Laboratory Manual

ASTR 24100 Physics of Stars

The University of Chicago

Winter 2022

Labs

LAB 1

Inventing Color

[1]

Everything glows (gives off electromagnetic radiation) when it has a temperature — particles are wiggling around randomly (faster if it's hotter) and giving off energy as they wiggle. This **thermal radiation**, also called blackbody radiation, is the same for any object at the same temperature. The spectral density B of this radiation at wavelength λ and temperature T is given by Planck's Law:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}, \tag{1.1}$$

where h is the Planck constant, c is the speed of light in the medium, and k_B is the Boltzmann constant. The peak of this distribution λ_{peak} is given by Wien's displacement law,

$$\lambda_{peak} = \frac{b}{T}, \tag{1.2}$$

where $b \approx 2898\ \mu$m·K is Wien's displacement constant. This universal behavior means we can take something's temperature by looking at the spectrum of light it gives off. This is great for learning about something's temperature at a distance, like stars.

Team roles

1. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. Some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflect on how the team executed each function.

[1]Contributions from Brent Barker, Mike Gladders, Amanda Pagul, and others.

1.1 Exploring thermal radiation

2. Load the interactive thermal radiation simulation at https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html
3. Play with the controls and see what happens. Discuss among your team.
4. Find a pattern — how does the shape change as the temperature goes up? For example, how does the peak wavelength change, and how about the total radiated power (area under curve)? **Record your observations.**
5. At what peak wavelength do you radiate? Use the sim to determine this and estimate your uncertainty. **Record your findings.**
6. Using your estimated temperature, calculate your peak wavelength according to Wien's law.
7. Comparing the spectra of the light bulb, Sun, and Sirius A, how can you use the spectrum to determine what color each one appears as?

1.2 CCDs and filters

Cameras have come a long way since the days of developing film, but sensors are not as smart you might think. They register light, but they can't usually tell the color (i.e. wavelength) of the incoming photons. So then how do you get a color image? In astronomy (and in your cell phone!) color images are generated by measuring the amount of light at specific colors and then combining these measurements to create a colorful image. A filter is used to select bands of color to allow through. In consumer cameras and phone cameras, these filters are permanently attached to the front of individual pixels. In astronomical imaging, there is no permanent filter, and different filters are moved into place.

Filters

Light is composed of photons with energies that determine their wavelengths (shorter wavelength $\implies$ higher energy). Thus every light source exhibits a **spectrum** of energies based on its energetic components, determined by the physics of the light emission process. Thus, observing the energetic constituents of light from astronomical objects - a.k.a. observing the spectrum of emitted radiation - is a fundamental tool in observational astrophysics. However, obtaining the specific intensity of radiation as a function of energy from an astronomical source is challenging. An easier way to asses the electromagnetic energies observed is to image them in different filters: materials that are transparent to a known range of wavelengths and opaque to all others. Thus, one can image the same object with multiple different filters to get a sense of the wavelength regimes that make the strongest contributions to the overall electromagnetic output.

A filter is characterized by its **transmission function**: a function that characterizes the amount of light that is transmitted by the filter at each wavelength. Figure 1.1 shows the transmission functions for standard astronomical filters that are used in the Sloan Digital Sky Survey and now used as standard filters elsewhere.

8. Open the Google Sheet found here: https://docs.google.com/spreadsheets/d/1qgTqiqWOFCzYuSmHeT-5hwy_lBYOOTvu24kM955nfq4/edit?usp=sharing
9. Make a copy of this sheet for your group to use.

This spreadsheet calculates the same theoretical thermal radiation curve (according to Planck's Law) as the PhET simulation above, seen in the first two columns. Then it applies the transmission functions of the SDSS filters and plots the remaining power spectrum that gets through each filter. Finally, it sums up each filter's power spectrum and gives a total pixel value and magnitude. The pixel value is the relative number of counts a pixel with that filter would read. This is proportional to the brightness it sees.

