# Laboratory Manual

PHSC 12620 The Big Bang

The University of Chicago

Spring 2019

# Labs

LAB 1

# Measuring distant objects with parallax

## 1.1 Introduction

Since it takes time for light to travel to us from objects in the universe, the further out an object is, the further back in time we see it. So for us to have an accurate picture of how the universe was in the past, we need to know how far away things are. For things that are nearby on Earth, we can travel there and see how far we went, or how long we took to get there. For things further away like the moon, we can use Kepler's laws, or we can bounce a beam of light off of it and see how long it takes to get back. For objects outside of our solar system, it would take too long, and the light would disperse too much, for us to use this last technique. For those objects that are still relatively nearby, we can use the parallax technique as the first rung on our distance ladder.

## 1.2 Procedure

Go to the fifth floor of Eckhardt Research Center. Near the elevators on the north side, there are two telescopesend. The target is a mock-up of the night sky, but with colors and relative sizes of objects greatly exaggerated to facilitate observations.

You will now use the telescope apparatus to measure the parallax of a "foreground star" (actually a nearby building), with the Chicago skyline serving as "background stars." Make sure that it is positioned so that sightlines from both telescopes pass through both the "foreground star" and the "background stars," (an overlapping background of the skyline). With your smartphone, take an image of the "foreground" and "background" stars through each telescope. Also measure the distance between the two telescopes using a measuring tape and record this value in your lab notebook. See Figure 1.1 for example data.

You have now finished collecting data, so now it's time to analyze it. Upload your photographs to a computer. The simplest way to do this may be to email the images to yourself from your phone. Give each file a descriptive name (e.g. "`parallax_left_telescope`").

You will now convert the images from their default format (likely .png or .jpeg) into .fits files, a format commonly used by astronomers. This format will be readable by SAO Image DS9, an astronomical image analysis tool. Open the first file in GIMP ("GNU Image Manipulation Program"). From the FILE menu, select EXPORT AS, change the file extension to ".fits," and then click EXPORT. Repeat this procedure for each of your images.

Open a saved .fits image of the star field in DS9. Your first task is to measure the size of the field of view in pixels. Adjust the contrast so you can clearly see the field of view. From the menu at the

Figure 1.1: Example images. My foreground “star” was the point defined by the left intersection of the lower cable and the white pillar on top of the gym on campus. One of my reference “stars” was the top right corner of the building in the background. Note that the images produced by this telescope are upside down.

top of the screen, select REGION, SHAPE, LINE. On the first row of buttons in the DS9 window, click EDIT then on the second row click REGION. Draw a line across the field of view. On the first row of buttons, click REGION then on the second row click INFORMATION. A window should pop up that will give you the length of the line in physical units, that is, in pixels. Record this value in your lab notebook.

Now open the first parallax image. Measure the distance from the foreground star to several reference background stars and record these values in your lab notebook. Make sure to record both the X- and Y- offsets. Repeat these measurements for the second parallax image using the same background stars.

## 1.3 Calculations

Given that the field of view of the Galileoscope is 1.5°calculate the *plate scale* of your images, in arcsec/pixel.

Select a reference star from your parallax measurements. Using your plate scale, determine the angular separation between the reference star and the target star. Record the value for the total separation, $r$, and for both the horizontal ($x$) and vertical ($y$) components. You can check your measurements against each other by inputting these values into the Pythagorean formula: $r^2 = x^2 + y^2$. Do your measurements agree?

Repeat the above calculations for all reference stars in both of your parallax images. You can now calculate the distance to the foreground star. The distance to a star using parallax comes from simple

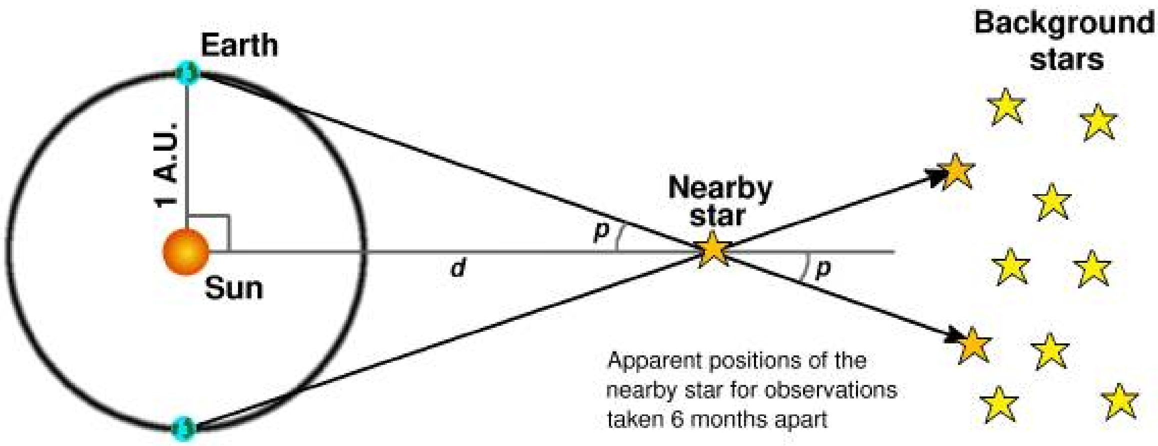


Figure 1.2: Illustration of the geometry involved in a parallax measurement to determine $d$, the distance to a nearby star.

trigonometry

$$\tan p \approx p = \frac{a}{d}, \tag{1.1}$$

where we have made use of the small angle approximation $\sin(p) \approx p$. In this formula, $p$ is the parallax angle in radians, $d$ is the distance to the star, and $a$ is half of the distance between the two measurement positions. Figure 1.2 illustrates this geometry.

Therefore, $p$ is half the angular distance the target star appeared to move. To find that distance, for each reference star

Use each reference star to calculate an independent measurement of parallax. To do this, for each reference star, subtract the two displacement vectors from each other by components and find the magnitude of that difference vector. Divide by 2 and convert to radians to find $p$. After finding $p$ from each reference star, average these values together and estimate your uncertainty by finding the standard deviation of these measurements. You can perform this calculations in Excel using the functions AVERAGE() and STDEV(). Report your measurements in a table in your lab report.

Also use a map to find the distance from you to your foreground star (with uncertainty). Compare these quantities with their uncertainties using the procedure found in Appendix B.3, to see the degree to which they agree.

## 1.4 Questions

These should be included in your lab report.

1. Your parallax measurements depend on an incorrect implicit assumption. What is this assumption, and how will it bias your results? How would you change the procedure in order to minimize this bias?

2. How precise were your parallax measurements? What were the primary sources of uncertainty? How would you improve the procedure for future measurements?

## 1.5 Report checklist

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points, with an additional 10 points given for attendance and participation, for a total of 60 points for this lab.

- A figure with your two images.
- The table of reference stars and displacement vectors.
- A statement of your final determined value of the distance, with uncertainty, and comparison with the distance found with a map. Show your work (see Appendix A).
- Answers to the questions in Section 1.4, with justification.
- Your reflection on the experiment, as detailed in Appendix A.

LAB 2

# Measuring the distance to a galaxy using globular clusters

In this lab, you will measure the distance to the Virgo galaxy, which we will use for next week's laboratory as the first rung in our distance ladder measuring the distance to other galaxies.

We will use a globular cluster in the Milky Way called M5 (see Figure 2.1) as the first step of our distance ladder to other galaxies. By comparing this globular cluster to another in a nearby galaxy called M87 in the Virgo cluster, we will estimate its distance, adding another rung to our distance ladder. Ideally, we should compare many globular clusters in the Milky Way to M87, here we will use just one cluster, which is enough to demonstrate the principle.

