In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from scipy import integrate

plt.rcParams['figure.dpi']= 200
plt.rcParams['axes.facecolor']='white'
plt.rcParams['savefig.facecolor']='white'
```

In [ ]:
```
## 1a ##
def f(t,y):
    x = -3*y*np.sin(t)
    return x

def fe(dt, start, stop, y_0):
    y = np.zeros(int(np.round((stop-start)/dt) + 1))
    t_arr = np.linspace(start,stop,y.size)
    y[0] = y_0
    for i in range(len(t_arr)-1):
        y[i+1] = y[i] + dt*f(t_arr[i], y[i])
    return y

dts = np.array([2**-2, 2**-3, 2**-4, 2**-5, 2**-6, 2**-7, 2**-8])
E_fe = np.zeros((len(dts)))

t_0 = 0
tN = 5
y_0 = np.pi/np.sqrt(2)

for j, dt in enumerate(dts):
    y_fe = fe(dt, t_0, tN, y_0)
    y_true = np.pi*np.exp(3*(np.cos(5) - 1))/np.sqrt(2)
    E_fe[j] = np.abs(y_true - y_fe[-1])

l_fe = np.polyfit(np.log(dts), np.log(E_fe), deg = 1)

A1 = y_fe.reshape(-1,1)
A2 = E_fe.reshape(1,-1)
A3 = l_fe[0]
```

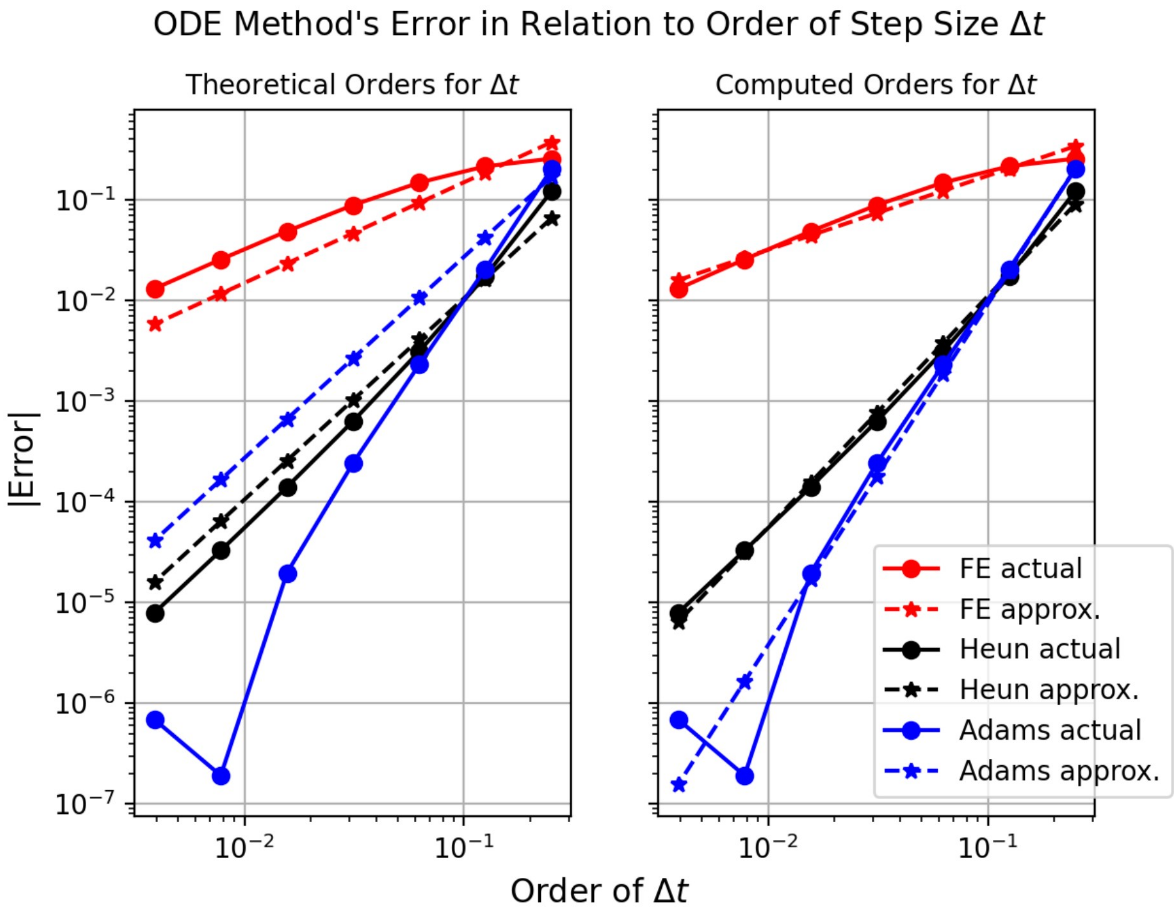
In [ ]:
```
## 1b ##
def f(t,y):
    x = -3*y*np.sin(t)
    return x