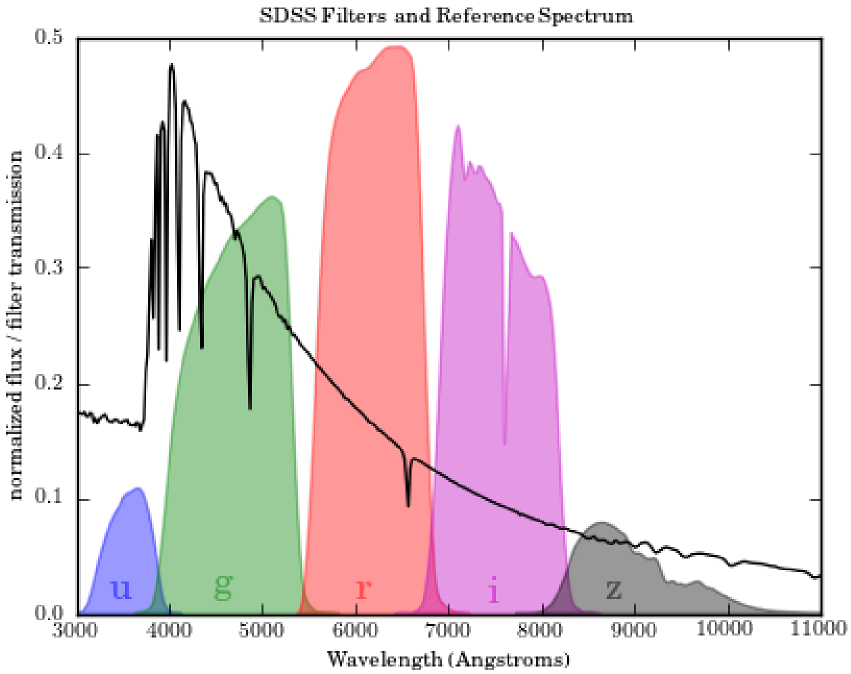

Figure 1.1: Filter Transmission Functions for the Sloan Digital Sky Survey, overlaid on a stellar spectrum (in this case the spectrum is probably an A-type star). The magnitude observed by each filter will be proportional to the integrated spectrum multiplied by the filter transmission. It's clear in this image that the underlying spectrum of the star will cause the different filters to have different magnitudes. How would these magnitudes change if the spectrum was, say, of an M-star instead? (Image source: `http://www.astroml.org/_images/fig_sdss_filters_1.png`)

Magnitude

Astronomers measure brightness of stars using a measurement system called *magnitude*. About two millenia ago, astronomers categorized stars into 6 different brightness classes, 1st class through 6th class. When brightness began to be more quantified, it was found that first magnitude stars were about 100 times brighter than 6th magnitude stars. Then it was decided to scale the 6 classes logarithmically, to account for the several orders of magnitude involved. The relationship between intensity (power per unit area) I and magnitude m is defined as

$$m \equiv -2.5 \log_{10} \left(\frac{I}{I_{\text{ref}}} \right) , \tag{1.3}$$

where I_{ref} is the intensity of some standard reference object. In the spreadsheet, these values marked as magnitudes are not technically magnitudes, since we are not using the dimensions of intensity, but it will help your analysis to treat them as such.

Color index

In Section 1.1, you learned that the color of a thermally radiating object is related to its temperature. Here, you will develop a quantitative color index and use it to determine the temperature of the Sun.

10. In the spreadsheet, change the temperature of the blackbody (edit cell `B1`) and watch how the spectrum changes, and how the filter magnitudes change with respect to each other. **Record your observations.**

The magnitude depends not just on the temperature, but also on the size of the star and the distance away from us. So to characterize a star's color quantitatively, independent of the size and distance, astronomers use a ratio of brightnesses of different filters. This is equivalent to subtracting the magnitudes. The subtraction of two broadband filter magnitudes is called the *color index*.

Correlating color index and temperature

In order to use the color index of a star to find its temperature, you need to determine how the two related to each other. If you assume the EM radiation emitted by the star is entirely due to thermal radiation, then you can find the theoretical color index of a blackbody at various temperatures, and then find where the star's color index sits on that graph.

