Laboratory Manual

PHSC 12610 Black Holes

The University of Chicago

Winter 2019

Labs

LAB 1

Behavior of gravity waves in water (the ripple tank)

1.1 Introduction

The Michelson interferometer, named after University of Chicago professor Albert A. Michelson (Nobel prize in Physics 1907), is an extremely sensitive instrument capable of measuring incredibly tiny displacements. A modern version of the Michelson interferometer has been developed by The Laser Interferometer Gravitational-Wave Observatory (LIGO) experiment to detect changes in distance of 10^{-19} m (much less than the size of the nucleus of an atom!). This displacement is sensed between mirrors separated by 4 km (see Figure 1.1). There are two sites for LIGO — one in Hanford, WA and the other in Livingston, LA. The LIGO interferometer has recently detected gravitational waves for the first time (September 15, 2015); the first announced gravitational wave detection fits, with remarkable precision, the expected signal from the merging of two black holes, 29 and 36 solar masses, located 410 Mpc away. The reported signal and the comparison to the fitted model are shown in Figure 1.2.

The working principle of the Michelson interferometer is the interference of light. In this lab, you will first explore the concepts of interference with waves produced in water, in a device known as a ripple tank. In particular, in this first portion of the lab you will experimentally verify a relationship between wave frequency and wavelength, and then demonstrate constructive and destructive wave interference. You will then extend that understanding of interference to a wave geometry more appropriate to the second portion of the lab. The final measurement with the ripple tank will allow you to show that plane waves propagating through a slit behave as though the slit were a new source of waves, propagating radially (i.e. in a circular pattern).

Next week, you will measure interference phenomena with light, with a modern version of the famous double-slit experiment performed by Thomas Young in 1801. You will show that the interference properties of waves established in the first section of the lab apply to light as well, thus experimentally demonstrating that light behaves in a wavelike manner.

Having established the wavelike nature of light, you will then finally use a table-top Michelson interferometer to measure changes in distances smaller than a human hair (not quite LIGO sensitivity, but still pretty impressive!).

Figure 1.1: An aerial view of the two LIGO sites.

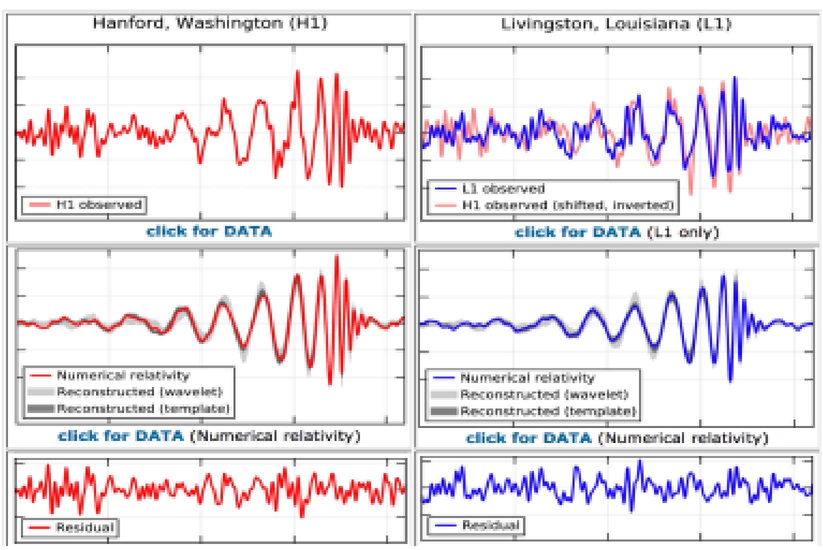

Figure 1.2: The left panels show the LIGO signal at the Hanford site (top) and the best-fit model (middle) and the residual of the model minus the data (bottom). The residuals are consistent with noise. The right panels show the same for the Livingston site, with the Hanford signal plotted in red in the top panel to demonstrate the similarity of the two measurements (as expected in the event of a true gravitational wave signal). This first LIGO detection of a gravitational wave event marks a significant transformation in our collective ability to measure and understand black holes, and since that first detection, more black hole merger events have been detected and reported.

1.2 Learning Goals

- Learn how to conduct an observational experiment, including collecting data and analyzing the data to find and describe a pattern quantitatively.
- Discover the relationship between frequency and wavelength of waves.
- Learn how to conduct a testing experiment, including identifying a hypothesis, designing an experiment, making a prediction, and comparing it to an experimental outcome.
- Gain familiarity with wave interference.

1.3 An aside: picking a project topic

By the end of lab today, ensure that you have chosen a topic for your presentation+paper project, and that it has been approved by your TA.

1.4 The Scientific Cycle[1]

One way of describing science is the process of incrementally improving a shared model of how our universe works. In different fields of science, different methods and cycles are used, so there is no "One True Scientific Method." One can still create a model for the process of science, and we describe here one such cycle (the hypothetico-deductive cycle), summarized in Figure 1.3.

In this cycle, there are three types of experiments, each one representing a different stage of the scientific effort. One stage, often started when encountering a novel phenomenon, is the **observational experiment**. This is an experiment that consists of deciding what to observe and how to observe it, collecting data, finding a pattern, and brainstorming possible explanations for what is observed (also called "hypotheses").

Once one has some trial explanations, one can test one or more of those with a **testing experiment**. Here, one designs a new experimental procedure and uses each hypothesis to predict what will happen. Then the prediction is compared to the procedure's outcome. If they are different, then the hypothesis is judged to be not a helpful explanation for that phenomenon. If they are the same, then it is still helpful. Throughout this stage, one may make various assumptions that would need to be validated, as they can effect the prediction or outcome.

Once a hypothesis has been tested enough for people to find it useful, then it can be applied to solve practical problems, or to determine properties of particular situations, in an "application experiment."

[1] adapted from [1]

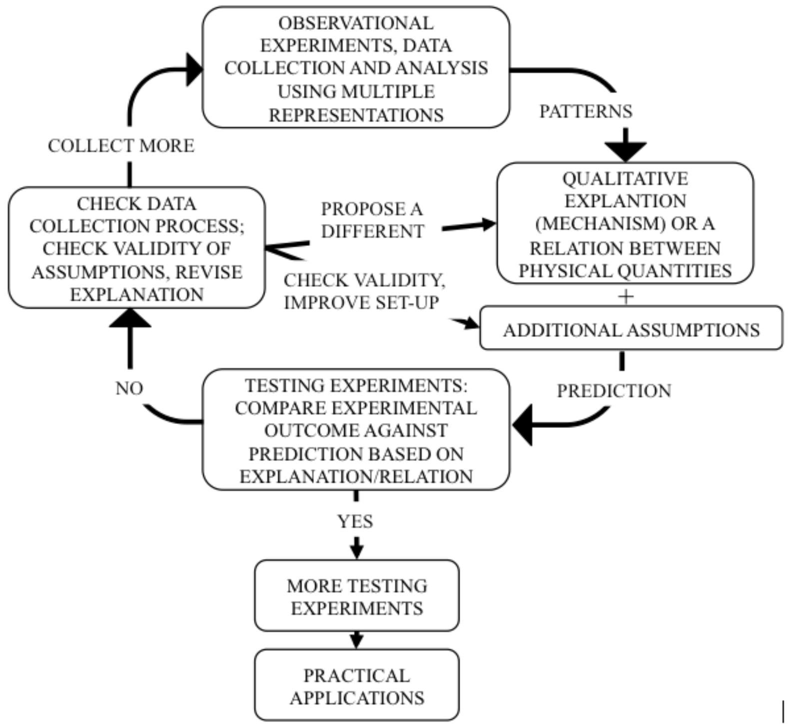

Figure 1.3: A model of the process some scientists go through to create knowledge.[2]

1.5 Experiment 1: Observation of frequency and wavelength

Goal: Observe gravity waves in a ripple tank and determine a mathematical relationship between frequency and wavelength.

Available equipment: ripple tank with strobe light and ripple generator, plane wave attachment, 2 dippers (narrow plastic rods), 1 short wall, 1 medium wall, 2 long walls for ripple tank, flashlights or desk lamps, digital camera (e.g. your smartphone), computer with ImageJ installed (can be your device), object of known size to be submerged

Caution: Flickering Lights! You will be using a stroboscopic light in this lab. Such light is known to trigger reactions in some individuals (e.g. photosensitive epilepsy). If you are worried that you may be sensitive to strobe light, speak to the TA and skip attending this part of the lab. In any case, avoid staring directly at the light.

Self-assessment: To help you improve your scientific abilities, we provide you with self-assessment rubrics. A rubric is a scoring system. Self-assessment is determining how well you performed a particular task. So, these self-assessment rubrics are designed to help you evaluate your performance while you are designing and performing your experiment.

The complete set of rubrics is available in Appendix B. In each lab, your report will be assessed using Rubric F, found in Table B.5, as well as 5 additional rubric rows listed in that lab. Each week, read through these and use them to evaluate your work as you design and perform the experiment. Your instructor will use the same rubrics to determine part of your grade for the lab.

Rubrics to focus on during this experiment: B7, B8, F1, F2. See Appendix B for details.

The ripple tank and generator

In this section you will explore interference phenomena using a ripple tank. The tank — 42.5 cm x 42.5 cm and 2.5 cm deep — is filled with water, and is equipped with a ripple generator. The generator uses voice coil actuators to produce the precise and quiet up-and-down motion of the rippler arms. Waves are generated in the tank by the moving dippers that touch the surface of the water. The generator also controls a light source that produces a bright, clear image of the wave patterns in the ripple tank. The light can be used as a steady source or as a strobe to 'freeze' the motion of the wave patterns (in this case the flashing light and the generator are driven with the same frequency). The ripple generator frequency ranges from 1.0 to 50 Hz adjustable in 0.1 Hz increments. You will work with frequencies in the range 16–32 Hz. A mirror placed below the tank and working in conjunction with a projection screen provide a magnified image of the wave patterns in the water; you will record patterns seen on this screen by photographing them with a digital camera. The ripple generator terminates in a bar with numerous clips in which you can place various "dippers".

