ME 41100 System Dynamics and Control

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Homework 5

Problem 1. A block diagram of a control system is shown in Fig. 1.

1) If r is a unit step function and the system is closed-loop stable, what is the steady-state tracking error? (Hint: $E(s) = 0.5R(s) - 0.5Y(s)$.)

2) What is the system type?

3) What is the steady-state error to a ramp input $r(t) = 5t$ if $K_2 = 2$ and $K_1 = 10$?

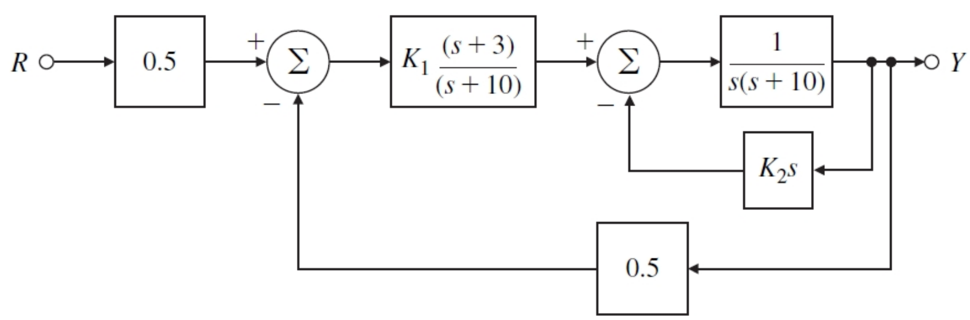

Figure 1: Block diagram of a system.

Problem 2. You are given the system shown in Fig. 2, where the feedback gain β is subject to variations. You are to design a controller for this system so that the output $y(t)$ accurately tracks the reference input $r(t)$. Let $\beta = 1$. You are given the following three options for the controller $D_{ci}(s)$:

$$D_{c1}(s) = k_P, \quad D_{c2}(s) = \frac{k_P s + k_I}{s}, \quad D_{c3}(s) = \frac{k_P s^2 + k_I s + k_2}{s^2}$$

Choose the controller (including particular values for the controller constants) that will result in a Type 1 system with a steady-state error to a unit reference ramp of less than 0.1. (Hint: Use Routh's test to determine conditions for stability.)

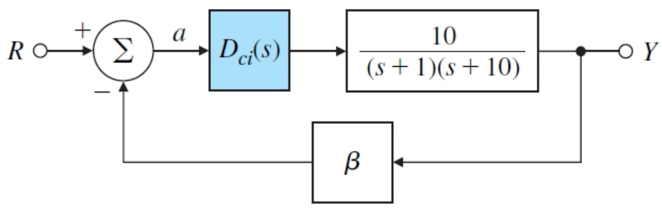

Figure 2: Control system.

Problem 3. Consider the second-order plant with transfer function

$$G_p(s) = \frac{1}{(s+1)(5s+1)}$$

and in a unity feedback structure. Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for P [$G_c = k_P$], PD [$G_c(s) = k_P + k_D s$], and PID [$G_c(s) = k_P + k_I/s + k_D s$] controllers. Let $k_P = 19, k_I = 0.5$, and $k_D = 4/19$.

Problem 4. Roughly sketch the root loci for the pole-zero maps as shown in Fig. 3 without the aid of a computer. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole-zero map is from a characteristic equation of the form

$$1 + K\frac{b(s)}{a(s)} = 0$$

where the roots of the numerator $b(s)$ are shown as small circles $\circ$ and the roots of the denominator $a(s)$ are shown as x's on the s-plane. Note that in Fig. 5(c) there are two poles at the origin; there are two poles on the imaginary axis in (f), slightly off the real axis.

