MLM Final Project Part 1

April 30 2020

Team Members and division of work:

Question 0.

Load classroom.csv and create MATH1ST (fit all models using REML)

```
classroom <- foreign::read.dta("/Users/mbp/Documents/NYU/APSTA 2042 - Multi-level Models (Nested)/Datas
classroom <- classroom %>% mutate(math1st = mathkind + mathgain)
```

Question 1.

Estimate UMM model with random intercepts for both schools and classrooms.

```
lm_umm <- lmer(math1st ~ 1 + (1|schoolid/classid), data = classroom)
summary(lm_umm)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ 1 + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.47   9.245
##  schoolid         (Intercept)  280.69  16.754
##  Residual                      1146.79  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.403   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

a. Report the ICC for schools and classrooms:

Response:

Based on the summary output of the UMM model, the ICC for schools and classrooms are:

$$ICC_{school} = \frac{280.69}{85.47 + 280.69 + 1146.79} = 0.185525$$

$$ICC_{classroom} = \frac{85.47}{85.47 + 280.69 + 1146.79} = 0.05649228$$

b. Write out the model:

Model 1 Equation:

$$MATH1ST_{ijk} = b_0 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

Question 2.

Add all school-level predictors:

Model 2 Equation:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

```
lm2 <- lmer(math1st ~ housepov + (1|schoolid/classid), data = classroom)
summary(lm2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math1st ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.96  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.807 159.023   <2e-16 ***
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

a. Report if the additional predictors are justified:

```r
anova(lm_umm, lm2, refit = F)
```

```
## Data: classroom
## Models:
## lm_umm: math1st ~ 1 + (1 | schoolid/classid)
## lm2: math1st ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm_umm  4 11953 11973 -5972.3    11945
## lm2     5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the school-level predictor is justified according to the ANOVA between the unconditional means model, and the model with the school-level predictor HOUSEPOV. The chi-square test results in a p-value of approximately 0.

b. Report the change to school variance:

Response:

Change in $\sigma^2_\zeta = 250.93 - 280.69 = -29.76$

Question 3: Add all class-level predictors

Model 3 Equation:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$$\text{with } \zeta_k \sim N(0, \sigma^2_\zeta),\ \eta_{jk} \sim N(0, \sigma^2_\eta),\ \text{and } \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon), \text{ independent of each other}$$

$$\text{and } k = \text{schools},\ j = \text{ classrooms and } i = \text{ students}$$

```r
lm3 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep + (1|schoolid/classid), data = classroo
summary(lm3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                     1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
```

```
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29852    5.20495 228.85767 102.268  < 2e-16 ***
## housepov    -41.62116   14.08834 109.83230  -2.954  0.00383 **
## yearstea      0.06193    0.14717 223.76570   0.421  0.67432
## mathknow      2.55143    1.44530 231.06560   1.765  0.07883 .
## mathprep     -0.75440    1.42809 203.20755  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw
## housepov -0.568
## yearstea -0.264  0.077
## mathknow -0.052  0.082  0.030
## mathprep -0.666  0.032 -0.175  0.004
```

a. Report if adding the predictors is justified:

```
linearHypothesis(lm3, c("mathknow", "mathprep", "yearstea"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## mathknow = 0
## mathprep = 0
## yearstea = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + yearstea + mathknow + mathprep + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 3.4804     0.3233
```

Response:

Based on the Wald test above, adding the classroom-level predictors as a block is not needed, at the 0.05 level of significance. The p-value is 0.3233.

b. Report changes in class-level variance and individual variance:

Response:

- Change in $\sigma^2_\eta = 94.36 - 82.36 = 12.00$
- Change in $\sigma^2_\epsilon = 1136.43 - 1146.96 = -10.53$

c. Give a potential reason to explain why individual variance but not class variance is reduced:

Response:

Adding the classroom-level predictors shows a potential misspecification of the model. The classroom level predictors can make it difficult to estimate the individual level variance (i.e overstated) due to individual outliers in classrooms that have a very small amount of students.

Question 4.

Add all student-level predictors except mathgain and mathkind:

```
lm4 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (1|schoolid/classid), data = classroom)
summary(lm4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.689
##  schoolid         (Intercept)  169.45  13.017
##  Residual                      1064.96  32.634
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63041    5.31209  275.39010 101.585  < 2e-16 ***
## housepov     -17.64850   13.21755  113.87814  -1.335    0.184
## yearstea       0.01129    0.14141  226.80861   0.080    0.936
## mathknow       1.35004    1.39168  234.49768   0.970    0.333
## mathprep      -0.27705    1.37583  205.27111  -0.201    0.841
## sex           -1.21419    2.09483 1022.42110  -0.580    0.562
## minority     -16.18676    3.02605  704.47787  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56211   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