11. Choose the filters from which to make your color index. Do you want to choose filters that detect wavelengths right next to each other, or farther apart? If you're not sure, try both and see which is better for this application. You can call the resulting color index "$x - y$", for example $g' - r'$ if you are subtracting the r' filter value from the g' value.
12. Make a table and graph of temperature vs. color index. **Record your table and graph.**

Taking the Sun's temperature

13. Switch to the "Sun" tab in the spreadsheet. This has experimentally determined values for the intensity of EM radiation impinging on the Earth from the Sun, alongside the same filtering system as in the theoretical tab.
14. Design a procedure to use the filter magnitudes of the Sun to determine its temperature and your uncertainty of that temperature. Run this procedure by your TA to get feedback on it before starting.
15. Conduct your experiment and keep a log.
16. Look up the effective temperature of the Sun's photosphere on Wikipedia.
17. Calculate the percent difference between your value and the one that Wikipedia references:

$$\text{percent difference} = \frac{|a-b|}{\frac{a+b}{2}} \times 100\% \tag{1.4}$$

18. Use the t' statistic described in Appendix A.3 to compare the two values and interpret the result — do these two measurements really measure the same thing? If they are very different, check your procedure, and if the difference stays, discuss what could be different about the methods used to find the temperature, or different assumptions used. **Record your discussion.**

1.3 Creating a color image

To create the beautiful color astronomy images we like to see, one must manually combine images taken with different filters. There are always choices to be made that will change how the combined image looks.

19. Using the directions below, create a false color image of either the Orion nebula (M42), a stellar nursery, or the Crab nebula (M1), the remnant of a supernova. You can find the relevant images on Canvas in the Lab module.

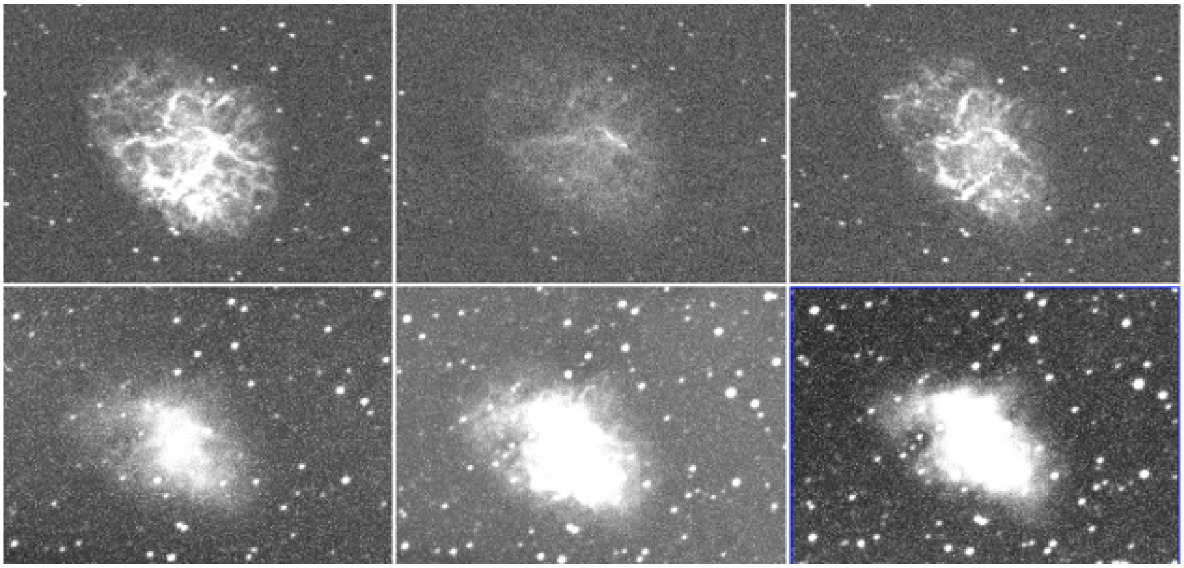

Figure 1.2: Images of M1, the Crab Nebula, in different wavelength filters, revealing different features. Top row from left to right are the narrowband h-alpha, oiii, and sii filters. Bottom row is the broadband g', r', and i'. The structure comes out nicely in the narrowband, while there are more stars in broadband.

