Laboratory Manual

PHSC 12710 Galaxies

The University of Chicago

Winter 2023

Labs

LAB 1

Measuring distant objects with parallax

1.1 Introduction

Since it takes time for light to travel to us from objects in the universe, the further out an object is, the further back in time we see it. So for us to have an accurate picture of how the universe was in the past, we need to know how far away things are. For things that are nearby on Earth, we can travel there and see how far we went, or how long we took to get there. For things further away like the moon, we can use Kepler's laws, or we can bounce a beam of light off of it and see how long it takes to get back. For objects outside of our solar system, it would take too long, and the light would disperse too much, for us to use this last technique. For those objects that are still relatively nearby, we can use the parallax technique as the first rung on our distance ladder.

1.2 Forming Groups

If you are attending the lab session live and do not yet have a group, one way the TA could assist is to arrange "speed networking" among those who still need a group. This would involve the TA organizing Zoom Breakout Rooms, where each room is 2-3 students, and each group talks about how they work and what they are looking for in a group member. Then after 5 minutes or so, the Rooms are changed so people are with different people. This could help people get to know each other enough to form lab groups.

1. Once you have a group, meet with each other and decide a) what tools you will use to communicate and collaborate, b) when you will meet, c) what you will do when you need to change an agreement, and d) what you will do when you a person has an issue with how the group is functioning.

1.3 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

1.4 Worksheet

Complete the worksheet "The Parsec" on the following pages. You can draw the diagrams needed and include a picture of your diagrams, or use a drawing program to draw on them.

The Parsec

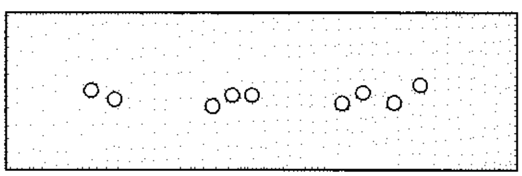
Part I: Stars in the Sky

Consider the diagram to the right.

1) Imagine that you are looking at the stars from Earth in January. Use a straightedge or a ruler to draw a straight line from Earth in January, through the Nearby Star (Star A), out to the Distant Stars. Which of the distant stars would appear closest to Star A in your night sky in January? Circle this distant star and label it "Jan."

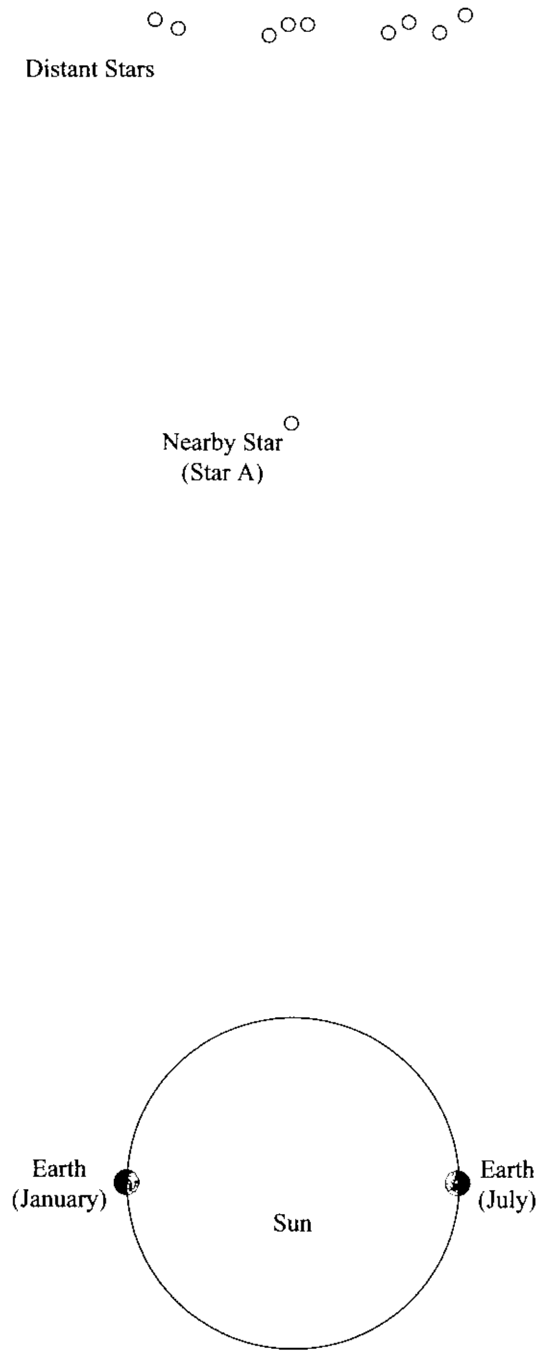
2) Repeat Question 1 for July and label the distant star "July."

3) In the box below, the same distant stars are shown as you would see them in the night sky. Draw a small × to indicate the position of Star A as seen in January and label it "Star A Jan."



4) In the same box, draw another × to indicate the position of Star A as seen in July and label it "Star A July."

5) Describe how Star A would appear to move among the distant stars as Earth orbits the Sun counterclockwise from January of one year, through July, to January of the following year.



The apparent motion of nearby objects relative to distant objects, which you just described, is called **parallax.**

6) Consider two stars (C and D) that both exhibit parallax. If Star C appears to move back and forth by a greater amount than Star D, which star do you think is actually closer to you? If you're not sure, just take a guess. We'll return to this question later in this activity.

Part II: What's a Parsec?

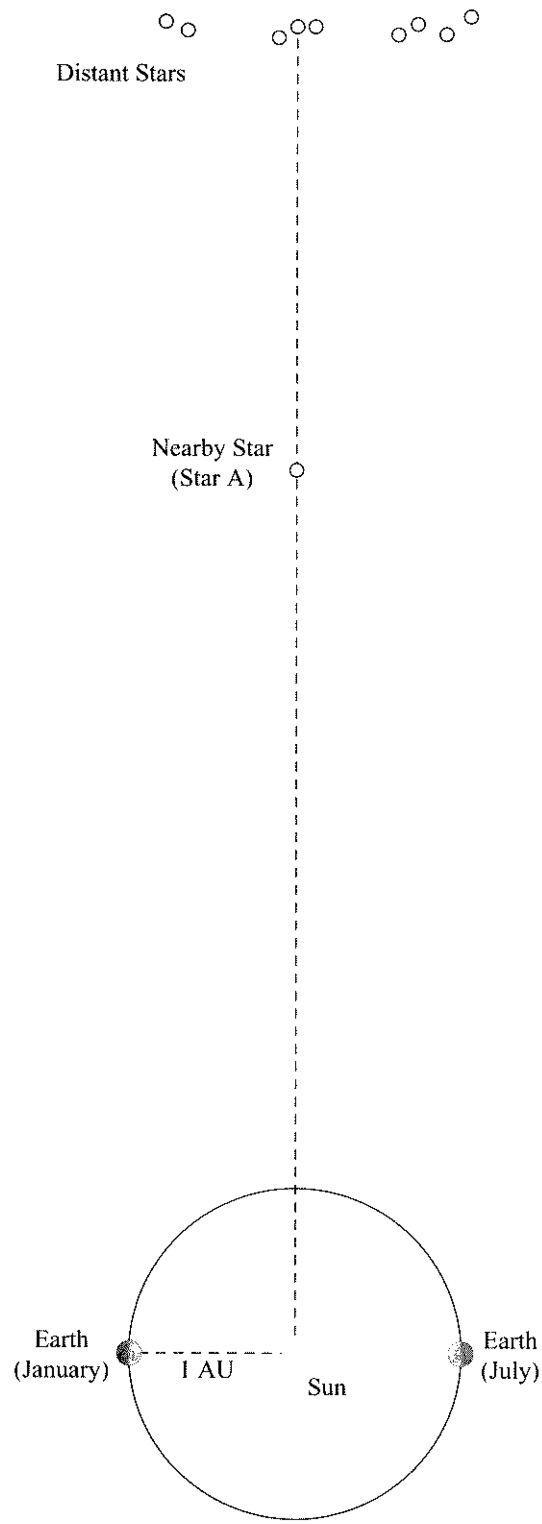
Consider the diagram to the right.

7) Starting from Earth in January, draw a line through Star A to the top of the page.

8) There is now a narrow triangle, created by the line you drew, the dotted line provided in the diagram, and the line connecting Earth and the Sun. The small angle, just below Star A, formed by the two longest sides of this triangle is called the **parallax angle** for Star A. Label this angle **"p_A."**

Knowing a star's parallax angle allows us to calculate the distance to the star. Since even the nearest stars are still very far away, parallax angles are extremely small. These parallax angles are measured in "arc-seconds" where an arcsecond is 1/3600 of 1 degree.