Suggestions for your experiment

1. You may want to decide on roles for each group member. Example roles include Facilitator (ensures time and group focus are efficiently used), Scribe (ensures work is recorded), Technician (oversees apparatus assembly, usage), Skeptic (ensures group is questioning itself). Note that each role is responsible for ensuring that the thing happens, rather than necessarily doing it themselves. **Decide if you are using these roles, and if so, assign them and note them in the lab report.**

2. Ensure that every group member knows what the terms frequency and wavelength mean, in relation to waves. Use whatever means at your disposal to do this.

3. This is an "observational experiment." Review Rubric B (Table B.2) and discuss any unclear expectations with your group and the instructor. Note that your lab report will be graded, in part, on demonstration of Abilities B7 and B8.

4. Ensure that one of the ripple tank's ripple generator is set up with 1 dipper fixed in the center clip of the bar that extends from the box, and that the height of the generator is such that the dipper just touches the top of the water. You can make coarse adjustments by moving the generator along the support rod, and fine adjustments with the two red knobs on it.

5. Brainstorm different methods you could use to determine the relationship between wavelength and frequency. Feel free to play with the ripple tank as you do so, seeing what the frequency and amplitude knobs do. Notice that for different frequencies, different amplitudes produce the clearest image. Here are some things to consider:

   - Which variable will you control (and thus will be the independent variable) and which will you measure?
   - What is the range of the independent variable that you will use? How many different settings will you choose?
   - You will need to use several settings of the independent variable, and then plot the data in a graph, decide on what pattern you see, and give some justification for that pattern. You can use words like "proportional", "linear", "parabolic", "exponential", "logarithmic", and so on, if they fit. Ensure you use the mathematical definition of these.
   - How will you measure the wavelength?
     - Is it a more precise measurement if you measure several of them at once and divide to get a single wavelength?
     - The reflected image might magnify the ripple tank, so it can be helpful to place an object of known size in the tank, like a coin, so you can determine the correct scaling.

One way to take careful measurements of the wavelength is to take a picture of the projected tank, then use a program like ImageJ to measure the lengths you need. If you do so, one way to keep track of what settings go with what image is to mark a card with the settings and place it in view of the camera. See the section below on measuring lengths with ImageJ.

6. Decide on your measurement and analysis method and discuss it with an instructor before you begin. They will help increase the chances that your method will lead to successful results, or at least that the unhelpful path that you choose will take a short enough amount of time for you to change it when you discover it does not work. We want you to have productive failure that you have time to learn from.

7. Perform your experiment. Your lab report for this experiment should include:

    - A labeled sketch or photo of the setup, and a description of the experimental procedure (see Rubric F1).
    - A plot of wavelength vs. frequency (with the independent variable on the horizontal axis)
    - A description of the pattern found. This can be done with a line (straight or curved) showing the pattern you see (either drawn manually or using the curve fitting function of the plotting program, e.g. LibreOffice Calc or Microsoft Excel) and with words describing what you found. (B7)
    - An equation to represent the pattern. This can be taken from a curve fit or found by hand. Make sure there is some discussion of how well the equation agrees with the data, but you don't need to be very precise about it. (B8)
    - A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2)

Measuring lengths using ImageJ

ImageJ (`http://imagej.nih.gov/ij/download.html`), which is installed on the lab computers, is useful for measuring lengths in images. To do so, load your image, then follow these steps to calibrate the ruler — that is, to tell ImageJ how long something is in the image, so it knows how many pixels correspond to what length).

1. Start with an image that has an object in it that you know one of the lengths of (e.g. the length of side, or a diameter).

2. Open that image in ImageJ.

3. Select the icon with the straight line on it, and click and drag along the known length.

4. From the drop-down menu, select "Analyze" > "Set Scale...".

5. Set "Known distance" to the value of the known length.

6. Set "Unit of length" to the unit you are using, for example "mm" for millimeters.

7. Record the pixel scale given at the bottom of the box for future use.

8. Now when you use the straight line tool, it will give the length in physical units in ImageJ's toolbar.

1.6 Experiment 2: Testing the conditions for constructive interference

Goal: Test the hypothesis that constructive interference between two waves occurs at positions where the distance from each source differs by a half-integer number of wavelengths, or

$$\Delta d = (m + \frac{1}{2})\lambda \,, \tag{1.1}$$

where Δd is the "path length difference", λ is the wavelength, and m is any integer.

Available equation: Same as in the previous experiment.

Setup: Instead of 1 dipper, use two dippers mounted with 3 empty clips between them on the bar. Ask the TA for assistance in setting this up. Adjust so that the dippers are just resting in the surface of the water. Adjust the frequency and amplitude to get clear, sharp waves.

Rubric rows to be assessed in this experiment: C1, C4, C7, F1, F2. See Appendix B for details.

Testing this hypothesis

In general, one tests a hypothesis by using it to make a prediction about what will happen in a certain experimental procedure. With this hypothesis, it asserts a relationship between path length difference, wavelength, and constructive interference. But there are only certain points on the ripple tank image where it is easy to see constructive interference — the bright spots at the intersection of waves originating from both sources. For an example, see Figure 1.4.

In this case, it is easier to start by finding those locations, measuring the Δd, finding the wavelength for the given frequency using the relationship you found in Experiment 1, and solving for m. The hypothesis predicts that m should always be an integer. As a result, your experiment becomes this: find out how close the experimentally determined m's are to integers.

Brainstorm your experimental procedure, decide on it, discuss with your TA, then perform the experiment.

Your lab report for this experiment should include:

- A clear description of the hypothesis (see Rubric C1).
- A labeled sketch or photo of the setup, and a description of the experimental procedure (F1).
- A clear statement of the prediction that the hypothesis makes for this particular procedure (C4).
- A table of path lengths, path length differences, and measured m values.
- An analysis of how close the measured m values are to the prediction. Use some quantitative measure of this, but don't worry about being precise about uncertainties (C7).
- A judgment about the hypothesis. Is it supported, disproved, or undetermined? (C8, though not assessed this time)
- A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2).

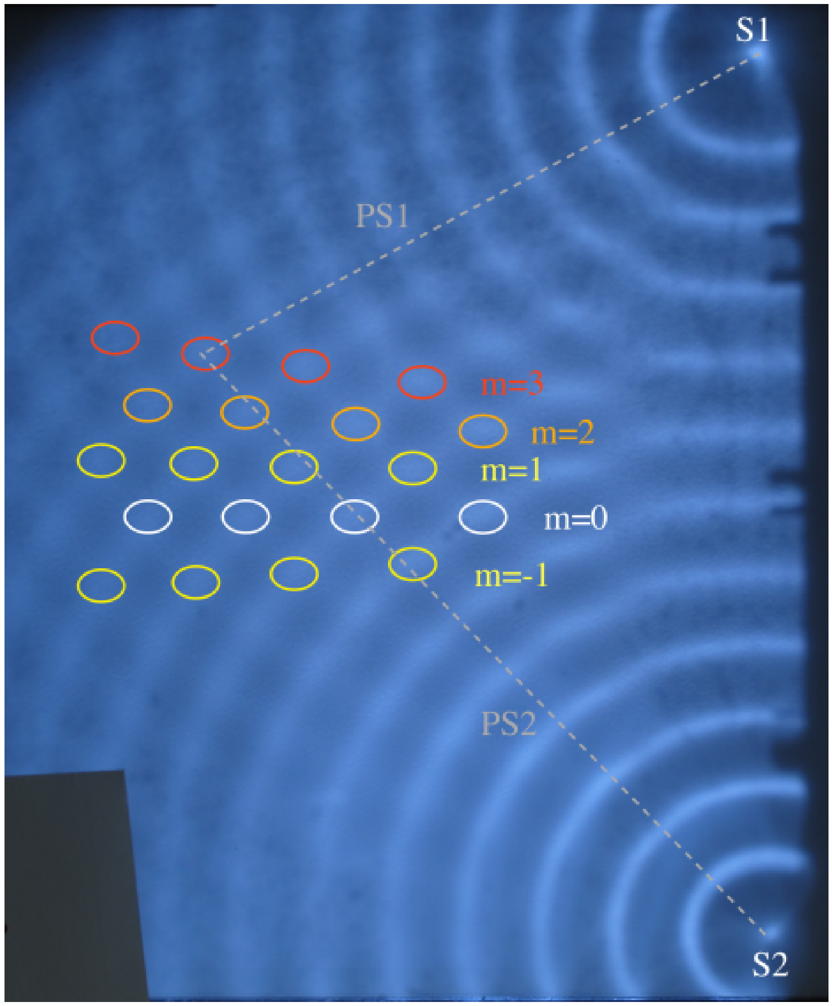

Figure 1.4: Example interference pattern for 2 dippers. The bright spots are circled. For a particular bright spot of constructive interference, the two path lengths PS1 and PS2 are drawn.

1.7 Experiment 3: Observing plane waves encountering narrow gaps

This experiment does not clearly follow the model of the scientific cycle, but is closest to an observational experiment. In next week's lab, you will investigate the properties of light traveling through small slits. Ripples in water are more obviously waves, so it is helpful to observe what happens here first.

Instead of dippers, remove them and position the bar so that it is resting in the water. This will produce straight line waves, or, in two dimensions, "plane waves". This is the same kind of waves we will use next week with light.