Now you'll create a color image from three separate images of the same target. You can use the broadband filters that you used in the previous section, and you can also try using the narrowband filters h-alpha, oiii, and sii. These filters only allow specific wavelengths through, that correspond to particular electronic excitations of hydrogen, oxygen, and silicon, respectively. Different features can be seen with different filters, as seen in Figure 1.2.

Since there is not a 1-to-1 correspondence between the red-green-blue options in DS9 and the filters available, you will assign a different, arbitrary filter to each color in DS9. This makes the image a *false color image*, since the colors we will see in it do not correspond to the actual wavelength of the light captured.

Loading and manipulating images in ds9 consists of:

- loading an image (file > open)
- setting lower, upper limits (z1,z2) on an image (scale > various algorithms; use scale > scale parameters for full control). See Figures 1.3–1.4 for examples.
- controlling the intensity mapping within those bounds (mouse right click-and-hold and drag)

You can change the zoom and center location in an image by

- moving around in image (mouse middle click if edit>point is set[the default], or edit>pan and mouse left click)
- zooming in and out (mouse wheel, zoom> +,- etc.)

To build a color image

- Identify and download from SEO your three different filter image FITS files.
- Open a color (rather than monochrome) frame: Frame > new rgb
- Open the red, green, infrared files using the rgb subwindow to select which channel you are working in, and then scale and control intensities on each one.