a. Report if the block of predictors is justified:

```
linearHypothesis(lm4, c("sex","minority", "ses"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## sex = 0
## minority = 0
## ses = 0
##
## Model 1: restricted model
## Model 2: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##
##   Df  Chisq Pr(>Chisq)
## 1
## 2  3 85.055  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the student-level predictors as a block is justified (at the 0.05 significance level) according to the Wald test comparing the previous model containing school & classroom-level predictors, to the current model including school, classroom, and student-level predictors. The chi-square test results in a p-value of approximately 0.

b. Report change in all variance components

Response:

- Change in $\sigma^2_\zeta = 169.45 - 223.31 = -53.86$
- Change in $\sigma^2_\eta = 93.89 - 94.36 = -0.47$
- Change in $\sigma^2_\epsilon = 1064.96 - 1136.43 = -71.47$

c. Give a potential reason as to why the school variance drops from the last model:

Response:

The student-level predictors explain some variance at the school level. SES & Minority Status, and SEX composition of children vary between different schools and may impact math scores. For example, some schools located in poorer areas with a different demographic of students will have much different individual math scores than those in more affluent areas.

d. Write this model out:

Model 4 Equation:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + b_5 SEX_{ijk} + b_6 MINORITY_{ijk} + b_7 SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with $\zeta_k \sim N(0, \sigma^2_\zeta)$, $\eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, independent of each other

and $k =$ schools, $j =$ classrooms and $i =$ students

Question 5.

a. Try to add a random slope for each teacher level predictor (varying at the school level; one by one separately - not all together)

b. Report the models and their fit.

```
lm5 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + yearstea | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00805439
## (tol = 0.002, component 1)
```

```
summary(lm5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8482 -0.6147 -0.0322  0.5979  3.6603
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.247e+01  9.6159
##  schoolid         (Intercept) 1.684e+02 12.9758
##  schoolid.1       yearstea    1.008e-02  0.1004
##  Residual                     1.065e+03 32.6361
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.59885    5.30780  266.47952 101.662  < 2e-16 ***
## housepov     -17.72082   13.21686  113.58577  -1.341    0.183
## yearstea       0.01128    0.14192  122.87733   0.079    0.937
## mathknow       1.33106    1.39155  234.33195   0.957    0.340
## mathprep      -0.26584    1.37588  204.90504  -0.193    0.847
## sex           -1.21060    2.09480 1022.21558  -0.578    0.563
## minority     -16.16715    3.02635  702.61831  -5.342 1.24e-07 ***
## ses           10.04528    1.54492 1066.09816   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.450
## yearstea -0.258  0.070
## mathknow -0.082  0.057  0.028
## mathprep -0.632  0.037 -0.172  0.003
```

```
## sex      -0.191 -0.007  0.015  0.006 -0.006
## minority -0.320 -0.179  0.023  0.115  0.001 -0.010
## ses      -0.121  0.082 -0.027 -0.007  0.053  0.020  0.162
## convergence code: 0
## Model failed to converge with max|grad| = 0.00805439 (tol = 0.002, component 1)
```

```
anova(lm4,lm5, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm5: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm5:     ses + (0 + yearstea | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm5 12 10754 10813 -5364.8    10730 0.007      1     0.9336
```

Response:

The addition of random slope on the *yearstea* variable is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm6 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathknow | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.389e+01  9.689914
##  schoolid         (Intercept) 1.694e+02 13.016328
##  schoolid.1       mathknow    1.700e-06  0.001304
##  Residual                     1.065e+03 32.633705
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63047    5.31204  275.40357 101.586  < 2e-16 ***
## housepov     -17.64821   13.21718  113.88792  -1.335    0.184
## yearstea       0.01129    0.14141  226.81110   0.080    0.936
## mathknow       1.34993    1.39168  234.50059   0.970    0.333
```

```
## mathprep      -0.27708    1.37583  205.27196  -0.201    0.841
## sex           -1.21417    2.09483 1022.42010  -0.580    0.562
## minority     -16.18681    3.02603  704.47306  -5.349 1.20e-07 ***
## ses           10.05075    1.54485 1066.56262   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm6, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm6: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm6:     ses + (0 + mathknow | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm6 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *mathknow* variable is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm7 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + mathprep | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm7)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
```

```
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.388e+01 9.689e+00
##  schoolid         (Intercept) 1.694e+02 1.302e+01
##  schoolid.1       mathprep    2.171e-07 4.659e-04
##  Residual                     1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63039    5.31207  275.39222 101.586  < 2e-16 ***
## housepov     -17.64851   13.21749  113.87941  -1.335    0.184
## yearstea       0.01129    0.14141  226.80838   0.080    0.936
## mathknow       1.35003    1.39167  234.49786   0.970    0.333
## mathprep      -0.27705    1.37582  205.27063  -0.201    0.841
## sex           -1.21419    2.09483 1022.42070  -0.580    0.562
## minority     -16.18676    3.02605  704.47629  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56201   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm7, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm7: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm7:     ses + (0 + mathprep | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm7 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *mathprep* variable is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

c. Why is it a bad idea to include a random slope on the housepov effect?

Response:

It is not a good idea to add a random slope on the housepov effect because housepov is a school-level predictor and cannot vary at the classroom or individual level. Every individual would have the same *housepov* level within a school.

d. Retry the above models, allowing the slopes to be correlated with the random intercepts (still one by one):

