   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
```

```
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
summary(lm7)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.388e+01 9.689e+00
##  schoolid         (Intercept) 1.694e+02 1.302e+01
##  schoolid.1       mathprep    2.171e-07 4.659e-04
##  Residual                     1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39222 101.586  < 2e-16 ***
## housepov     -17.64851   13.21749  113.87941  -1.335    0.184
## yearstea       0.01129    0.14141  226.80838   0.080    0.936
## mathknow       1.35003    1.39167  234.49786   0.970    0.333
## mathprep      -0.27705    1.37582  205.27063  -0.201    0.841
## sex           -1.21419    2.09483 1022.42070  -0.580    0.562
## minority     -16.18676    3.02605  704.47629  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56201   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
### QUESTION: Do we add each separately, then run separate anovas for each random slope
### addition comparing to no random slopes? or run an anova on all of them included?
```

b. Report the model fit or lack of fit

```
# Run ANOVA on each random slope model comparing to the model with no random slopes:
anova(lm4,lm5, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm5: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm5:     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm5 12 10754 10813 -5364.8    10730 0.007      1     0.9336
```

```
anova(lm4,lm6, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm6: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm6:     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm6 12 10754 10813 -5364.8    10730     0      1          1
```

```
anova(lm4,lm7, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm7: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm7:     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm7 12 10754 10813 -5364.8    10730     0      1          1
```

> The ANOVAs for each model with a random slope addition show that the addition of random slopes on the teacher level predictors, varying by school, is not significant.

c. Why is it a bad idea to include a random slope on the housepov effect?

d. Retry the above, allowing the slopes to be correlated with the random intercepts (still one by one)

e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any

Question 6

a. Try to add a random slope for each student level predictor (varying at the classroom level; one by one - not all together)

```
# Add random slope effects varying at the classroom-level, for each student-level predictor:
lm8 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | classid) + (1|schoolid/classid), data = classroom)

lm9 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
lm10 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | classid) + (1|schoolid/classid), data = classroom)

summary(lm8)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid          sex         3.310e-05  0.005754
##  classid:schoolid (Intercept) 9.387e+01  9.688824
##  schoolid         (Intercept) 1.695e+02 13.017987
##  Residual                     1.065e+03 32.633681
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63033    5.31211  275.37965 101.585  < 2e-16 ***
## housepov     -17.64878   13.21784  113.87028  -1.335    0.184
## yearstea       0.01129    0.14141  226.80606   0.080    0.936
## mathknow       1.35013    1.39167  234.49478   0.970    0.333
## mathprep      -0.27702    1.37582  205.26984  -0.201    0.841
## sex           -1.21421    2.09483 1022.41564  -0.580    0.562
## minority     -16.18672    3.02607  704.48078  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56152   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
```

```
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

```
summary(lm9)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          minority       0.00   0.00
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38908 101.585  < 2e-16 ***
## housepov     -17.64848   13.21758  113.87764  -1.335    0.184
## yearstea       0.01129    0.14141  226.80896   0.080    0.936
## mathknow       1.35004    1.39168  234.49773   0.970    0.333
## mathprep      -0.27705    1.37583  205.27155  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47894  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56222   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          ses           49.60   7.043
##  classid:schoolid (Intercept)   87.11   9.333
##  schoolid         (Intercept)  171.02  13.077
##  Residual                     1043.44  32.302
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.44881  -1.325    0.188
## yearstea       0.01103    0.14117  226.97682   0.078    0.938
## mathknow       1.36796    1.38563  229.40643   0.987    0.325
## mathprep      -0.27938    1.37171  204.89333  -0.204    0.839
## sex           -1.37733    2.09334 1022.81814  -0.658    0.511
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.070
## mathknow -0.082  0.058  0.029
## mathprep -0.631  0.040 -0.172  0.005
## sex      -0.190 -0.007  0.014  0.006 -0.005
## minority -0.321 -0.180  0.025  0.111  0.002 -0.011
## ses      -0.108  0.081 -0.026  0.002  0.050  0.020  0.145
```

b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.