Adjust the amplitude and frequency, with the frequency in the range 20–25 Hz, until you see clear well-defined vertical parallel lines. Now, insert the two large "walls" in the tank, parallel to the rippler bar and perhaps 5cm away; allow a small (few mm) opening between the two wall sections, placed so that opening is vertically centered in the projected image. Adjust the amplitude upward until you see a clear wave pattern radiating from that opening. Take a picture. Repeat this with two apertures instead of one; do this by adding a smaller wall between the two larger sections, with all sections parallel to the rippler bar, and a small gap between each larger wall and the central smaller portion. Again, take a picture, adjusting amplitude as necessary to get well-defined waves.

The analysis of this will be done as individual homework.

1.8 Individual Homework

These questions are to be answered individually and your answers should be submitted under the Lab 1 Homework assignment on Canvas.

Both questions concern the last two situations recorded in the lab: the case of 2 walls (1 gap or "aperture") and the case with 3 walls (2 apertures).

1. What wave pattern do you see in the case of a single aperture? What do you see In the case of a double aperture? How do these patterns compare to the data you took using dippers on the rippler bar?

2. Is the wavelength of the pattern you observe consistent with the relationship between frequency and wavelength you measured with the dippers? Include any measurements and calculations you make in answering this question in your homework response, and be quantitative.

LAB 2

Behavior of electromagnetic waves in space (lasers and slits)

2.1 Introduction

In 1801, Thomas Young's "double-slit" experiment demonstrated the wave nature of light by showing that two coherent light sources produce interference patterns. You will perform a modern version of Young's experiment using a laser as light source. The laser illuminates two thin slits, each of width a separated by a distance d, which act as two coherent sources of light. This is analogous to what you have observed with water waves in the previous lab section, in which you saw a plane wave combined with an aperture (a slit) acting as a circular source of waves. An interference pattern appears on a viewing screen, placed at a distance L from the double slit, in the form of bright and dark regions corresponding to maxima and minima of interference. You will use the interference pattern to measure the wavelength λ of the laser, and show that the same framework of equations that is derived in the introduction to the previous lab holds for light too.

2.2 Experiment 1: Observing patterns made by 1 and by 2 slits

Goal: Describe the patterns made by a laser that is incident on 1 slit and on 2 slits, and the differences and similarities between them.

Available equipment: optical bench, viewing screen, blank white paper, PASCO "Multiple Slits" assembly, red laser with mounting clamp, support stand, optionally computer with ImageJ installed

> **Warning: Laser Hazard!** The power of our lasers is low enough that the normal human blink reflex is sufficient to protect against incidental eye exposure.
>
> That being said, the following rules reduce the risk of eye exposure to laser light:
>
> 1. Do not direct the laser beam into anyone's eye.
> 2. Be aware of the laser reflecting off of mirror-like surfaces and where that beam goes.
> 3. Turn off the laser when not in use.
> 4. Keep the laser pointing horizontally and near the plane of the table, while keep your eyes above that plane.

5. To determine whether the laser is on, put your hand or a light-colored object in front of the beam, rather than looking into the laser aperture.

Rubrics to be assessed for this section: F1, F2. See Appendix B for details.

Setup

Ensure the red laser is turned on and pointed at the slit assembly. Rotate the head of the slit assembly so that the active slit is the "Comparison" slit location with both a single and double slit furthest counter-clockwise. Adjust the laser so that the active slit is well illuminated by the laser spot. Note the laser should be pointed slightly upward if the laser head is 10cm off the table, and pointed so it hits the screen 5cm from the top edge, and so moving the slit assembly closer to the laser will move the spot on the slit assembly lower, and moving it further will move the spot higher. With the slits aligned with the laser, you will see light on the screen, but no longer a simple spot. Instead, you will see a vertical feature, the details of which depend on whether the laser is illuminating the single slit, or the double slit. Nudge the rail end back and forth to see the difference.

Include the following in your report:

1. A labeled photo or sketch of the experimental setup.
2. An image of each pattern from the first slit assembly setting, taken from the same camera location.
3. A written description of each pattern and how they are alike and different. Do you see same pattern in the double slit as you do in the single slit (in addition to another pattern)?
4. What happens to the double-slit image if the slit separation is wider, like in the second setting in the "Comparisons" section of the assembly?
5. How about when the slits are wider, like in the third setting?

2.3 Experiment 2: Testing the wave hypothesis

Goal: Determine whether light can be described as a wave. Note that if this is true, then light from a laser would be a plane wave.

Available equipment: Same as in Section 2.2, plus a green laser with mounting clamp

Rubrics to be assessed for this section: C4, C7, C8, G2, G4, F1, F2. See Appendix B for details.

Behavior of a plane wave incident on single and double slits

The following equation describes the location, y_m (measured relative to the center of the pattern), of the mth interference minimum (dark spot) seen on a screen when a plane wave is incident on a single slit.

$$y_m = \frac{m\lambda L}{a}\,, \tag{2.1}$$

where L is the distance from the slit to the screen, and a is the width of the slit.

For a double slit, the following equation describes the location y_n of the nth interference maximum (bright spot) seen on a screen when a plane wave is incident on a double slit.

$$y_n = \frac{n\lambda L}{d}\,, \tag{2.2}$$

where d is the distance between the two slits.

Suggestions for your experiment

- **REQUIRED:** Use both the green laser and the red laser for this experiment, and ensure that you keep the data (image and setup parameters) for use in the individual homework.
- For measuring the interference minima and maxima, you can do so by putting a paper on the screen and marking the locations directly, then measuring the marks with a ruler or with ImageJ. You could also take a picture of the pattern directly. Ensure that you take a reference photo with a known length on the screen, and take the image as face-on as possible, from the same location every time if you are taking multiple images.
- The green laser has a wavelength of 532 nm. You can assume that this value is exact, with zero uncertainty.
- The stated uncertainty in the slit size and separation according to the manufacturer (PASCO) is ± 0.005 mm for the slit width (a), and ± 0.01 mm for the slit spacing (d).
- If you use a value with an uncertainty in a calculation, if you want to use that value for comparison, you must propagate the uncertainty through to the final value. See Appendix A.2.
- To compare your outcomes to your predictions, get a value with uncertainty for each, then compare them using the t' test, described in Section A.3.

Items to include in your report

Relevant rubric rows from Appendix B are listed in parentheses.

1. Statement of the hypothesis (C1).
2. Description of the experimental setup and procedure (C2, F1).
3. The quantitative prediction that the hypothesis makes about what will happen during the experimental procedure (C4). Ensure that uncertainty is handled correctly (G2).
4. A report of the experimental outcome (results), neatly organized (G4).
5. Determination of whether / how much the prediction agrees with the outcome, comparing using uncertainties (C7, G2).
6. Judgment about the hypothesis — based on this experiment, does it lead you to support the hypothesis more or less, about how much (qualitative)? (C8)
7. A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2).

2.4 Individual homework

The tolerances in the slit manufacturing make a direct computation of the laser wavelength somewhat uncertain, as the uncertainties in the slit spacing are at best a few percent (0.01mm/0.5mm = 2%). Unfortunately, the red lasers we have in the lab could be quite a few different wavelengths, and we don't have a manufacturers record of the exact value. Diode lasers like this can be found online with "red" values of 633, **635**, 637, 638, 639, 640, 642, **650**, 653, **655**, **658**, **660**, **670**, and 680 nm (bolded values are more common — the laser is likely one of these). A 2% uncertainty in the slit spacing translates to a ±13nm uncertainty at 650nm, and so is useless for selecting the actual laser wavelength from the choices above.

However, we can do better. The ratio of the computed wavelengths for the red and green laser measurements of a given slit configuration (i.e. the $a = 0.04$ mm and $d = 0.50$ mm case, since you recorded data for both) is a number that doesn't include the slit manufacturing uncertainty (or for that matter any uncertainty in your measurement of the distance between the slit and the screen) because both numbers cancel when you compute the ratio. Thus, with a green laser of known wavelength (532nm) and that ratio you can compute the red laser wavelength with greater accuracy.

Use this method to determine the red laser's wavelength. What do you get? What, of the choices above, is the most likely actual wavelength for the red laser?

LAB 3

Using light waves to measure small distance changes (Michelson interferometer)

With the basic properties of waves and wave interference established (via the ripple tank) and the same behavior demonstrated in light (via the laser-based modern version of Young's double slit experiment) we are now finally ready to look at a Michelson interferometer. This technology is the basis of the LIGO experiment. You may want to refer back to the introduction of Lab 1 to remind yourself of some details. LIGO itself is a large experiment that has been constructed over several decades of work and technology development, and so is many orders of magnitude more precise and sensitive than what we can do in an hour on a lab bench. Nevertheless, the basic principles are the same.

Figure 3.1 shows a diagram of a Michelson interferometer. A beam of light from the laser source of wavelength λ strikes the beam-splitter. The beam-splitter B is designed to reflect 50% of the incident light and transmit the other 50%. The incident beam therefore splits into two beams; one beam is reflected toward mirror M_1, the other is transmitted toward mirror M_2. M_1 and M_2 reflect the beams back toward the beam-splitter. Half the light from M_1 is transmitted through the beam-splitter to the viewing screen and half the light from M_2 is reflected by the beam-splitter to the viewing screen.

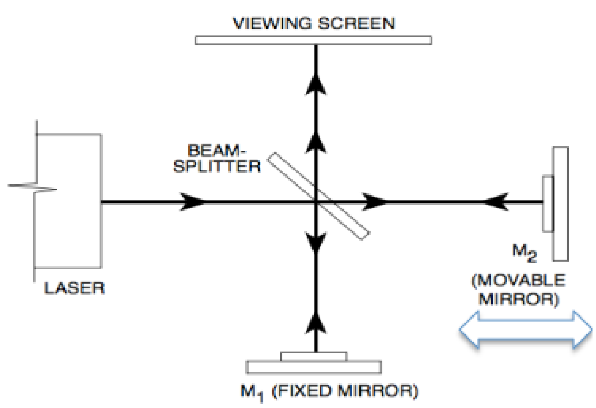

Figure 3.1: Schematic of a Michelson interferometer.