def heun(dt, start, stop, y_0):
    y = np.zeros(int(np.round((stop-start)/dt) + 1))
    t_arr = np.linspace(start,stop,y.size)
    y[0] = y_0
    for i in range(len(t_arr)-1):
        ytilde = y[i] + dt*f(t_arr[i], y[i])
        y[i+1] = y[i] + (dt/2)*(f(t_arr[i],y[i]) + f(t_arr[i+1], ytilde))
    return y

dts = np.array([2**-2, 2**-3, 2**-4, 2**-5, 2**-6, 2**-7, 2**-8])
E_heun = np.zeros((len(dts)))

t_0 = 0
tN = 5
y_0 = np.pi/np.sqrt(2)

for j, dt in enumerate(dts):
    y_heun = heun(dt, t_0, tN, y_0)
    y_true = np.pi*np.exp(3*(np.cos(5) - 1))/np.sqrt(2)
    E_heun[j] = np.abs(y_true - y_heun[-1])

l_heun = np.polyfit(np.log(dts), np.log(E_heun), deg = 1)

A4 = y_heun.reshape(-1,1)
A5 = E_heun.reshape(1,-1)
A6 = l_heun[0]
```

In [ ]:

```python
## 1c ##
def f(t,y):
    x = -3*y*np.sin(t)
    return x

def adams(dt, start, stop, y_0):
    y = np.zeros(int(np.round((stop-start)/dt) + 1))
    t_arr = np.linspace(start,stop,y.size)
    y[0] = y_0
    y[1] = y[0] + dt*f((t_arr[0] + dt/2), (y[0] + (dt/2)*f(t_arr[0],y[0])))
    for i in range(1,len(t_arr)-1):
        yp = y[i] + (dt/2)*(3*f(t_arr[i],y[i]) - f(t_arr[i-1], y[i-1]))
        y[i+1] = y[i] + (dt/2)*(f(t_arr[i+1],yp) + f(t_arr[i],y[i]))
    return y

dts = np.array([2**-2, 2**-3, 2**-4, 2**-5, 2**-6, 2**-7, 2**-8])
E_adams= np.zeros((len(dts)))

t_0 = 0
tN = 5
y_0 = np.pi/np.sqrt(2)

for j, dt in enumerate(dts):
    y_adams = adams(dt, t_0, tN, y_0)
    y_true = np.pi*np.exp(3*(np.cos(5) - 1))/np.sqrt(2)
    E_adams[j] = np.abs(y_true - y_adams[-1])

l_adams = np.polyfit(np.log(dts), np.log(E_adams), deg = 1)

A7 = y_adams.reshape(-1,1)
A8 = E_adams.reshape(1,-1)
A9 = l_adams[0]
```

In [ ]:

```python
## 1plot ##
names = ['FE', 'Heun', 'Adams']
intercepts = [4e-1,5e-2,1]
orders = [1,2,2]
slopes = [l_fe, l_heun, l_adams]
color = ['r','k','b']
fig, ax1 = plt.subplots(1,2, sharey = True)
for i, E in enumerate([E_fe, E_heun, E_adams]):
    ax1[0].loglog(dts, E, f'o{color[i]}-', label = f'{names[i]} actual')
    ax1[0].loglog(dts, (np.exp(intercepts[i])*dts**orders[i]), f'*{color[i]}--', label = f'{names[i]} approx.')
    ax1[1].loglog(dts, E, f'o{color[i]}-')
    ax1[1].loglog(dts, (np.exp(slopes[i][1])*dts**[slopes[i][0]]), f'*{color[i]}--')

ax1[0].set_title('Theoretical Orders for $\Delta t$', fontsize = 10)
ax1[1].set_title('Computed Orders for $\Delta t$', fontsize = 10)
ax1[0].grid()
ax1[1].grid()
fig.subplots_adjust(top = 0.87)
fig.supxlabel('Order of $\Delta t$')
fig.supylabel('|Error|')
fig.suptitle('ODE Method\'s Error in Relation to Order of Step Size $\Delta t$')
fig.legend(loc = [0.73,0.14])
plt.savefig('hw1_1.png')
plt.show()
```



In [ ]:

```
## 2b ##
def pol_one(t, y):
    return np.array([y[1], -y[0] - (y[0]**2 - 1)*y[1]])

methods = ['RK45','RK23', 'BDF']
tols = np.array([1e-4,1e-5,1e-6,1e-7,1e-8,1e-9,1e-10])
slopes = np.zeros((3))

y0 = [2,np.pi**2]

for i, method in enumerate(methods):
    tavg = np.zeros((tols.size))
    for j, tol in enumerate(tols):
        sol = sp.integrate.solve_ivp(pol_one, [0,32], y0, method = method, atol = tol, rtol = tol)
        blah = np.diff(sol.t)
        tavg[j] = np.mean(np.diff(sol.t))

    slopes[i] = np.polyfit(np.log(tavg), np.log(tols), deg = 1)[0]

A11 = slopes[0]
A12 = slopes[1]
A13 = slopes[2]
```