If sources 1 and 2 have the same luminosity, then their distances $d$ and apparent brightness $b$ are related by

$$\frac{b_1}{b_2} = \left(\frac{d_2}{d_1}\right)^2 . \tag{2.1}$$

Since the numbers we will extract from the images are either in brightness or magnitudes, it is convenient to re-cast this relation in terms of magnitudes. Magnitudes $m_1$ and $m_2$ are related to brightness $b_1$ and $b_2$ by

$$m_2 - m_1 = 2.5 \log \left[ \left(\frac{b_1}{b_2}\right)^2 \right] . \tag{2.2}$$

Combining the two equations, we get

$$\log(d_1/d_2) = 0.2(m_1 - m_2) . \tag{2.3}$$

This says that once we have measured magnitudes $m_1$ and $m_2$ for two sources, then we can derive the ratio of their distances from us, *as long as they have the same luminosity.*

Figure 2.1: Messier 5 (M5) is a globular cluster (a gravitationally bound collection of stars) of more than 100,000 stars in the Milky Way Galaxy. Located at Right Ascension (RA) = 229.640°, and Declination (Dec) = 2.075°. The above image is 2.85 arcmin on a side, or about 1/20th of a degree. Image source: ESA/Hubble & NASA, http://www.spacetelescope.org/images/potw1118a/

## 2.1 Road Map

To keep track of the steps in this lab, we will fill in Table 2.1. In this table, the entry for the magnitude of M5 refers to the sum of all of its stars. In principle, we could measure this ourselves wit the roof-top telescope, but for this lab we take a value from a catalog of such numbers. The SDSS data cannot be used because the stars are too crowded together for an accurate measurement.

The first step is to make a *color-magnitude diagram* for the stars in M5 to find a star that has similar to the Sun; we assume that such a star has the same luminosity as the Sun. The magnitude of the star (specifically its $r$-band magnitude) gets entered into the above table, and you derive the distance to M5.

| Object | Magnitude | Distance (AU) |
|---|---|---|
| Sun | -26.89 | 1 |
| Sun-like stars in M5 | | |
| M5 itself | 5.65 | |
| M87 globular clusters | | |

Table 2.1: Table of magnitude and distance.

The second step is to identify faint things surrounding the galaxy M87 that are likely to be globular clusters associated with it, and get their magnitudes (again the $r$-band magnitude) from the database. Some value that properly represents the ensemble gets entered into the above table and you derive the distance to M87.

*To summarize:* the distance to the Virgo cluster depends on two assumptions: 1) stars with Sun-like colors in the globular cluster M5 have the same luminosity of the Sun. 2) Globular clusters like M5 in the Milky Way have luminosities that are comparable to the globular clusters in M87. Neither of these assumptions is necessarily well justified based on information available to you, but there are checks that reassure us that the assumptions are good enough for at least a first estimate of distance.

## 2.2 Analyzing the M5 globular cluster

In the window at http://skyserver.sdss.org/dr13/en/tools/search/sql.aspx, enter the following query:

```
SELECT TOP 200
   objid,ra,dec,u,g,r,i,z
FROM Star
WHERE
   r BETWEEN 10 AND 23
   AND ra between 229.50 and 229.78
   AND dec between 2.2 and 2.3
```

### Questions and results for your report

1. From the data above, create a .csv file, rename it M5.csv. Read it into a spreadsheet, and make columns for the colors $g - r$, $r - i$, and $g - i$. The Sun has colors $g - r = 0.44$, $r - i = 0.11$, and $g - i = 0.55$. Plot the $r$ magnitude vs. one of the colors (e.g. $g - r$), and reverse the $r$ magnitude axis, since lower magnitudes represent brighter objects. On this color-magnitude diagram, identify the *main sequence* of stars. This plot is called a color-magnitude diagram, which is similar to an H-R diagram as seen in Figure 2.2.

2. Begin to fill out Table 2.1 with the magnitude and distance to a Sun-like star in M5. When you finish this lab and turn in this lab report, this table will be completely filled out.

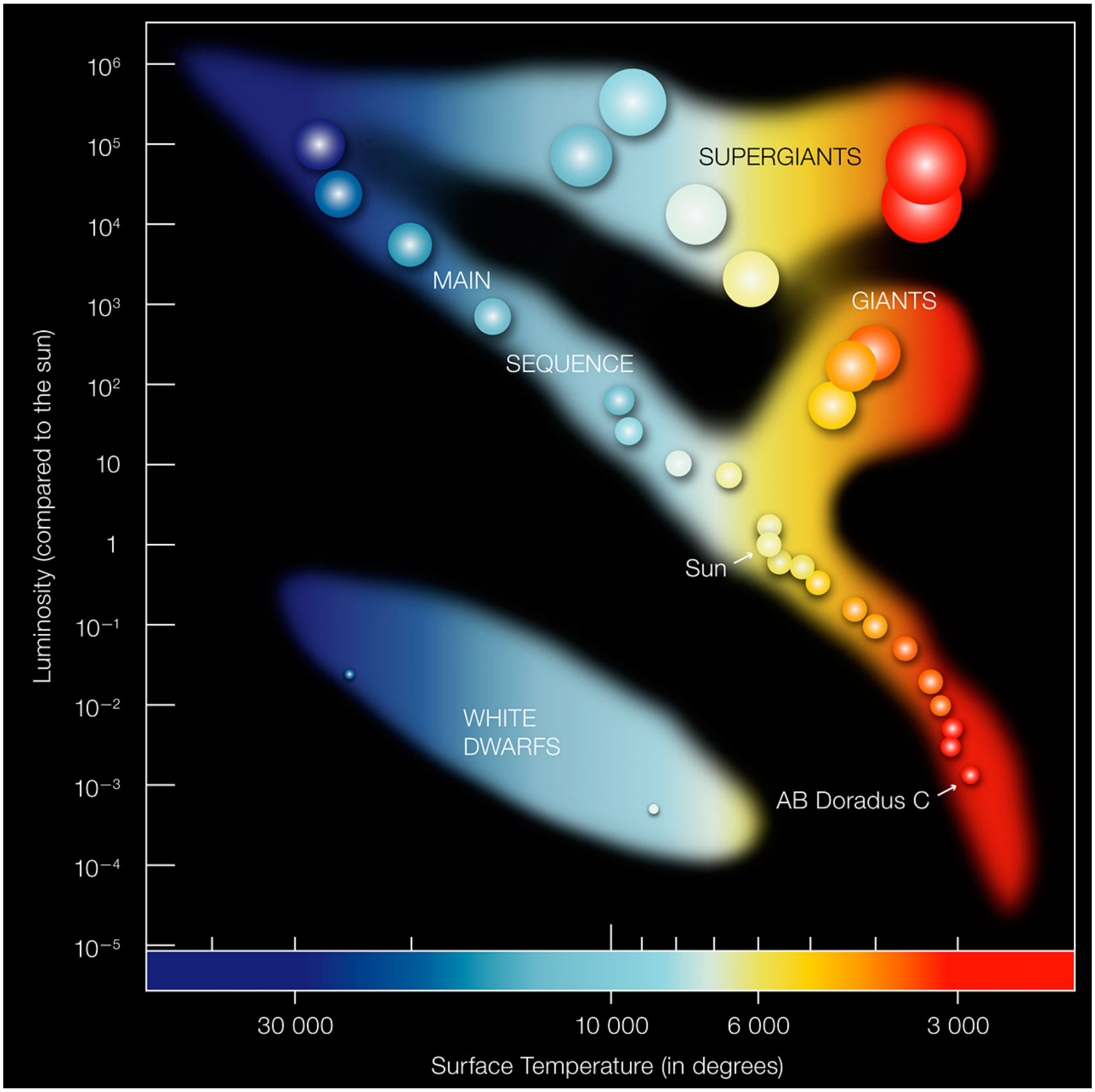


Figure 2.2: Herzsprung-Russell (H-R) diagram, plotting stars according to their luminosity and surface temperature. Luminosity is related to magnitude, and surface temperature is related to color. Image Source: ESO (https://www.eso.org/public/images/eso0728c/

## 2.3 Analyzing the globular clusters near the M87 galaxy

Figure 2.3 shows the field surrounding the giant Virgo galaxy M87, also known as NGC 4486.