In this way the original beam of light splits, and portions of the resulting beams are brought back together. The beams are from the same source and their phases are hence highly correlated. When mirror M_2 is moved (closer to or further from the laser source) the *difference* in the path length of the light beams ($B\mathrm{M}_1 - B\mathrm{M}_2$) changes, resulting in changed in interference fringes. For a clear visualization of the effect, a lens placed between the laser source and the beam-splitter spreads out the beam. An interference pattern of dark and bright rings, or *fringes*, is seen on the viewing screen. The rings are generated by interference of different portions of the laser beam, expanded to easy visibility by the lens.

3.1 Setup and Alignment of the interferometer

Before you do an experiment with the interferometer, you'll need to ensure that it is aligned and an interference pattern (a set of concentric alternating light and dark rings) is clearly seen on the viewing screen when the laser is turned on. If that's true, then you can skip ahead to the next section.

Our laboratory setup is shown in Figure 3.2.

Warning: Laser Hazard! Lasers can cause temporary and permanent damage to eyes when exposed directly or through reflective surfaces.

The following rules reduce the risk of eye exposure to laser light:

1. Do not direct the laser beam into anyone's eye.
2. Be aware of the laser reflecting off of mirror-like surfaces and where that beam goes.
3. Turn off the laser when not in use.
4. Keep the laser pointing horizontally and near the plane of the table, while keep your eyes above that plane.
5. To determine whether the laser is on, put your hand or a light-colored object in front of the beam, rather than looking into the laser aperture.

1. The interferometer itself (this is part that has the optics) should be bolted to an optical rail at one end, with the beamsplitter mirror facing the long end of the rail. Do so, if this isn't already in place.
2. An aluminum block, with upward facing magnets, should also be bolted into the rail near the other end.
3. A steel plate, with an upturned edge, should also be bolted to the rail, with the flat edge tight against edge of the interferometer.
4. A 3/4" thick steel block, with two V-shaped grooves (one large and one small) should be placed on top of the aluminum block with magnets, with the V-shaped grooves facing upward; the magnets will keep the steel block in place.
5. To begin, orient the block so the V-shaped grooves are aligned with the long axis of the rail, and the larger groove is toward the side of the rail opposite from the position of M_1 in the interferometer. The grooves are mount points for lasers, of two different barrel widths.
6. Place a laser in one of the V-shaped grooves, pointed toward the interferometer, and turn it on. If necessary, you may secure the laser to the block using an elastic band or similar, taking advantage of the small grooves on the underside of the block that allow easy passage of a securing band.

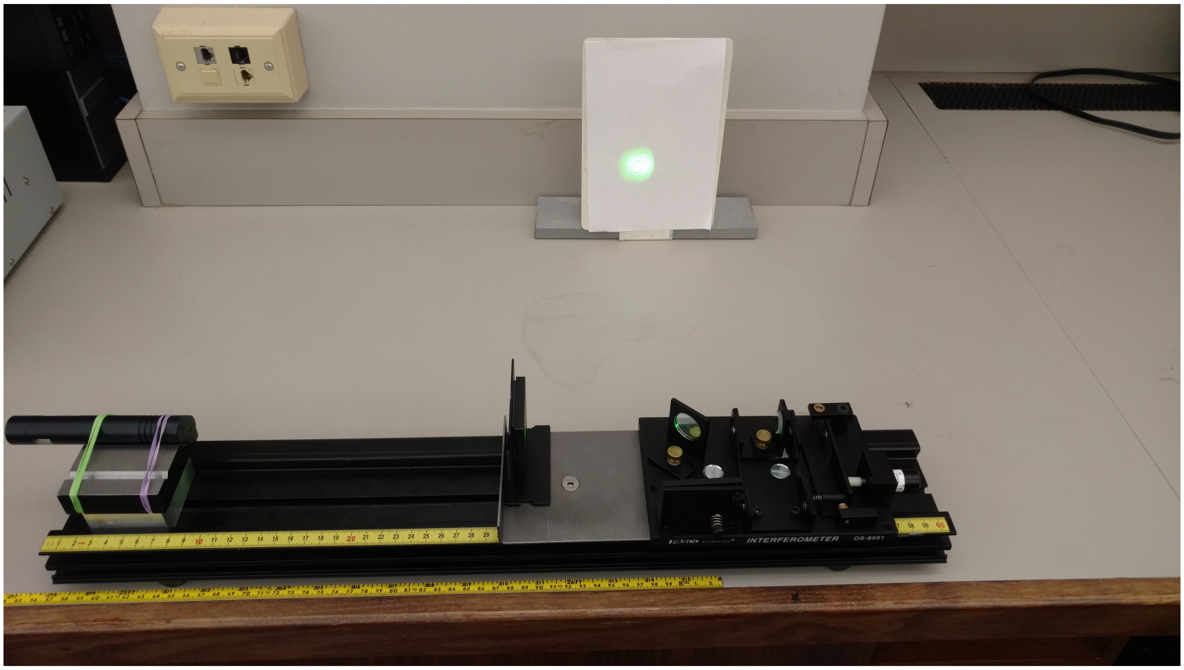

Figure 3.2: Our particular classroom setup, fully assembled and aligned, showing an interference pattern on the viewing screen.

7. Place a viewing screen so that it is opposite M_1, or use a convenient light-colored wall.
8. Loosen the thumbscrew that holds the beam-splitter and rotate the beam-splitter so it is out of the beam path of the laser as shown in Figure 3.3.
9. Align the steel block holding the laser so that the beam hits M_2 as well-centered as possible; you can slide the steel block, rotate it (the magnets hold it in place but allow freedom of movement), and place paper in the groove under the laser to adjust the height or angle. Your goal should be to have the laser beam parallel to the long axis of the optical rail, and centered on M_2.
10. The reflected beam should return back to the laser head. (The reflected beam need not be — and likely won't be — at the same height as the incident beam, but it should return along the same path when viewed from exactly above. Hold your hand or piece of paper near the laser head — without blocking the outgoing beam — to see where the return beam is going.) If the return beam is not going where you want, you may loosen the thumbscrew that holds M_2 and adjust the rotation of M_2 so the laser beam is reflected directly back toward the laser head. Once satisfied with the alignment, hold M_2 in position and tighten the thumbscrew.
11. Adjust the alignment screws on the mount for the mirror M_2, so that the mount plate does not appear tilted (see Figure 3.4 for the location of these screws). When viewed from above there is gap between the plate holding the mirror and a second plate behind it. Adjust the screws so the plates appear parallel.
12. Rotate the beam-splitter so its surface is at an angle approximately 45 with the incident beam from the laser (see Figure 3.4). You will see two sets of laser spots on the viewing screen, corresponding to the two paths that the beam takes in reaching the screen. (Each path results in more than one laser spot because of multiple reflections within the beam- splitter.) Adjust the beam-splitter so the two sets of laser spots are as close as possible, then tighten the thumbscrew to secure the beam-splitter.

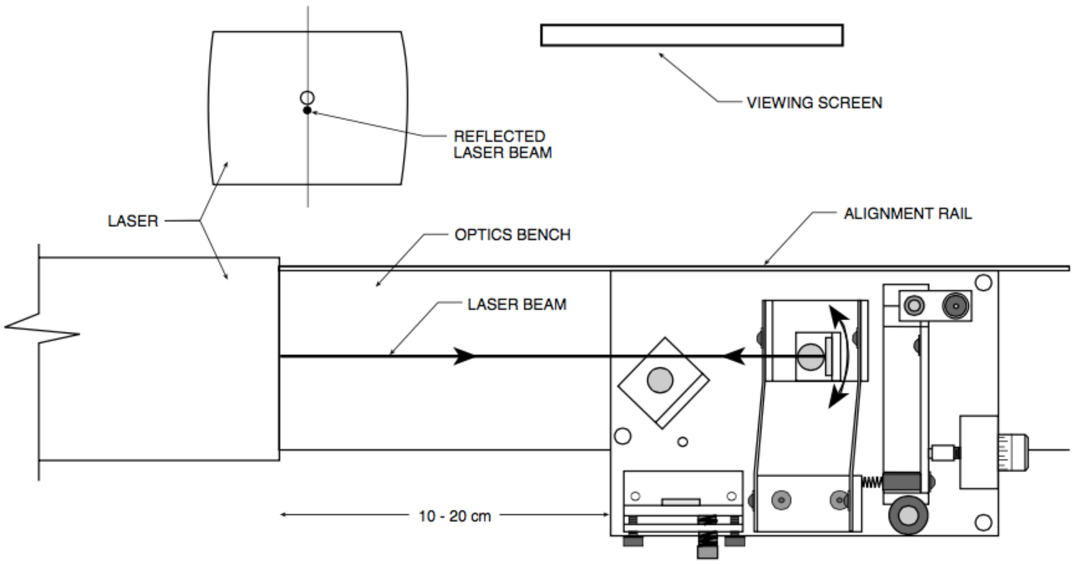

Figure 3.3: Adjusting the M_1 mirror.