To describe the distances to stars, astronomers use a unit of length called the **parsec.** One parsec is defined as the distance to a star that has a **par**allax angle of exactly 1 arc**sec**ond. The distance from the Sun to a star 1 parsec away is 206,265 times the Earth–Sun distance or 206,265 AU. (Note that the diagram to the right is not drawn to scale.)



1.5 A quick measurement with hand tools

You will now use the parallax technique to measure the distance to an object in your environment without needed to travel to it.

2. Identify an object to be your "nearby star" and another as your "distant star", by analogy to the figure in the worksheet. Also choose the positions to view from as "Earth (January)" and "Earth (July)". When picking the stars and your Earth positions, check to see that from each Earth position, the distant star does not appear to move much, compared to the nearby star, and your guessed distance to the nearby star is at least 10 times greater than the distance between your two Earth positions.

You'll first use crude measuring tools to compute the nearby star's distance with the parallax technique, and then use a more precise method. First, you'll use the little finger on your outstretched arm as a measurement of angular size — the width of the index finger covers ("subtends") about 1 degree.

3. Identify an object to be your "nearby star" and another as your "distant star", by analogy to the figure in the worksheet. Find a place where you can stand and move a meter or two side to side and still see both "stars". The movement will simulate the Earth moving from its January to its July position.
4. Looking from just one eye, move so that the two stars appear to be directly overlapping. Mark your current position as Earth (January). Hold up your smallest finger at arm's length and move to your left or right until your finger fits just in between the two stars. This means that they are 1 degree away from each other in angular separation. Mark this position as Earth (July).

> The equations we will use later require the following. Make sure these are true. If not, choose different stars.
>
> - The background star is at least 10 times as far away as the nearby star.
> - The distance to the nearby star is at least 10 times the distance between your July and January Earth positions.

5. Draw a diagram, similar to the second figure in the worksheet, and find your own Earth-Sun distance (half the distance between your Earth positions). Calculate the parallax angle in radians, which is half the 1 degree you measured with your finger. For the distance measurement, you can use a ruler, measuring tape, or objects that have standard lengths like coins, paper money, or sheets of paper (or Google Maps if the distance is very long, like for mountains or tall buildings in the distance).

Now the distance to the nearby star can be found using the triangle formed by the line segments Sun-Earth, Earth-Star, and Star-Sun (see Figure 1.1). Trigonometry relates these lengths to each other according to

$$\tan p = \frac{a}{d}\,, \tag{1.1}$$

where p is the parallax angle in radians, d is the distance to the star, and a is half of the distance between the two measurement positions. Since the length d is much greater than a, the angle p is very small, and so we can use the small angle approximation $\tan u \approx u$, and therefore

$$p = \frac{a}{d}\,. \tag{1.2}$$

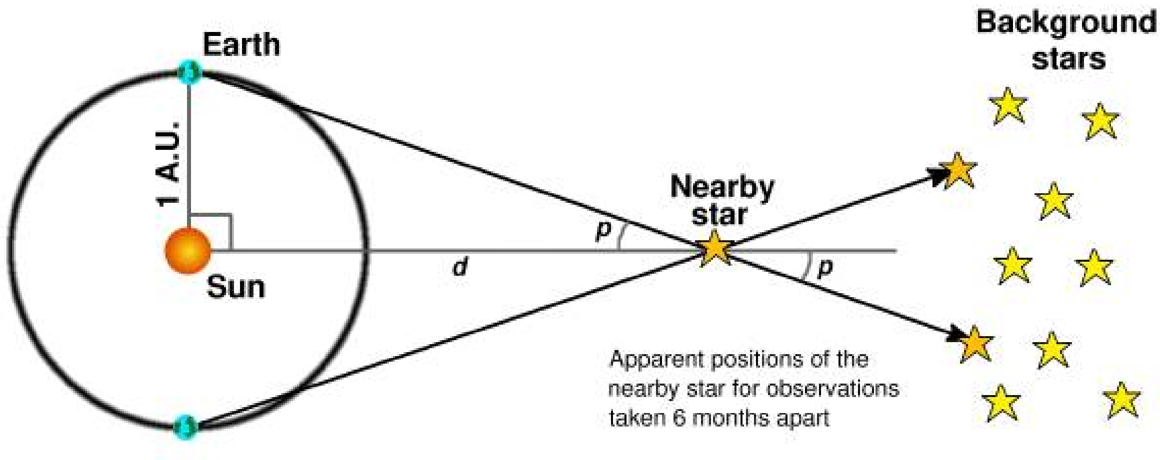


Figure 1.1: Illustration of the geometry involved in a parallax measurement to determine d, the distance to a nearby star.

6. Use Equation 1.2 to calculate the distance to the nearby star.
7. Conduct the data collection twice more to get two more Earth-Star distances. You can use this to calculate the *measurement uncertainty* — the uncertainty is half the range of values, and the distance is the average of the 3 distances you found.
8. Find the distance to the nearby star in a more direct way — measure the distance with a measuring tape or pieces of paper, or for more distant objects, find them on Google Maps. For this value, also measure this distance 3 times and find the value and uncertainty as in the previous step.

Notation for uncertain values

To be precise about how imprecise our measurements are, scientists often express the quantities as "⟨value⟩ ± ⟨uncertainty⟩", followed by the units. For example, if you measured your average distance to be 3.3 meters, and your uncertainty of that distance to be 0.4 meters, then to express in a succinct way, you can write the distance as "3.3 ± 0.4 meters".

One way of describing how different two values are, without considering the uncertainty of those values, is to calculate the percent difference:

$$\text{percent difference} = \frac{d_1 - d_2}{\left(\frac{d_1+d_2}{2}\right)} \times 100\% \tag{1.3}$$

9. How close is your parallax distance measurement to the direct measurement? Report the percent difference.

In the next section, you will compare these values with each other using their uncertainties as well.

1.6 Measuring with more precise equipment

Using a finger for measuring angular separation is not very precise. Here you'll use the parallax technique to determine the same distance to the nearby star, but using a camera and analysis software instead.

Figure 1.2: Example images. My foreground "star" was the point defined by the left intersection of the lower cable and the white pillar on top of the gym on campus. One of my reference "stars" was the top right corner of the building in the background. Note that the images produced by this telescope are upside down.

10. Take a picture from a digital camera (likely your phone camera) from the vantage point of each Earth position used above. See Figure 1.2 for example data.
11. Upload your photographs to a computer. The simplest way to do this may be to email the images to yourself from your phone. Give each file a descriptive name (e.g. "`parallax_left_telescope`").

Finding the pixel scale

Notice that with the finger, I told you that your little finger, outstretched, is about 1 degree wide. This was a conversion between the linear size of your finger, for example 1 cm, to an angular size, 1 degree. With the camera, we need to find out the similar conversion — how many pixels in an image corresponds to what angular size, also known as the *pixel scale* of the image. To do this, you can take a known angular size and measure its length in pixels.

12. Find an object of known length and place it a known distance from the camera (distance to camera should be 10 times or more the length of the object). Take a picture of that object.

You will use an image analysis program to measure lengths in pixels and find the pixel scale. These instructions are for using ImageJ, a free downloadable software tool, but you can use your own if you have a different preference.

13. Open ImageJ on a lab computer, or install it on your own computer (`https://imagej.nih.gov/ij/download.html`) and open it there.

14. Open your image using the File > Open. . . menu.
15. Select menu option Analyze > Measure to open the Results menu, which will give the results of your length measurements.
16. Select the line tool by clicking the 5th button from the left.
17. Click and drag on the image to draw a line that you want to measure.
18. Press "m" on your keyboard to take a measurement of that line. The length in pixels will be among the numbers given in the Results window.
19. Use this length to calculate the pixel scale in radians per pixel.
20. Find the angular size of the known object. Since the object is far away, we can again use the small angle approximation for the triangle involved and find that the angular size of the object is equal to its length divided by the distance to the object. This angle is in radians.
21. Find the pixel scale by dividing the number of pixels in the length by the angular size of the known object. This gives the pixel scale in pixels per radian.

Finding the parallax angle

Now that you have the pixel scale of the image, you can use that to measure the parallax angle of the your nearby star and thus find the distance to that object like you did with the little finger method.