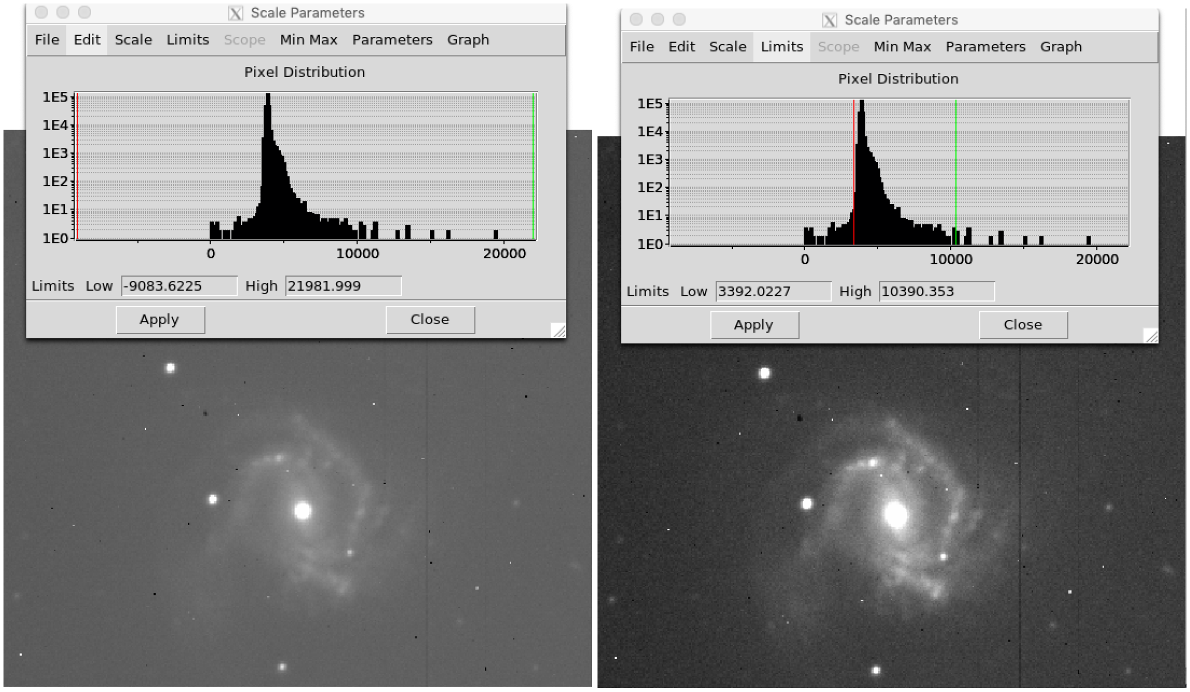

Figure 1.3: Proper choice of data ranges is important. The default for ds9 is often the min/max values in the image, which can be a poor choice if there are outlier pixels, as shown here on the left. The red line shows the lower limit z1, which is mapped to no color (black here), and the green line is the upper limit z2, mapped to full color (white here). Pixel values between these are shown in various brightnesses of the color. On the right, z1 and z2 are more tuned to the distribution of pixel values, which more effectively uses the dynamic range of the display for pixel values where there are significant amounts of data.

- There are many possible ways to scale the images. Some testing suggests that choosing Scale > ASINH (or Linear or Square Root as other choices) and Scale > 99.5% (or maybe 99% or 98%) produces reasonable results. Experiment!
- One thing to note: the rgb subwindow allows you to control how the images are aligned spatially via the "align" menu at the top. There are three relevant choices: "WCS", "Image" or "Physical". The latter two should give the same result in this instance. "WCS" alignment uses information in the image header that has been added by the processing pipeline, that establishes a World Coordinate System (this tells ds9 and other programs how pixel x,y values are mapped into the sky coordinates - typically Right Ascension [east/west] and Declination [north/south]). The default is WCS and should work fine, if the images processed correctly. If that doesn't look good, you could try the others. If that still doesn't look good, note that you tried your best, and give an example of how it didn't work will in either mode. Examples of good and bad image alignment are shown in Figure 1.5.

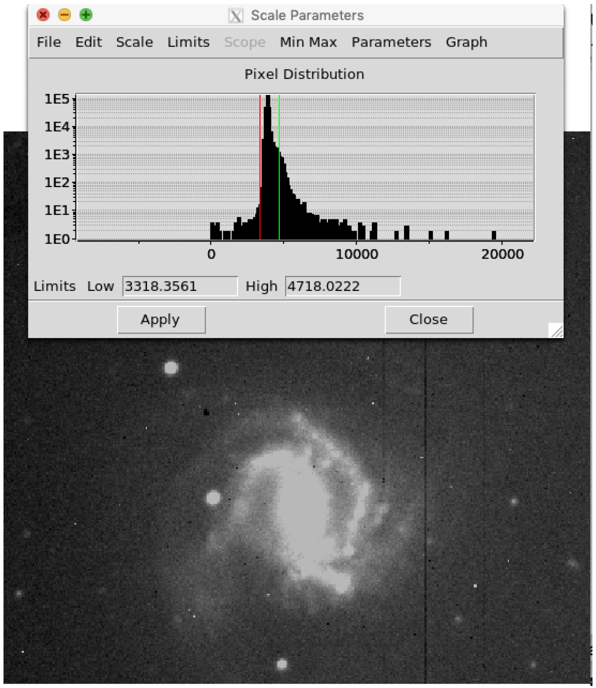

Figure 1.4: Smaller values of z2 will emphasize fainter values in the target. Compare the image to the left to the right-hand image above.