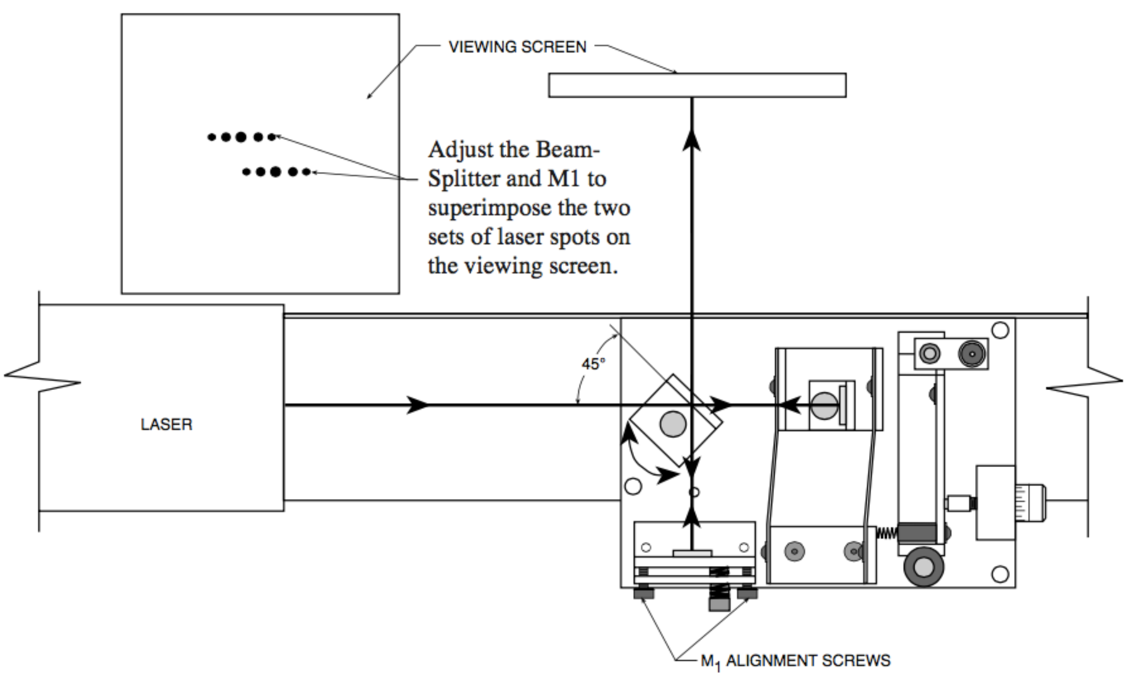

Figure 3.4: Aligning the laser spots.

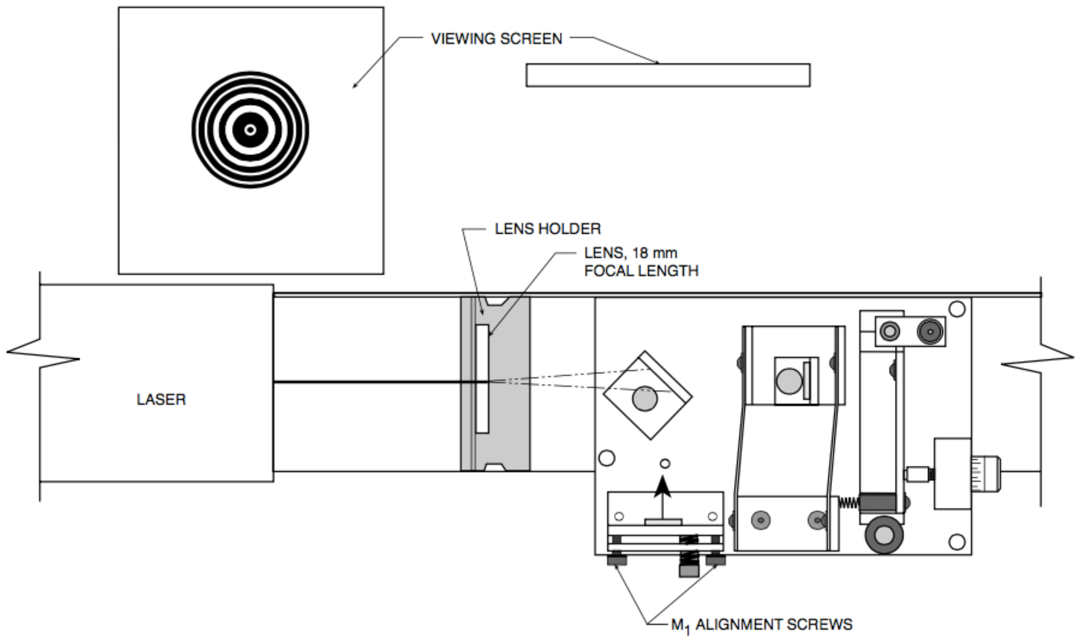

Figure 3.5: Positioning the lens, and fine alignment of M_1.

13. Now, using the alignment screws, adjust the angle of M_1 until the two sets of laser spots are superimposed on the viewing screen (the two brightest spots must be superimposed).

14. Place the 18 mm focal length lens on the optical bench on the steel plate between the laser mount and the interferometer (see Figure 3.5 for setup and resulting desired pattern). The lens is in a holder that is magnetically coupled to a base; align one long edge of the base along the upturned edge of the steel plate. The lens should be about 10cm from the beamsplitter. Adjust the position of the lens on the holder so the light from the laser, now spread out by the lens, strikes the center of the beam-splitter. Move the lens vertically by sliding the lens holder vertically against the base (the magnets again allow freedom of movement here. The simplest way to move lens horizontally is to just slide the base lone the edge of the steel plate below. You should see an illuminated oval (or at least a partially illuminated oval) of laser light on the viewing screen. Adjust the lens position until the oval is as uniformly illuminated as you can achieve. Now, if you have performed the alignment correctly, you will see not just an illuminated oval, but a interference pattern of concentric rings on the viewing screen. If the alignment is not just right, the center of the fringe pattern may not be visible on the screen. Adjust the alignment screws on M_1 very slowly as needed to center the pattern. *NOTE: aligning the interferometer so that you get fringes can be fiddly...if necessary, try a few times, and seek help from your TA if you cannot make it work.*

3.2 Relating wavelength to distance changes

By moving the mirror M_2, the path length of one of the beams can be varied. Since the beam traverses the path between M_2 and the beam-splitter twice, moving M_2 1/4 wavelength nearer to the beam-splitter will reduce the optical path of that beam by 1/2 wavelength. The interference pattern will change; the radii of the maxima will be reduced so they now occupy the position of the former minima. If M_2 is moved an additional 1/4 wavelength closer to the beam-splitter, the radii of the maxima will again be reduced so maxima and minima trade positions. However, this new arrangement will be indistinguishable from the original pattern. So when moving the position of M_2, you will observe the fringes "moving" reproducing the original pattern.

The movement distance d of the mirror M_2 and the corresponding number of times m the fringe pattern is restored to its original state are related by

$$m\lambda = 2d\,, \tag{3.1}$$

where λ is the wavelength of the incident light. Thus, very small displacements d can be measured by counting the number m. Conversely, the wavelength of the light can be accurately determined if d is known. In your interferometer, a knob with a micrometer scale can be used to move M_2. M_2 is help on a lever spring arm that is anchored to the baseplate of the interferometer at another location. The micrometer knob pushes on another arm that applies tension to a strap that is coupled to the lever arm holding M_2.

3.3 Experiment 1: determining the wavelength of the laser

Goal: Determine the wavelength of a laser.

Rubric rows to be assessed: D1, D4, F1, F2, G2, G4, G5.

Available equipment: Michelson interferometer mounted on optical rail, laser

Since this is such a sensitive measurement, we provide a measurement procedure for you. In order to determine the wavelength, you'll measure the number of fringes moved and the distance the mirror moved and use Equation 3.1 to calculate the wavelength of the laser.

Procedure

1. Adjust the micrometer knob so the lever arm is approximately parallel with the short edge of the interferometer baseplate. In this position the relationship between knob rotation and mirror movement is most nearly linear.

2. Turn the micrometer knob one full turn counterclockwise. Continue turning counterclockwise until the zero on the knob is aligned with the index mark. *(NOTE: Whenever you reverse the direction in which you turn the micrometer knob, there is a small amount of give before the mirror begins to move. This is called mechanical backlash, and is present in any mechanical system involving reversals in direction of movement. By beginning with a full counterclockwise turn, and then turning only counterclockwise when counting fringes, you can eliminate backlash in your measurement.)*

3. Place a sheet of paper on the viewing screen, secure it with tape, and make a reference mark on the paper between two of the fringes. This will help you in keeping count of the fringes.

4. Now turn counterclockwise the knob until you have counted about 40 movements of the fringes.

5. Record the measurement on the knob as distance d and record the number of fringe movements m.

6. Repeat Steps 1–5 2 more times, for a total of 3 measurements.

Use your findings to determine the wavelength of the laser, including an estimate of the uncertainty in your reported value. Include the following in your report:

1. A statement of the problem you are solving (D1).

2. A clear, concise description of the experimental setup and procedure (F1).

3. A table of the data that you took (G4).

4. A description of your analysis that led you to find the wavelength (G5).

5. A description, with calculations shown, of your determination of the uncertainty in the wavelength (G2).
6. A final judgment of what your team thinks the wavelength of the laser is, based on your experimental results, including uncertainty (D4).
7. A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2).

3.4 Experiment 2: Measuring distance changes

In your individual homework, you will determine distance changes in one of the arms of the interferometer, not caused by gravitational waves, like in LIGO, but by thermal expansion and the slight bending of the baseplate of the apparatus. During lab, take the following data, both without turning the knob to move the mirror:

1. The clear strap that pulls the mirror back and forth will expand and contract with heating and cooling (like most solids). Measure the number of fringe movements that happen when you hold your finger very close to the strap, within a few millimeters, for 20–30 seconds.
2. The baseplate is very sturdy, yet still bends when uneven pressure is applied, even if imperceptible to our senses. Measure the number of fringe movements that happen when you press lightly on the baseplate between one of the mirrors and the beam-splitter.

3.5 Individual homework

For this homework, use the data taken during Experiment 2 to determine the path length changes in each case (include calculation of uncertainty). Report on if this is surprising to you or not, either that the distance change is as large or small as it is, or the fact that you can measure such a small distance change.