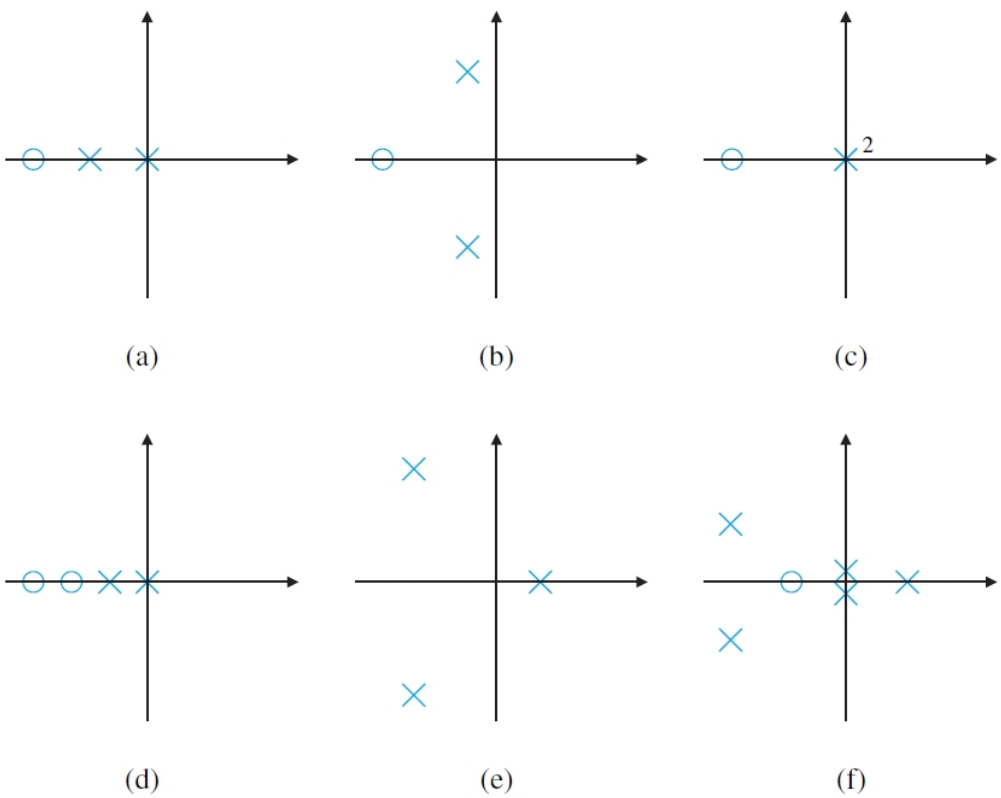

Figure 3: Pole-zero maps.

Problem 5. For the system shown in Fig. 4, determine the characteristic equation and sketch the root locus of it with respect to positive values of the parameter c. Give $L(s)$, $a(s)$, and $b(s)$, (recall that $L(s) = b(s)/a(s)$), and be sure to show with arrows the direction in which c increases on the locus.

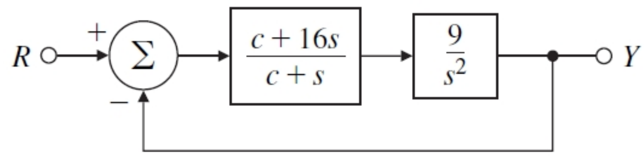

Figure 4: Unity feedback system.

Problem 6. Suppose the unity feedback system of Fig. 5 has an open-loop plant given by $G(s) = \frac{1}{s^2}$. Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in series with the plant so that the dominant poles of the closed-loop system are located at $s = -2 \pm 2j$, and the other pole of the closed-loop system is located at $s = -a$.

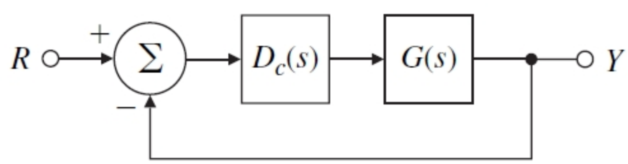

Figure 5: Unity feedback system.

Problem 7. [MATLAB] Assume that the unity feedback system of Fig. 5 has the open-loop plant

$$G(s) = \frac{1}{s(s+3)(s+6)}$$

Design a lag compensation to meet the following specifications:

- The step response settling time is to be less than 5 sec.
- The step response overshoot is to be less than 17%.
- The steady-state error to a unit-ramp input must not exceed 10%.

(Hint: Use MATLAB to help you finish the design.)