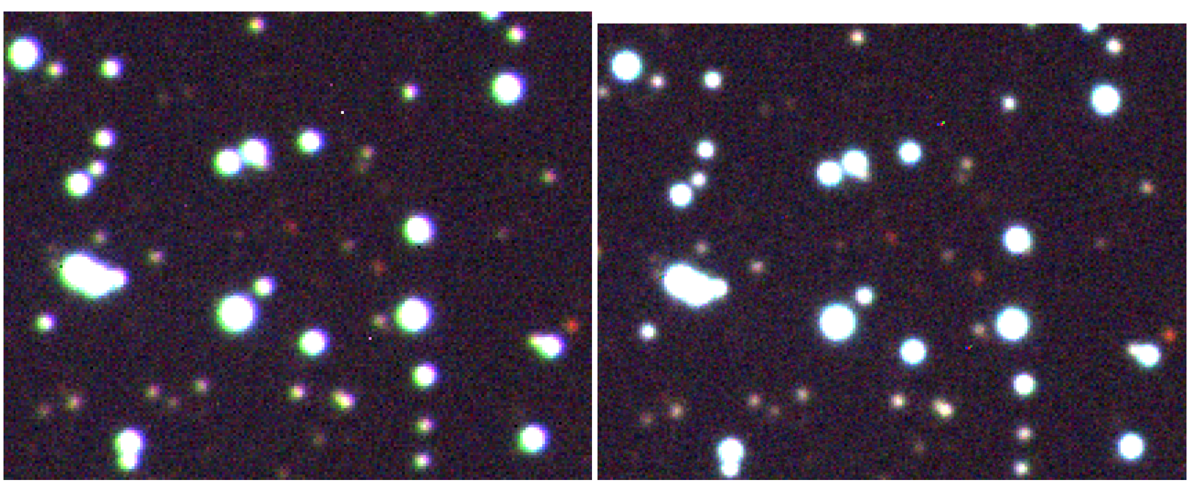

Figure 1.5: Zoom-in on a color image, showing poor (left) and good (right) image alignment across filters. Note how objects are shifted between different color channels in the poorly aligned image.

1.4 Report checklist and grading

Each item below is worth 10 points. See Appendix B for guidance on writing the report and formatting tables and graphs.

1. List of team role assignments and agreements about group communication
2. Pattern of how shape of spectrum changes with temperature, your peak wavelength, and how to use spectrum to predict color (Steps 2–7).
3. Observations of how spectrum and filter magnitudes change with temperature (Step 10).
4. Table and graph of color index versus temperature (Step 12).
5. Procedure, calculations, and determination of solar temperature and uncertainty.
6. Comparison with percent difference, t' statistic, and discussion (Steps 14–18).
7. Your beautiful color image (Step 19).
8. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").
9. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

LAB 2

Characterizing stars with H-R diagrams

2.1 Introduction

In this lab you will use a Hertzsprung-Russell (H-R) Diagram to learn about a star cluster. This will give us insight into the stellar populations in the clusters we observe, and allow us to estimate the age of their constituent stars.

2.2 Learning goals

- Gain practice performing basic calculations on datasets and making informative scientific figures from those data
- Retrieve data from astronomical databases.
- Learn where different stellar populations lie on the HR diagram, and understand the physical reasons behind these localizations
- Estimate stellar cluster ages using the predictions of stellar evolutionary models.

2.3 Scientific background

See the slides from Lectures 1 and 2 on Canvas.

For the basics of H-R diagrams, you can also read this textbook chapter: https://openstax.org/books/astronomy/pages/18-4-the-h-r-diagram

2.4 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage

- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

2.5 Identifying trends in color-magnitude diagrams

Four unlabeled color-magnitude diagrams (another term for H-R diagrams) are shown in Figure 2.1. What trends do you see among them? They have been modified so that the vertical axis is the absolute magnitude rather than apparent magnitude so that they can be compared. These four clusters were not all formed at the same time. In fact, they have distinct ages that are quite different from one another. **Determine the sequence from youngest to oldest, and provide an explanation.**