LAB 4

Black hole at the Galactic center? (stellar orbits)

4.1 "Observing" Black Holes

Black Holes are robustly predicted by Einstein's theory of General Relativity, the current state-of-the-art in mathematical explanations of gravitational phenomenon. It has also been demonstrated theoretically that such extreme objects naturally arise in the late-stages of stellar evolution for the most massive stars. However, they are difficult to observe for a simple reason: by their very nature (as suggested by their name), they do not emit light. Nonetheless, their presence can be inferred indirectly through their gravitational effect on other objects. Astronomers and physicists have discovered so many independent and corroborating lines of evidence of this type detected that the existence of black holes is now considered a well-established scientific fact.

The detection of gravitational waves by the Laser Interferometer Gravitational-Wave Observatory (LIGO) in 2017 is arguably the most stunning and conclusive evidence of this to date. In a beautiful demonstration of the validity of General Relativity, the perturbations in space and time (Gravitational Waves) characteristically generated by the coalescence of a binary black hole pair were detected on Earth after having traveled at the speed of light from their source roughly one billion light years away. The clarity of this signal and its agreement with predictions have all but conclusively determined the existence of black holes.

However, less direct (but nonetheless extremely convincing) evidence for the existence of black holes had been well known in observational astronomy for some time. In this lab, you will explore one of the most striking examples of these observational signatures: the orbits of stars around Sag A*, the radio source that is collocated with the 'dark attractor' at the center of our galaxy that is almost certainly a Super-Massive Black Hole (SMBH). Astronomers now know that almost all galaxies have such behemoths at their centers, and that they likely play a fundamental role in galaxy evolution. The discovery of one at the center of the Milky Way (roughly 8 kpc distant) was an important step in the development of this SMBH paradigm. In this lab, you will partially reproduce a simplified version of the analysis that led astronomers to this exciting conclusion.

Rubric Rows to be assessed: C4 and C5 (for Kepler's 2nd Law); D8, D9, and G1 (for Kepler's 3rd Law mass estimation); F1 and F2 (for all parts)

4.2 Newtonian Dynamics and Orbital Dynamics Basics

The trajectories of objects moving under the influence of gravity are generally called *orbits*. Newton's Law of Gravity, while not an adequate description for extreme gravitational fields, is nonetheless a good approximation generally and in the specific cases of orbits we'll be examining. It is given by

$$F_{\rm grav} = G\frac{m_1\,m_2}{r^2}\,, \tag{4.1}$$

where $F_{\rm grav}$ is the force of gravity between any two objects of mass m_1 and m_2 a distance r from each other. G is Newton's Gravitational Constant, whose value in CGS (Centimeters Grams Seconds) units is $6.67\times10^{-8}\,{\rm cm^3\,g^{-1}\,s^{-2}}$. The gravitational force is attractive: it pulls objects together. This, coupled with Newton's Second Law of Motion relating the force acted upon an object F and its acceleration a,

$$F = ma\,, \tag{4.2}$$

tells us that *the acceleration of an object due to gravity will be greater in stronger gravitational fields*. These two fundamental physical laws can be used to derive the properties of Newtonian gravitational orbital motion. We won't do so here, but instead just cover some of the key implications, concisely stated by Kepler's Laws of Orbital Motion. These Laws, which are concise mathematical descriptions of how the planets move in the Solar System, preceded Newton's work by nearly a century, and were based solely on observational data. Newton's work on gravity and motion explained, at a deeper level, why Kepler's Laws are as they are.

Kepler's Laws

Kepler's First Law

Kepler's First Law states that **A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse.** This is true generally for any mass orbiting a much more massive object. An *ellipse* is a generalization of a circle, allowing for the circle to be stretched along a certain direction. See Figure 4.1 for details.

Kepler's Second Law

The second law states that **a line connecting a planet to the Sun sweeps out equal areas in equal time intervals**. This concept is demonstrated in Figure 4.2. As a consequence of this, when a planet is closer to the sun, it orbits with a faster velocity. Furthermore, planets that have more circular orbits will have more uniform velocities over the course of their orbit than those with very eccentric orbits.

Kepler's Third Law

Kepler's Third law addresses the path of an object m as it orbits a much more massive object of mass M. Specifically, it relates the orbital period P (the time it takes for one complete orbit to occur) to the semi-major axis a of the ellipse according to

$$P^2 = \frac{4\pi^2}{GM}a^3\,, \tag{4.3}$$

where other objects are also considered to be not affecting the orbit of m. **This is the equation you will be using in the central calculation of the lab.**

An important qualification to be made about Kepler's Laws is that they apply only to two-body systems. Kepler's Third law breaks down when you have more than 2 orbiting objects in a system. However, they are nonetheless a very good approximation for the orbits of small masses around a much larger mass, in which case the gravitational force of the massive object dominates over the intra-small-object interactions, and thus each smaller body approximately behaves independently from the other small objects. And so the motion of each small object, to a good approximation, can be modeled by Kepler's Laws.

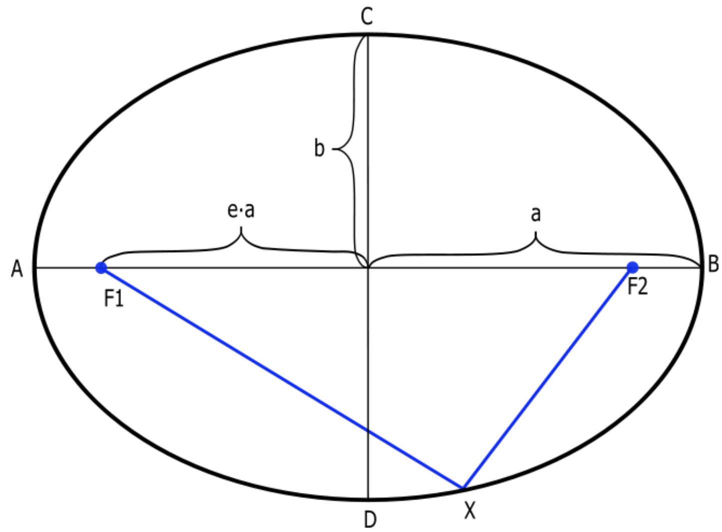

Figure 4.1: The geometry of an ellipse: a is the semi-major axis of the ellipse, F1 and F2 are each a focus of the ellipse, b is the semi-minor axis, and e is the eccentricity. The eccentricity describes the extent to which the ellipse is oblong: an ellipse with $e = 0$ is just a circle. The foci are defined such that the distance from F1 to X, added to the distance from F2 to X, is the same no matter where X is located on the ellipse. For a circle, the foci coincide at the center. The Newtonian generalization of Kepler's First Law tells us that a small mass will orbit a much larger mass on an ellipse, and the larger mass will be located at one of the foci.

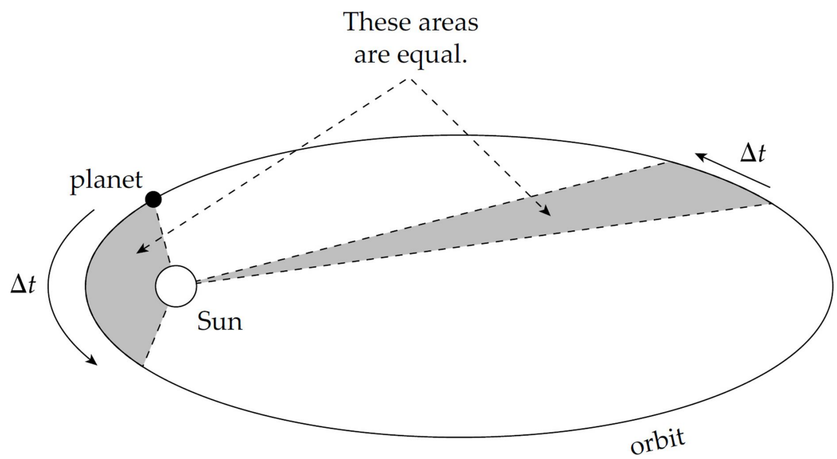

Figure 4.2: Illustration of Kepler's Second Law. The shaded regions are of equal area, so by Kepler's Second Law, the planet passes through each region of the orbit in the same amount of time. Since the region of the orbit where the planet is closest to the Sun is a much further distance, and speed is change in position over change in time, the planet will move with a much faster average speed than at the other portion of the orbit, far away from the sun.

Escape speed

Another important concept in orbital dynamics — which comes from the general Newtonian understanding of orbits — is that of the escape speed. This is the minimum speed that an object must reach to break out of its orbit at a certain radius from the central massive object. The escape speed v_{escape} at a distance r from the center of a mass M is given by

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}\,. \tag{4.4}$$

An orbiting object that achieves a speed greater than this is unbound to the object it is orbiting, and will escape its gravitational pull, flying out of the system entirely.

4.3 General Relativity

Despite the great success of Newtonian dynamics in explaining a wide variety of the celestial motions that we observe, this picture of gravity is incomplete and inaccurate in the most extreme regimes, where General Relativity is required. A fully self-consistent description of relativity is well beyond the scope of these notes (or this course), but the fundamental conceptual premise is easily stated: gravity is not a magical "attraction at a distance" between objects with mass, but rather the result of curved space-time, which is an effect of all mass. A helpful analogy is that of a sheet of fabric being depressed by a massive object (see Figure 4.3). Other objects placed on this depressed fabric will begin orbiting around the massive object.