The task is to find the magnitudes for the faint speckles surrounding M87 that are barely visible in Figure 2.3, namely its globular clusters. We set up a similar query to that used for M5, except of course the coordinates (RA, Dec) are different. Enter the following query:

```
SELECT TOP 200
   objid,ra,dec,u,g,r, i,z
FROM Star
WHERE
   r BETWEEN 10 AND 23
   AND ra between 187.591 and 187.821
   AND dec between 12.278 and 12.504
```

The globular clusters are so far away that they appear as, and are categorized as, stars in the SDSS data. As a cross-check, also run the above query in a random piece of sky at least 5°away.

### Questions and results for your report

3. Make color-magnitude diagrams for both samples (M87 and random-sky) and compare them. Does either either color-magnitude diagram show any evidence for a correlation between brightness (magnitude) and color for the plotted points?

4. Once you have identified which of the sources on your M87 color-magnitude diagram can be identified with a population of globular clusters surrounding M87, argue which apparent magnitude should be selected to enter into the table, and do so. Why is there a range of magnitudes? How would you make this process more precise? What other sources of uncertainty do you think there are?

5. Calculate the distance to M87 using Equation 2.3. If you have not already converted AU to parsecs, do so now to get the distance in mega-parsecs ($1\,\mathrm{Mpc} = 2.06 \times 10^{11}$ AU). The accepted value for the distance to the Virgo cluster is 16.4 Mpc. From your uncertainties above, how well do these two values agree within your expected level of uncertainty? See Appendix B.3 for details of how to determine this.

Figure 2.3: Messier 87 (M87) is a nearby elliptical galaxy in the constellation Virgo. It is known for having a large population ($\sim 10,000$) of globular clusters, about 100 times more than the Milky Way Galaxy. Centered at RA=187.706°and Dec=12.391°, the above image is 97 arcminutes across. Image source: Chris Mihos (Case Western Reserve University)/ESO, http://www.eso.org/public/images/eso1525a/.

## 2.4 Include in your report

Each of the following items will be graded out of 10 points, with 10 additional points for attendance and participation, for a total of 50 points possible for this lab.

- Completed table
- Three color magnitude graphs
- Questions 1–2 from the first section
- Questions 3–6 from the second section

LAB 3

# Galactic distances and the Hubble diagram

## 3.1 Introduction

In 1929, Edwin Hubble measured that distant galaxies were systematically redshifted relative to galaxies that were closer. From this data, Hubble inferred that the universe was expanding, an idea initially worked out by Georges Lemaitre using Einstein's theory of gravity.

In this lab, you will conduct a measurement similar to Hubble's and will produce your own version of his famous Hubble diagram shown below.

## 3.2 Building intuition

The graph in Figure 3.1 illustrates the impact of an expanding universe of photons emitted from distant objects. Because the speed of light is constant, photons that we measure today were emitted in the past, with photons originating from objects that are further away being emitted earlier in time. This means that photons from objects that are further away are older, and thus, those photons have experienced more expansion by the universe.

1. Sketch the plot from Figure 3.1, then sketch on the plot two more lines corresponding to the relationship for 1) a contracting universe, and 2) a static universe.

From this relationship, we can determine whether the universe is expanding, contracting, or static by looking at a number of galaxies and measuring their distance (corresponding to the horizontal axis) and the expansion experienced by their photons (corresponding to the vertical axis).

For this lab, we will use galaxy images and spectra listed in an online table.

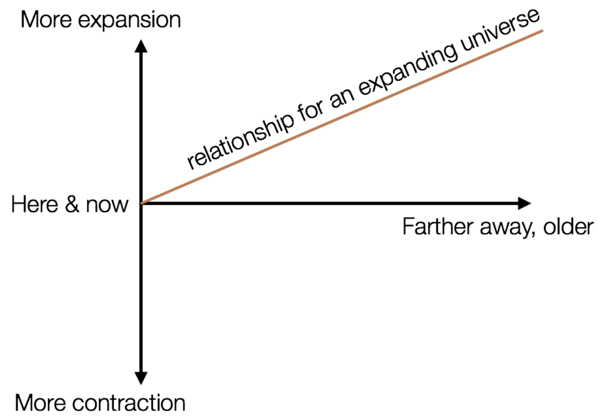


Figure 3.1: Schematic of a Hubble diagram plot. It illustrates the relationship between the expansion experienced by a photon and the distance of its emitter.

## 3.3 Measuring distance

We will use geometry to measure the distance of our galaxies. Galaxies that are closer will look bigger and will subtend a larger angle whereas galaxies that are further will looks smaller and will subtend a smaller angle. This relationship between the angular size of the galaxy and its distance is illustrated in Figure 3.2.

Our galaxies will all be nearly the same size (22 kpc). Using the geometry shown in the illustration, we can arrive at the following relationship:

$$\text{angular size} = \frac{22\text{ kpc}}{\text{distance}}\,. \tag{3.1}$$

So, by measuring the angular size of our galaxy images, we can use the above equation to determine the distance to the galaxy.

From the Files section of the Canvas site, download and extract to a folder `HubbleDataWebpage.zip`. In that folder, open `HubbleDataPage.html`. You will find a list of galaxy names. Click on the "Image" link for the first galaxy, NGC 1357. In the new tab, you will see an image of galaxy NGC 1357 similar to Figure 3.3.

2. What kind of galaxy is it (spiral, elliptical, unclear)? Note this in your spreadsheet.
3. Are there any noteworthy features in the image? Use your spreadsheet to record your answers.

We want to measure the angular size of NGC 1357, which you can do by measuring the angular separation between two appropriate points spanning the entire galaxy. In the lower left hand corner

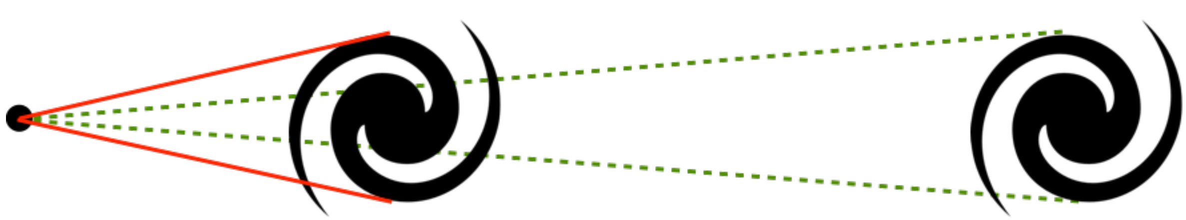

Figure 3.2: Looking from the dot on the left, there are two galaxies that are the same size, one further away than the other. The more distant galaxy subtends a smaller angle (dashed green lines) than the closer galaxy (solid red lines).

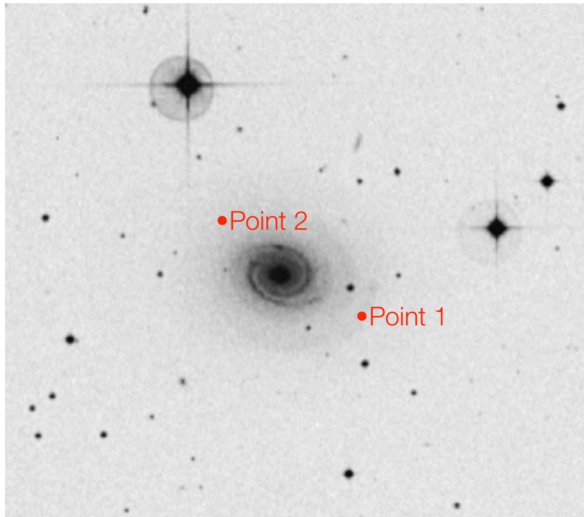


Figure 3.3: Example of galaxy image. Colors are inverted here, and Points 1 and 2 mark the furthest extent of the galaxy

of google skymaps, google shows you the coordinates of your cursor. Record the coordinates (RA and DEC) for two points spanning the galaxy. Then use an online calculator (e.g. `http://cads.iiap.res.in/tools/angularSeparation`) to calculate the angular separation between the points. Be careful and make sure that you don't choose points that are too far outside or inside the galaxy image.

