AMATH 482: Home Work 1

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AMATH 482 Wi 22
28 January, 2022

Abstract

24 hours of noisy acoustic observation has revealed the imprecise location of a new, advanced submarine patrolling the Puget Sound. To deploy a submarine-tracking aircraft, the noisy data has to be de-noised and its location and path must be determined. The following paper averages the Fourier Series of the noisy spatial-temporal acoustic data to determine the submarine's main frequency[1], [47, 38, 9] and [17, 26, 55]. By employing a basic Gaussian filter situated at the submarine's main frequency on the Fourier Series of the noisy data, the position for each time point can be revealed after inversion and is plotted in Fig. 3.

1 Introduction and Overview

The purpose of this assignment is to determine the precise path and location of the submarine from noisy, spatial-temporal pressure data. The data provided is 49, half-hour acoustic observations of a $64 \times 64 \times 64$ region of the Puget Sound which the submarine is known to travel through. The isosurface of the data at a single time point, Fig. 1, reveals that the submarine's imprecise location can be determined, but for a precise location, localized filtering must be employed.

To begin the filtering, the reshaped data will be converted from the spatial-temporal domain to the frequency domain through the Discerete Fourier Transform. The equipment used has already specified the spatial-temporal and frequency parameters to convert the data between the separate domains. The frequency data can then be averaged over the length of the temporal dimension of the data, i.e., by the 49 different time cuts. This will reduce some noise allowing the frequency of the submarine to be precisely found by calculating the maximum value of the averaged frequency data. Next, a simple Gaussian filter can be applied to each individual frequency time cut localized around the submarine's frequency. By reverting this filtered data from the frequency domain to the spatial domain, the maximum

[1] Unless explicitly stated, all frequencies will be stated as the index of the frequency grid rather than the frequency grid itself for brevity and clarity; however, the code attached only uses the frequency grid for computation.

acoustic value can be precisely determined which corresponds to the submarine's location at each time segment.

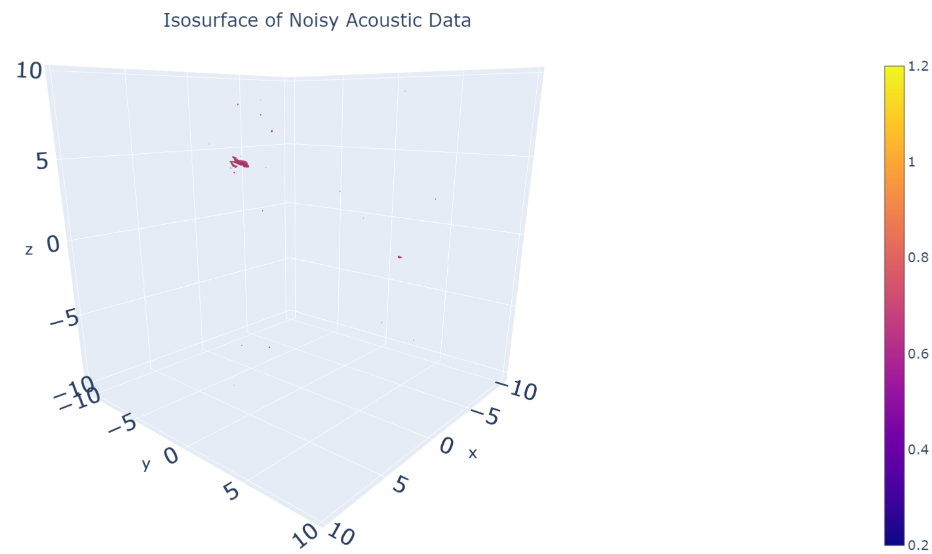

Figure 1: The large red blob is the approximate location at one of the readings of the submarine while the smaller, distal blobs are noisy data that needs to be filtered out.

