CS70–Fall 2011 — Solutions to Homework 7

1. **Sample Space and Events**

   (a) Ω has 16 total outcomes.
   HHHH, HTHH, THHH, TTHH, HHHT, HTHT, THHT, TTHT,
   HHTH, HTTH, THTH, TTTH, HHTT, HTTT, THTT, TTTT

   (b) A has 8 possible outcomes.
   HHHH, HTHH, HHHT, HTHT, HHTH, HTTH, HHTT, HTTT

   (c) B has 8 possible outcomes.
   HHHH, HTHH, THHH, TTHH, HHHT, HTHT, THHT, TTHT

   (d) C has 4 possible outcomes.
   HHHH, HTHH, HHHT, HTHT

   (e) D has 12 possible outcomes.
   HHHH, HTHH, THHH, TTHH, HHHT, HTHT THHT, TTHT,
   HHTH, HTTH, HHTT, HTTT

   (f) Like you can see from the outcomes, A and B are not disjoint. The relationships are

   $$\text{C} = \text{A } \textbf{AND} \text{ B}$$
   $$\text{D} = \text{A } \textbf{OR} \text{ B}$$

   (g) I leave the details of this up to you.

   $$|\Omega| = 2^n$$
   $$|A| = 2^{n-1}$$
   $$|B| = 2^{n-1}$$
   $$|C| = 2^{n-2}$$
   $$|D| = 2^n - 2^{n-2}$$

(h) $\frac{2}{3}$. If you see carefully, this problem is essentially the Monty Hall problem in disguise.
An outcome consists of us selecting a coin, and then choosing one of its sides. Since there are 3 coins each with 2 sides, our sample space Ω has 6 elements

$$\Omega = \{(HH, H1), (HH, H2), (HT, H), (HT, T), (TT, T1), (TT, T2)\}$$

where $(HH, H1)$ refers to the outcome of drawing the coin with two heads and of looking at the first side. Similarly, $(HH, H2)$ refers to the outcome of choosing the coin with two heads and of looking at the second side, (HT, T) refers to the outcome of choosing the coin with both heads and tails and of looking at the tails side, etc. Let A be the event that we choose the HH coin, and let B be the event that we see a heads when we put the coin down on the table. We wish to compute $Pr[A|B]$. Since $A = (HH, H1), (HH, H2)$, $B = (HH, H1), (HH, H2), (HT, H)$, and A $\cap$ B = A, we see

$$Pr[A|B] = \frac{Pr[A]}{Pr[B]} = \frac{2}{3}$$

2. **Monty Hall Revisited**
Solution: $\frac{2}{3}$. Consider three doors. Each of these doors has a probability of $\frac{1}{3}$ of having the prize, because they are independent from one another. Lets call them A, B and C. So,

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

Suppose you select one of these doors, say A. As mentioned, the probability that A has the prize is mere $\frac{1}{3}$. Therefore, the probability that the prize is in B or C(not in A is $\frac{2}{3}$. Now the host opens either B or C, if it doesnot have the prize. Suppose he opens B. Now because B doesnot have the prize, the

$$P(B) = 0$$

but we already know that

$$P(A) = \frac{1}{3}$$

because total probability has to be 1. Thus,

$$P(C) = \frac{2}{3}$$

Note: For more explanation, please refer to This and This for more elaborate explanation.

3. **Rolling Dice**

   The details are left to you. These are pretty straight forward problems btw.

   (a) $\frac{1}{6}$
   (b) $\frac{1}{3}$
   (c) $\frac{11}{36}$
   (d) $\frac{1}{3}$