22. Open the first parallax image. Measure the pixel length from the nearby star to the background star (this length is zero if the two stars are completely overlapping). Convert the length to angular separation in radians using the pixel scale you found earlier. Repeat these measurements for the second parallax image using the same background star.
23. Find the total angular distance the star moved between images by subtracting the two separations. However, if the stars switched orientations, for example the background star switched from being on the left to being on the right of the nearby star, then you should add the two separations.
24. Divide this by two to get the parallax angle.
25. Use Equation 1.2 to find your new calculation for the distance to the nearby star.
26. Calculate the percent difference between this and the direct measurement like you did in the previous section.
27. Use the t' statistic described in Appendix A.3 to compare the two values and interpret the result — do these two ways of distance measurement really measure the same thing?

1.7 Report checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Your group's agreements about communication.
2. The completed worksheet "The Parsec".
3. Work and final answer for your distance measurements using your finger, with uncertainty.
4. Work and final answer for direct distance measurement, along with percent difference.

5. A figure with your three images (pixel scale image and two parallax images).
6. The displacement vectors from distant star to nearby star.
7. Final determined value of the distance and comparison with the direct distance using percent difference and t' statistic. Show your work (see Appendix B).
8. A 100–200 word reflection on group dynamics and feedback on the lab manual. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what would you keep and change about the lab write-up?

LAB 2

Measuring the distance to a galaxy using globular clusters

Measuring the distance to another galaxy is not straightforward, since it would take too long to fly there, and it would take too long to bounce light off of it (among other difficulties). In this lab, you will use the apparent brightness of star clusters orbiting another galaxy to estimate the distance. After all, things that are further away from us appear dimmer.

In this lab, we will use a globular cluster in the Milky Way called M5 (see Figure 2.1) as the first step of our distance ladder to other galaxies. First, we will compare the brightness of the Sun to the brightness of a sun-like star in the globular cluster M5 (which is in the Milky Way and close enough that we can resolve individual stars) to find the distance to M5. Then we will compare the brightness of the entire M5 cluster to a cluster orbiting the galaxy M87 (that cluster is so far away that we see it as just one bright mass). This will allow us to find the distance to that galaxy! Ideally, we should compare many globular clusters in the Milky Way to M87, here we will use just one cluster, which is enough to demonstrate the principle.

Confusing nomenclature alert!

M5 is a globular cluster in our galaxy, while M87 is an entire galaxy. They share the prefix 'M', even though they are very different objects, because these numbers are part of the Messier catalogue, a numbered list of astronomical objects seen in the sky by Charles Messier, who was searching for comets and wanted to record these objects so that when he saw them during his comet search, he could ignore them. So don't forget to consider the negative space, and the work you do along the way!

Figure 2.1: Messier 5 (M5) is a globular cluster (a gravitationally bound collection of stars) of more than 100,000 stars in the Milky Way Galaxy. Located at Right Ascension (RA) = 229.640°, and Declination (Dec) = 2.075°. The above image is 2.85 arcmin on a side, or about 1/20th of a degree. Image source: ESA/Hubble & NASA, http://www.spacetelescope.org/images/potw1118a/

2.1 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the “Groups” tab. Scroll to a group called “L2 Cluster [number]” that isn’t used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

2.2 Brightness and distance

If two identical lightbulbs are placed with one close to you and another farther away, the more distant one will appear dimmer. This is because the light from a spherical emitting source spreads out over a spherical shell that gets larger as the light gets more distant from the source. So if sources 1 and 2 have the same luminosity, then their distances d and apparent brightness b are related by

$$\frac{b_1}{b_2} = \left(\frac{d_2}{d_1}\right)^2 . \tag{2.1}$$

Since the numbers we will extract from the images are either in brightness or magnitudes, it is convenient to re-cast this relation in terms of magnitudes. Magnitudes m_1 and m_2 are related to brightness b_1 and b_2 by

$$m_2 - m_1 = 2.5 \log \left[\left(\frac{b_1}{b_2}\right)^2 \right] . \tag{2.2}$$

Combining the two equations, we get

$$\log(d_1/d_2) = 0.2(m_1 - m_2) . \tag{2.3}$$

This says that once we have measured magnitudes m_1 and m_2 for two sources, then we can derive the ratio of their distances from us, *as long as they have the same luminosity*.

2.3 Road Map

To keep track of the steps in this lab, we will fill in Table 2.1. In this table, the entry for the magnitude of M5 refers to the sum of all of its stars. In principle, we could measure this ourselves with the rooftop telescope, but for this lab we take a value from a catalog of such data. The SDSS data cannot be used because the stars are too crowded together for an accurate measurement.

The first step is to make a *color-magnitude diagram* for the stars in M5 to find a star that has similar to the Sun; we assume that such a star has the same luminosity as the Sun. The magnitude of the star (specifically its r-band magnitude) gets entered into the above table, and you derive the distance to M5.

Object	**Magnitude**	**Distance (AU)**
Sun	-26.89	1
average Sun-like star in M5		
M5 itself	5.65	
average M87 globular cluster		

Table 2.1: Table of magnitude and distance.

The second step is to identify faint things surrounding the galaxy M87 that are likely to be globular clusters associated with it, and get their magnitudes (again the r-band magnitude) from the database. Some value that properly represents the ensemble gets entered into the above table and you derive the distance to M87 by comparing the magnitudes of M5 and M87.

To summarize: the distance to the M87 galaxy depends on two assumptions: 1) stars with Sun-like colors in the globular cluster M5 have the same luminosity of the Sun. 2) Globular clusters like M5 in the Milky Way have luminosities that are comparable to the globular clusters in M87. Neither of these assumptions is necessarily well justified based on information available to you, but there are checks that reassure us that the assumptions are good enough for at least a first estimate of distance.

2.4 Analyzing the M5 globular cluster

First you'll retrieve from an online database the magnitude of stars in the region of sky where M5 is. In the window at http://skyserver.sdss.org/dr13/en/tools/search/sql.aspx, enter the following query:

```
SELECT TOP 200
   objid,ra,dec,u,g,r,i,z
FROM Star
WHERE
   r BETWEEN 10 AND 23
   AND ra between 229.50 and 229.78
   AND dec between 2.2 and 2.3
```

Questions and results for your report

2. From the data above, create a .csv file, rename it M5.csv. Read it into a spreadsheet, and make columns for the colors $g - r$, $r - i$, and $g - i$. The Sun has colors $g - r = 0.44$, $r - i = 0.11$, and $g - i = 0.55$. Plot the r magnitude vs. one of the colors (e.g. $g - r$), and reverse the r magnitude axis, since lower magnitudes represent brighter objects. On this color-magnitude diagram, identify the *main sequence* of stars. This plot is called a color-magnitude diagram, which is similar to an H-R diagram as seen in Figure 2.2.

3. Begin to fill out Table 2.1 with the magnitude and distance to a Sun-like star in M5. When you finish this lab and turn in this lab report, this table will be completely filled out.