```
lm8 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (yearstea | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.262269
## (tol = 0.002, component 1)
```

```
summary(lm8)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (yearstea | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7452 -0.6033 -0.0289  0.6043  3.8441
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid.schoolid (Intercept)   38.078  6.1707
##  schoolid         (Intercept)  125.458 11.2008
##  schoolid.1       (Intercept)  239.710 15.4826
##                   yearstea       0.553  0.7436  -0.96
##  Residual                     1066.380 32.6555
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95138    5.48737  222.90272  98.217  < 2e-16 ***
## housepov     -17.15034   13.45738  119.67099  -1.274    0.205
## yearstea       0.02201    0.15774   75.83443   0.140    0.889
## mathknow       1.04334    1.34417  209.95200   0.776    0.439
## mathprep       0.05066    1.34588  191.07819   0.038    0.970
## sex           -1.33486    2.08775 1024.46979  -0.639    0.523
## minority     -16.44275    2.99653  669.47476  -5.487 5.80e-08 ***
## ses           10.14923    1.53875 1062.66218   6.596 6.66e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.455
## yearstea -0.370  0.084
## mathknow -0.085  0.049  0.012
## mathprep -0.606  0.050 -0.139  0.014
## sex      -0.184 -0.004  0.009  0.008 -0.004
## minority -0.305 -0.170  0.031  0.122 -0.007 -0.012
```

```
## ses      -0.119  0.079 -0.019 -0.001  0.049  0.022  0.168
## convergence code: 0
## Model failed to converge with max|grad| = 0.262269 (tol = 0.002, component 1)
```

```
anova(lm4, lm8, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm8: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm8:     ses + (yearstea | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm8 14 10752 10822 -5361.8    10724 5.8252      3     0.1204
```

Response:

The addition of random slope on the *yearstea* variable, allowing the slope to be correlated with the intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm9 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (mathknow| schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -7.0e-02
```

```
summary(lm9)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (mathknow | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups           Name        Variance  Std.Dev. Corr
##  classid.schoolid (Intercept) 9.391e+01  9.69057
##  schoolid         (Intercept) 6.516e+01  8.07238
##  schoolid.1       (Intercept) 1.041e+02 10.20458
##                   mathknow    1.287e-03  0.03588 1.00
##  Residual                     1.065e+03 32.63430
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63990    5.31192  275.40403 101.590  < 2e-16 ***
## housepov     -17.64168   13.21225  104.51644  -1.335    0.185
## yearstea       0.01115    0.14141  226.85047   0.079    0.937
```

```
## mathknow       1.35429    1.39197  215.14576   0.973    0.332
## mathprep      -0.27753    1.37595  201.49289  -0.202    0.840
## sex           -1.21330    2.09486 1021.82795  -0.579    0.563
## minority     -16.19342    3.02606  703.83461  -5.351 1.18e-07 ***
## ses           10.04804    1.54488 1062.35212   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.082  0.057  0.029
## mathprep -0.631  0.038 -0.173  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm9, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm9: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm9:     ses + (mathknow | schoolid) + (1 | schoolid/classid)
##     Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4 11 10752 10806 -5364.8    10730
## lm9 14 10758 10827 -5364.8    10730 3e-04      3          1
```

Response:

The addition of random slope on the *mathknow* variable, allowing the slope to be correlated with the intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm10 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (mathprep| schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm10)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (mathprep | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8541 -0.6035 -0.0222  0.5914  3.6472
```

```
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.  Corr
##  classid.schoolid (Intercept) 7.863e+01  8.867217
##  schoolid         (Intercept) 5.405e-06  0.002325
##  schoolid.1       (Intercept) 5.528e+02 23.511360
##                   mathprep    1.590e+01  3.987197 -1.00
##  Residual                     1.064e+03 32.621849
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60920    5.60863  159.87223  96.032  < 2e-16 ***
## housepov     -14.01017   12.88650  116.09965  -1.087    0.279
## yearstea      -0.02589    0.13951  223.54922  -0.186    0.853
## mathknow       1.29832    1.37215  229.72100   0.946    0.345
## mathprep       0.04050    1.34875  138.98576   0.030    0.976
## sex           -1.16743    2.08695 1023.15678  -0.559    0.576
## minority     -16.46498    2.99519  663.80011  -5.497 5.51e-08 ***
## ses           10.14137    1.53960 1060.94309   6.587 7.05e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.460
## yearstea -0.260  0.089
## mathknow -0.071  0.027  0.048
## mathprep -0.692  0.107 -0.155  0.012
## sex      -0.183  0.003  0.022  0.003 -0.008
## minority -0.275 -0.187  0.025  0.107 -0.035 -0.013
## ses      -0.121  0.095 -0.033 -0.001  0.061  0.024  0.161
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm10, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm10: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm10:     ses + (mathprep | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm10 14 10753 10822 -5362.3    10725 4.8144      3     0.1859
```

Response:

The addition of random slope on the *mathprep* variable, allowing the slope to be correlated with the intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

e. Report anything unusual about the variance components (changes that are in a direction you didn't expect) and any potential explanation for why those changes occured (hint: what did you add to the model?).

Response:

Question 6.

a. Try to add a random slope for each student level predictor (varying at the classroom level; one by one - not all together)