For the purpose of this lab, the most important consequence of this paradigm shift is the fact that gravity thus affects everything, including even massless objects like light, in exactly the same way that it does massive objects, since they also travel through space-time. Thus, there might exist objects with such extremely strong gravitational fields (and thus extremely contorted spacetimes) that even light cannot escape. Since Special Relativity (Einstein's original theory of relativity, which he later built off

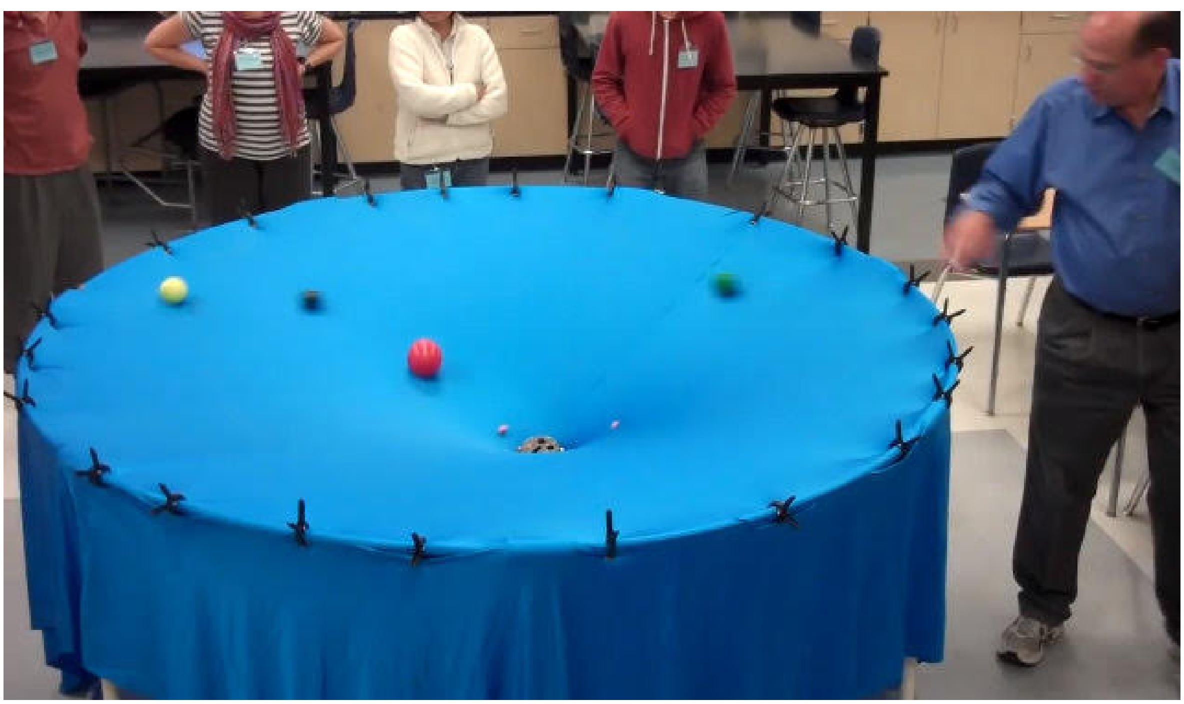

Figure 4.3: General Relativity Analogy.

of to develop General Relativity) tells us that light sets the cosmic speed limit, nothing else can escape either. These are termed Black Holes. We should note that, while we are assessing observational evidence for a black-hole in this lab, we will nonetheless use the Newtonian gravitational framework to understand our observations, sine the gravitational fields experienced by the stars we are observing are — to an excellent approximation — well-described by Newtonian gravity.

4.4 Schwarzschild Radii

One can extend Newtonian dynamical arguments to estimate the relevant scales for relativistic objects. For example, in the context of Newtonian Gravitation, one can think of a Black Hole as an object that has an escape velocity greater than the speed of light. Thus one can substitute the speed of light into the equation for the escape speed (Equation 4.4) to estimate the radius of a Black Hole,

$$R_{\text{Schwarzschild}} = \frac{2GM}{c^2} \tag{4.5}$$

where $c = 2.998 \times 10^{10}$ cm/s is the speed of light. $R_{\text{Schwarzschild}}$ is called the Schwarzschild Radius, and is actually the correct value predicted by General Relativity for a certain class of black holes, so it's a very good estimate. Any object dense enough to be contained inside of its Schwarzschild radius must be a black hole.

4.5 Lab Tasks

For each of the following objects, a) calculate the Schwarzschild radius for that object's mass, b) list an everyday object of comparable size and c) find the ratio of the Schwarzschild to physical radius of the object: (hint: do this in excel so you don't have to repeat the calculation ad museum)

1. One of your group members.
2. the Earth.
3. the Sun.
4. the Solar System.
5. The Milky Way Galaxy.

4.6 Orbit Simulator and 3D model

First we will explore some interactive animations of orbital motion, then a three-dimensional rendering of the real scientific data that we will analyze in the second part of this lab.

4.7 Lab Tasks

1. Go to
   https://www.windows2universe.org/physical_science/physics/mechanics/orbit/orbit_shape_interactive.html. This website allows one to adjust the shape of an orbit for a hypothetical planet in the solar system.

   a) Set the semi-major axis at 1AU (the radius of Earth's nearly-circular orbit), and the eccentricity to it's maximum value (0.9). By observing the motion of the planet with respect to Earth, verify (qualitatively, no need for numerical calculations here) each of Kepler's Three Laws.

b) Based on the orbital dynamics your observe, what simplifying approximations can you tell are being made in the calculation that goes into this animation? Particularly notice when your test planet comes close to another planet in its orbit. Do their orbits remain the same? Should they?

2. Now go to www.astro.ucla.edu/~ghezgroup/gc/animations.html, the website for the UCLA Galactic Center Group, where you will find a number of informative graphics that relate to the content of this lab. On the bottom-right under the section "3D Movie of Stellar Orbits in the Central Parsec" is a volume-rendered video of stars orbiting Sag A* — the radio and X- ray source at the center of the galaxy, the mass of which you will be measuring in the next section.

3. Expand the video to full-screen and pay attention to one or two of the stellar orbits, which are show in full by the video. Pause the video at several points during the video. How do the 2D shapes of the orbits on the image change as the camera angle moves? How does the inclination of the orbital plane in our field-of-view modify the observed orbital shape? How might this confuse the determination of orbital parameters? How can we disentangle this? (hint: think about Kepler's Laws and the location of the foci). What kind of errors in our measurement will result from an assumption that we are looking at an orbit directly from above? Will this cause the inferred mass to be over- or under-estimated? (See Scientific Ability Rubric Row D9 in Table B.4).

4. As well, consider the spatial orientation of the orbits here with reference to those of the solar system, considered in the previous task. Is there any coherent organization of stellar orbits around the black hole in this video? How does this compare to the orientation of planets in the solar system? What might be a physical reason for this difference (hint: think about their respective formation mechanisms).

4.8 Sag A* Mass Estimation

Now you will roughly measure the mass of Sag A* by extracting the positions of several stars orbiting the galactic center from a published YouTube video that illustrates the stellar dynamics observed by the UCLA Galactic Center research group. Their data comes from the largest research telescopes on the planet, observing the Galactic Center using sophisticated imaging cameras that give images even sharper than those produced by the Hubble Space Telescope. We will "observe" the video of their measurements, to illustrate the process of data → measurement → analysis → conclusion (a mass!).

Lab Tasks

1. Go to https://www.youtube.com/watch?v=7vcSKbXnLJA. This movie shows observations of Sag A* and the surrounding stellar cluster taken over more than a decade. Watch the video, and record your impressions in light of your previous activities on orbital dynamics and the introductory information. What is going on here? How might this be used as evidence for the existence of a SMBH at the galactic center? What other corroborating data would you want/need to substantiate that claim?

2. After watching several times and recording your initial impressions, you will now take some crude "measurements" of the data in the video. You will then use these to both verify Kepler's Second Law and make an order-of-magnitude estimate of Sag A*'s mass using Kepler's Third Law.

   Data acquisition:

   a) Restart the video and take a screen-grab of the first frame using the PrintScreen function (you will be doing this several times so make a folder in which to save your screenshots).

frame	d (arcseconds)	θ (radians)	$\Delta\theta$ (radians)	A (arcseconds^2)
1				
2				
3				
4				
5				
6				
7				
8				
9				

Table 4.1: Data table for one stellar orbit. Note that you will not have any calculations for the gray cells for frame 1, since there is no previous frame to compare to.

b) Let the animation advance by one second (so that the stars have begun moving in their orbits), pause, and capture another screen image. Do this for each second of the animation, giving you 9 separate images of the stars in different positions on their orbits.

c) For Kepler's Second Law, you will need to measure several areas traced out by an orbit in the same time duration. Open the first frame in the ImageJ software. Note the white arrow on the left of the screen indicating the angular scale of the image. Notice that Kepler's law describes areas, while we are limited, at present, to angular areas, since we are not including how far away the stars are to us. This approximation should be just fine for testing the law, as long as the angle is small.

d) On the menu tab click the straight line icon located at the 5th position to the left. Click one end of the arrow and drag the line from one end of the angular scale to the other. Now go to the Analyze dropdown menu and select Set Scale. Set the known distance to be the distance of the scale, 0.1 arcseconds. (Look up how to convert from arcseconds to degrees, and then to radians).

e) Once the scale is set, select the straight line icon again and drag the line from the star symbol (representing Sag A*) to the orbiting star you want to measure. The measure hotkey is "m." This will now give you the angular distance from Sag A* to the orbiting star in arcseconds. It will also give you an angle value that will allow you to measure the angular rotation of the star in its orbit. Record these values for both S0-2 and S0-37 in the first 2 columns of a data table set up like Table 4.1, one table for each of these stars, one row for every frame you captured.