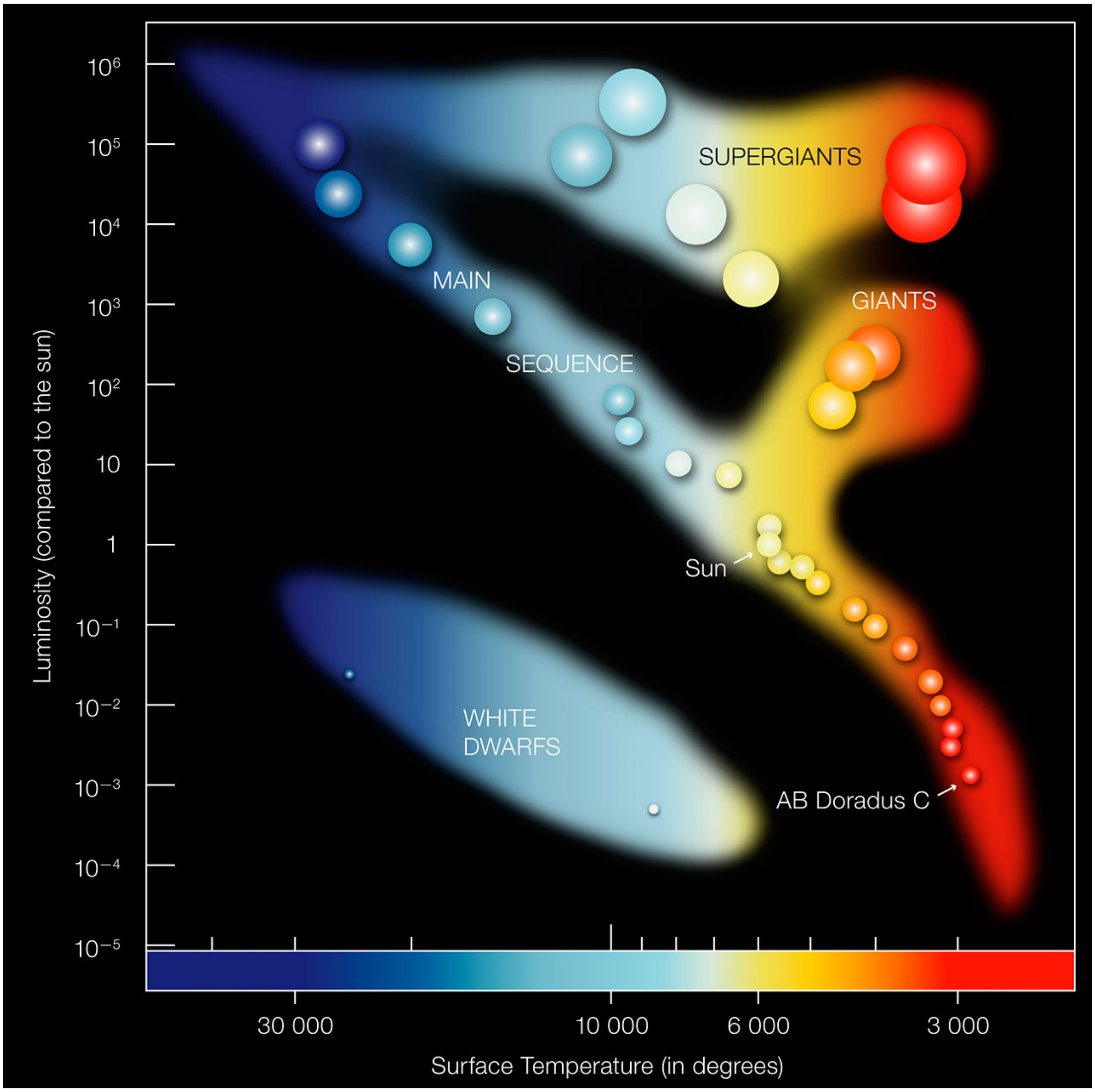


Figure 2.2: Herzsprung-Russell (H-R) diagram, plotting stars according to their luminosity and surface temperature. Luminosity is related to magnitude, and surface temperature is related to color. Image Source: ESO (https://www.eso.org/public/images/eso0728c/

2.5 Analyzing the globular clusters near the M87 galaxy

Figure 2.3 shows the field surrounding the giant Virgo galaxy M87, also known as NGC 4486.

The task is to find the magnitudes for the faint speckles surrounding M87 that are barely visible in Figure 2.3, namely its globular clusters. We set up a similar query to that used for M5, except of course the coordinates (RA, Dec) are different.

4. Enter the following query in SkyServer:

```
SELECT TOP 200
   objid,ra,dec,u,g,r, i,z
FROM Star
WHERE
   r BETWEEN 10 AND 23
   AND ra between 187.591 and 187.821
   AND dec between 12.278 and 12.504
```

The globular clusters are so far away that each cluster of stars appears as, and is categorized as, stars in the SDSS database.

5. As a cross-check, also run the above query in a random piece of sky at least 5° away.

Questions and results for your report

6. Make color-magnitude diagrams for both samples (M87 and random-sky) and compare them. Does either color-magnitude diagram show any evidence for a correlation between brightness (magnitude) and color for the plotted points?

7. Once you have identified which of the sources on your M87 color-magnitude diagram can be identified with a population of globular clusters surrounding M87, argue which apparent magnitude should be selected to enter into the table, and do so. Why is there a range of magnitudes? How would you make this process more precise? What other sources of uncertainty do you think there are?

8. Calculate the distance to M87 using Equation 2.3, comparing its magnitude to that of M5 (the entire cluster M5, not the individual star you used earlier). If you have not already converted AU to parsecs, do so now to get the distance in mega-parsecs (1 Mpc $= 2.06 \times 10^{11}$ AU). The accepted value for the distance to the Virgo cluster (the galaxy cluster that M87 is in) is 16.4 Mpc. From your uncertainties above, how well do these two values agree within your expected level of uncertainty? See Appendix A.3 for details of how to determine this.

9. Based on the distance you found, for the light that arrived at the sky survey's telescope from M87, when was it emitted? What was happening on Earth at that time?

Figure 2.3: Messier 87 (M87) is a nearby elliptical galaxy in the constellation Virgo. It is known for having a large population ($\sim 10,000$) of globular clusters, about 100 times more than the Milky Way Galaxy. Centered at RA=187.706°and Dec=12.391°, the above image is 97 arcminutes across. Image source: Chris Mihos (Case Western Reserve University)/ESO, http://www.eso.org/public/images/eso1525a/.

2.6 Report checklist and grading

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Completed table of magnitudes and distances
2. Color magnitude diagram for M5
3. Calculation and determination of magnitude and distance of a Sun-like star in M5 (Step 2)
4. Color-magnitude diagrams for M87 and random-sky
5. Questions in Steps 5–6
6. Calculation and determination of magnitude and distance of M87, with analysis of uncertainty and comparison (Step 7)
7. Age of light and what was happening on Earth (Step 8)
8. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

9. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

LAB 3

Measuring mass using Gravitational Lensing

> "The observation of such a gravitational lens effect promises to furnish us with the simplest and most accurate determination of nebular [sic] masses. No thorough search for these effects has yet been undertaken." — Fritz Zwicky, 1937

3.1 Introduction

Gravitational lensing is the bending of the paths of photons in the distortion of space-time caused by mass (i.e., by gravity). Gravitational lensing manifests in the universe on a range of mass and length scales. At the smallest scales — the Solar System — the mass of the Sun can cause an subtle shift in the apparent positions of background stars along lines of sight close to the Sun. This effect, only visible during an eclipse (otherwise the light from the Sun precludes observing the background stars) was first observed by Sir Arthur Eddington in 1919, and provided crucial early support to Einstein's theory of General Relativity, because the observed effect agreed with his predictions.

In the 1930s the astronomer Fritz Zwicky showed that galaxies (then still referred to as nebulae, as you can see in the quote above) and clusters of galaxies might be *the* best place in the universe to search for gravitational lensing, and argued that observation of this effect would be the best and cleanest way to measure the mass of these objects.

In this lab, you will find examples of gravitational lenses on cosmological scales, using data from the Hubble Legacy Archive (HLA) database. Then you will use the image of a distant object created by the galaxy lens to find the mass of the galaxy or galaxy cluster that is acting as the lens.

3.2 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L3 Lensing [number]" that isn't used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

3.3 Gravitational lensing theory

The basics of gravitational lensing can be understood from a simple diagram shown in Figure 3.1, coupled with the knowledge that mass bends the paths of photons. Consider an observer, O, a lensing mass, L, and a source of photons, S. We will consider the cosmological scale case here, so S might be a distant galaxy or quasar, and L a cluster of galaxies or a lone massive galaxy, situated between O and S.

Photons are being emitted by S in all directions, and normally in the absence of an intervening lens, O will see the source at a position corresponding to the direction from which photons are arriving from the source (case a). With an intervening lens L in place, other photons which normally would not arrive at O have their paths diverted as they pass by L, and end up being observed by O (cases b,c). Under this circumstance the observer 'sees' images of S not at its actual location but at virtual locations corresponding to the direction from which the diverted photons are arriving (images S'). If the alignment between S, L and O is perfectly along a line, and L is circularly symmetric on the sky, then the image S' is a ring on the sky. This is called an Einstein Ring – a sketch of this from the observer's perspective, and actual examples from Hubble Space Telescope observations of lensing systems found in the Sloan Digital Sky survey (SDSS) are shown in Figure 3.2.

Note that the degree to which the photon's path is deviated is proportional to the mass of L, with a larger mass providing a greater deflection. *T*he apparent radius of the Einstein ring on the sky thus provides a measure of the mass of the lens interior to the ring. The size of the ring is also related to the distance along the line of sight between S, L and O; to robustly determine the mass of the lens we must also know these distances.

Also note that the alignment between S, L and O is rarely perfect, and the lens L is rarely perfectly circular on the sky; the result of this is that complete Einstein rings are very rare and most lensed images S' are arcs which look like portions of a ring image.