```
lm11 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | classid) + (1|schoolid/classid), data = classroom)
summary(lm11)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid          sex         3.310e-05  0.005754
##  classid:schoolid (Intercept) 9.387e+01  9.688824
##  schoolid         (Intercept) 1.695e+02 13.017987
##  Residual                     1.065e+03 32.633681
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63033    5.31211  275.37965 101.585  < 2e-16 ***
## housepov     -17.64878   13.21784  113.87028  -1.335    0.184
## yearstea       0.01129    0.14141  226.80606   0.080    0.936
## mathknow       1.35013    1.39167  234.49478   0.970    0.333
## mathprep      -0.27702    1.37582  205.26984  -0.201    0.841
## sex           -1.21421    2.09483 1022.41564  -0.580    0.562
## minority     -16.18672    3.02607  704.48078  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56152   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
```

```
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
```

```
anova(lm4, lm11, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm11: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm11:     ses + (0 + sex | classid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm11 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *sex* variable, varying by classrooms, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm12 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm12)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid          minority       0.00   0.00
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38908 101.585  < 2e-16 ***
## housepov     -17.64848   13.21758  113.87764  -1.335    0.184
## yearstea       0.01129    0.14141  226.80896   0.080    0.936
```

```
## mathknow       1.35004    1.39168  234.49773   0.970    0.333
## mathprep      -0.27705    1.37583  205.27155  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47894  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56222   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm12, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm12: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm12:     ses + (0 + minority | classid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm12 12 10754 10813 -5364.8    10730     0      1     0.9999
```

Response:

The addition of random slope on the *minority* variable, varying by classrooms, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm13 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
               sex + minority + ses + (0 + ses | classid) + (1|schoolid/classid), data = classroom)
summary(lm13)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
```

```
##  Groups           Name        Variance Std.Dev.
##  classid          ses           49.60   7.043
##  classid:schoolid (Intercept)   87.11   9.333
##  schoolid         (Intercept)  171.02  13.077
##  Residual                     1043.44  32.302
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46506 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.44881  -1.325    0.188
## yearstea       0.01103    0.14117  226.97682   0.078    0.938
## mathknow       1.36796    1.38563  229.40643   0.987    0.325
## mathprep      -0.27938    1.37171  204.89333  -0.204    0.839
## sex           -1.37733    2.09334 1022.81814  -0.658    0.511
## minority     -16.29362    3.02464  703.33746  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39731   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.070
## mathknow -0.082  0.058  0.029
## mathprep -0.631  0.040 -0.172  0.005
## sex      -0.190 -0.007  0.014  0.006 -0.005
## minority -0.321 -0.180  0.025  0.111  0.002 -0.011
## ses      -0.108  0.081 -0.026  0.002  0.050  0.020  0.145
```

```
anova(lm4, lm13, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm13: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm13:     ses + (0 + ses | classid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm13 12 10752 10812 -5364.0    10728 1.5969      1     0.2063
```

Response:

The addition of random slope on the *ses* variable, varying by classrooms, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Why is it a bad idea to include a classroom-level variable with random slopes at the classroom level?

Response:

c. Retry the above, allowing the slopes to be correlated with the random intercepts. Report findings.

```
lm14 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (sex | classid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0172129
## (tol = 0.002, component 1)
```

```
summary(lm14)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7561 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid          (Intercept)  105.51  10.272
##                   sex           31.52   5.614   -0.74
##  classid:schoolid (Intercept)   24.76   4.976
##  schoolid         (Intercept)  169.80  13.031
##  Residual                     1056.36  32.502
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.724e+02 101.284  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.146e+02  -1.383    0.169
## yearstea     3.052e-03  1.416e-01  2.270e+02   0.022    0.983
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## mathprep    -3.462e-01  1.374e+00  2.013e+02  -0.252    0.801
## sex         -1.197e+00  2.123e+00  2.157e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.61e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.452
## yearstea -0.258  0.072
## mathknow -0.085  0.060  0.029
## mathprep -0.629  0.040 -0.174  0.005
## sex      -0.204 -0.005  0.015  0.003 -0.008
```

```
## minority -0.321 -0.178  0.024  0.116  0.003 -0.009
## ses      -0.123  0.083 -0.027 -0.005  0.054  0.020  0.164
## convergence code: 0
## Model failed to converge with max|grad| = 0.0172129 (tol = 0.002, component 1)
```

```
anova(lm4, lm14, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm14: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm14:     ses + (sex | classid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm14 14 10757 10827 -5364.5    10729 0.5003      3     0.9188
```