4. Enter the value for the angular size (in radians, not degrees) in a spreadsheet.

5. Divide up the remaining galaxy images between your groupmates and repeat Steps 2–4 for each of the galaxies recording your notes and measurements in the spreadsheet.

6. Once you have measured the angular size of all the galaxies, use Equation 3.1 to estimate the distance for each galaxy and record the values in a "distance" column.

## 3.4 Measuring expansion

The wavelength of light changes as the universe expands, an effect known as cosmological redshifting. If the universe expands, the wavelength is stretched, becoming longer and redder. For a contracting universe, the wavelength will be compressed becoming bluer. We define the redshift, $z$, as

$$z = \frac{\lambda_{\text{measured}} - \lambda_{\text{original}}}{\lambda_{\text{original}}}, \tag{3.2}$$

where $\lambda$ represents the wavelength. The redshift is a measure of how much the wavelength has been stretched or compressed.

We can measure the redshift by examining spectra (the energy emitted in different wavelengths) of the same galaxies we just measured. For NGC 1357, click on the link "Ca Spectra." The link will show spectra associated with Ca absorption which produces dips at wavelengths of 3933.7 Angstroms and 3958.5 Angstroms. You should see two prominent dips in the data. See Figure 3.4 for example spectra.

Go back to the galaxy list and click on the link "H-alpha spectrum" to bring up the spectra associated with Hydrogen alpha emission of light with wavelength 6562.8 Angstroms. You should see a clear peak in the data. The H-alpha peak is the leftmost of the distribution. Since we know the original wavelengths for these processes, we can compare the measured wavelength of these features with their original wavelength to determine how much the light has stretched.

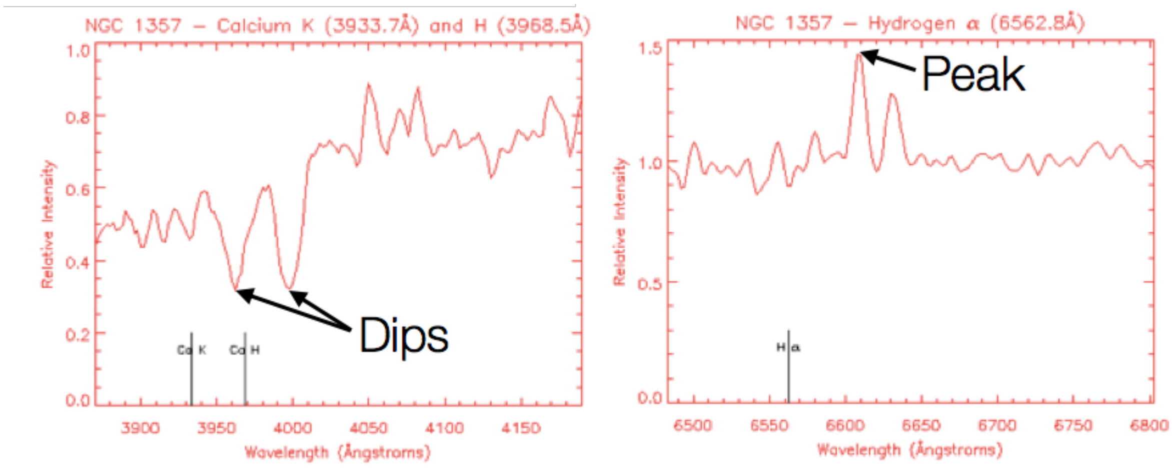


Figure 3.4: Spectrum of light detected from NGC 1357. The dips and peak that you will use to identify redshift are identified.

7. For each of the spectra estimate the value for the two Ca dips and the H-alpha peak. Record the values for the two Ca absorption lines and the H-alpha emission line in the excel worksheet.
8. Use Equation 3.2 to calculate the redshift for each of the lines and take the average to estimate the redshift for the galaxy. Record the redshift in an "average redshift" column in the spreadsheet.
9. Repeat this process for each of your galaxies.

## 3.5 The Hubble diagram

10. Using the measurements in your worksheet, make a plot of redshift versus distance for your galaxies.
11. Is there a trend in your data? Is the trend clear?
12. Compare your plot with the sketches from Step 1. Does your data indicate that the universe is expanding? Contracting? Static? Why?

Hubble's constant $H_0$ gives a relation between the recessional speed $v$ of an object and its distance $D$, according to the equation

$$v = H_0 D\,. \tag{3.3}$$

You will determine Hubble's constant from your data. You have the distances of the galaxies already. To find the velocities, multiply the redshift by the speed of light $c$,

$$v = zc\,. \tag{3.4}$$

This equation is valid for low values of redshift.

13. Make a Hubble diagram by plotting velocity (in km/s) vs. distance (in Mpc).
14. Use this plot and Equation 3.3 to fit a line to the data and find Hubble's constant, which should be the slope of that line. If you are doing a linear fit, you need to force the $y$-intercept of the line to be zero. You may also need to specify that the fit equation be displayed on the plot.
15. Do some research on the history and background of Hubble's measurement. Write a one paragraph summary of this lab and discuss the following:

a) What was the historical context of Hubble's measurement?

b) Why was it important?

c) What did you do in this lab and how do your measurements and conclusions compare with Hubble's?

## 3.6 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

- Data table
- Sketch of predicted relations for expanding, contracting, and static universes
- Your Hubble diagram (velocity vs. distance)
- Your Hubble constant
- Answers to Questions 11 and 15.

LAB 4

# The accelerating universe

## 4.1 Introduction

In 1929, Edwin Hubble discovered that the universe was expanding. At the end of the 20 th century, astronomers made another stunning discovery associated with the expansion of the universe. In 1998 two independent projects obtained results showing that the expansion of the universe was accelerating, a result that eventually led to Nobel prizes for Perlmutter, Riess, and Schmidt in 2011. The two teams used the same technique: measuring the distance and redshift of Type Ia supernova. In this lab, you will work through some supernova data looking for evidence of the accelerated expansion of the universe.

A supernova is essentially an exploding star. This "explosion" produces a tremendous amount of light which then slowly fades over a period of weeks to months. Measuring the light associated with the supernova over time produces what is known as the supernova light curve. You can use the following link to build some intuition about supernovae and their light curves: https://youtu.be/TY6Y5_7xQ8o. An example of a supernova light curve is shown in Figure 4.1. The vertical axis is in "magnitudes," the standard astronomical measure of brightness where a larger magnitude corresponds to a fainter object.

Supernova are used to measure distance in a manner similar to how we measured distance in the Hubble lab. In the Hubble lab, we used the fact that objects that are father away look smaller, that is, they subtend a smaller angle on the sky. So, if we know the physical size of the object, we can measure the object's angular size and from that determine its distance. For supernova, astronomers utilize the fact that objects that are closer, look brighter, and objects that are farther away, look dimmer. So, if we know the intrinsic brightness of an object (called its absolute luminosity), we can then compare that to our measured brightness (called its apparent brightness) to determine the object's distance. This concept for measuring distance is illustrated in Figure 4.2 showing how an object of a known brightness (or size) looks fainter (smaller) when it is farther away.

The 1998 supernova teams studied Type Ia supernova because it is possible to use the shape of the supernova light curve to deduce the intrinsic brightness of the supernova. By comparing our inferred intrinsic brightness with our measured apparent brightness, we can then use the supernova to measure the distance.

