Laboratory Manual

PHSC 12720 Exoplanets

The University of Chicago

Spring 2022

Labs

LAB 1

Scale of the solar system and local stellar environment

> *Space is big. You just won't believe how vastly, hugely, mind-bogglingly big it is. I mean, you may think it's a long way down the road to the chemist's, but that's just peanuts to space.* Douglas Adams, The Hitchhiker's Guide to the Galaxy

Why can't we just see planets orbiting other stars with normal observational techniques like looking through larger and larger telescopes? Through this lab, we hope you gain a felt sense appreciation for the scale of star systems and the distance between the Sun and our closest stellar neighbor, and see why we need to use special techniques to detect exoplanets.

1.1 Forming Groups

If you are attending the lab session live and do not yet have a group, one way the TA could assist is to arrange "speed networking" among those who still need a group. This would involve the TA organizing Zoom Breakout Rooms, where each room is 2-3 students, and each group talks about how they work and what they are looking for in a group member. Then after 5 minutes or so, the Rooms are changed so people are with different people. This could help people get to know each other enough to form lab groups.

1. Once you have a group, meet with each other and decide a) what tools you will use to communicate and collaborate, b) when you will meet, c) what you will do when you need to change an agreement, and d) what you will do when a member has a concern about how the group is functioning. **Record your agreements.**

Team roles

2. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage

- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. Some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

Add members to Canvas lab report assignment group

3. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L1 Scale [number]" that isn't used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

1.2 Your current intuition

In order to gauge your current sense of where things are in relation to each other, each member of your group will first make three different small, qualitative scale models. **For this section, do not use a book, or the Internet, other people, or any other resource, to guide your efforts.** This will help you see where your current intuition lies. And when you compare to others, do not change your guess — it is expected that you might not have an accurate conception yet.

4. Individually, without looking at your groupmates' work, take a blank sheet of paper and **draw** a horizontal line across the page. On the left end of it, place a dot and mark it "Sun". On the right end, place a dot and mark it "Neptune". Now place and label a dot for your best guess of the orbital distance for each of the seven other planets orbiting the Sun.

5. Next, **make a similar scale model drawing**, this time placing the following objects on it: the Sun, Neptune, the Voyager I robotic probe (the furthest human-made object from Earth that started its journey in 1977), our nearest star Proxima Centauri, and its planet, Proxima Centauri b.

6. Finally, **draw** the Sun and all 8 planets with your best guess of their relative sizes — you should end up with 9 circles. The distances do not need to be scaled for this estimate.

7. Compare your drawings with your groupmates. **Record any surprises or big differences that you had.**

1.3 Making an accurate scale model

Now that you have your current sense of it, your lab section will make accurate scale models of the three scales you made above. Bigger is better, so you will use the hallway that extends on the second floor from the south end of Kersten, across the skywalk, through Eckhardt, and into the Accelerator Building.

Available equipment: measuring wheel, meter stick, tape measure, paper, masking or label tape, scissors, markers

Tips

- To make a scale model, you will need to gather the size or distance information for the objects in your model using any resource you'd like, then divide each of those by the same number to create your scale. You can use a spreadsheet to make this task easier.
- For the two distance scale models, you will need to measure the distance you have to work with in the hallway. Ensure that you can see from one end to the other. Prop open the skywalk doors if needed. You should place some kind of upright sign, perhaps taped down, at each spot where an object is in your model.
- For the size scale model, since all the objects are mostly spherical, you can create your model by cutting out circles of paper. For the larger objects, feel free to tape sheets of paper together to make a bigger circle.

8. Once all groups are ready to give their tour, take turns visiting each other's scale models and walking through them.
9. Make a table of the real distances and sizes and your scaled down versions. **Include this in your report.**
10. **Document your scale model** with pictures and/or video (you can upload your video somewhere and include a link in your lab report).

1.4 Appreciating distances

Answer the following questions:

1. Using a nominal speed of a car on Earth, how long would it take to drive:
   a) once around the equator?
   b) from the Earth to the Sun?
   c) from the Sun to Neptune?
   d) from the Sun to Proxima Centauri?
2. If you wanted to travel each of these distances in 1 year, how fast would you need to go in each case?
3. For the longest distance, how does that speed compare to the speed of light, which is the fastest anything can go?

1.5 Report checklist

Include the following in your lab report. See Appendix A for formatting details.

1. Your intuition estimates from Section 1.2 and your reflection of how they compare to your group mates'.
2. A table of the distances/sizes and scaled version that you created in Section 1.3.
3. Pictures or video of your scale model (Step 9).
4. Worked solutions to the questions in Section 1.4.
5. A 100–200 word reflection on the scale — was there anything that surprised you? How do you see your place in the universe, given the scale of the solar system and how far away even the nearest star is?