Response:

The addition of random slope on the *sex* variable, varying by classrooms and allowing for correlation between the slope and coefficient, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm15 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | classid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -3.0e-01
```

```
summary(lm15)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9039 -0.6220 -0.0293  0.6030  3.4571
##
## Random effects:
##  Groups           Name        Variance  Std.Dev. Corr
##  classid          (Intercept) 2.256e+02 15.01953
##                   minority    1.705e+02 13.05674 -0.82
##  classid:schoolid (Intercept) 1.114e-03  0.03338
##  schoolid         (Intercept) 1.572e+02 12.53793
##  Residual                     1.045e+03 32.33362
## Number of obs: 1081, groups:
```

```
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73593    5.37927  270.80086 100.336  < 2e-16 ***
## housepov     -17.34850   12.90777  103.40177  -1.344    0.182
## yearstea      -0.01634    0.14283  234.17118  -0.114    0.909
## mathknow       1.45686    1.39331  233.96331   1.046    0.297
## mathprep      -0.13518    1.36995  203.84076  -0.099    0.921
## sex           -1.01001    2.08971 1015.71358  -0.483    0.629
## minority     -16.48509    3.21682  182.51990  -5.125 7.55e-07 ***
## ses            9.89393    1.54595 1062.81068   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.435
## yearstea -0.265  0.080
## mathknow -0.079  0.061  0.038
## mathprep -0.618  0.037 -0.171 -0.006
## sex      -0.188 -0.009  0.015  0.009 -0.005
## minority -0.368 -0.171  0.025  0.108 -0.004 -0.009
## ses      -0.117  0.085 -0.023  0.001  0.051  0.021  0.149
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 1 negative eigenvalues
```

```
anova(lm4, lm15, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm15: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm15:     ses + (minority | classid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm15 14 10754 10824 -5363.2    10726 3.1966      3     0.3623
```

Response:

The addition of random slope on the *minority* variable, varying by classrooms and allowing for correlation between slope and intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm16 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (ses | classid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -2.7e-02
```

```r
summary(lm16)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (ses | classid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid          (Intercept)   82.922  9.106
##                   ses           44.096  6.640   0.76
##  classid:schoolid (Intercept)    3.127  1.768
##  schoolid         (Intercept)  173.161 13.159
##  Residual                     1048.330 32.378
## Number of obs: 1081, groups:
## classid, 285; classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.52096    5.26660  269.60380 102.442  < 2e-16 ***
## housepov     -16.29035   13.13434  111.28885  -1.240    0.217
## yearstea       0.01605    0.14080  227.60392   0.114    0.909
## mathknow       1.37995    1.37293  222.44360   1.005    0.316
## mathprep      -0.37731    1.34601  182.86005  -0.280    0.780
## sex           -1.32181    2.08795 1017.08436  -0.633    0.527
## minority     -16.09273    3.03497  717.65834  -5.302 1.52e-07 ***
## ses           10.05540    1.64508  171.15056   6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.450
## yearstea -0.266  0.074
## mathknow -0.078  0.059  0.030
## mathprep -0.625  0.036 -0.165 -0.001
## sex      -0.186 -0.009  0.013  0.007 -0.009
## minority -0.325 -0.181  0.021  0.108  0.004 -0.014
## ses      -0.084  0.078 -0.024  0.015  0.056  0.022  0.142
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate  Hessian with 1 negative eigenvalues
```

```r
anova(lm4, lm16, refit = F)
```

```
## Data: classroom
```

```
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm16: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm16:     ses + (ses | classid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm16 14 10754 10824 -5362.8    10726 3.8395      3     0.2793
```

Response:

The addition of random slope on the *ses* variable, varying by classrooms and allowing for correlation between the slope and intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

Question 7.