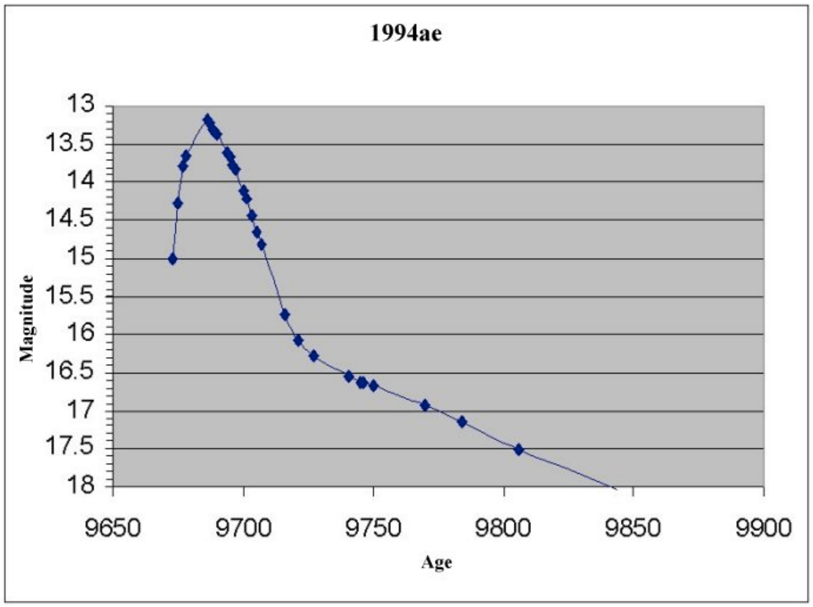


Figure 4.1: Light curve for supernova 1994ae. The age is given in days.


Figure 4.2: A method to determine the distance of an object. For objects with the same absolute luminosity, those are are further away will appear dimmer.

## 4.2 Making a supernova Hubble diagram

The first activity for this lab is to make a Hubble diagram, like in the Hubble lab, using Type Ia supernovae. Then you will analyze your Hubble diagram to qualitatively measure cosmic acceleration. Your data is in the Excel spreadsheet (available on Canvas in the Files section in the compressed file DarkEnergyLab.zip) named SNe_Reiss_2004, and consists of distance moduli and redshifts for 186 supernova from Reiss, et al., 2004.

As discussed above, the apparent magnitude of the supernova can be used as a measurement of distance if we know the intrinsic brightness (also called absolute brightness) of the supernova. In astronomy, we call the difference between apparent and absolute brightness the "distance modulus," $\mu$, which is defined as

$$\mu = m - M\,, \tag{4.1}$$

where $m$ is the apparent brightness (in magnitudes), and $M$ is the absolute brightness (in magnitudes). The distance to the object is related to the distance modulus by

$$\text{distance} = 10^{(\mu-25)/5}\ \text{Mpc}\,. \tag{4.2}$$

1. In the spreadsheet, in a new column, calculate the distance for each supernova from the supernova's distance modulus.

After determining distances for objects, the next part of the Hubble diagram involves making a measurement of the expansion of space. As with the Hubble lab, we will estimate the expansion using the redshift distortion of an object's spectrum. Recall that we measured the redshift by looking for a specific spectral feature (either a dip or a peak) which occurs at a known wavelength. A larger redshift indicates a faster recession velocity.

A constant expansion rate for the universe means that the amount of expansion is proportional to the amount of time. Mathematically, we can write this as

$$\text{expansion} \propto \text{time}\,. \tag{4.3}$$

As expected, light emitted by more distant (and therefore older) objects will undergo more expansion. This expansion leads to the redshifting we discussed above. So, we can write that for a constant expansion rate, the relationship between redshift and distance is

$$\text{redshift} = \frac{H}{c} \times \text{distance}\,, \tag{4.4}$$

where $c$ is the speed of light (300,000 km/s) and $H$ is called the Hubble constant. This relation is known as "Hubble's Law." It describes a linear relationship between redshift and distance, the two axes of the Hubble diagram.

We can measure the expansion rate of the universe by using our Hubble diagrams and fitting the data to Hubble's law. The slope of the line that fits the data will give a measurement of the Hubble constant, H, which parameterizes the expansion rate of the universe.

2. In the spreadsheet, sort data from closest to furthest. Then separately fit linear relationships to the 40 nearest and 40 furthest supernovae to obtain two Hubble constant measurements (Note that in Equation 4.4, the $y$-intercept is set to zero, so be sure to do this in your fit as well). Plot both linear relationships, as well as the data being fit, on a single graph. In order to see both sets of data more easily, set both axes to log scale to make a log-log plot.

3. What value of the Hubble constant best fits the data for the 40 closest supernova?

4. What value of the Hubble constant best fits the data for the 40 farthest supernova?

5. How do these two results show that the expansion rate is accelerating?

6. If the expansion rate is accelerating, what do you predict you should see if you fit data for 40 supernovae at intermediate distances?
7. What value of the Hubble constant best fits the data for 40 intermediate distance supernovae? Add this to your plot.

## 4.3 Finding and measuring supernovae

Supernova are random events. In order to use supernova for cosmological studies, astronomers must find them first. In this final section of the lab, you will look at real data from the Dark Energy Survey. You will search for a supernova and measure its light curve. Dr. Daniel Scolnic, a KICP fellow at the University of Chicago, has graciously provided the images for this section.

8. In the file you downloaded earlier, find the folder "SNe_search" and open the two image files SNe1_search.jpeg and SNe2_search.jpeg. Compare these two images. These images correspond to pictures taken of the same patch of sky on two different nights. A supernova is in one of the images. Can you see it?

In general, it is difficult to find supernova in a raw image of the sky because of all the other objects in the image. Take a moment to look at the different objects. Almost all of them correspond to stars and galaxies.

Since the majority of objects in an image of the sky are not supernovae, astronomers can try to remove them by generating a template image for that patch of sky and then subtracting it from the image. Once these non-supernova objects are removed (or mostly removed), it becomes easier to search for supernovae.

9. Open the file SNe1_template.jpeg and compare it to SNe1_search.jpeg.

SNe1_template.jpeg is the template file and it should look very similar (though not exactly the same) to SNe1_search.jpeg. Subtracting the two yields a "difference" image which is file SNe1_diff.jpeg.

10. Open SNe1_diff.jpeg.

You should notice two things, 1) most of the features are now gone, and 2) the subtraction is imperfect. The imperfect subtraction introduces some artifacts in the differenced image such as the spiderweb-like patterns and perfect geometric shapes (like uniformly dark or bright squares and circles). You will note that most of the artifacts from the imperfect subtraction are in locations where there were very bright objects in the original image.

A supernova in the difference images will look like a round solid blob that is very bright. Because a supernova is transient, the brightness of the blob will not be the same in all images. In fact, there should be a few images where there is no blob at all.

11. Open the two difference images SNe1_diff.jpeg and SNe2_diff.jpeg. Compare these two difference images and identify the supernova. Remember, the supernova will look like a bright solid round blob in one of the images.

Once you have found the supernova, the next step is to measure the supernova light curve. We will do this using the DS9 software on your computer.

12. Open DS9 and press the "File" button to bring up the file menu. Press "open" and open the file 1-25-2014.diff.fits. This is the original file for the image with the supernova above.

In DS9, you will need to press the "scale" button followed by the "zscale" button so that the image will appear properly. You can drag the box to look more closely at different parts of the image.

13. Play around with the settings associated with the zoom, scale, and color buttons and examine different parts of the image.

14. Find the supernova in this file (it is in the same location as in the earlier jpeg images). As you move the mouse around on top of the supernova, the "value" field will show you the value of the pixel underneath the mouse. Find the location on the supernova where this value is maximaland record the (X,Y) coordinates (the numbers for Image X and Y) below. We will use these (X, Y) coordinates as the coordinates for the supernova. Also record the pixel value.

15. To measure the supernova's light curve, load up each of the other *.fits files. Move the mouse to the (X, Y) coordinates you determined for the supernova and record the date and pixel value in your spreadsheet. Remember, the supernova is a transient object and it may not be present in all images. The filename tells you the date the image was taken.

16. In the spreadsheet software, make a plot of the pixel value versus time. Make sure you use the date of the image to put your data in chronological order and to determine the time between each of the images.

## 4.4 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

- Your Hubble diagram with all three sets of 40 supernovae and the three best-fit lines with their equations.
- Your three determinations of the Hubble constant (near, far, and mid), with work showing how you found them.
- Answers to questions 5 and 6.
- The table and plot of your experimental supernova light curve.
- Write one paragraph summarizing this lab. What is the significance and historical context of the discovery of cosmic acceleration (also called Dark Energy)? Why was it important? How does your analysis of the data in this lab demonstrate that the expansion rate of the universe is accelerating?