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6. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

LAB 2

Detecting exoplanets with the radial velocity method

2.1 Introduction

In the Fall of 1995, two Swiss astronomers announced evidence for a planet orbiting the star 51 Pegasi, a groundbreaking discovery since the star, 51 Pegasi, is very similar to our own Sun. Since 1995, thousands of potential exoplanets have been detected and the field of exoplanet science has become a pillar of modern astronomy.

In this lab, we will explore the radial velocity technique, the method used to detect the first exoplanets and a powerful technique for studying exosolar planetary systems.

2.2 Team roles

1. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used
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- Skeptic: ensures group is questioning itself

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Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

2.3 Add members to Canvas lab report assignment group

2. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L2 Radial [number]" that isn't used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

2.4 Building Intuition

We usually say that a planet orbits a star, but this isn't quite true — both planet and star orbit the center of mass of the system. This is called the *barycenter*. This results in the star wobbling in a circular motion. The motion is too small to detect from Earth directly. But when the source of waves moves towards or away from an observer, the wavelength of that wave is shifted lower or higher, respectively. Since stars have characteristic gaps in the spectrum of light they emit, we can detect this *Doppler shift*, and thus determine the *radial velocity* of the star. From this and our knowledge of orbital dynamics, we can deduce characteristics of exoplanets orbiting that star.

3. Open the link for the Center of Mass webpage (https://astro.unl.edu/mobile/center-of-mass-simulator/index.html). Use the sliders to explore how varying separation and relative mass changes the Center of Mass.
4. Open the link for the radial velocity simulator (https://www.bu.edu/astronomy/visualizations/AlienWorlds/simulation-radial.html).
5. Vary the mass of the planet, the mass of the star, and the distance. Note how the radial velocity graph changes. Vary the other parameters (viewing angle, star radius, planet radius) and see how the radial velocity changes. **Include your findings in your report.**

2.5 Building quantitative intuition

For this lab, the following values will be useful:

- 1 AU $= 1.496 \times 10^{11}$ m (approximate distance between the Earth and the Sun)
- 1 $\mathrm{M_J} = 1.898 \times 10^{27}$ kg (mass of Jupiter)
- 1 $\mathrm{M_{Sun}} = 1.989 \times 10^{30}$ kg (mass of the Sun)
- $G = 6.674 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$ (Newtonian constant of gravitation)

Mathematical tools for orbital mechanics

Equation for finding the center of mass position x_{CM} for two masses, m_1 and m_2, at positions x_1 and x_2:

$$x_{\mathrm{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \tag{2.1}$$

An object in a circular orbit of radius r experiences an acceleration $\vec{a}$ directed inward with magnitude

$$a = \frac{v^2}{r}\,. \tag{2.2}$$

In our case, the object is experiencing this acceleration as a result of the force exerted on it through gravity by the other object according to

$$F = G\frac{m_1\, m_2}{r_{12}^2}\,, \tag{2.3}$$

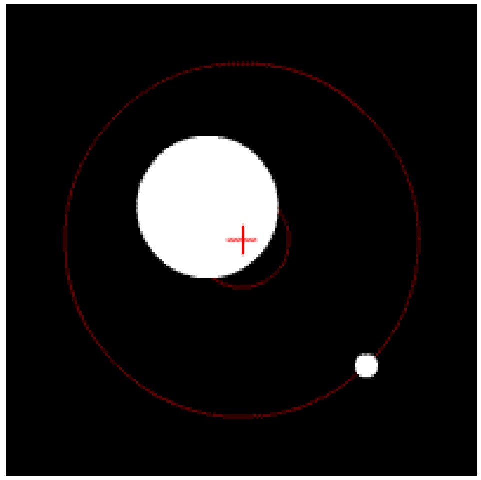

Figure 2.1: Schematic of a star (upper-left) and planet (lower-right) orbiting a common center-of-mass (marked with a red cross).

where r_{12} is the distance between the two objects, which is different from the orbital radius, since the object orbits the center of mass (or barycenter), rather than the other object.

To relate velocity in the first equation with the masses of the objects in the second equation, we can use the fact that the acceleration that an object experiences is proportional to the force on it and inversely proportional to its mass — or in equation form,

$$a = \frac{F}{m}\,, \tag{2.4}$$

also known as Newton's second law of motion.

So in this equation, if we substitute in a from Eq. 2.2 and F from Eq. 2.3, we can solve for the orbital velocity of Object 1 like so:

$$\frac{v_1^2}{r_1} = \frac{G\frac{m_1 m_2}{r_{12}^2}}{m_1} \tag{2.5}$$

$$\frac{v_1^2}{r_1} = G\frac{m_2}{r_{12}^2} \tag{2.6}$$

Analyzing a sample orbital system

Consider the picture in Figure 2.1 and define $M_\mathrm{p} = 0.5M_\mathrm{J}$ as the mass of the planet (the smaller object), $M_\mathrm{S} = 1.0M_\mathrm{Sun}$ as the mass of the star (the larger object), and $d = 0.0527$ AU be the separation between the two. Assume the system is in a circular orbit.