2 Theoretical Background

The Fourier Transform (FT) of time series data conveniently expresses complicated data as a sum of sines and cosines, weighted by the coefficient c_k, for various frequencies, $k \in (-\infty, \infty)$. One definition of the Fourier Transform is:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k \exp(\frac{i\pi kx}{L}),$$

$$c_k = \frac{1}{2L}\int_0^{2L} f(x)\exp(-\frac{i\pi kx}{L})dx.$$

The Discrete Fourier Transform, or DFT, works for discrete time series data and is what is being implemented through the Fast Fourier Transform (FFT) which truncates the Fourier Series into a calculable Riemann sum. The DFT also changes some conventions, but the general form is the same as the Fourier Transform.

By computing the Fourier Series of time series data for each time point then averaging the Fourier Series for different time points, mean-zero noise will be reduced leaving a clearer Fourier Series. The highest amplitude component corresponds to the the three dimensional spatial frequency which best describes the data with the least information. From this dominant frequency, a simple, localized Gaussian filter can be created in the form of

$$g(x, y, z) = \exp(-\frac{1}{2\sigma^2}[(x - k_{max(x)})^2 + (y - k_{max(y)})^2 + (z - k_{max(z)})^2]) \tag{1}$$

where k_{max} refers to the different dimensional components of the largest amplitude in the frequency domain. A uniform meshgrid is generated for the Gaussian filter and applied to the Fourier Series which, when inverted, will return a clarified signal.

3 Algorithm Implementation and Development

To begin to apply the FFT to the submarine data, the data must first be reshaped using the *np.reshape*[2] command for each time slice. This reshapes the flattened data provided into a cube of $64 \times 64 \times 64$ pressure values for each of the 49 time slices. A *for* loop is implemented in order to sequentially reshape individual slices of data where the FFT is computed, shifted, and added together into a total where the average maybe taken. The FFT is implemented using *np.fft.fftn* then shifted using *np.fft.fftshift*. Shifting is required as numpy does not center its indices at 0. From this sum of the Fourier Series along all time slices, the average may be taken and the central frequency can be found. Using *np.amax* over the averaged Fourier Series returns the value of the largest, averaged c_k. Using this value in conjunction with *np.where* command will extract the x, y, and z spatial frequencies of the submarine. The outputted x and y for 3D spatial frequencies must be manually swapped as *np.where* switches them; this does not apply for 2D *np.where* searches. Because all the values of the acoustic data are real valued, the mirrored Fourier Transform can also be accounted for by subtracting the coordinates of the central frequency of the submarine from the length of the index dimension, 64. The central frequency of the submarine can also be located qualitatively by plotting slices of the averaged Fourier series with different z frequency heights using *plt.imshow*[3].

By substituting the k_{max} and the mirrored k_{max} into two separate Gaussians described in Equation (1) with the frequency values associated with each dimensional index calculated before, a nested *for* loop will generate the two localized Gaussian filters which can be added together to form the actual Gaussian employed. This filter can be be applied to the Fourier Series for each time slice and reverted back into the spatial domain through the *np.fft.ifftshift* and then *np.fft.ifftn* commands. The command *np.real* must be used on the output of *np.fft.ifftn* as very small imaginary values will leak through based upon minor computational errors in the algorithm. For each of these de-noised data cubes, another round of using *np.amax* and *np.where* will locate the indices of the submarine's position at each time point. Applying these triplets of indices for each time step to the provided spatial domain of the equipment will reveal the precise location of the submarine. Using *plt.plot* on the x and

[2]np refers to the numpy package

[3]plt refers to the matplotlib.pyplot package

y components of the submarine's position will generate the x-y path of the submarine. For graphing the 3D position of the submarine, the *plt.axes(projection = '3d')* command must be used.

Wherever *for* loops have been used for inputting data, first, generate arrays of zeros for the size of data required in order to prevent incorrect data inputting. This has been executed with the *np.zeros* command.