LAB 5

# Measuring the Cosmic Microwave Background

## Introduction

In this experiment we will perform a measurement first carried out in the mid 1960s by Arno Penzias and Robert Wilson, for which they shared the 1978 Physics Nobel Prize for the discovery of the Cosmic Microwave Background Radiation.[1] We will measure the temperature of the Cosmic Microwave Background Radiation by comparing the power of this emission directly with the emission from thermal sources in the lab.

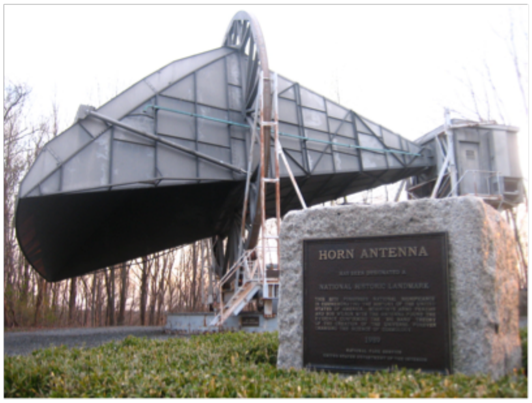


Figure 5.1: A photo of the horn antenna with which Arno Penzias and Robert Wilson discovered the Cosmic Microwave Background radiation. This lab uses a smaller horn working at a much higher frequency (shorter wavelength), so that the "beam patterns" are similar (recall the Small Radio Telescope lab and beam measurements to see why).

[1] A note on the Penzias and Wilson Nobel-prize winning measurement can be found at: http://www.bell-labs.com/project/feature/archives/cosmology/.

## 5.1 Measuring Temperatures with Thermal Radiation

We use the basic fact that at sufficiently long wavelengths, the power emitted by a perfect thermal emitter is proportional to the temperature of the emitter:

$$P = \alpha T \tag{5.1}$$

where $P$ is the power emitted, $\alpha$ is a constant that depends on area, wavelength, and other factors, and $T$ is the physical temperature of the emitter in Kelvin.

To measure the power we use an instrument that can measure the incoming radiation power and it has input optics arranged so that it "views" a small range of angles in front of the instrument. This is the basis of a radiometer. You can imagine this as a reverse flashlight. In the case of the flashlight, the filament of the light bulb is hot and emits through an arrangement of mirrors to form a narrow beam. For the radiometer, the "heat radiation" from a narrow range of angles funnels into the instrument and the power is detected and the amount of power read out on a meter.

In practice, the receiver used to measure radiation is not perfect and generates some 'noise' power of its own. So even if we pointed the receiver at an object with $T = 0$, it would still measure some power. This extra power is denoted in terms of the temperature, $T_{\mathrm{rx}}$, you would measure if you looked at a zero temperature emitter and did not correct for this fact:

$$P = \alpha(T + T_{\mathrm{rx}}). \tag{5.2}$$

To compute temperature of an emitter, $T$, from the power, $P$, measured by receiver, we need to know $\alpha$ and $T_{\mathrm{rx}}$. It would be hard to calculate the factor $\alpha$ from first principles, especially because in order to be useful the calculation must include all of the details of the measuring equipment. However, it is straightforward to obtain an accurate measurement of $\alpha$ experimentally by measuring the power for different temperatures.

We will determine the factor $\alpha$ by placing first an emitter of one known temperature in front of the radiometer, noting the power reading on the meter, then placing a different emitter with a different known temperature in front of the radiometer and noting the new power level. According to equations 5.1 and 5.2, these two measurements can be used to calculate $\alpha$:

$$\alpha = \frac{P_1 - P_2}{T_1 - T_2} \tag{5.3}$$

Once $\alpha$ is calculated, we can solve for $T_{\mathrm{rx}}$ from measurement of one of the known temperature emitters:

$$T_{\mathrm{rx}} = \frac{P_1}{\alpha} - T_1 \tag{5.4}$$

With $\alpha$ and $T_{\mathrm{rx}}$ measured, we can "point" the radiometer anywhere and remotely measure the temperature of any emitter corresponding to the received power $P$ by solving eq. 5.2 for $T$,

$$T = \frac{P}{\alpha} - T_{\mathrm{rx}} \tag{5.5}$$

Note that when you will be measuring the temperature of the "hot" load you will need to convert the temperature in degrees Celsius or Fahrenheit it reports into Kelvins.

## 5.2 The receiver and experimental set up

Figure 5.2 shows the experimental set up you will be using in this lab. The 30 GHz receiver is enclosed in a cryostat enclosure that keeps it very cold to reduce its noise. It is mounted on a cart in a contraption that allows to change its pointing for different elevations. Radiation measured by the receiver enters through receiver window at the top of the enclosure. The power measured by the receiver is displayed in digital form on the power readout screen. Pointing of the receiver can be changed using the elevation control lever and current elevation can be read on the electronic angle meter mounted on the enclose (this meter should be calibrated at the zero angle at the beginning of the lab with the help of your TA).

Figure 5.3 shows details of the receiver when it is taken out of the cryostat enclosure.

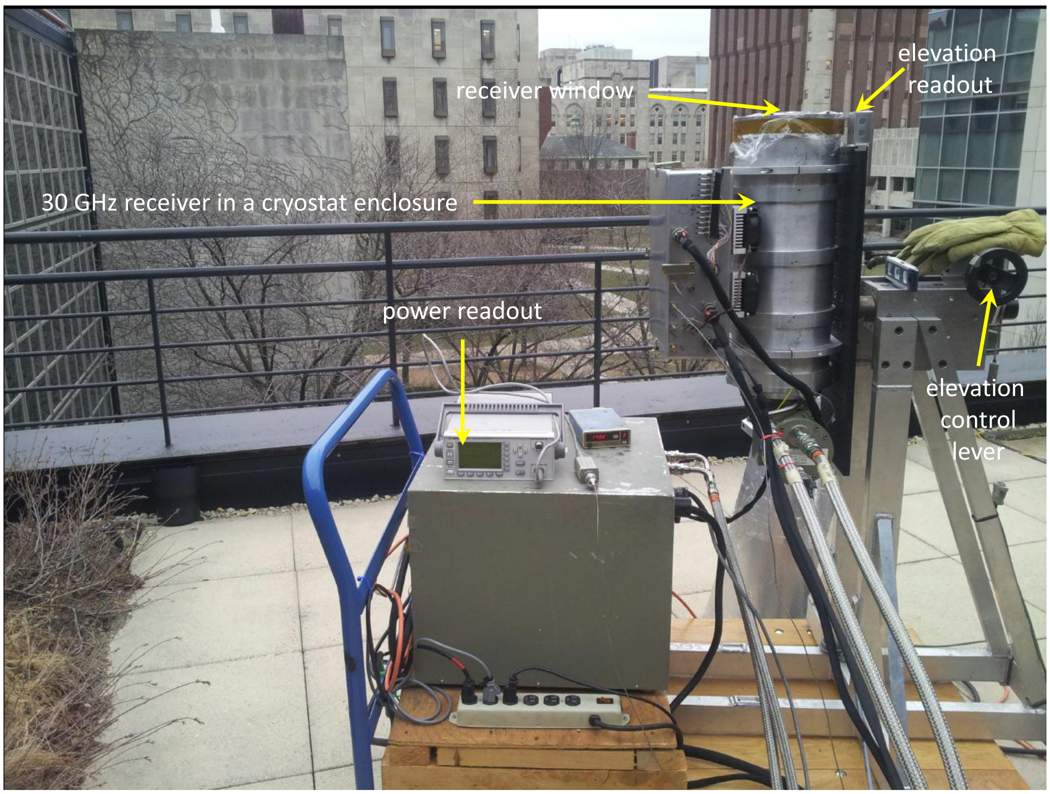


Figure 5.2: Photo of the 30 GHz receiver enclosed in a cryostat enclosure mounted on the cart. This set up on a patio of KPTC 312 will be used in this lab for measurements of temperature of the sky.