6. How far is the Center of Mass from the center of the star (in AU)?

7. What is the star's orbital radius (in AU)?

Since the star is moving around the Center of Mass, it has a non-zero velocity. Let the orbital period of the system be $T = 4.23$ days.

8. What is the orbital velocity of the host star?

9. Sketch the figure, then on that sketch, draw arrows showing the velocity at different parts of the orbit. What happens to the direction of the star's velocity as the planet traces out its orbit?

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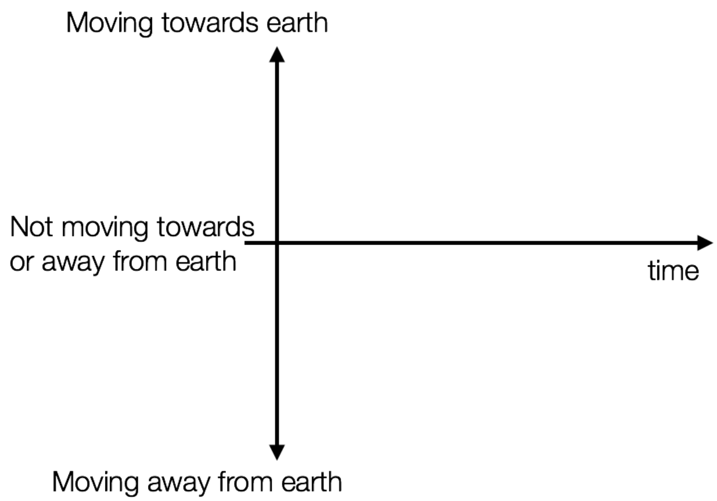

Figure 2.2: Graph to draw for radial velocity estimation.

Astronomers can use spectroscopic techniques for measuring the velocity of stars along the line of sight between the earth and the star. In the illustration in Figure 2.1, suppose the earth is to the right of the system so that we are observing the system edge on from the right side of the page.

10. **Draw a graph** similar to Figure 2.2 and then draw on it the "line of sight" velocity for the star as a function of time (hint: it will be a trigonometric function).

11. How are the period and amplitude related to the orbital period and the star's velocity?

Consider a similar system only with a planet that is twice the mass ($M_{\mathrm{p}} = M_{\mathrm{J}}$, $M_{\mathrm{S}} = M_{\mathrm{Sun}}$, $d = 0.0527$ AU, and $T = 4.23$ days).

12. What is the system's host star orbital radius (in AU) and orbital velocity (in m/s)?

13. How do these compare with the values for the previous system with a less massive planet?

14. **Sketch the line of sight velocity** for this more massive system on the same plot that you already drew.

15. How does the line of sight velocity depend on the planet's mass?

2.6 Radial velocity of 51 Peg

Table 2.1 lists measured line of sight velocities for the star 51 Pegasi measured by Marcy & Buttler in 1995 at the Lick Observatory in California. You will use this data to determine the mass and orbital distance of the exoplanet orbiting 51 Peg.

16. Open the Google Sheet found here: `https://docs.google.com/spreadsheets/d/1aTp-UxxgGf3wzgCrGr6HHANdEkv edit?usp=sharing`

17. Make a copy of this sheet for your group to use.

Day	v (m/s)	Day	v (m/s)	Day	v (m/s)	Day	v (m/s)	Day	v (m/s)	Day	v (m/s)	Day	v (m/s)
0.6	-20.2	3.6	-35.1	5.6	45.3	7.7	-22.6	9.6	25.1	10.8	51	13.6	2.7
0.7	-8.1	3.7	-42.6	5.7	47.6	7.8	-31.7	9.7	35.7	11.7	-2.5	13.7	17.6
0.8	5.6	4.6	-33.5	5.8	56.2	8.6	-44.1	9.8	41.2	11.8	-4.6		
1.6	56.4	4.7	-27.5	6.6	65.3	8.7	-37.1	10.6	61.3	12.6	-38.5		
1.7	66.8	4.8	-22.7	6.7	62.5	8.8	-35.3	10.7	56.9	12.7	-48.7		

Table 2.1: Line-of-sight velocities for 51 Pegasi measured over time.

This spreadsheet contains the data in Table 2.1, as well as a model for the velocity as a function of time. Circular motion, projected to 1 dimension, can be described by a sine function. The most general sine function has free parameters that can be changed to represent any sinusoidal motion. For example, the velocity function can be described with

$$v(t) = \mathrm{A}\sin\left[\frac{2\pi}{\mathrm{B}}\left(t+\mathrm{C}\right)\right] + \mathrm{D}\,, \tag{2.7}$$

where A is the amplitude of the motion and B is the period of the motion. C is a time offset, to account for where the star is in its oscillation cycle at $t = 0$. D is constant velocity offset, to account for any constant line of sight velocity that was not already compensated for (for example, the motion of the Sun relative to the star).

