AMATH 351 Au 21 HW1

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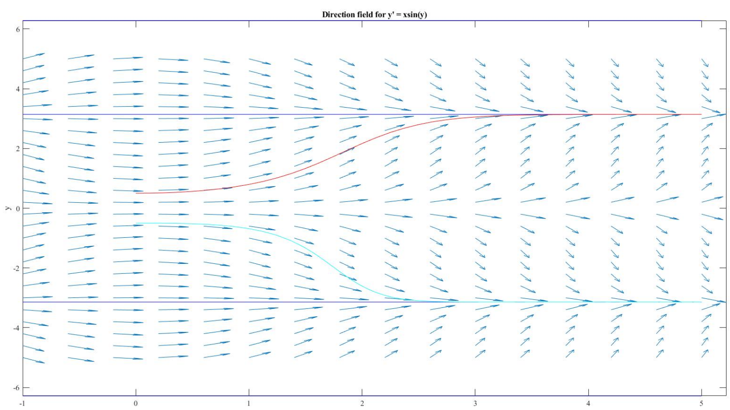
1 Problem 1

1.1 (a)

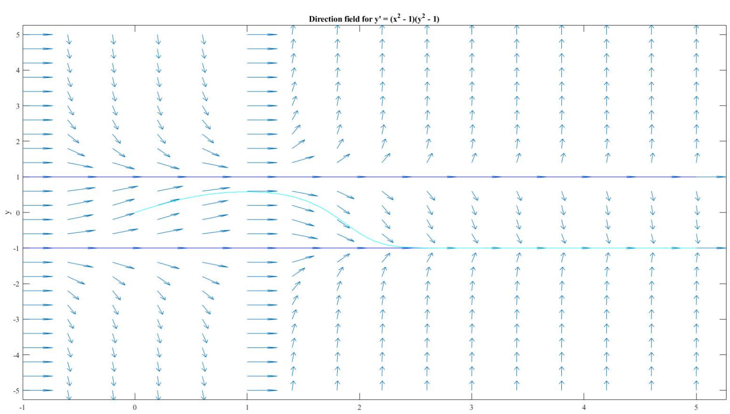
$\boxed{\text{If } y(0) = 1/2,\ \lim x \to \infty\ x\sin(y) = \pi}$.

$\boxed{\text{If } y(0) = -1/2,\ \lim x \to -\infty\ x\sin(y) = -\pi}$.

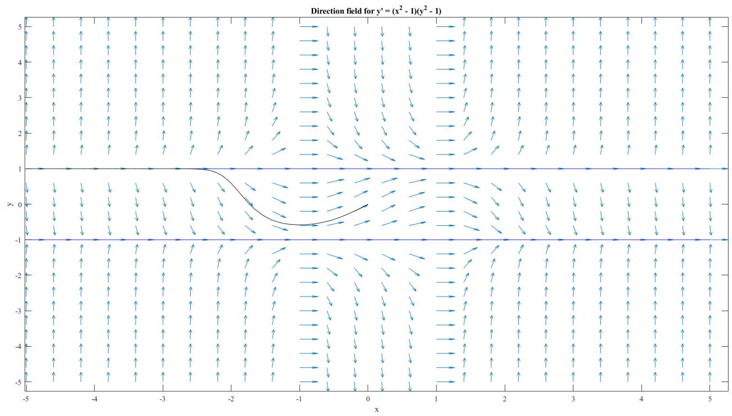


1.2 (b)

(i) $\boxed{\text{If } y(0) = 0,\ \lim x \to \infty\ (x^2-1)(y^2-1) = -1\ .}$



(ii) $\boxed{\text{If } y(0) = 0,\ \lim x \to -\infty\ (x^2-1)(y^2-1) = 1\ .}$

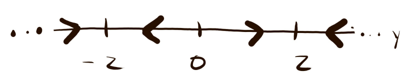


2 Problem 2

2.1 (a)

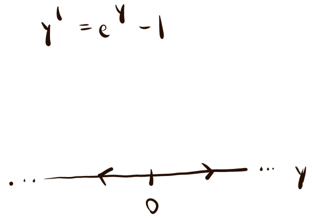
$y' = 4y - y^3 \to y' = y(4-y^2)$.
$y' = 0$ when $y = 0, \pm 2$.
$y = 0$ is an **unstable solution** as any initial value close to it wander away from $y = 0$. $y = 2$ is a **stable solution** as initial values close to said value settle to $y = 2$. $y = -2$ is a **stable solution** as initial values close to said value settle to $y = -2$.

$y' = y(4-y^2)$



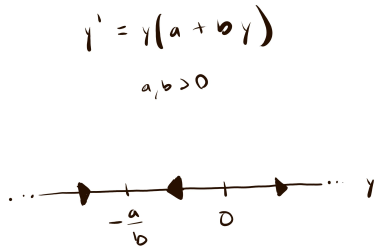
2.2 (b)

$y' = e^y - 1$.
$y' = 0$ when $y = 0$.
$y = 0$ is an **unstable solution** as any initial value close to it blow up in either direction.



2.3 (c)

$y' = y(a + by)$.
$y' = 0$ when $y = 0, -\frac{a}{b}$.
$y = 0$ is an **unstable solution** for the same reasons previously stated in section 2.2. $y = -\frac{a}{b}$ is a **stable solution** as any initial value close to it will correct itself to the solution $y = -\frac{a}{b}$.



