ME 41100 System Dynamics and Control Dr. B. Wang

Spring 2024

Homework 3

Problem 1. (Electrical systems)

1) Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical circuit shown in Figure 1.

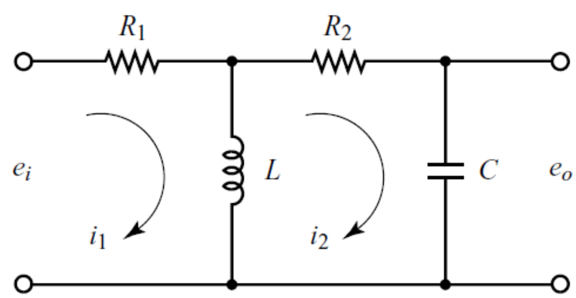

Figure 1: Electrical circuit.

2) Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical circuit shown in Figure 2.

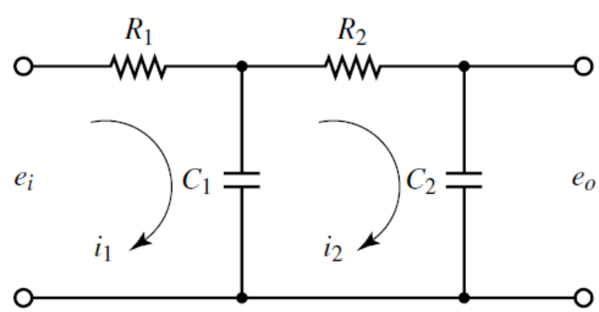

Figure 2: Electrical circuit.

3) For the system $E_o(s)/E_i(s)$ shown in Figure 2, suppose that $R_1 = R_2 = C_1 = C_2 = 1$, and that all initial conditions are zero. Is the system overdamped, critically damped, or underdamped? What is the dominant pole?

Problem 2. (Electro-mechanical systems) Consider the system shown in Figure 3. An armature-controlled DC servomotor drives a load consisting of the moment of inertia J_L. The torque developed by the motor is $T = Ki_a$, where K is the motor torque constant and i_a is the armature current. The moment of inertia of the motor rotor is J_m. The angular displacements of the motor rotor and the load element are θ_m and θ, respectively. The gear ratio is $n = \theta/\theta_m$.

1) Assume that the output is the angular velocity $\dot{\theta}$. Obtain the transfer function $s\Theta(s)/E_i(s)$.

2) Suppose that $n = 0.1$, $K = 10$, $L = R = 1$, $J_m = 0.1$, $J_L = 10$ the back emf constant of the motor $K_b = 0.1$. Is the system overdamped, critically damped, or underdamped? What is the damped natural frequency? What is the unit-step response?

3) What are the rise time t_r, the peak time t_p, the maximum overshoop M_p, and the settling time t_s of the system?

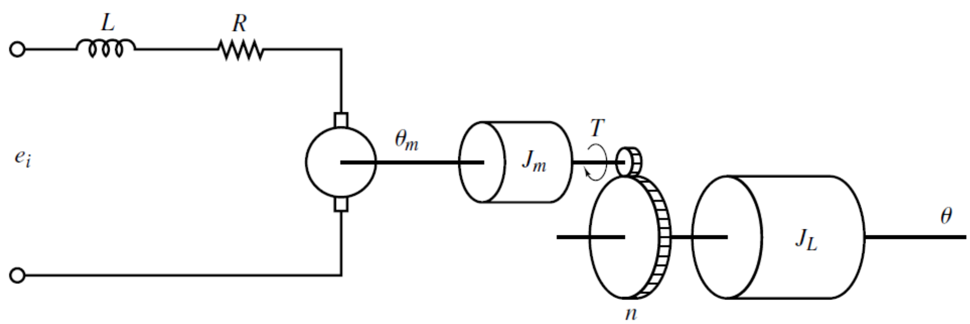

Figure 3: Armature-controlled DC servomotor system.

Problem 3. [Optional] (Thermal systems) Consider the thermal system shown in Figure 4. Considering small deviations from steady-state operation, assume that the heat loss to the surroundings and the heat capacitance of the metal parts of the heater are negligible. Let us define $\bar{\Theta}_i$ the steady-state temperature of inlet liquid, $\bar{\Theta}_o$ the steady-state temperature of outlet liquid, G the mass flow rate of liquid through the heating chamber, R the thermal resistance, C the thermal capacitance of liquid contained in the heating chamber, $\bar{H}$ steady-state heat input. Let us assume that the heat input is suddenly changed from $\bar{H}$ to $\bar{H} + h$ and the inlet liquid temperature is suddenly changed from $\bar{\Theta}_i$ to $\bar{\Theta}_i + \theta_i$. Then the outlet liquid temperature will be changed from $\bar{\Theta}_o$ to $\bar{\Theta}_o + \theta_o$.

1) Obtain the transfer functions $\Theta_o(s)/H(s)$ and $\Theta_o(s)/\Theta_i(s)$ of the thermal system.

2) Assume that the mass of liquid contained in the heating chamber $M = 10$, the specific heat of the liquid $c = 0.1$, and the thermal resistance $R = 2$. What are the time constant T, rise time t_r, settling time t_s of the system?

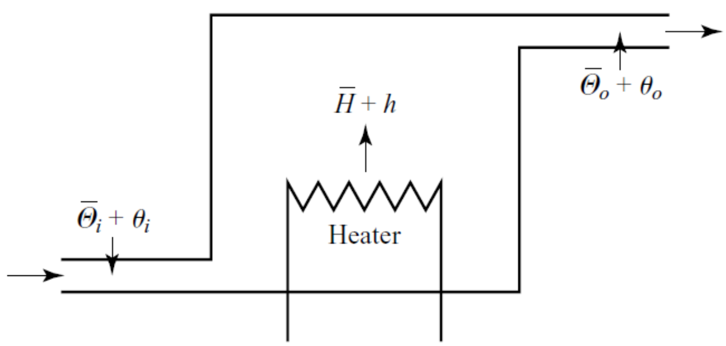

Figure 4: Armature-controlled DC servomotor system.

Problem 4. Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical system shown in Figure 5. What is the order of the system? What are the zeros and the poles? What are the rise time and the settling time of the step response of the system?

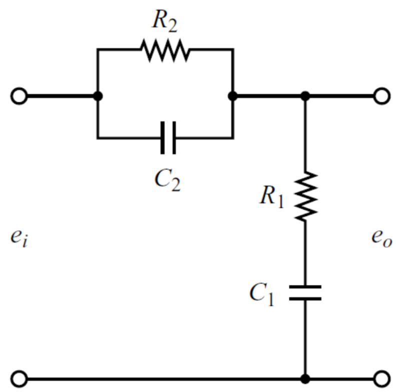

Figure 5: Electrical system