18. In your copy of the spreadsheet, explore changing the fit parameters A–D and see what effect it has on the model velocity.
19. For your first pass at fitting the model, find a set of parameters that makes the model velocity curve visually similar to the data.

To get a more precise estimate for the parameters, you can use a common statistic called the *rms deviation*. "rms" stands for root-mean-square. For N data points $y_{\mathrm{data},i}$, the rms deviation from the theoretical model points $y_{\mathrm{model},i}$ can be found by

$$\text{rms deviation} = \sqrt{\frac{\sum_{i=0}^{N-1}\left(y_{\mathrm{data},i} - y_{\mathrm{model},i}\right)^2}{N}}\,. \tag{2.8}$$

The smaller the rms deviation, the closer the fit. The rms deviation is already calculated for you in cell `D6` in the spreadsheet.

20. Starting with your first pass estimate of parameters, vary them to minimize the rms deviation and find your best fit parameters. **Record these parameters, the resulting plot, and your final rms deviation.**
21. From your fit to the data, what is the orbital period (days) and host star orbital velocity (m/s)?
22. What is the orbital radius (in AU) for the host star?

Kepler's third law relates the orbital period to the system's semimajor axis. In the case where the planet's mass is much smaller than the star's mass, Kepler's third law is

$$P^2 = \frac{4\pi^2}{GM}a^3\,, \tag{2.9}$$

where P is the orbital period, G is Newton's constant, M is the mass of the star, and a is the semimajor axis. The mass of 51Peg is $1.1M_{\mathrm{Sun}}$.

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23. According to your fitted orbital period and Kepler's third law, what is the system's semimajor axis?

The host star orbital radius and the semimajor axis are related by the Center of Mass.

24. Using the system's semimajor axis, the mass of 51Peg, and the host star's orbital period, determine the planet's mass (in units of M_{J}).

2.7 Radial velocities for other systems

25. Go to the webpage for SystemicLive (http://www.stefanom.org/systemic-live/). Note that while the graphics within the tutorial are currently not visible on the website, the data visualization and analysis software are still functional.

26. Click "Open Systemic" to start the program. Then on that first page, scroll down to "Tutorials and Resources" and click on the link "51 Pegged: Rediscovering the first exoplanet with Systemic Live".

27. Follow the steps of that tutorial to analyze radial velocity data for 51Peg.

Note that one key analysis tool you will use in the remainder of the lab is the power spectrum of the radial velocity data. We know that we can create any periodic function from the sum of sinusoids of different frequencies and phases. The power spectrum tells us which frequencies dominate that decomposition. Therefore, peaks in the power spectrum correspond to periodicities in the data, and hence point to possible planetary periods in the radial velocity measurements.

28. How do the mass and separation results from Systemic compare with your earlier 51Peg numbers? Print out a plot of the "Phased Radial Velocity" and write on it the Period and Mass for the system.

29. Use Systemic to determine orbit parameters for the following systems: 47Uma, 70Vir. Print out plots for your "Phased Radial Velocity" for each system. Write on your plots the Period and Mass for each system orbit.

30. Use Systemic to analyze the data for the Upsilon Andromedae system (noted as "upsand"). Hint, there are multiple planets for this system. How many planets do you find? What are their masses and orbital periods?

2.8 Reflection

Do some research on the history and background of the systems you analyzed (51Peg, 47Uma, 70Vir, and upsand). Write a one paragraph summary of this lab and discuss the following:

31. What was the historical context and significance of these measurements?

32. How do they relate to what you did in this lab?

2.9 Report checklist

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. Discussion of how radial velocity changes based on available parameters (Step 5)

2. Answers to Questions 6–9, including figure with sketch of velocities

3. Graph of line of sight velocity vs time for two stars (Steps 10 and 14)
4. Answers to Questions 11–13, 15
5. Your plots for line-of-sight velocities and fit parameters (Steps 19–20)
6. The exoplanet's orbital period and the host star's orbital velocity and orbital radius (Steps 21–22)
7. The star-exoplanet system's semimajor axis and the exoplanet's mass (Steps 23–24)
8. Orbit parameters and comparison for 3 star-planet systems (Steps 28–30)
9. Reflection on the systems you analyzed (Steps 31–32)
10. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

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LAB 3

Detecting exoplanets with the transit method

In the previous lab, we studied the Radial Velocity technique which is used for both detecting and studying systems of exoplanets. In this lab, we will explore another technique, called transit photometry, which complements the radial velocity technique. The transit method is employed by instruments like the Kepler satellite to discover new exoplanets and to measure their properties including the orbital size and the size of the planet. In turn, these properties can be combined with the temperature of the star to estimate the planet's characteristic temperature to answer the question as to whether an exoplanet is habitable (capable of supporting biological life similar to that of Earth).