Lab Tasks

2. To visualize how the lensing works, watch the animation at the following link:

   https://public.nrao.edu/gallery/animation-of-a-gravitational-lens-creating-an-einstein-ring/

   Note that real lenses move much more slowly and are actually stationary on the human timescale, and that only precise alignment yields full rings and that partial arcs are much more typical.

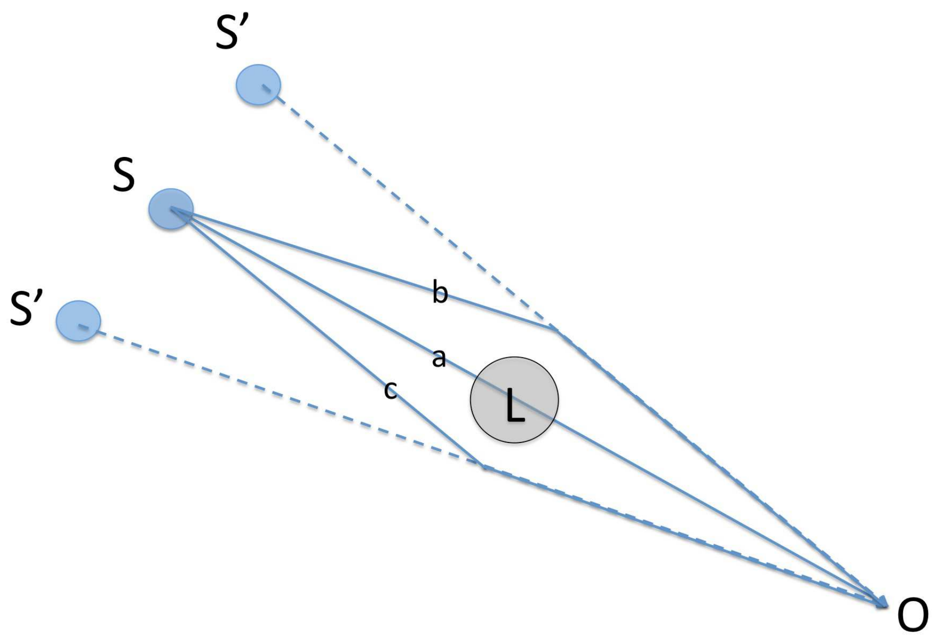


Figure 3.1: Diagram showing the geometry of source (S), observer (O), and gravitational lens (L). The source that would appear along direction OS without the lens, in the presence of lens appears along directions S′ instead.

3. Take a few minutes to explore some images of gravitational lensing recorded by instruments on the Hubble Space Telescope. An extensive gallery of images can be found at http://hubblesite.org/images/news/18-gravitational-lensing

The type of lensing we are exploring in this lab is called strong lensing, and is the most obvious manifestation of the bending of photon paths by gravity. Other types of lensing have also been observed.

When the lens involved has the mass of a star (apart from the specific case of the Sun, discussed above) rather than a galaxy or cluster of galaxies, then the radius of the Einstein ring is typically micro-arcseconds. In that case the lensing cannot be observed as a spatial distortion directly in the image (no telescope produces images with a resolution fine enough to see micro-arcsecond structures) however the focusing effect of lensing still produces an enhancement in brightness which can be observed. This type of lensing is often referred to as micro lensing.

A further manifestation of lensing is the weak distortion of the images of background objects which are well away from the observer to lens line-of-sight. In that case the background source is not imaged into a ring or arc or multiple images, but there is still a discernible pattern of distortion around the massive object. This pattern can be analyzed statistically, and provides another mass estimate for the lens. This effect, not surprisingly, is known as weak lensing.

Gravitational lensing as a phenomenon has a wide range of manifestations in the universe, and it is now an important technique for study mass in the universe on a range of scales, from extrasolar planets through to the largest structures. What began as a theoretical curiosity in the early 20th century is now one of the most powerful tools that we have to learn more about the cosmos. In the remainder of this lab you'll find a few astronomical lenses to work with and then apply some of the basics of strong lensing to measures the masses of galaxies and clusters of galaxies.

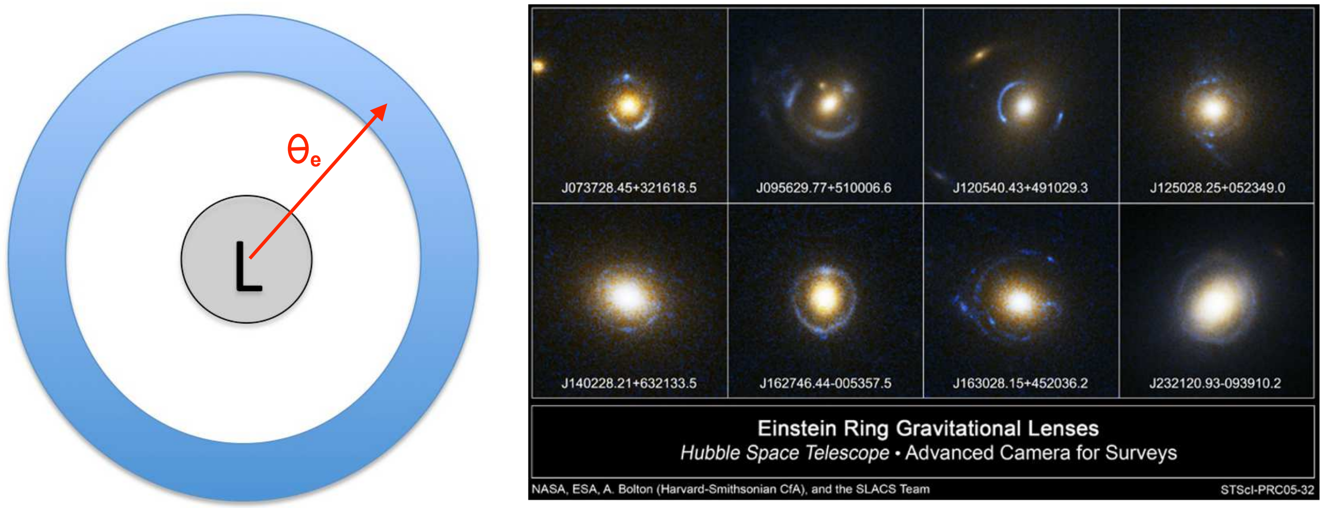


Figure 3.2: A sketch of the Einstein ring from the observer's perspective (left), and actual examples from Hubble Space Telescope observations of lensing systems found in the SDSS (right). The distance from the center of the ring to the ring itself is called *the Einstein radius* noted as θ_e in the sketch above, and in the equations below. It is this quantity which you will be estimating from the images, once you have identified several systems showing strong lensing.

3.4 Measuring Masses

In this section of the lab, you will use gravitational lensing to measure masses of galaxies that bend the rays of light. Theory predicts that the angular size, θ_e, of the Einstein radius (i.e. angle on the sky from the center of the lens to the ring or arc) is related to the mass within this angular radius, $M_{<\theta_e}$, by the following equation:

$$\theta_e = \left(\frac{4GM_{<\theta_e}}{c^2}\frac{D_{\rm LS}}{D_{\rm OL}D_{\rm OS}}\right)^{1/2}, \tag{3.1}$$

where $D_{\rm OL}$ is the distance from the observer to lens, $D_{\rm OS}$ is the distance from the observer to the lensed source galaxy, and $D_{\rm LS}$ is the distance from the lens to the source galaxy, $c = 2.999 \times 10^{10}$ cm/s is the speed of light in vacuum, and $G = 6.673 \times 10^{-8}\,\mathrm{cm^3\,g^{-1}\,s^{-2}}$ is Newton's gravitational constant.

The source galaxies are too faint for the SDSS to have measured their redshift, so we do not know the redshifts of the lensed sources in these cases. However, galaxies are lensed most efficiently when the lens is half-way between source and observer. Therefore, in the absence of better information we will make a reasonable assumption that the sources are at twice the distance to the lens: i.e., $D_{\rm OS} = 2D_{\rm OL} = 2D_{\rm LS}$, which is in agreement with the typical redshift of sources in known gravitational lenses. Using this assumption and solving for the mass we get:

$$M_{<\theta_e} = \frac{\theta_e^2\,c^2 D_{\rm OL}}{2G}. \tag{3.2}$$

Note that the angular size of the Einstein radius, θ_e, here should be in radians, not in arcseconds.

4. From the gallery of lenses in Figure 3.2, select two of them. They are identified by their name, for example "J073728.45+321618.5".