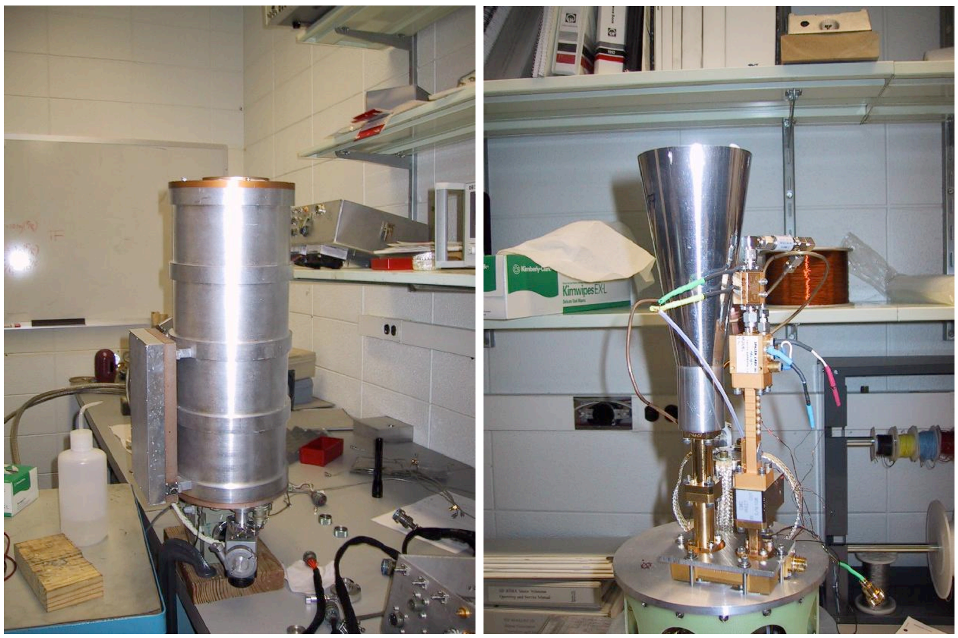

Figure 5.3: *Left:* The CMB lab 30 GHz receiver closed in the lab. *Right:* A photo of the inside of the cryostat showing the horn and electronics of the receiver. The outer shell of the cryostat provides the vacuum seal for the interior components that are cooled to roughly 15 K (-258 C) inside of thermal radiation shields (not shown) to lower the noise generated by the electronics, and even the noise due to the radiation from the horn. The electronics amplify the signal by roughly a factor of 10,000. By cooling and using a specially designed amplifier for this research, the receiver only adds a small amount of noise, roughly equivalent to that which would result by placing 10 K at the input of an idealized noiseless receiver (if only such things existed!). This receiver was built here at the university and is also used to at the CARMA radio astronomy observatory, see http://www.mmarray.org/, for which The University of Chicago is a partner. It is one of a handful of the most sensitive 30 GHz receivers in the world.

## 5.3 Measuring the Temperature of Items in the Lab

We will first measure the temperature of several items using the radiometer.

Some things you could measure:

1. The wall or ceiling
2. The sky through the door of the lab
3. Your cupped hands above the receiver or your face

For each item, you should make three or more measurements quickly in succession of the cold load, the hot load, and the source for which you want to measure temperature. The hot load is a piece of material that is a good emitter and the temperature is near room temperature and is measured with a a digital thermometer located at the top of the load (see left panel of Fig. 5.4). The cold load is a similar material that is first immersed completely in liquid Nitrogen (which has temperature $T_{\text{cold}} = 77.4$ K),[2] as shown in the middle panel of Fig. 5.4. When you are ready to make a measurement with the cold load, take it out of the dewar, let the liquid Nitrogen drip for $\sim$5 seconds and place it in front of the receiver window. Be careful to fully cover the window, but do not rest the cold load on the cryostat. Then comes the source of unknown temperature. For each of these objects you will need to read the power reading on the power meter attached to the receiver (see Fig. 1).

Find the unknown temperature using equation 5.2 to find $\alpha$, equation 5.4 to find $T_{\text{rx}}$, with $T_1$ and $P_1$ being the temperature and power for the hot load, $T_2$ and $P_2$ being the temperature and power for the cold load. Then using equation 5.5, find the unknown temperature using the power measured while the unknown was in front of the receiver.

Figure 5.4: *Left:* The "hot" load with digital thermometer at the top. *Center:* "cold" load submerged in a dewar with liquid Nitrogen. Make sure that the cooper part of the load is fully submerged with the liquid surface at a wooden part of the load. *Right:* ground shield which will be used to minimize radiation from nearby buildings and the ground when measuring radiation from the sky.

## 5.4 Measuring the Temperature of the CMB

Measuring the temperature of the CMB is conceptually the same as measuring the temperature of the items in the lab. We will compare the power we get from the sky to the hot and cold loads.

Because the CMB temperature is very low (just a few degrees K) we have to take a more careful account of a few things than we did for the previous section. First, we will have to get $T_{\text{rx}}$ pretty accurately because the CMB temperature, $T_{\text{CMB}}$ is less than $T_{\text{rx}}$. Second, there are hot sources (compared to the CMB) all around the experiment. You can imagine that even a small bit of one of

[2] It is important that the entire cone is immersed in the liquid Nitrogen or else the effective temperature of the load will be poorly determined and this will lead to incorrect measurement of the source temperature.

these hot sources in part of the beam would change the oncoming power a lot and cause a big change in what we infer for the CMB temperature.

We will also have to worry about two other sources. One is the hot ground and buildings around us. We will guess about these sources by placing a ground shield on the receiver, which keeps the ground emission out of the input (see right panel of Fig. 5.4 to see how the ground shield looks like). We will check how much difference this makes and subtract that source if it seems to be an important factor.

The second, and more difficult extraneous source to remove is emission from our own atmosphere. While the atmosphere is *almost* transparent (and therefore almost non-emissive), it is not perfect. What's more, the main emitter is water-vapor which varies a lot. There is not much water vapor on those clear, crisp days when the sky is deep blue. On the other hand, when it's wet and cloudy, there is a lot of water vapor, and consequently, if we point our radiometer up at the sky we will get a hotter temperature on humid or wet days.

As you also know water vapor is not well mixed in our atmosphere (e.g. clouds), so on a poor day you will see the output power of the radiometer changing quickly as the winds blows different blobs of water vapor in front of the radiometer. Try comparing the stability of the radiometer when staring at the warm load compared to staring at the sky. On a good day the power when staring at the sky is as stable as that when staring at the load.

How can we estimate the amount of power coming from the atmosphere when we point the radiometer at the sky? We need to do this to get a good estimate of $T_{\rm CMB}$. We do this by measuring the effective temperature as a variety of angles from the vertical. We can calculate how much atmosphere we are going through for each angle and from that, extrapolate the temperature we would get if were looking through zero atmosphere. The atmosphere contribution approximately follows

$$T_{\rm atm}(z) = T_0 A(z) \tag{5.6}$$

where $A(z)$ is the number of airmasses you are looking through. $A(z) = \sec(z)$, where $\sec(z) = 1/\cos(z)$ is the secant function – reciprocal of the cosine function and is equal to 1 when looking straight up ($z = 0$). $T_0$ is the atmosphere temperature at the zenith and $z$ is zenith angle (the angle between straight up and where the radiometer is looking). The total temperature we will measure when looking at the sky (after subtracting $T_{\rm rx}$ according to equation 5.5) will be

$$T_{\rm total} = T_{\rm CMB} + T_{\rm atm}(z) \tag{5.7}$$

To get an estimate of $T_{\rm CMB}$, we will measure the $T_{\rm total}$ at a variety of zenith angles. We then plot $T_{\rm total}$ vs $A(z)$ to get a straight line for $A(z)$ going from 1 to about 2 ($z$ going from 0 to 60°). Then we can extrapolate the straight line to $A = 0$ and read off $T_{\rm CMB}$.