Over the next two weeks, you will learn how to use the transit method to detect exoplanets. You will use one of the MicroObservatory telescopes, built and maintained by the Harvard-Smithsonian Center for Astrophysics and located at the Whipple Observatory in Amado, Arizona to take a series of images of a "target" star in order to calculate a light curve for that star, which could be used to learn about the planet(s) orbiting them. These images will form the basis of your subsequent investigation in the DIY Planet Search.

3.1 Team roles

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Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

3.2 Add members to Canvas lab report assignment group

2. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L3 Transit [number]" that isn't used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

3.3 Scheduling observations

First, schedule the remote observations. The observations are made at night in Arizona, and they can be scheduled during the day before those observations.

3. As a group, create a single account that you will all log in to at the DIY Planet Search website at `https://waps.cfa.harvard.edu/microobservatory/diy/index.php`.
4. Navigate to About and read the "About DIY Planet Search" and "About MicroObservatory" sections.
5. Navigate to "DIYTools", watch the "Schedule Target Tutorial", and schedule observations of at least two different star systems. For both, choose "All hours" and 60 second exposures.

You will only analyze one star system, but it may be cloudy on one night, so observing two different ones makes it more likely you get data to analyze.

3.4 Analysis of sample data

You will now analyze the simulated transit data. Complete as much of this as you can the first day. If you aren't able to finish to analysis, complete it when you return the second day. The properties for the two star-planet systems are given in Table 3.1. Some are already filled in, and you will calculate the remaining ones.

6. Download the transit light curve data file from Canvas, in the Labs module. Open this in a spreadsheet program. The two planets are in separate tabs.

	Planet 1	Planet 2
star radius	$0.85 \times R_\odot$	$0.202 \times R_\odot$
star mass	$0.9 \times M_\odot$	$0.15 \times M_\odot$
star luminosity	$0.656 \times L_\odot$	$0.00292 \times L_\odot$
planet radius (in R_J or R_E)		
orbital period	12.164 days	
orbital radius (in AU)		
irradiance at planet (W/m^2)		
power absorbed by planet (W)		
planet temperature (K)		
planet temperature (°F)		

Table 3.1: Properties of two simulated star-planet systems.

7. For Planet 1, plot the light curve (flux versus time) and include it in your report.
8. On the plot, mark where the transit starts and stops. Estimate the flux of the star for when the planet is transiting and when it isn't. Write down your values on the plot.
9. From these numbers, calculate the radius of Planet 1 and record it in the table. Report the radius in the appropriate units (either R_J, if the planet is closer in size to Jupiter, or R_E, if the planet is closer in size to Earth).
10. Repeat the analysis for Planet 2 using only the first two days of data.
11. For Planet 2, plot the light curve for the first 14 days of data and estimate the orbital period for Planet 2.

We can use the orbital periods together with the mass of the host star to determine the distance of the planet from the star (the orbital radius) using Kepler's third law,

$$P^2 = \frac{4\pi^2}{GM} a^3 \,, \tag{3.1}$$

where P is the orbital period, a is the orbital radius, M is the mass of the host star, and $G = 6.674 \times 10^{-11}\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$ is Newton's gravitational constant.

12. Use Kepler's law to determine the orbital radius for Planet 1 and Planet 2 and write down your values in the table.

The irradiance at the planet corresponds to the radiated flux (power per unit area) by the host star at the planet's orbital radius. It is calculated as

$$\text{irradiance} = \frac{\text{star luminosity}}{4\pi a^2} \,, \tag{3.2}$$

13. Use the above formula to fill in the table for the two planets.

If we assume that the planet absorbs all of the light hitting it from its host star, we can then calculate the total power absorbed by the planet by multiplying the irradiance by the cross sectional area of the planet

$$\text{power absorbed} = \text{irradiance} \times \pi r^2 \,, \tag{3.3}$$

where r is the radius of the planet.

14. Calculate the absorbed power and enter it in the table.

The planet not only absorbs radiation, but it also emits thermal radiation. Assuming that the planet radiates as a perfect blackbody (as in it does not reflect anything), the flux (power per unit area) radiated is related to the planet's temperature by the Stefan-Boltzmann law

$$\text{thermal flux} = \sigma \times T_p^4 \,, \tag{3.4}$$

where T_p is the temperature of the planet (in kelvins) and $\sigma = 5.67 \times 10^{-8}\ \mathrm{W} \cdot \mathrm{m}^{-2}\mathrm{K}^{-4}$ is the Stefan-Boltzmann constant.

The total power radiated by the planet is then given by

$$\text{thermal radiation} = 4\pi r^2 \times (\text{thermal flux}) \,. \tag{3.5}$$

If the planet has a stable temperature, then the power absorbed must equal the thermal power emitted, that is,

$$\text{thermal radiation} = \text{power absorbed} \,. \tag{3.6}$$

15. Use this relationship to calculate the temperature of both planets (in kelvins) and record the value in the table.
16. To get a better sense of the temperature for each planet, convert your calculated temperature into Fahrenheit and record the values in the table. How do these temperatures compare with the weather outside?