For each of the two selected lens systems, complete the following steps to find their masses by measuring their Einstein radius, looking up their redshifts, finding the distances to them using a calculator, then using the above equation.

5. Search for the system by name in the Hubble Legacy Archive at `https://hla.stsci.edu/hlaview.html` . You need to add "SDSS" to the beginning of the name identified in the previous step.

Immediately below the green tabs, it shows the system name followed by its equatorial coordinates RA and Dec (as well as RA and Dec in the alternate units `[hour:arcminute:arcsecond degree:arcminute:arcsecond]`. Each row in the table is a different observation of this system.

6. Pick one of the observations and select "Display", which opens a new window.

This new window is a low-resolution view of the image taken by the Hubble Space Telescope. The box in the upper left gives the coordinates of the pixel that the cursor is hovering over. In order to more easily see different features, you can lighten or darken the image using the buttons in the top left of the window.

7. Find the lens system by clicking and dragging the image around, using the system's coordinates as a guide.
8. Use the zoom, lighter, and darker buttons to create the best zoomed in view of the lens and Einstein ring. **Record a screenshot of this image of the ring.**
9. To measure the radius of the ring, move your cursor and record the coordinates of the center and the edge of the ring. Input these into the angular separation calculator at `https://cads.iiap.res.in/tools/angularSeparation` to get the angular radius. Record these values of θ_e for each lens in degrees. Convert the values in degrees to radians and **record the result**.

10. To find the distance to the lens systems, you will need to look up its redshift in the SIMBAD database. Go to `http://simbad.u-strasbg.fr/simbad/sim-fbasic` and search for your system's name. The redshift is the number after "`z(spectroscopic)`". **Record this number.**

11. Start a browser and go the Ned Wright cosmology calculator. You can google it or use this link: `http://www.astro.ucla.edu/~wright/CosmoCalc.html`. Use default values for all parameters, but enter the appropriate value of the redshift for each of your two lenses in the input field marked `z` and click on the button `General`. The window on the right will display a variety of information for the input redshifts and assumed cosmological parameters. **Record the "angular size distance" $\mathbf{D_A}$ in Mpc and and the scale, i.e. how many kiloparsecs correspond to 1 arcsecond (kpc/ ″).**

12. Compute mass within θ_e, $M_{<\theta_e}$, using the distance D_{OL} that you calculate from redshift with the cosmology calculator, i.e., $\mathrm{D_A}$.**Include the measured masses in your report.** Express masses M_{θ_e} in solar masses ($1 M_\odot = 1.989 \times 10^{30}$ kg).

13. The mass of *stars* in typical galaxies known in the universe is about $10^{10}\, M_\odot$. **How do your measured masses compare to this value of the maximum mass of visible stars? What could cause the difference?**

3.5 Report Checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. For each lens selected, the image of the lens, a table of name, θ_e, redshift, and angular size distance D_A (Steps 7–10)

2. Calculation and final value for masses (Step 11)

3. Comparison to known maximum galaxy mass (Step 12)

4. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

5. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

LAB 4

How fast is the galaxy rotating and what does it mean?

4.1 Introduction

Observational astronomy using light at radio wavelengths, i.e., meter through millimeter wavelengths, is a relatively new field, starting with Karl Jansky's discovery of radio wavelength emission from the sky in 1933. Jansky worked at Bell Labs and was trying to understand the excess noise in trans- Atlantic radio communications. It was a remarkable discovery, making headlines in the New York Times and leading to the birth of new field. However, Bell Labs did not pursue radio astronomy further at that time. An enthusiast Grote Weber did, however. As a hobby, he built in 1933 the first modern radio telescope in his backyard in Wheaton, IL and used it to map the sky, identifying many interesting astrophysical sources. He was detecting continuum emission, most of the sources were later found to be highly energetic active galactic nuclei emitting a type of radiation called synchrotron emission. In 1944, Hendrik van de Hulst worked out the theory of the hydrogen spin-flip transition, often simply called 21 cm hydrogen emission, which soon became a powerful tool for astronomy. It still is today. The 21 cm line (1420.5 MHz frequency) is used to map the cold neutral hydrogen gas in the Galaxy and other galaxies. It was used to show unambiguously the spiral arms in the Milky Way. It also allows astronomers to measure the rotation curves of galaxies and solve for the enclosed mass. Such measurements have led to the discovery that large halos of an invisible, "dark" matter dominate the mass of galaxies. In this lab you will make such a measurement. In so doing you will learn the basics of radio astronomical observations and use them to measure the rotation of the Milky Way.

4.2 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L4 Rotation [number]" that isn't used and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

4.3 Masses and Orbital Velocity

Here you will learn how to use the rotation curve (plot of velocity vs distance from center) of a gravitationally bound rotating system to deduce information about the distribution of mass in it. You'll do this by deriving the relationship between the velocity of an orbiting body and the mass of everything within its orbit, and then exploring what the velocity looks like for several different theoretical mass distributions.

Equipment

- Desmos Graphing Calculator: https://www.desmos.com/calculator

Steps

2. First, we know from Newton's first law that the force on an object is equal to its mass times its acceleration, $F = ma$. We also know that an object moving in a circle will experience an acceleration towards the center of its path which is given by $a = \frac{v^2}{r}$, where v is the velocity of the object and r is the radius the circle it travels along. Combine these two equations to find an equation for the force experienced by an object undergoing circular motion. **Record your derivation and final equation.**

3. Objects in orbit also move in an approximately circular path and the force of gravity they experience is given by $F = G\frac{Mm}{r^2}$, where M and m are the masses of the two objects and G is the gravitational constant. Using the equation you found in the previous step, derive an equation for the velocity of the orbiting object as a function of its radius.*Hint: plug equations into each other to remove variables that are common in both equations. Your final equation should be in terms of v, M, r, and G.* **Record your derivation and final equation.**

4. Create a plot of your equation for the velocity as a function of radius, for example using the online graphing calculator listed in the equipment. To plot it, type the equation in the left side of the screen. It will graph y on the vertical, x on the horizontal axis, so use y in place of your v and x in place of your r. For the variables M and G, click the "add slider" button so you can plot the shape of the curve. You can leave them set to equal 1, since we are using this to see the shape of the curve, not the absolute value. **Include this graph in your report.**

5. Adjust the mass slider to larger values and observe what happens to the curve. How does it change? **Record your answer.**

The graph you created is called a *rotation curve*, which plots the orbital velocities of objects in an orbital system against their distance from the center. Using these curves, it is possible to learn a lot about the system it describes.

When you first derived the equation for rotational velocity, you assumed that M represented the mass for a single object at the center. Now we will assume that this equation holds true in the case that M represents the total mass contained within the orbital radius. This assumption is valid for a spherically symmetric system, which is a good enough approximation for us. Let's explore some cases for different mass distributions.

6. For the case where the mass is uniformly distributed in a disk, the total enclosed mass increases as r^2. To graph this, delete the M slider and create a new expression $M = cx^2$. How is this curve different from the previous one? **Include your answer and plot in your report.**

7. Play with different mass distributions by changing the equation for M and observing how the curve changes.

8. The rotation curve for a sample orbital system is shown in Figure 6.1. Change the mass formula in your plot to visually match the shape of this curve. How is the mass distributed in this orbital system? **Record your plot, mass equation, and answer.**

9. Another rotation curve is shown in Figure 6.2 (line B). Change the mass formula in your plot to visually match the shape of this curve, ignoring the initial steep rise of the velocity at small distance. How is the mass distributed in this orbital system? **Record your plot, mass equation, and answer.**

10. Look up information on the masses of the planets in the solar system as well as the Sun. Which of the rotation curves would you expect the solar system to have? *It might help to add up all the masses and see how much each planet contributes to the total mass.* **Record your answer.**