A flow chart of the measurement strategy is shown in Figure 5.5. It is important to understand the quality of the data as you take it – your TA will help you with this. There are many reasons the data could be corrupted, e.g., poor cold load temperature if not well immersed in liquid Nitrogen, the radiometer not being leveled, not centering the loads on the window or aligning the reflecting shield along the beam, buildings in the way, poor and unstable atmospheric transmission, etc. Good zenith angles to give well sampled air mass measurements are: $0^o$, $15^o$, $25^o$, $35^o$, $40^o$, $45^o$, $48^o$, $52^o$, $54^o$, $56^o$, $58^o$, $60^o$, although you may find that the buildings do not allow good results at the higher zenith angles, even after correcting for the sidelobe response (see below). Be sure to measure $T_{total}$ at each of the zenith angles with the hot load – cold load – hot load method as shown in the flow chart.

Because the receiver has some sensitivity to emission from angles far outside of its beam referred to as the beam sidelobe response (and at extreme angles as the far sidelobe response), it is important to account for the excess power picked up by the sidelobe response from the warm buildings, the ground and also the Sun. We do this by making measurements at each zenith angle with the "ground shield" (see right panel of Fig. 5.4). Once the sky and load measurements are obtained at a given zenith angle, take several measurements in quick succession of the power on the sky with the ground shield off and placed along the beam axis. If the shield is well aligned (use two spotters to make sure it is), then the receiver output power will be lower when the shield is in place, especially at lower elevations (higher zenith angles) due to emission from the warm buildings. Determine the percentage change to the sky power for each zenith angle. You then correct the power in Eq. 5.5 before calculating the $T_{total}$ for Eq. 5.7.

Note, because of the sidelobe response, it is important that everyone is far from beam of the receiver when you are taking measurements. It is best for everyone to stay behind the plane defined by the front of the receiver.

During the measurements, different students in the lab will take on a particular role to handle hot or cold load, read off power, put on ground shield, record the data.

1. Your measurements should be recorded in a table, which has elevation, power for the cold load, power for the hot load, temperature of the hot load read off at each measurement, power of the sky without shield, power of the sky with shield. You will take the measurements jointly but will analyze them to measure $T_{\text{CMB}}$ on your own.

 

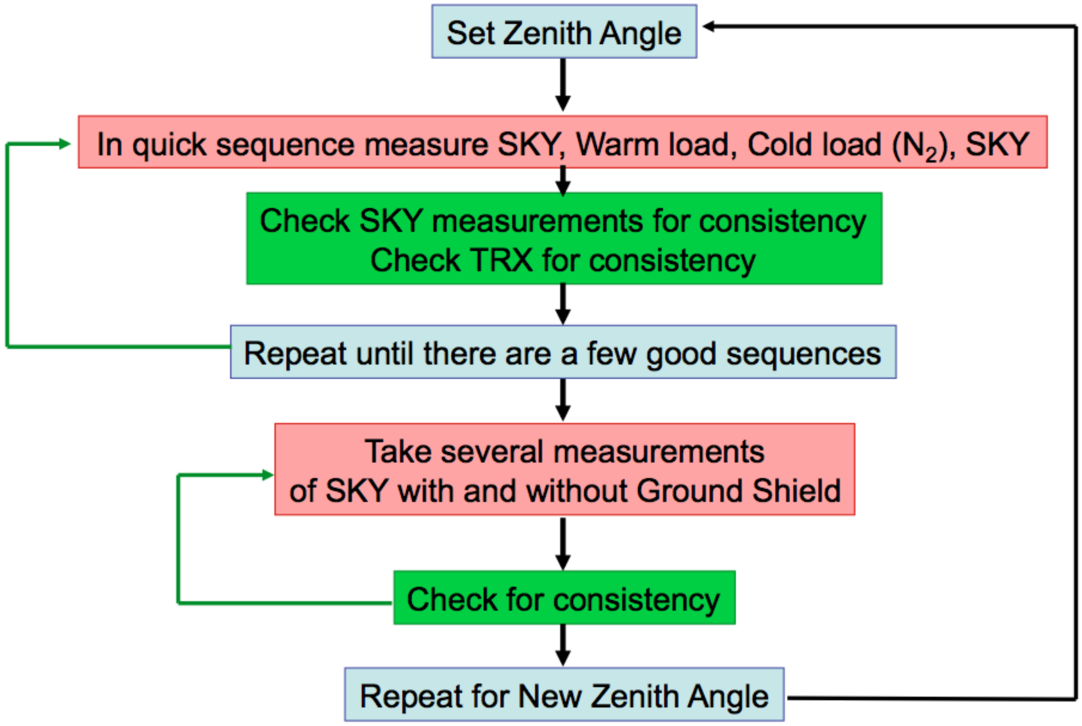


Figure 5.5: Flow chart for taking sky data. It is important to make sure you obtain consistent results for each elevation angle. The ratio of the power on ambient to liquid nitrogen load should be very stable, since the scale factor $\alpha$ in Eq. 5.1 cancels in the ratio. If the ratio is not stable there is a problem with the instrument (or liquid nitrogen load was not full immersed) and you will not get reliable results for Cosmic Microwave Background results. The ratio of the output power for the ambient load to the nitrogen cooled load should be about 3.4 to 3.6, depending on the temperature of the ambient load. The typical receiver temperature should be around 11 K. On a cloudy day the sky measurements may not be stable enough to achieve reliable results. Why is that? *Hint, light at cm wavelengths is also known as microwave radiation. What is it that your microwave oven is heating up? And, if it absorbs the microwave radiation, then it must also emit it.*

## 5.5 Analysis

2. Plot your data, $T_{\text{Sky}}$ vs. airmass, with airmass on the horizontal axis. Airmass is defined as

$$A(z) = \sec(z), \tag{5.8}$$

   where $z$ is the elevation angle.

3. Fit your data to a line and record the slope and intercept for the line. *Tip: double check your units. For elevation angle, we usually measure angles in degrees, but most calculators expect units of radians. Also, we will measure all temperatures in Kelvin.*

## 5.6 Report checklist and grading

Each item below is worth 10 points, and there is an additional 10 points for attendance and participation.

- Data and determination of temperature of two items inside the lab.
- Data table from Step 1.
- The plot and fit parameters from Steps 2 and 3.
- Does your data agree with the model where the emission is dominated by the atmosphere? What value to get for any residual temperature ($T_{\mathrm{CMB}}$) and how do you interpret it?
- What are some of the uncertainties associated with your measurement? What steps did you take to measure or mitigate these uncertainties?
- Write a paragraph discussing the quality of your data and the historical significance of the Penzias and Wilson measurement. Include a discussion of the following: What was the context of the Penzias and Wilson measurement? Why was this measurement important? What were some of the competing cosmological theories at that time and how did the Penzias and Wilson measurement fit in to that context?

APPENDIX A

# Lab Report Format

## A.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

## A.2 Organizing the report

The report should follow the sequence of the lab manual. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- Any procedure that you performed that is different from what is described in the lab manual.
- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.
- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$.
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.
- At the end of each experiment, you should discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

## A.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

### Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

### Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

APPENDIX B

# Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

## B.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

### Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1]The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

### Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{B.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and $N$ is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{B.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as $52 \pm 3\,\text{m}$.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

## B.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{B.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{B.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{B.5}$$

For other calculations, there is a more general formula not discussed here.

| Expression | Implied uncertainty |
|---|---|
| 12 | 0.5 |
| 12.0 | 0.05 |
| 120 | 5 |
| 120. | 0.5 |

Table B.1: Expression of numbers and their implied uncertainty.

#### What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference $C$ of a circle is given by $C = 2\pi r$. Here, the coefficient, $2\pi$, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table B.1.

#### How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel $11.3 \pm 0.1$ km in $350 \pm 10$ s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "$0.032 \pm 0.001$ km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

## B.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a $t'$ value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{B.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if $\delta a$ and $\delta b$ are uncorrelated, random uncertainties, then $t'$ represents how many standard deviations the difference $a - b$ is away from zero.

# Bibliography

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