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3.5 Analyzing your own data

You will measure the brightness of the target star in each image that was taken. Since this will be around 80–100 images, you will probably want to split up the work among group members.

17. Log in to DIY Planet Search, navigate to "DIYTools", and select "Measure Brightness" from the menu at the left.
18. Watch the three tutorials at the right.
19. Select a "dark" image to use for calibration by opening it from the My Requests menu.
20. Follow the five steps listed for each image. In the first step, ensure that you calibrate your image by selecting "Calibrate"
21. If you cannot find the target star using the Finder, and your instructor cannot either, follow these instructions or ask your TA to follow these instructions to locate the star:
    a) Save the image locally as a FITS file using the Images menu.
    b) Navigate to `astrometry.net`, go to "Upload", and upload the FITS file. This website will "plate solve" the image, finding where in the sky the telescope was pointing, and assigning an RA,Dec coordinate to each pixel.
    c) Once it finishes analyzing, click "go to results page" and select "new-image.fits" to download a FITS file with the coordinates overlaid.
    d) Look up the star's name in Wikipedia or elsewhere to find its RA,Dec coordinates.
    e) Open the new FITS file in SaoImage DS9. Right-click on the image and drag around to adjust the contrast so you can see the stars.
    f) The RA, Dec coordinates of the mouse pointer are displayed at the top of the window. Move the pointer so that it points to the coordinates of the star. This should identify the target star.
22. Once all images have been analyzed, select "Interpret and Share" from the left-hand menu.
23. Use your data to estimate the transit depth and optionally share your comments and results to the whole DIY Planet Search community.
24. Using the guidance in the other tabs, estimate how big the planet is, whether it is tilted, and its distance to its star.

3.6 Report checklist and grading

Each item below is worth 10 points.

1. Data table from Part 1.
2. Plots of your light curves for Planet 1 and Planet 2.
3. Discuss how different features of the light curve connected to physical properties of the orbital system.
4. Your calculated temperature for Planet 1 and Planet 2, and your interpretation of the temperatures for the simulated planets.
5. Plot of your light curve from DIY Planet Search.

6. Your estimation of the transit depth, planet size, tilt, and star-planet distance of your observed star.
7. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

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LAB 4

Using Kepler's Laws to find the density of Jupiter

4.1 Introduction

So far, you have been using several techniques to learn about the properties of planets orbiting other stars based one what we can observe from Earth, including using orbital properties to discover the mass of planets. The same physical laws can be used to study planets and their moons.

To study the composition of a planet, it is useful to know its density — then one can learn more about whether it is rocky or gaseous. In this lab, you will use Kepler's Laws to find the density of Jupiter, given orbital properties of its moons. In this case, we can use actual images of the these using Stone Edge Observatory, a remotely operated telescope in California.

Under nominal circumstances, we would have you schedule the observations yourself, so you could analyze the data that you, yourself, took. However,Jupiter is not above the horizon at night right now in the northern hemisphere, where the telescope is, so we cannot observe it at this time of year. Fortunately, we have archival images that were taken in 2017 by this observatory that you can analyze.

4.2 Team roles

1. **Decide on roles** for each group member.

   The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. Some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

4.3 Add members to Canvas lab report assignment group

2. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L4 Jupiter [number]" that isn't used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

4.4 Using Kepler's laws

Keplers third law relates the orbital period to the systems semi-major axis. In the case where the planets mass is much smaller than the stars mass, Kepler's third law is:

$$P^2 = \frac{4\pi^2}{GM} a^3 \,, \tag{4.1}$$

where P is the orbital period, G is Newton's constant (6.674×10^{-11} N m^2/kg^2), M is the mass of Jupiter, and a is the semi-major axis.

We can rewrite the third law as:

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}\left(\frac{4}{3}\pi R^3 \rho\right) = \rho \frac{GR^3}{3\pi} \tag{4.2}$$

where R is the radius of Jupiter and ρ (the Greek letter pronounced "row") is the density of Jupiter. Solving for ρ gives:

$$\rho = \left(\frac{a}{R}\right)^3 \frac{3\pi}{GP^2} \tag{4.3}$$

The goal for this lab is to use our data to determine the ratio (a/R) for each moon. We can then combine that with the moons period to estimate the density of Jupiter.

To determine the semi-major axis, a, for a moon's orbit, we can assume that the orbit is actually a circular orbit, and we know that we are viewing the orbits edge-on. So what we are seeing is a projection of a circular motion onto one dimension, which results in a sinusoidal motion, with the amplitude of that motion being the radius of the orbit (and thus the semi-major axis). So here we will analyze images taken at different times, plot the angular position of the moon over time, fit those data points to a sine (or, equivalently, a cosine) function, and the amplitude will be the semi-major axis, a.