4 Computational Results

By averaging the Fourier Series of the noisy acoustic data and then finding the maximum frequency corresponding to the submarine, as described in Section 3, the center frequency was determined to be [47, 38, 9] and [17, 26, 55]. These indices correspond to their appropriate position in the frequency grid provided by the equipment. Using the central frequency of the submarine, the Gaussian filter could be generated with $\sigma = 4$ in the form $\exp(-\frac{1}{2(4^2)}[(x-47)^2+(y-38)^2+(z-9)^2])$. The value of σ was chosen by inspecting final trajectory so any σ value of equal magnitude would be appropriate. This can be observed graphically through Fig. 2 which shows a strong peak at approximately the appropriate indices as *plt.imshow* only allows real plotting and not imaginary also.

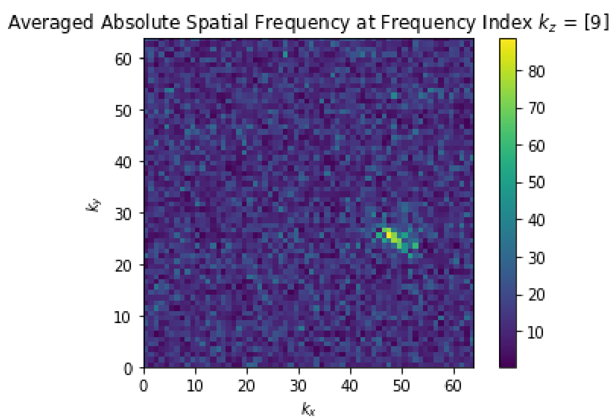

Figure 2: The bright yellow dot corresponds to the largest amplitude of the averaged Fourier Series.

After applying the filter iteratively as described in Section 3, this results in the final position of the submarine at [-5.3125, 5.9375, 0.625] (x,y,z) with the 2D and 3D path plotted in Fig. 3.

5 Summary and Conclusions

Through the averaging of the Fourier Series of noisy acoustic data and then implementing a Gaussian filter at the center frequency of said average, [47, 38, 9] and [17, 26, 55], the

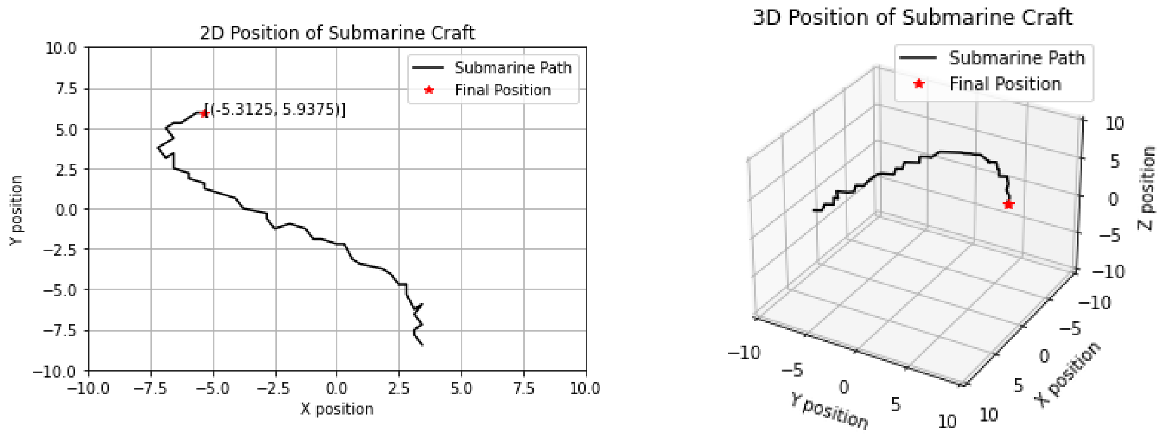

Figure 3: The 2D and 3D plots of the position of the Submarine with the final position marked.

path of the submarine, Fig. 3, could be generated in order to deploy a submarine-tracking aircraft for future monitoring of said submarine. We suggest that the submarine task force use extrapolation methods on the path of the submarine in order to accurately track the advanced submarine in the future. Observing the submarine's location for longer and in more frequent, shorter periods of time along with implementing a finer Gaussian filter or different filter will generate a more precise path which the submarine can be tracked along.

6 Acknowledgements

The author would like to graciously thank his classmates for allowing him to discuss the problem with them as a means to help them and test his own understanding of the material.