a. Try to add a random slope for each student level predictor varying at the school level:

```
lm17 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + sex | schoolid) + (1|schoolid/classid), data = classroom)
summary(lm17)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5556
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   96.08   9.802
##  schoolid         (Intercept)  161.63  12.713
##  schoolid.1       sex           35.85   5.987
##  Residual                      1054.36  32.471
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43513    5.30741  272.54817 101.638  < 2e-16 ***
## housepov     -16.77631   13.22883  112.39531  -1.268    0.207
## yearstea       0.01448    0.14163  226.44545   0.102    0.919
## mathknow       1.40068    1.39464  234.45910   1.004    0.316
## mathprep      -0.27193    1.38011  205.78600  -0.197    0.844
## sex           -1.33538    2.18749  138.10017  -0.610    0.543
## minority     -16.16537    3.02862  704.25875  -5.338 1.27e-07 ***
## ses            9.98475    1.54243 1058.28030   6.473 1.46e-10 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.259  0.070
## mathknow -0.081  0.055  0.028
## mathprep -0.633  0.036 -0.172  0.004
## sex      -0.179 -0.010  0.013  0.007 -0.004
## minority -0.320 -0.178  0.024  0.114  0.001 -0.015
## ses      -0.120  0.081 -0.029 -0.007  0.052  0.020  0.161
```

```
anova(lm4, lm17, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm17: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm17:     ses + (0 + sex | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm17 12 10753 10813 -5364.4    10729 0.6137      1     0.4334
```

Response:

The addition of random slope on the *sex* variable, varying by schools, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm18 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + minority | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
summary(lm18)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance  Std.Dev.
##  classid.schoolid (Intercept) 9.388e+01  9.689369
##  schoolid         (Intercept) 1.694e+02 13.017176
##  schoolid.1       minority    1.777e-06  0.001333
##  Residual                     1.065e+03 32.633690
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
```

```
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63040    5.31208  275.39129 101.586  < 2e-16 ***
## housepov     -17.64850   13.21752  113.87885  -1.335    0.184
## yearstea       0.01129    0.14141  226.80855   0.080    0.936
## mathknow       1.35003    1.39168  234.49782   0.970    0.333
## mathprep      -0.27705    1.37582  205.27091  -0.201    0.841
## sex           -1.21419    2.09483 1022.42090  -0.580    0.562
## minority     -16.18676    3.02605  704.47638  -5.349 1.20e-07 ***
## ses           10.05076    1.54485 1066.56207   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.259  0.071
## mathknow -0.083  0.058  0.029
## mathprep -0.631  0.038 -0.172  0.004
## sex      -0.190 -0.007  0.016  0.007 -0.006
## minority -0.320 -0.178  0.024  0.115  0.001 -0.011
## ses      -0.121  0.082 -0.028 -0.007  0.053  0.020  0.162
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm18, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm18: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm18:     ses + (0 + minority | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm18 12 10754 10813 -5364.8    10730     0      1          1
```

Response:

The addition of random slope on the *minority* variable, varying by schools, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm19 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (0 + ses | schoolid) + (1|schoolid/classid), data = classroom)
summary(lm19)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
```

```
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0289  0.5798  3.7130
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid.schoolid (Intercept)   88.56   9.411
##  schoolid         (Intercept)  168.00  12.961
##  schoolid.1       ses           72.50   8.515
##  Residual                      1035.11  32.173
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13752    5.27926  270.52802 102.124  < 2e-16 ***
## housepov     -16.94575   13.21161  112.81447  -1.283    0.202
## yearstea       0.03079    0.14052  223.94252   0.219    0.827
## mathknow       1.35586    1.38461  232.19737   0.979    0.328
## mathprep      -0.19799    1.35995  198.59551  -0.146    0.884
## sex           -1.40187    2.08169 1011.29089  -0.673    0.501
## minority     -16.52526    3.02191  700.07600  -5.468 6.32e-08 ***
## ses            9.78982    1.82216   79.01650   5.373 7.61e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.260  0.070
## mathknow -0.079  0.056  0.028
## mathprep -0.628  0.041 -0.172  0.002
## sex      -0.190 -0.007  0.018  0.006 -0.007
## minority -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses      -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
```

```
anova(lm4, lm19, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm19: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm19:     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm19 12 10749 10809 -5362.4    10725 4.6972      1    0.03021 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *ses* variable, varying by schools, is significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

b. Retry the above, allowing the slopes to be correlated with the random intercepts.

```
lm20 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (sex | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.00699638
## (tol = 0.002, component 1)
```

```
summary(lm20)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (sex | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8048 -0.6095 -0.0223  0.5970  3.5524
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid.schoolid (Intercept)   97.35   9.867
##  schoolid         (Intercept)  160.82  12.682
##  schoolid.1       (Intercept)   45.52   6.747
##                   sex           84.10   9.170   -0.92
##  Residual                      1041.74  32.276
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.364e+00  2.626e+02 100.658  < 2e-16 ***
## housepov    -1.742e+01  1.326e+01  1.135e+02  -1.314    0.191
## yearstea     6.875e-03  1.418e-01  2.277e+02   0.048    0.961
## mathknow     1.380e+00  1.396e+00  2.364e+02   0.988    0.324
## mathprep    -2.795e-01  1.378e+00  2.062e+02  -0.203    0.840
## sex         -1.340e+00  2.301e+00  8.743e+01  -0.583    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.97e-08 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.449
## yearstea -0.258  0.072
## mathknow -0.082  0.060  0.027
## mathprep -0.627  0.038 -0.172  0.004
## sex      -0.222 -0.003  0.014  0.006 -0.005
## minority -0.319 -0.178  0.024  0.114  0.004 -0.011
## ses      -0.121  0.083 -0.028 -0.006  0.053  0.018  0.163
```