4.5 The set of observations

The images can be found on Canvas, in the Files section, in the compressed file "jupiter_data.zip".

Observations were taken on four separate dates between April 14 and 23, 2017, as stated in the names of the files. Images were taken in each of the g, r, i, and *clear* bands. These refer approximately to the color filters used in each case: green, red, infrared, and clear (no filter). See Figure 4.1 for the frequency response of such a filter set. All images were taken with a 0.05 second exposure time.

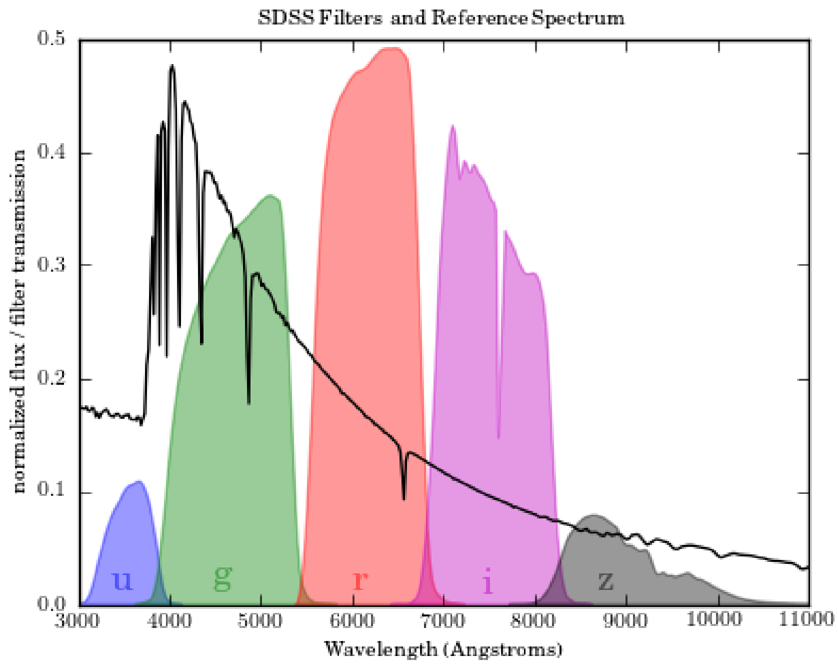

Figure 4.1: Typical transmission rates of various filters used in astronomy. This is from the Sloan Digital Sky Survey.

4.6 Analysis

You will use the *clear* images to measure the position of the moons relative to Jupiter and *gri* images to measure the radius of Jupiter. Since we will be using the ratio of moon position to Jupiter radius, we can use whatever units are convenient to measure the distance, as long as we are consistent, as the unit divides by itself in the calculation.

3. Download and install DS9 from http://ds9.si.edu/site/Download.html. SAOImage DS9, or DS9 for short, is an image viewer, analyzer, and processor written and used by astronomers for working with astronomical images.

   If you click the link to download, it might say "redirecting" while never actually redirecting. In this case, copy the link into the address bar directly.

> **For MacOS**, unless you know otherwise, choose from the top set of choices (to the right of the blue apple logo). To find your version, from the Apple menu in the corner of the screen, choose "About This Mac".
>
> If it displays a warning and prevents you from installed from an unidentified developer, follow the instructions at the following link to create an exception:
>
> https://support.apple.com/guide/mac-help/open-a-mac-app-from-an-unidentified-developer-mh4061 mac

4. Open the Google Sheet found here: https://docs.google.com/spreadsheets/d/1y1vfi3l1KzGoEOcOctwA0Z5s7io edit?usp=sharing

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5. Make a copy of this sheet for your group to use. Explore the parts of the sheet. Notice the model equation and fit parameters at the top. Notice the data columns, including the time, distance from the center of Jupiter `x`, and the radius of Jupiter `R_Jupiter`. There is also a plot of the measured and model x/R. There is a different sheet for each moon you are observing. Notice the different tabs at the bottom of the window for each of the four moons.

6. Download the Jupiter data zip file from Canvas and extract it to a folder on your computer.

7. Start DS9 and open the file from the earliest timestamp, with the clear filter.

8. **Record the time** the image was taken by going to the *File* menu and then *Display Fits Header*. Towards the bottom of the header will be listed the *DATE-OBS* which will be in the UTC timezone.

9. Convert the time given to decimal time in days after 2017-04-14T00:00:00. It may help to use the converter below the plot in the spreadsheet.

In the data table, there is an entry for each tenth of a day, so that the model x/R can be computed and plotted.

10. To include your measurement, insert a new row for the image's timestamp by right-clicking the row number of the time just before the image's timestamp and selecting "Insert 1 below".