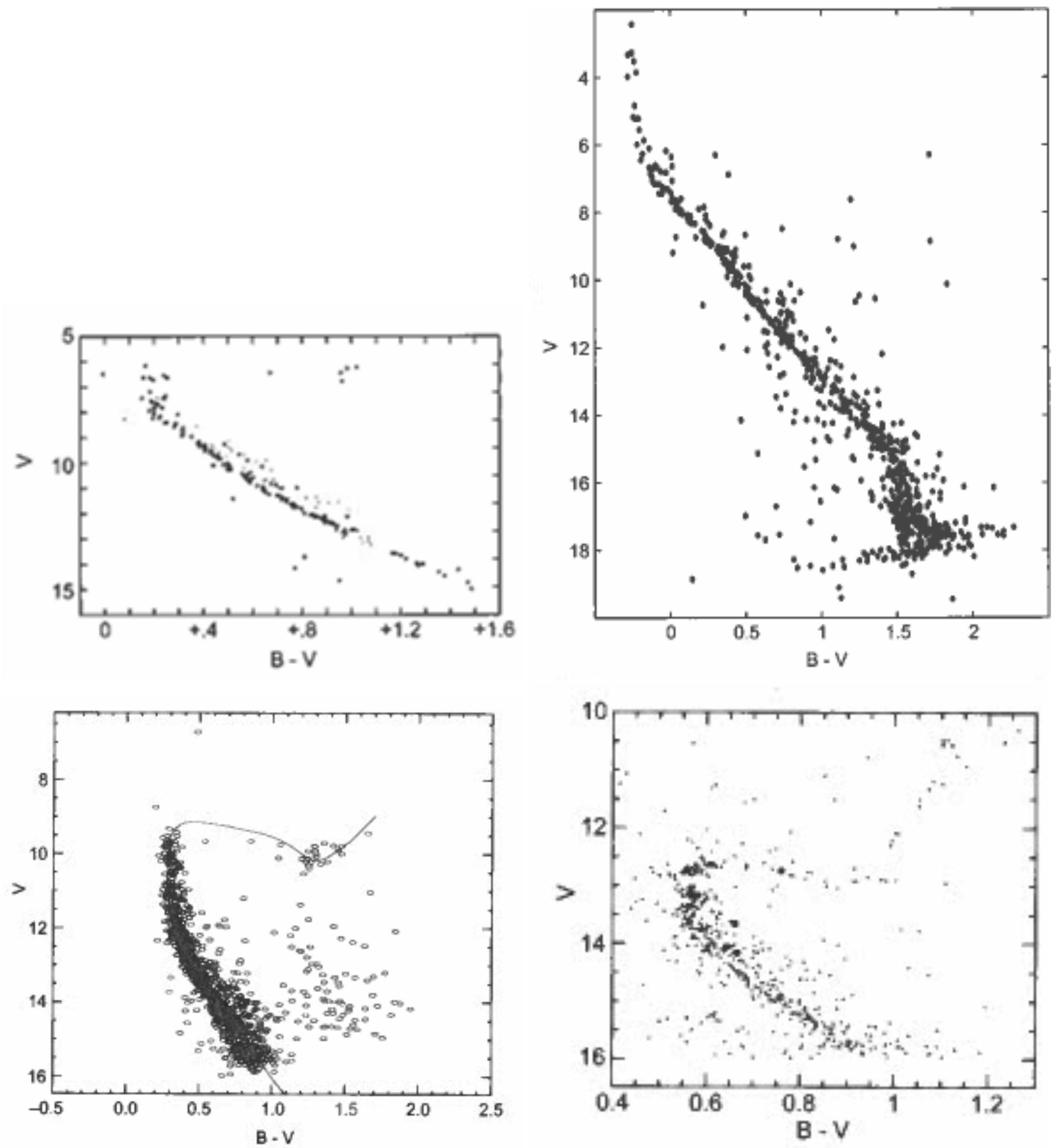

Figure 2.1: H-R diagrams for several clusters of stars. The vertical axes have been changed to absolute magnitude for comparison purposes.

2.6 Making an HR Diagram

To learn about a star cluster, you'll make a color-magnitude diagram of the stars in that cluster. You'll be analyzing M15, a globular star cluster in the Pegasus constellation. To make the diagram, you need magnitudes in at least two different filters for many stars in the cluster. Since this would be incredibly time-consuming to do by hand, you will retrieve this information from an online database.