```
## convergence code: 0
## Model failed to converge with max|grad| = 0.00699638 (tol = 0.002, component 1)
```

```
anova(lm4, lm20, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm20: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm20:     ses + (sex | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm20 14 10756 10825 -5363.8    10728 1.863      3     0.6013
```

Response:

The addition of random slope on the *sex* variable, varying by schools and allowing for correlation between the slope and intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm21 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (minority | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0151558
## (tol = 0.002, component 1)
```

```
summary(lm21)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (minority | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid.schoolid (Intercept)   86.71   9.312
##  schoolid         (Intercept)   72.88   8.537
##  schoolid.1       (Intercept)  308.35  17.560
##                   minority     343.11  18.523   -0.92
##  Residual                      1039.37  32.239
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.397  < 2e-16 ***
## housepov    -1.606e+01  1.258e+01  9.998e+01  -1.277    0.204
## yearstea    -4.371e-03  1.377e-01  2.172e+02  -0.032    0.975
```

```
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.629e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.825e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.394
## yearstea -0.253  0.091
## mathknow -0.078  0.061  0.024
## mathprep -0.576  0.037 -0.167 -0.002
## sex      -0.172 -0.013  0.014  0.010 -0.005
## minority -0.494 -0.157  0.027  0.099 -0.002 -0.014
## ses      -0.105  0.089 -0.021 -0.005  0.052  0.024  0.113
## convergence code: 0
## Model failed to converge with max|grad| = 0.0151558 (tol = 0.002, component 1)
```

```
anova(lm4, lm21, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm21: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm21:     ses + (minority | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm21 14 10746 10815 -5358.8    10718 11.967      3   0.007497 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of random slope on the *minority* variable, varying by schools and allowing for correlation between the slope and intercept, is significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

```
lm22 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep +
              sex + minority + ses + (ses | schoolid) + (1|schoolid/classid), data = classroom)
```

```
## boundary (singular) fit: see ?isSingular
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -8.3e-01
```

```
summary(lm22)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
##     ses + (ses | schoolid) + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7131
##
## Random effects:
##  Groups           Name        Variance Std.Dev. Corr
##  classid.schoolid (Intercept)   88.55   9.410
##  schoolid         (Intercept)  167.99  12.961
##  schoolid.1       (Intercept)    0.00   0.000
##                   ses           72.51   8.515    NaN
##  Residual                      1035.12  32.173
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.1374     5.2791  269.0650 102.126  < 2e-16 ***
## housepov     -16.9457    13.2113  110.2922  -1.283    0.202
## yearstea       0.0308     0.1405  219.4601   0.219    0.827
## mathknow       1.3558     1.3846  230.3407   0.979    0.328
## mathprep      -0.1980     1.3599  196.3254  -0.146    0.884
## sex           -1.4019     2.0817 1009.9276  -0.673    0.501
## minority     -16.5252     3.0219  665.7853  -5.469 6.43e-08 ***
## ses            9.7898     1.8222   76.9573   5.373 8.01e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv yearst mthknw mthprp sex    minrty
## housepov -0.451
## yearstea -0.260  0.070
## mathknow -0.079  0.056  0.028
## mathprep -0.628  0.041 -0.172  0.002
## sex      -0.190 -0.007  0.018  0.006 -0.007
## minority -0.323 -0.180  0.024  0.110  0.001 -0.010
## ses      -0.091  0.076 -0.019  0.006  0.042  0.017  0.124
## convergence code: 0
## boundary (singular) fit: see ?isSingular
```

```
anova(lm4, lm22, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm22: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm22:     ses + (ses | schoolid) + (1 | schoolid/classid)
##      Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4  11 10752 10806 -5364.8    10730
## lm22 14 10753 10823 -5362.4    10725 4.6972      3     0.1954
```

Response:

The addition of random slope on the *ses* variable, varying by schools and allowing for correlation between the slope and intercept, is not significant (at the 0.05 level of significance), according to the ANOVA LRT comparing the model with and without the random slope addition.

c. Report anything unusual about the variance components (changes that are unexpected)

Response:

Question 8.

a. Take the two predictors that had significant random slopes, in the forms in which they worked (indep. or correlated) and add both to the model, and test for need of one conditional on needing the other.