11. Enter your image's decimal time in the time column.

12. Identify the moons in your image by doing the following:

    a) Adjust the contrast and zoom in your image until you can clearly see Jupiter and four moons. To get a better view, select "Scale >> Log" from the drop-down menus, and also right-click-drag up, down, left, and right anywhere on the image to adjust the contrast dynamically.

    b) In another browser tab, open http://www.shallowsky.com/jupiter/, enter the observation time, and press the circular arrow "refresh" symbol. It automatically adjusts for your local timezone (in Chicago it adds "-5" to the end of the timestamp.), so to unadjust back to UTC, do that arithmetic operation to the entered time. In the case of Chicago and "-5", subtract 5 hours from the entered time.

    c) Ensure the images match and correct the time if necessary. If the image seems flipped or rotated, select a different orientation from the drop-down.

13. **Use** DS9 to measure the distance from the center of Jupiter to each moon using the Ruler function described in the following paragraph. Keep track of whether the moon is on the East or West side of Jupiter by making all locations on the left side positive and locations on the right side negative. **Record your value** in the data table for each moon.

You can use the DS9 *Ruler* function to measure the distance (in pixels) between two points in the image. To use the *Ruler*, first go to the Region menu from the toolbar at the top. Select the *Shape* sub-menu and click *Ruler*. Then, click the *Edit* menu button and select *Region*. You can then use the mouse to draw a line between two points and DS9 will calculate the distance for you. If you have trouble using the *Ruler* feature, you can also read off the (x, y) coordinates for each point and calculate the distance yourself using Pythagorean's theorem.

14. Repeat the above analysis for each of the four timestamps.

The apparent radius of Jupiter can be different for each image, since the apparent brightness can change depending on local weather conditions. So if available, you will measure its radius for each timestamp.

15. For each timestamp, if its available, load the image taken with the "i" filter, listed in the filename, and measure Jupiter's radius, recording it in the table. If there is no such image, take an average of the other image's measured radii.

You should now have 4 distances for each of the four moons recorded in their respective data tables. In this next section, you will plot them and fit the sine functions, to find the semi-major axis of each one.

4.7 Plotting and fitting

16. For each row of measured data, enter the formula for the "x/R measured" row. In Google Sheets, the procedure is as follows:
    a) Select the cell in which you want to enter the formula (to the right of your measured `R_Jupiter`.)
    b) Enter "`=`", click the cell that contains your measured x to insert that cell's identifier, enter "/", click the cell that contains your measured R_J, and press enter. Check and verify that it divided the correct values.
17. In the next column, marked "x/R model", you need to continue the formula from the row above, in order to computer the model x/R for that timestamp. To do this, click on the cell in the row above, then click and drag the box on the lower right of the cell's border down by one cell.
18. Do the same for the `diff^2` column.

Now you should have all four data points displaying on the plot, for each moon, and you are ready to fit the sine curve for each moon, so you can find the ratio semi-major axis divided by Jupiter radius, a/R, as needed for applying Kepler's third law as in Equation 4.3.

19. For each moon, enter the period of its orbit (from Table 4.1) in the fit parameter for period, `B`.
20. For each moon, adjust the other fit parameters, amplitude and time offset, to visually match the measured points, and then fine-adjust to minimize the rms deviation.

The amplitude from your fit corresponds to the ratio, (a/R) which we will use in Kepler's Third law.

21. For each moon, use the amplitude of the curve together with Kepler's Third law to estimate the density of Jupiter. Make sure you use the right units for your values. *Hint: look at the units for the Gravitational constant. What units do you need for the moons orbital period?*
22. Take the average of the densities, and use the standard deviation of them for the uncertainty. Report this value, with uncertainty, in your report.
23. Compare your result to a value of the density you find online, using the uncertainties and the procedure in Appendix B.3. How close are they? What are possible sources of uncertainty of your measurement?

Moon	Period (days)	ρ_{Jupiter} (kg/m^3)
Ganymede	7.1546	
Io	1.7691	
Europa	3.5512	
Calisto	16.689	

Table 4.1: Four moons of Jupiter and their periods.

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24. Rocky planets in our solar system have densities ranging from 3000–5000 $\mathrm{kg/m^3}$, and water has a density of 1000 $\mathrm{kg/m^3}$ (atmospheric air at sea level on Earth has a density of about 1 $\mathrm{kg/m^3}$). Given these ranges and your result, would you conclude that Jupiter is rocky or gaseous, and why?

4.8 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

1. Data table including times and positions for each moon, as well as the radius of Jupiter.
2. Graphs of position (x/R) vs. time for each moon, with the fitted curve plotted as well.
3. Answers or evidence of completion for Steps 22–24.
4. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

APPENDIX A

Lab Report Format

A.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

A.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

A.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

APPENDIX B

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

B.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet (like Microsoft Excel, LibreOffice Calc, or Google Sheets), or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{B.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{B.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as $52 \pm 3\,\text{m}$.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

B.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{B.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{B.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{B.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table B.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table B.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

B.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{B.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2] Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

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