Complexity Worksheet

1. Do a "big-o" analysis of the following nested loops. Specify your analysis for each loop and also for the entire "algorithm" (as we did in class):

   (a)
   ```
   for (k=0; k<n; k++)
   {
       for (j=6; j<n; j++)
       {
           ...
       }
   }
   ```

   (b)
   ```
   for (k=0; k<n; k++)
   {
       j = n;
       while (j > 0)
       {
           ...
           j /= 2; // integer division
       }
   }
   ```

   (c)
   ```
   k = 1;
   do
   {
       j = 1;
       do
       {
           ...
           j *= 2;
       } while (j < n);
       k++;
   } while (k < n);
   ```

2. An algorithm has an efficiency of $O\left(n^2 \left|\sin(n)\right|\right)$. Is it any better than $O\left(n^2\right)$? Explain why or why not.

3. Suppose that each of the following expressions represents the number of logical operations in an algorithm as a function of n, the number of data items being manipulated. For each expression, determine the dominant term and then classify the algorithm in "big-o" terms:

   (a) $n^3 + n^2 \lg n + n^3 \lg n$

   (b) $n + 4n^2 + 4^n$

   (c) $48n^4 + 16n^2 + \log_8 n + 2n$

4. Consider the following nested loop construct. Categorize its efficiency in terms of the variable n using "big-o" notation. Suppose the statements represented by the ellipsis (...) require four main memory accesses (each requiring one microsecond) and two disk file accesses (each requiring one millisecond). Express in milliseconds the amount of time this construct would require to execute if n were 1000.

```
x = 1;
do
{
    y = n;
    while (y > 0)
    {
        ...
        y--;
    }
    x *= 2;
} while (x < n*n);
```