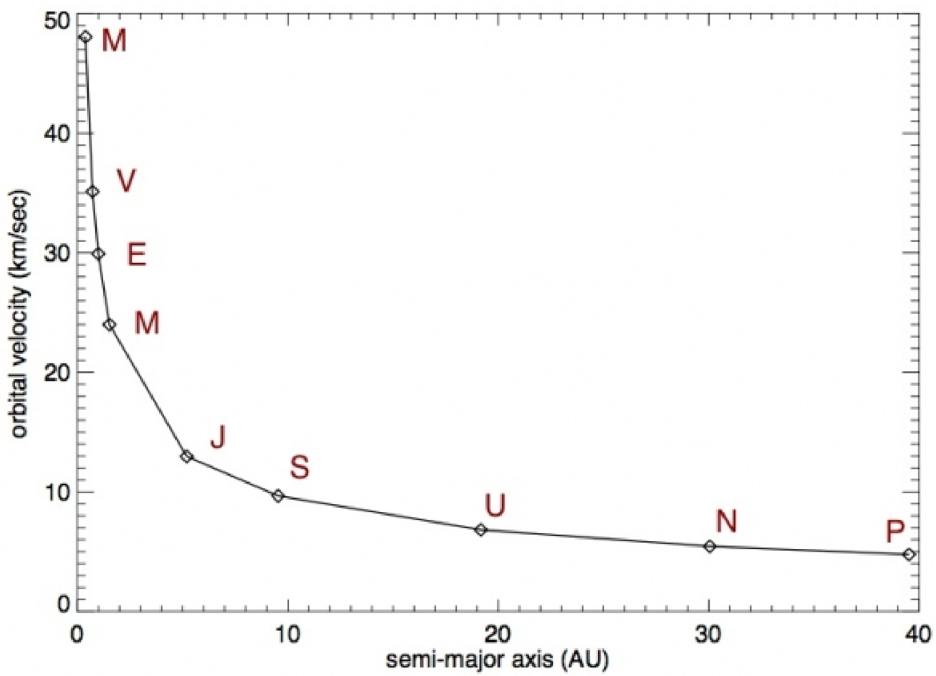


Figure 4.1: Rotation curve of an example orbital system, where each of the objects marked on the graph are orbiting the center of the system.

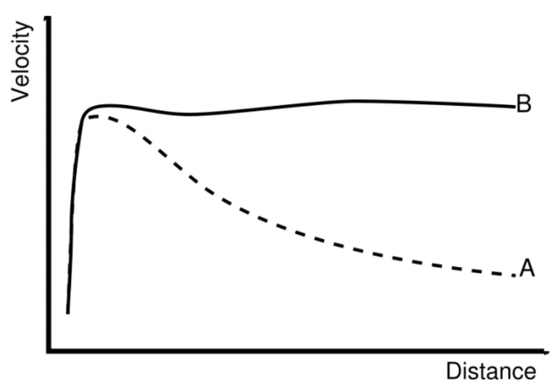


Figure 4.2: Rotation curves for two example orbital systems A and B.

4.4 Measuring the velocity of Hydrogen clouds in the galaxy

Hydrogen is the most common element in the universe. It exists in interstellar space as individual atoms, each atom consisting of a proton and an electron. Both particles have a property called spin and a hydrogen atom can exist either with the spins of the proton and electron parallel or anti-parallel. Sometimes the atom changes its spin state and, in doing this, emits a radio wave at a precisely known frequency of 1420.4 MHz corresponding to the wavelength of 21cm. By tuning the radio telescope receiver to this frequency, we can directly measure the amount of hydrogen in that direction and, importantly, its velocity. We can measure its velocity through the effect known as Doppler Shift. When an object is moving towards or away from an observer, the wavelengths of the light observed from the object get compressed or stretched. Since the wavelength and frequency of light are inversely related, frequencies are respectively increased or decreased. This is called Doppler Shift. Thus if we know the intrinsic frequency at which an object emits – in this case from the fundamental physics of the Hydrogen spin-flip transition – then we can calculate the velocity of that object with respect to our observational frame of reference – in this case the Earth. Your TA will go over this important concept with you in more detail during the first lab section. (Make sure you understand what's going on and ask questions if you are confused!) To do this, the radio telescope measures the flux at a number of very finely spaced frequencies and so can produce a spectrum of the hydrogen line. This allows us to measure the Doppler shift of the atomic hydrogen clouds in the interstellar space of the Milky Way emitting the radio waves and allows us to measure the rotation velocity of that gas around the center of our Galaxy.

11. Given what you have read and learned so far, **answer the following questions**:

    a) You spot a star which you know should be emitting a signal at 800nm. However, you instead detect a signal at 900nm. What does this tell you about the stars motion?

    b) You spot two gas clouds which should both be emitting at frequencies of around 1400hz. However, for cloud A you detect a signal of 1500hz and for cloud B you detect 1350hz. Which cloud is moving towards you? Away from you? Which one is moving faster?

    c) If an astronomical object is moving away from you, will its light become more red or more blue? What if its moving towards you?

Our star system, the Solar System, resides within the Milky Way Galaxy. When we observe it directly, it looks like the following:

> https://en.wikipedia.org/wiki/Milky_Way#/media/File:ESO-VLT-Laser-phot-33a-07.jpg.

The galaxy has a spiral disc shape, and this image is looking towards the center of the galaxy. While we can't move outside our galaxy to take a picture of it, based on what we know, it probably looks like the artist's rendition here:

> https://en.wikipedia.org/wiki/Galactic_coordinate_system#/media/File:Artist's_impression_of_the_Milky_Way_(updated_-_annotated).jpg

Locate the Sun in that image and notice the coordinate system that extends from it. This is the system of galactic longitude, shown as a schematic here:

> https://en.wikipedia.org/wiki/Galactic_coordinate_system#/media/File:Galactic_coordinates.JPG

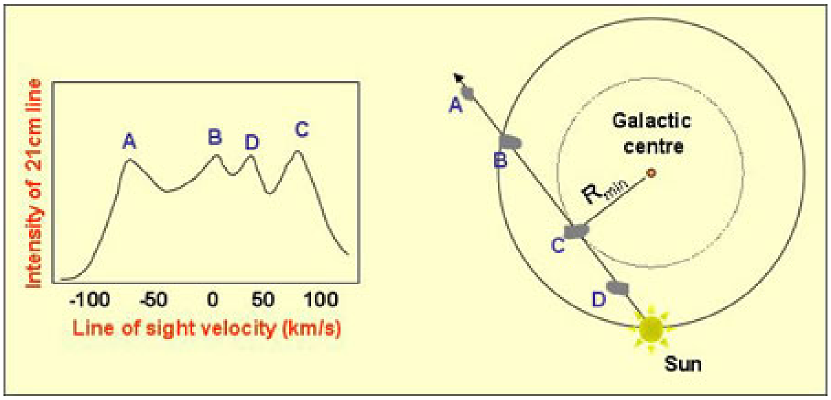


Figure 4.3: On the left, a graph of intensity vs line of sight velocity for 4 different gas clouds observed from Earth. The maximum line of sight velocity is from cloud C, since it's velocity orbiting the galactic center aligns with the line of sight. (Image from Swinburne University of Technology, https://astronomy.swin.edu.au/cosmos/H/HI+cloud)

Throughout the galaxy, there are neutral hydrogen gas clouds in the spiral arms. The neutral hydrogen clouds emit light with a spectral line at a wavelength of 21 cm (frequency of 1420.4 MHz). Since they are all moving at different velocities, when we observe 21 cm line in the galactic disc (galactic latitude $b = 0$), we see different distinct peaks at wavelengths close to 21 cm, caused by their differing Doppler shifts. So we can use Doppler shift to find the velocity of those clouds. A sample observation is found in Figure 6.3.

Calculating Rotational Velocity

When measuring the velocity of these objects, we have to keep in mind that we are moving relative to them. As such, we are not measuring their rotational velocity, but their velocity relative to us. We therefore have to do several calculations to get their orbital velocity. To do this calculation, we will need several components. First, we will need the velocity of the sun in the line of sight for the galactic longitude l. This is because the Sun is also moving along the galactic plane, and thus we need to be able to account for it in our measurements. We already know that the velocity of the Local Standard of Rest (the local stellar environment around the Sun) is 220km/s, so using some trigonometry, we can see that the line of sight velocity is given by

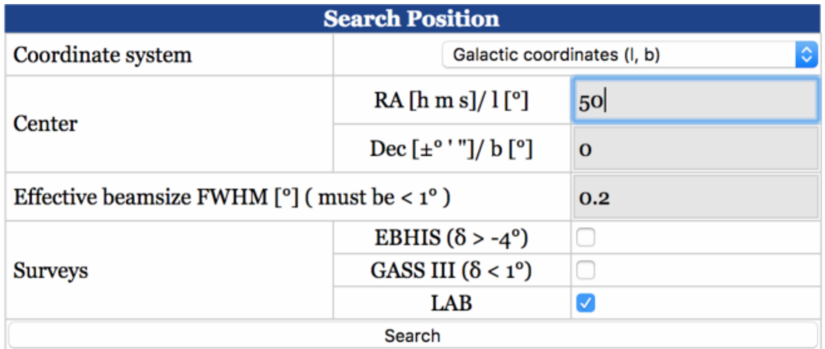
$$V_{\text{sun}}(l) = (220\,\text{km/s})\sin(l)\,. \tag{4.1}$$