```
# Model with only random intercepts: lm4
# Model with significant random slopes: lm_slopes

lm_slopes1 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
    ses + (0 + ses | schoolid) + (1 | schoolid/classid), data = classroom)
lm_slopes2 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
    ses + (minority | schoolid) + (1 | schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.0151558
## (tol = 0.002, component 1)
```

```
lm_slopes3 <- lmer(math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
    ses + (0 + ses | schoolid) + (minority | schoolid) +(1 | schoolid/classid), data = classroom)
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
```

```
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -1.7e-02
```

```
anova(lm_slopes3, lm_slopes2, refit = F)
```

```
## Data: classroom
## Models:
## lm_slopes2: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm_slopes2:     ses + (minority | schoolid) + (1 | schoolid/classid)
## lm_slopes3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm_slopes3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## lm_slopes3:     schoolid/classid)
##            Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm_slopes2 14 10746 10815 -5358.8    10718
## lm_slopes3 15 10742 10817 -5356.2    10712  5.12      1    0.02365 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(lm_slopes3, lm_slopes1, refit = F)
```

```
## Data: classroom
## Models:
## lm_slopes1: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm_slopes1:     ses + (0 + ses | schoolid) + (1 | schoolid/classid)
## lm_slopes3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm_slopes3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## lm_slopes3:     schoolid/classid)
```

```
##            Df   AIC   BIC  logLik deviance Chisq Chi Df Pr(>Chisq)
## lm_slopes1 12 10749 10809 -5362.4    10725
## lm_slopes3 15 10742 10817 -5356.2    10712 12.39      3    0.00616 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Response:

The addition of the random slopes on the two predictors *ses* and *minority* are needed based on the LRT comparing the the need of one random slope, conditional on needing the other.

b. Is the more complex model (with both random slopes in it) justified?

```
anova(lm4, lm_slopes3, refit = F)
```

```
## Data: classroom
## Models:
## lm4: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm4:     ses + (1 | schoolid/classid)
## lm_slopes3: math1st ~ housepov + yearstea + mathknow + mathprep + sex + minority +
## lm_slopes3:     ses + (0 + ses | schoolid) + (minority | schoolid) + (1 |
## lm_slopes3:     schoolid/classid)
##            Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## lm4        11 10752 10806 -5364.8    10730
## lm_slopes3 15 10742 10817 -5356.2    10712 17.087      4   0.001859 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Reponse:

The complex model with both random slopes is justified, with a p-value of 0.001859.

c. WRITE OUT THIS MODEL in your preferred notation

The model is:

$$MATH1ST_{ijk} = b_0 + b_1 HOUSEPOV_k + b_2 YEARSTEA_{jk} + b_3 MATHKNOW_{jk} + b_4 MATHPREP_{jk} + b_5 SEX_{ijk} + (b_6 + \zeta_{6k}) MINORITY_{ijk} + (b_7 + \zeta_{7k}) SES_{ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

with $\zeta_k \sim N(0, \sigma^2_\zeta)$, $\eta_{jk} \sim N(0, \sigma^2_\eta)$, $\zeta_{6k} \sim N(0, \sigma^2_{\zeta_6})$, $\zeta_{7k} \sim N(0, \sigma^2_{\zeta_7})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, independent of each other and $k =$ schools, $j =$ classrooms and $i =$ students

Question 9.

a. For UMM, write down: V_S, V_C, V_E for the three variance components (simply the estimates)

V_S = 280.69

V_C = 85.47

V_E = 1146.79

b. For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

V_S = 169.45

V_C = 93.89

V_E = 1064.96

c. By what fraction did these each decrease with the new predictors in the model?

V_S: 39.63% decrease

V_C: 9.85% increase

V_E: 7.14% decrease

Question 10. Now consider the model with a random slope in ses.

a. What are: V_C, V_S(ses=0), V_E ?

V_S(ses=0) = 168.00

V_C = 88.56

V_E = 1035.11

b. What are: V_S(ses=-0.50), V_S(ses=+0.5) ?

V_S(ses=0.5) =

V_S(ses=-0.5) =

Question 11.

Now consider the model with a random slope in minority.

a. What are: V_C, V_S(minority=0), V_E ?

V_S(minority=0) = 308.35

V_C = 86.71

V_E = 1039.37

b. What are: V_S(minority=0.25), V_S(minority=+0.50), V_S(minority=+0.75) ?

V_S(minority=0.25) =

```
# V_S(minority = 0.25)
# Insert code if you want to do the calculations in R
```

V_S(minority=0.5) =

```
# V_S(minority = 0.50)
# Insert code if you want to do the calculations in R
```

V_S(minority=0.75) =

```
# V_S(minority = 0.75)
# Insert code if you want to do the calculations in R
```

Question 12.

Now consider the model with a random slope in ses & minority.

a. What are: V_C, V_S(minority=0,ses=0), V_E ? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance

```
# If you want to look at your model/variance components insert code here or you can just do this in lin
```

```
V_S(sex=0, minority=0) = 401.766

V_C = 80.618

V_E = 1009.727
```

b. In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Response:

c. Can we make a similar statement about ζ_{0k}?

Response:

d. If you had a large value for η_{0jk}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Response:

e. If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Response: