ME 41100 System Dynamics and Control Dr. B. Wang

Spring 2024

Homework 1

Reading: FPE (Franklin, Powell, Emami-Naeini, 7th or 8th edition), Sections 1.1, 1.2, 3.1, Appendix A.

(The first two problems are designed to test your background.)

Problem 1. For each matrix and/or vector pair given below, compute their product $A \cdot B$ if possible, or explain why it is not possible.

a) $A = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

c) $A = \begin{bmatrix} 2 & 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 3 & 0 \end{bmatrix}$

Problem 2. Compute the magnitude and the phase of the following complex numbers:

a) $1 - 2j$

b) $2 + 3j$

c) $(1 - 2j)(2 + 3j)$

Problem 3. Find the Laplace transform of the function

$$f(t) = \begin{cases} 0, & t < 0, \\ te^{-2t}, & t \geq 0. \end{cases}$$

Problem 4. Find the Laplace transform of the following functions:

$$f_1(t) = \begin{cases} 0, & t < 0, \\ 3\sin(5t + 45^\circ), & t \geq 0. \end{cases}$$

$$f_2(t) = \begin{cases} 0, & t < 0, \\ 0.03(1 - \cos(2t)), & t \geq 0. \end{cases}$$

Problem 5. Find the Laplace transform of the function

$$f(t) = \begin{cases} 0, & t < 0, \\ t^2 e^{-at}, & t \geq 0. \end{cases}$$

Problem 6. Find the Laplace transform of the function

$$f(t) = \begin{cases} 0, & t < 0, \\ \cos(2\omega t) \cdot \cos(3\omega t), & t \geq 0. \end{cases}$$

Problem 7. Find the inverse Laplace transforms of the following functions:

$$F_1(s) = \frac{6s + 21}{(s+3)(2s+1)}.$$

$$F_2(s) = \frac{3(s+4)}{s(s+1)(s+2)}.$$

Problem 8. Find the inverse Laplace transforms of the following functions:

$$F_1(s) = \frac{6s+3}{s^2}$$

$$F_2(s) = \frac{5s+2}{(s+1)(s+2)^2}$$

Problem 9. Find the inverse Laplace transform of

$$F(s) = \frac{2s^2 + 4s + 5}{s(s+1)}.$$

Problem 10. Find the inverse Laplace transform of

$$F(s) = \frac{1}{s^2\left(s^2 + \omega^2\right)}.$$

Problem 11. Find the inverse Laplace transform of

$$F(s) = \frac{s}{(s+1)^2\left(s^2 + 4s + 5\right)}.$$

Problem 12. Using Laplace transforms find the solution $x(t), t \geq 0$, of the differential equation

$$\ddot{x}(t) + 4x(t) = 0, \quad x(0) = 5, \quad \dot{x}(0) = 0.$$

Problem 13. Using Laplace transforms find the solution $x(t), t \geq 0$, of the differential equation (Hint: You may used the result from Problem 10.)

$$\ddot{x}(t) + \omega_n^2 x(t) = t, \quad x(0) = 0, \quad \dot{x}(0) = 0.$$

Problem 14. Using Laplace transforms find the solution $x(t), t \geq 0$, of the differential equation

$$2\ddot{x}(t) + 2\dot{x}(t) + x(t) = u(t), \quad x(0) = 0, \quad \dot{x}(0) = 2,$$

where $u(t) = 1$.

Problem 15. Using Laplace transforms find the solution $x(t)$, $t \geq 0$, of the differential equation (Hint: You may used the result from Problem 7.)

$$2\ddot{x}(t) + 7\dot{x}(t) + 3x(t) = 0, \quad x(0) = 3, \quad \dot{x}(0) = 0.$$

Problem 16. Using Laplace transforms find the solution $x(t)$, $t \geq 0$, of the differential equation

$$\ddot{x}(t) + x(t) = u(t), \quad x(0) = 0, \quad \dot{x}(0) = 0.$$

where $u(t) = \sin(3t)$.