We also need to account for Earths rotation around the sun as well as relative motion of the solar system compared to the LSR. These are given to you by the SRT altogether as the VLSR or Velocity relative to the Local Standard of Rest. From the graph generated by the telescope, we simply need the maximum VLSR, as it corresponds to the hydrogen cloud directly in our line of sight. The circular velocity of the cloud can thus be obtained by

$$V_c(r) = v_{\max}(l) + v_{\text{sun}}(l)\,. \tag{4.2}$$

Finally, for the final data processing, you will need the distance of that cloud from the center of the galaxy. Once again, from the geometry of the graph above, we can see that this can be found from the distance of the Sun to the center $r_0 = 8.5$kpc (kiloparsec) and the galactic longitude l. The distance is then given by

$$r = r_0\sin(l) \tag{4.3}$$



Search Position		
Coordinate system	Galactic coordinates (l, b)	
Center	RA [h m s]/ l [°]	50
	Dec [±° ' "]/ b [°]	0
Effective beamsize FWHM [°] (must be < 1°)		0.2
Surveys	EBHIS (δ > -4°)	☐
	GASS III (δ < 1°)	☐
	LAB	☑
Search		

Figure 4.4: The input form for accessing the LAB spectra. Dec is always set to 0, only LAB is selected, FWHM is set to 0.2 and galactic coordinate system is used.

Goal

Measure the rotational velocity of the galactic plane along different orbital radii, create a rotation curve for the milky way and infer the mass distribution from it.

Equipment

- LAB sky survey: https://www.astro.uni-bonn.de/hisurvey/AllSky_profiles/

12. Open the link to the LAB sky survey provided in the equipment section.

13. In the search box, make sure that only the LAB survey box is checked and that Dec is always at 0. Figure 6.4 demonstrates how the display should look like.

14. Choose the galactic longitude to be 10 degrees and obtain a plot of brightness temperature vs VLSR (Velocity relative to Local Standard of Rest). The horizontal axis has already been translated from frequency of the hydrogen spin-flip line into relative velocity, using Doppler shift.

15. Interpret the plot according to Fig. 6.3. Measure the velocity of the fastest gas cloud, v_{max} (note these will be max positive for longitudes 10 to 90 deg. and max negative for longitudes -10 to -90 deg.). You will note that the spectrum does not provide a clean maximum velocity. Think about the best way to measure maximum velocity, given that you may be observing several hydrogen clouds all in a line, moving at different velocities. **Record your findings in a table formatted like Table 6.1**.

16. Record the spectrum by taking a screenshot of the spectrum plot. If necessary, crop the image in your preferred image editor such that only the spectrum viewer is visible. Your screenshot should look something like Figure 6.5.

17. Perform the same analysis for the other longitudes listed in Table 6.1.

18. Assuming a distance from the Sun to the galactic center of 8.5 kpc and a circular velocity of 220 km/s at this radius, make a spreadsheet that follows the template in Table 6.1 with the results of your calculations using your measurements of v_{max} and equations above.

19. Plot the orbital velocity versus the distance from the center in kpc using your plotting program of choice. **Include this graph in your report.**

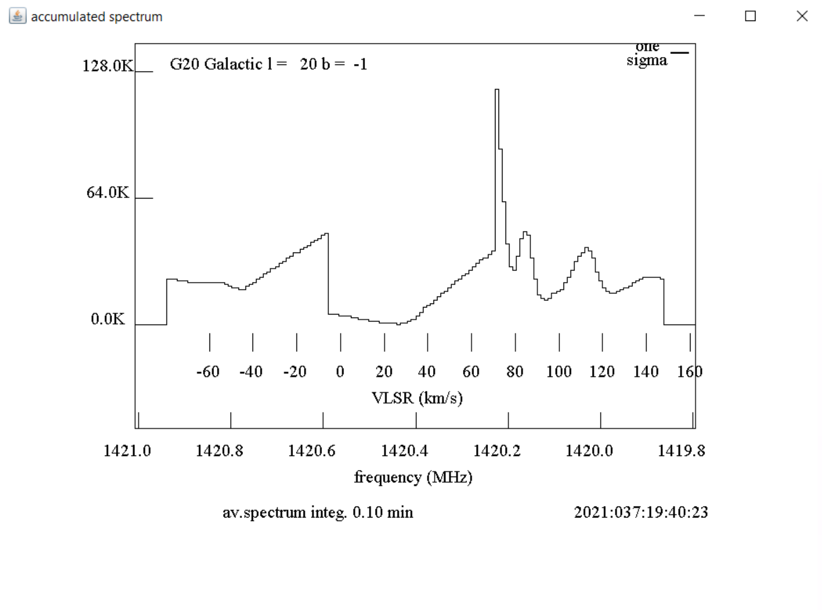


Figure 4.5: An example of a good capture of the spectrum plot. Note the different clear peaks that are seen.

20. From the rotation curve you obtain, how do you expect the matter to be distributed in the galaxy? **Record your answer.**

In our own solar system, the sun makes up most of the mass. As such, a safe assumption to make is that the mass in a given region of our galaxy is roughly proportional to its brightness (the brighter a region, the more stars and therefore the more mass there is). The brightness of our galaxy as a function of the radius roughly follows an exponential decay with radius,

$$I(r) = e^{-r/r_0}\,, \tag{4.4}$$

where $r_0 = 2.1$kpc is the characteristic length. To find the brightness within a given radius, we integrate that equation to find the total brightness within radius r, so that total brightness is proportional to:

$$I_{\text{within } r} = 1 + \left(\frac{r}{r_0} - 1\right) e^{-r/r_0} \tag{4.5}$$

21. In Desmos online graphing calculator, plot Equation 6.5.

22. Take a screenshot of the graph you obtain. Describe the behavior of the graph. How does brightness change as you move farther away from the center of the milky way?

23. Using the assumption above that mass density is roughly proportional to brightness, how would you expect mass to be distributed in the Milky way? How does it compare to what you found from the rotation curve.

24. Think of possible explanations for the discrepancy between the distribution of mass suggested by the brightness curve and the distribution suggested by the rotation curve. Think about the assumptions that went into the equations and how you came to your original conclusions for each curve. **Record your discussion.**

Galactic Longitude (degrees)	Tangential Distance r (kpc)	Maximum VLSR $v_{\max}$ (km/sec)	Line of Sight Solar Velocity V_{sun} (km/sec)	Circular Velocity v_c (km/sec)
10				
20				
30				
40				
50				
60				
70				
80				
90				
-10				
-20				
-30				
-40				
-50				
-60				
-70				
-80				
-90				

Table 4.1: Data Table for measurement of the rotation velocity of the Galaxy

4.5 Report Checklist

Include the following in your lab report. See Appendix B for formatting details. Each item below is worth 10 points.

1. Derivation and final equation for the speed of an orbiting body (Steps 2–3)
2. Graph of velocity vs radius with description of effect of different masses (Steps 4–5)
3. Graph and description of rotation curve for uniformm disk (Step 6)
4. Plot, mass equation, and description for first sample orbital system (Step 8)
5. Plot, mass equation, and description for second sample orbital system (Step 9)
6. Determination of rotation curve for the Solar System (Step 10)
7. Answers to Doppler shift conceptual questions (Step 11)
8. Plots of LAB spectra and table of maximum velocities of gas clouds at various longitudes (Steps 15–17)
9. Calculation of tangential distances, line of sight solar velocities, and circular velocities (Step 18)

10. Plot of rotation curve of the Milky Way Galaxy and interpretation of mass distribution (Step 19)
11. Plot and description of brightness vs radius of our galaxy (Step 21–22)
12. Determination of mass distribution as determined by brightness plot and comparison with determination from the rotation curve (Step 23)
13. Discussion of explanations for any differences found between the two mass distributions (Step 24)
14. Discuss the findings and reflect deeply on the quality and importance of the findings. This can be both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").
15. Write a 100–200 word paragraph reporting back from each of the four roles: facilitator, scribe, technician, skeptic. Where did you see each function happening during this lab, and where did you see gaps? What successes and challenges in group functioning did you have? What do you want to do differently next time?

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet (like Microsoft Excel, LibreOffice Calc, or Google Sheets), or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Lab Report Format

B.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

B.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

B.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.