1. Open the SkyServer Search Form in a browser: `http://cas.sdss.org/dr7/en/tools/search/form/Default.aspx`.
2. Select "Show me [stars] in the region [around:]". Note that you will need to know the celestial coordinates of the part of the sky that you want retrieve data about, as well as the radius of a circle that captures all the objects you want.
3. Find the J2000 equatorial coordinates (RA and Dec) of M15. You can do this by looking up M15 in the software or browser-based virtual planetarium "Stellarium" or another reference and record its RA and Dec. Note that "J2000" means the coordinates it had at 12:00 TT (terrestrial time, very close to UTC) January 1, 2000, so you will need to set the date accordingly. It may be helpful to turn off the ground and atmosphere to get a better view. Then, determine a radius in arcminutes that includes all of the cluster. You can either use the Field Of View (FOV) listed on the screen, or you can turn on the Equatorial Grid by selecting that option in the bottom toolbar or pressing 'e' and use the contour lines.
4. The Search Form wants the RA and Dec in decimal degrees. So convert the RA and Dec that you recorded to decimal degrees and enter this (for example, an RA of 15h30m20s converts to 232.5833 degrees). You can use this website to assist: `https://www.swift.psu.edu/secure/toop/convert.htm`.
5. In the Form, tell it to output 1000 objects, and the magnitudes of the objects.
6. Select "Generate" or "Update Query" and it will convert all the options into a database query that you should then "Submit". This opens a new tab with the output from the query.
7. If the outcome from the query has much fewer than 1000 objects, then increase the radius (and/or check to ensure you have the correct RA and Dec entered).
8. Once you have lots of objects, change the form to output a CSV file.
9. Plot the data from the CSV file using your favorite spreadsheet or programming environment. Specifically plot 'g' vs. 'g-r', with 'g' on the vertical axis, as a scatter plot.
10. Label the different stages of stellar evolution on your plot.

2.7 Comparing to stellar evolution models

The last portion of this lab involves estimating the age of these stellar populations by comparing their color magnitude diagrams with predictions from a stellar evolution model. Stellar physics is sufficiently well-understood that accurate evolutionary models have been constructed to calculate the observable properties of stars across their lifetimes. Because stars evolve in their position on the HR diagram, we can estimate the age of our observed cluster by comparing these models to our observations.

Several files containing model predictions for the magnitudes of stars at different ages are contained in files on the course website. These have been calculated using the observed metallicity of the star cluster to predict the positions of stars at ages 10^7, 10^8, 10^9, and 10^{10} years. These are called isochrones - stellar properties as a function of stellar mass for a fixed age (and metallicity). Note that these magnitudes were calculated using prior knowledge of the star cluster metallicity, and have been corrected for cluster distance and extinction due to intervening galactic dust (taken from [1] for M15 and [2] for NGC 869).

11. Plot these isochrones on your H-R diagrams and compare them to your data to estimate the cluster age. Be sure to estimate an error on this value and explain your method in your lab report.

12. What changes in stellar properties are represented by the different locations of isochrones of different ages?

13. What physical processes occuring inside the star underlie those changes?

2.8 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

1. Identification of trends in the four mystery H-R diagrams and determination of relative ages.

2. The coordinates of M15 and the radius you choose, along with how you found these (Step 3).

3. The plot of g vs. $g - r$ with labels of different stages of stellar evolution (Steps 9–10).

4. Plot of H-R diagram with isochrones (Step 11).

5. Analysis and determination of cluster age, with uncertainty (Step 11).

6. Answers to questions in Steps 12–13.

7. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

8. Write a paragraph (100–200 words) reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? Which did you do, and how did that go? what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet (like Microsoft Excel, LibreOffice Calc, or Google Sheets), or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a-b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Lab Report Format

B.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

B.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

B.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

Bibliography

[1] P. R. Durrell and W. E. Harris, "A color-magnitude study of the globular cluster M15", Astronomical Journal **105**, 1420 (1993).

[2] T. Currie, J. Hernandez, J. Irwin, S. J. Kenyon, S. Tokarz, Z. Balog, A. Bragg, P. Berlind, and M. Calkins, "The Stellar Population of h and χ Persei: Cluster Properties, Membership, and the Intrinsic Colors and Temperatures of Stars", The Astrophysical Journal Supplement **186**, 191 (2010).