ME 41100 System Dynamics and Control Dr. B. Wang

Spring 2024

Homework 3

Problem 1. (Electrical systems)

1) Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical circuit shown in Figure 1.

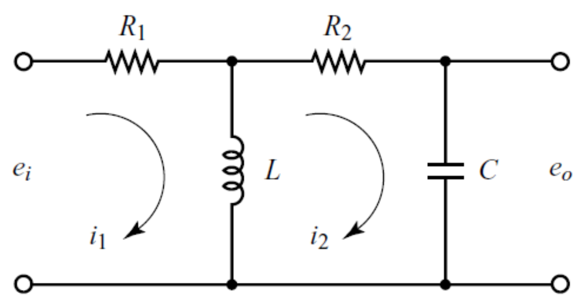

Figure 1: Electrical circuit.

2) Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical circuit shown in Figure 2.

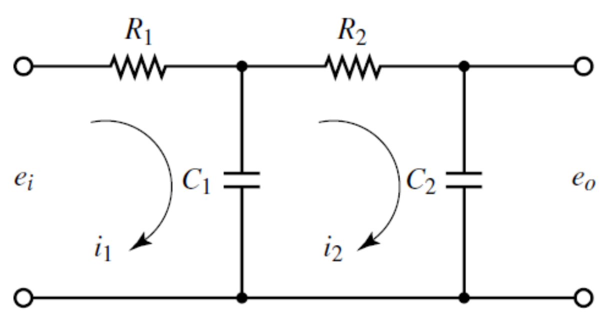

Figure 2: Electrical circuit.

3) For the system $E_o(s)/E_i(s)$ shown in Figure 2, suppose that $R_1 = R_2 = C_1 = C_2 = 1$, and that all initial conditions are zero. Is the system overdamped, critically damped, or underdamped? What is the dominant pole?

Problem 2. (Electro-mechanical systems) Consider the system shown in Figure 3. An armature-controlled DC servomotor drives a load consisting of the moment of inertia J_L. The torque developed by the motor is $T = Ki_a$, where K is the motor torque constant and i_a is the armature current. The moment of inertia of the motor rotor is J_m. The angular displacements of the motor rotor and the load element are θ_m and θ, respectively. The gear ratio is $n = \theta/\theta_m$.

1) Assume that the output is the angular velocity $\dot{\theta}$. Obtain the transfer function $s\Theta(s)/E_i(s)$.

2) Suppose that $n = 0.1$, $K = 10$, $L = R = 1$, $J_m = 0.1$, $J_L = 10$ the back emf constant of the motor $K_b = 0.1$. Is the system overdamped, critically damped, or underdamped? What is the damped natural frequency? What is the unit-step response?

3) What are the rise time t_r, the peak time t_p, the maximum overshoop M_p, and the settling time t_s of the system?

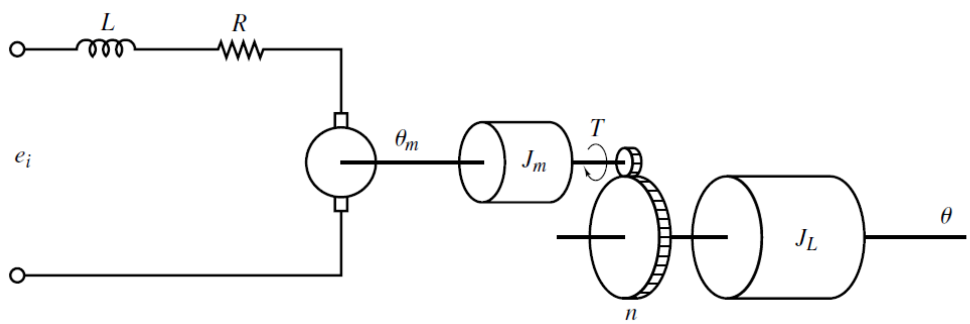

Figure 3: Armature-controlled DC servomotor system.

Problem 3. [Optional] (Thermal systems) Consider the thermal system shown in Figure 4. Considering small deviations from steady-state operation, assume that the heat loss to the surroundings and the heat capacitance of the metal parts of the heater are negligible. Let us define $\bar{\Theta}_i$ the steady-state temperature of inlet liquid, $\bar{\Theta}_o$ the steady-state temperature of outlet liquid, G the mass flow rate of liquid through the heating chamber, R the thermal resistance, C the thermal capacitance of liquid contained in the heating chamber, $\bar{H}$ steady-state heat input. Let us assume that the heat input is suddenly changed from $\bar{H}$ to $\bar{H} + h$ and the inlet liquid temperature is suddenly changed from $\bar{\Theta}_i$ to $\bar{\Theta}_i + \theta_i$. Then the outlet liquid temperature will be changed from $\bar{\Theta}_o$ to $\bar{\Theta}_o + \theta_o$.

1) Obtain the transfer functions $\Theta_o(s)/H(s)$ and $\Theta_o(s)/\Theta_i(s)$ of the thermal system.

2) Assume that the mass of liquid contained in the heating chamber $M = 10$, the specific heat of the liquid $c = 0.1$, and the thermal resistance $R = 2$. What are the time constant T, rise time t_r, settling time t_s of the system?

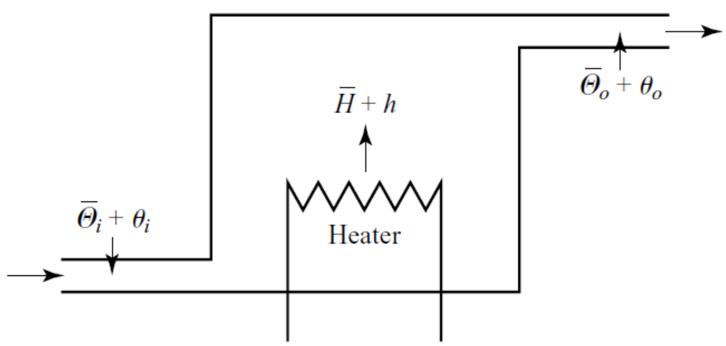

Figure 4: Armature-controlled DC servomotor system.

Problem 4. Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical system shown in Figure 5. What is the order of the system? What are the zeros and the poles?

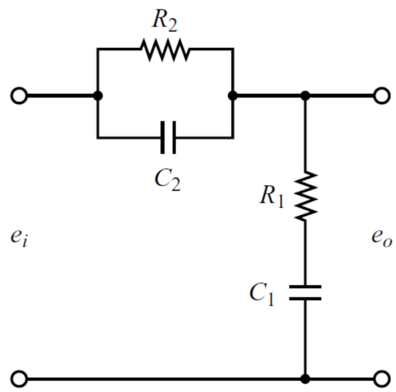

Figure 5: Electrical system