3 Problem 3

For $y' = x^2 - y$, if x^2 is treated as a constant, the equilibrium solution approaches $y = x^2$ which is a **stable solution**. If $y < x^2$, then the differential will be positive as, for all R, x^2 is a positive number, so the solution will rise to $y = x^2$. If $y > x^2$, then the equation will be negative, so the solution will fall to $y = x^2$
For **large values of x**, **the solution will shoot up to equally large positive y values** but maintain equilibrium as the differential corrects itself because x^2 is in front of $-y$ which pushes solutions to equilibrium.
Treating x^2 as a constant, the equilibrium solution for $y' = y - x^2$ is $y = x^2$, some constant, which is an **unstable solution** as if $y > x^2$ then the solution continues to grow past the equilibrium and if $y < x^2$ then the solution will continue to negatively grow away from the equilibrium solution.
At large values of x, solutions starting above x^2 **shoot up past the equilibrium solution** positively, and for below x^2, they **continually fall away** from the equilibrium solution.

4 Problem 4

4.1 (a)

$y' = \frac{1+\cos(t)}{1+3y^2}$
$(1+3y^2)y' = 1+\cos(t)$
$\int(1+3y^2)dy = \int(1+\cos(t))dt$
$\boxed{y + y^3 = t + \sin(t) + c}$

4.2 (b)

$y' = -3t^2y^2,\ y(2) = 1$
$\frac{1}{y^2}y' = -3t^2$
$\int y^{-2}dy = \int -3t^2dt$
$-\frac{1}{y} = -t^3 + c$
$-\frac{1}{(1)} = -(2)^3 + c$
$c = 7$
$\boxed{y = -\frac{1}{-t^3+7}}$

4.3 (c)

$y' = \frac{x(x^2+1)}{4y^3},\ y(0) = -\frac{1}{\sqrt{2}}$
$4y^3y' = x^3 + x$
$\int 4y^3\ dy = \int x^3 + x\ dx$
$y^4 = \frac{1}{4}x^4 + \frac{1}{2}x^2 + c$
$y = \pm(\frac{1}{4}x^4 + \frac{1}{2}x^2 + c)^{\frac{1}{4}}$
$(-\frac{1}{\sqrt{2}}) = -(\frac{1}{4}(0)^4 + \frac{1}{2}(0)^2 + c)^{\frac{1}{4}}$
$(-\frac{1}{\sqrt{2}})^4 = c \to c = \frac{1}{4}$
$\boxed{y = -(\frac{1}{4}x^4 + \frac{1}{2}x^2 + \frac{1}{4})^{\frac{1}{4}}}$

4.4 (d)

$y' - 3y = ye^{2x}$
$y' = y(e^{2x} + 3)$
$\int \frac{1}{y}dy = \int e^{2x} + 3$
$-\frac{1}{2y^2} = \frac{1}{2}e^{2x} + 3x + c$
$\boxed{y = \pm\sqrt{-\frac{1}{(e^{2x}+6x+C)}}}$

5 Problem 5

5.1 (a)

$(e^{2y} - y)\cos(x)\frac{dy}{dx} = e^y \sin(2x),\ y(0) = 0$
$e^y - ye^{-y}\frac{dy}{dx} = 2\sin(x)$
$\int e^y - ye^{-y}dy = \int 2\sin(x)dx$
$e^y + \frac{y}{e^y} - \int e^{-y}dy = -2\cos(x) + c$
$e^y + \frac{y}{e^y} + \frac{1}{e^y} = -2\cos(x) + c$
$e^{(0)} + \frac{(0)}{e^{(0)}} + \frac{1}{e^{(0)}} = -2\cos(0) + c$
$c = 4$
$\boxed{e^y + \frac{y}{e^y} + \frac{1}{e^y} = -2\cos(x) + 4}$

5.2 (b)

$\frac{dy}{dx} = \frac{\pi}{2}e^{2x}(1 + y^2),\ y(0) = 1$
$\int \frac{1}{(1+y^2)}dy = \frac{\pi}{2}\int e^{2x},\ y = \tan(\theta),\ dy = \sec^2(\theta)d\theta$
$\int \frac{\sec^2(\theta)}{1+\tan^2(\theta)}d\theta = \frac{\pi}{4}e^{2x} + c$
$\int 1d\theta = \frac{\pi}{4}e^{2x} + c$
$\theta = \frac{\pi}{4}e^{2x} + c$
$\tan^{-1}(y) = \frac{\pi}{4}e^{2x} + c$
$\tan^{-1}(1) = \frac{\pi}{4}e^{2(0)} + c$
$\frac{\pi}{4} = \frac{\pi}{4} + c$
$c = 0$
$\boxed{y = \tan(\frac{\pi}{4}e^{2x})}$

5.3 (c)

$y' = xe^x\sec(y),\ y(0) = \frac{\pi}{6}$
$\int \cos(y)dy = \int xe^x dx$
$\sin(y) = xe^x - \int e^x dx + c$
$\sin(y) = e^x(x - 1) + c$
$\sin(\frac{\pi}{6}) = e^0(0 - 1) + c$
$\frac{3}{2} = c$
$\boxed{\sin(y) = e^x(x - 1) + \frac{3}{2}}$