3. Kepler's Laws:

a) Kepler's Second Law: This data can now be used to test Kepler's Second Law. The area covered by a line connecting the planet and Sag A* between measurements i and $i-1$ can be given by

$$A_i = \pi \left(\frac{d_i + d_{i-1}}{2}\right)^2 \left(\frac{\theta_i - \theta_{i-1}}{2\pi}\right), \tag{4.6}$$

which approximates the swept area as the sector of a circle. Add this calculation for each segment and both stars to the data table. Does Kepler's Second Law hold? Discuss any discrepancies and how you might account for them (Rubric Rows C4, C7, C8).

b) Kepler's Third Law: Now you will use Kepler's Third Law, given in the introduction, to calculate the central mass around which these objects are orbiting. Go back to the final frame of the video and use the traced orbits to estimate the semi-major axis for both S0-2 and S0-37. This estimated length is an angular length in arcseconds and should be converted to a physical length in centimeters. First convert the arcseconds to radians, then multiply it by the distance from Earth to Sag A*, and use the timestamps on the video to estimate the orbital periods in seconds. Note that, while this is straightforward for S0-2 since it completes a full orbit in the span of the video, this is less obvious for S0-37. The key here is that S0-37 has a circular orbit. Use Kepler's Laws and the previous discussion of oribital inclination observational effects to determine how S0-37's period can be estimated from the given data. What approximations and assumptions need to be made to justify the use of this equation for the system we're observing (Rubric Rows D8, D9)?

c) Once you have done this calculation, compare your obtained value of M to the best-estimate mass of the central SMBH of $M_{\rm bh} = 4.0 \times 10^6\,{\rm M}_\odot$ (Boehle *et al.* 2016). How far off was your estimate? Was it within your estimated error (See Section A.3)? List all possible sources of error and bias, both procedural and physical, in this calculation. Particularly, think about the applicability of Kepler's Third Law to the physical system we are measuring (Rubric Rows G1, G2).

4. Sag A* Size estimation

a) Now, that we have a mass estimate for Sag A*, the next step is to place upper limits on its physical size. Using the last frame in the video, try to use the paths of the orbits that come closest to the star symbol indicating Sag A* to place an upper limit on its radius. To determine that Sag A* is a black hole, we need to make an estimate of its physical extent, which will allow us to rule out all other plausible forms of matter (at least as far as we know). What is the physical value of this upper limit? Given the mass you calculated, how does this compare to its Schwarzschild radius?

b) While this suggests that the central attractor is an extremely dense object, we can place even tighter constraints on its physical extent based on the timescales of its fluctuations in electromagnetic observation. Sag A* is lacking in any prominent optical emission, it is nonetheless observable in radio and X-ray frequencies. As well, we observe it's brightness in these frequencies to flare (Figure 4.4) somewhat regularly on the order of once per day, with actual flare events lasting much less time. Since the speed of light is finite, when an object flares, light from the side of the object opposite the observer will become visible later than that from the side closest to the observer. Thus, the time it takes for the object to flare will be limited by light-travel time across the object. This allows us to use the light-travel distance over the timescale of the flare as an upper-limit on the physical extent of the object. Use Figure 4.4 to do just this. Calculate the corresponding angular size of this distance as observed from earth (in arcseconds, for easy comparison with the video data). How does this measure compare with Schwarzschild radius of a black hole of the mass you measured?

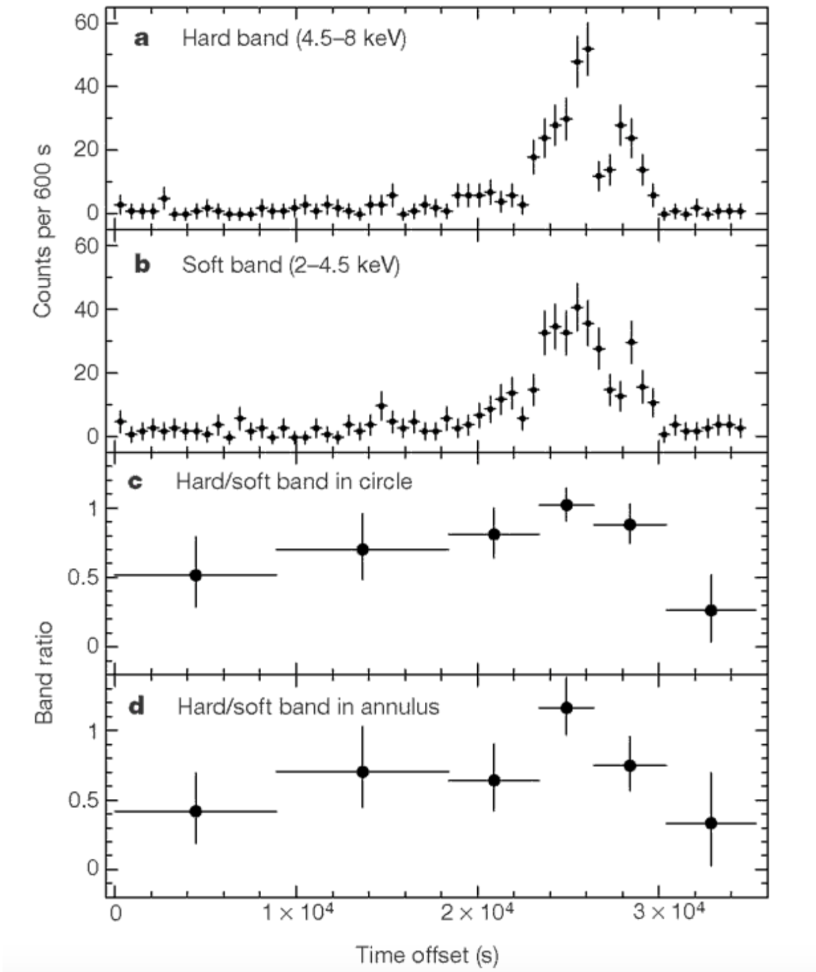

Figure 4.4: Sag A* light curve, showing significant flaring on the timescale of a day. (from https://heasarc.gsfc.nasa.gov/docs/objects/galaxies/sag-astar.html)

4.9 Individual Homework

Now that we have determined that Sag A* is an extremely dense, massive object — but not quite certainly a black hole — we are left to make plausibility arguments that allow us to eliminate all other possible forms of matter. For each category of astrophysical objects listed below, estimate the observational signatures of a hypothetical object with the mass you've measured for Sag A*. Decide whether or not the estimated observable properties can be used to rule out this alternative to a black hole.

1. Main-Sequence Star/Cluster (Hint: the luminosity of massive stars scales as M^4, so what luminosity would a Sag A*-star have? What would be the luminosity of a cluster of sun-like stars with a total mass equal to that of Sag A*?)

2. Brown Dwarf Star Cluster (Hint: A typical brown dwarf luminosity is $\sim 10^{-4}L_\odot$ and the maximum brown dwarf mass is $0.08M_\odot$. Brown dwarfs more massive than this are just stars, which you presumably ruled out above.)

3. White Dwarf Star Cluster (Hint: a typical White Dwarf luminosity is $0.03L_{\odot}$, and the maximum white dwarf mass is $1.4M_{\odot}$. There are fundamental physical arguments that preclude the stable existence of a white dwarf above this mass.)

4. Neutron Star Star Cluster (Hint: a typical Neutron Star luminosity is $10^{-6}L_{\odot}$, and the maximum neutron star mass is estimated to be $1.4M_{\odot}$. This maximum mass is also derived from fundamental physics considerations). Would such a configuration, even if hypothetically possible, be physically reasonable?

APPENDIX A

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1]The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as $52 \pm 3\,\text{m}$.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	**Implied uncertainty**
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a-b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Rubrics

Each scientific ability rubric row assessed is worth a possible 1 point, with "Missing" being 0 points, "Inadequate" 1/3 points, "Needs Improvement" 2/3 points, and "Adequate" 1 point.

The scientific abilities rubrics are found on the following pages.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
A11	**Graph**	No graph is present.	A graph is present but the axes are not labeled. There is no scale on the axes.	The graph is present and axes are correctly labeled, but the axes do not correspond to the independent and dependent variables, or the scale is not accurate.	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate.

Table B.1: Rubric A: Ability to represent information in multiple ways

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B1	**Is able to identify the phenomenon to be investigated**	No phenomenon is mentioned	The description of the phenomenon to be investigated is confusing, or it is not the phenomenon of interest.	The description of the phenomenon is vague or incomplete.	The phenomenon to be investigated is clearly stated.
B2	**Is able to design a reliable experiment that investigates the phenomenon**	The experiment does not investigate the phenomenon.	The experiment may not yield any interesting patterns.	Some important aspects of the phenomenon will not be observable.	The experiment might yield interesting patterns relevant to the investigation of the phenomenon.
B3	**Is able to decide what physical quantities are to be measured and identify independent and dependent variables**	The physical quantities are irrelevant.	Only some of physical quantities are relevant.	The physical quantities are relevant. However, independent and dependent variables are not identified.	The physical quantities are relevant and independent and dependent variables are identified.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B4	**Is able to describe how to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All chosen measurements can be made, but no details are given about how it is done.	All chosen measurements can be made, but the details of how it is done are vague or incomplete.	All chosen measurements can be made and all details of how it is done are clearly provided.
B5	**Is able to describe what is observed without trying to explain, both in words and by means of a picture of the experimental setup.**	No description is mentioned.	A description is incomplete. No labeled sketch is present. Or, observations are adjusted to fit expectations.	A description is complete, but mixed up with explanations or pattern. Or the sketch is present but is difficult to understand.	Clearly describes what happens in the experiments both verbally and with a sketch. Provides other representations when necessary (tables and graphs).
B6	**Is able to identify the shortcomings in an experiment and suggest improvements**	No attempt is made to identify any shortcomings of the experiment.	The shortcomings are described vaguely and no suggestions for improvement are made.	Not all aspects of the design are considered in terms of shortcomings or improvements.	All major shortcomings of the experiment are identified and reasonable suggestions for improvement are made.
B7	**Is able to identify a pattern in the data**	No attempt is made to search for a pattern.	The pattern described is irrelevant or inconsistent with the data.	The pattern has minor errors or omissions. Terms like "proportional" used without clarity, e.g. is the proportionality linear, quadratic, etc.	The pattern represents the relevant trend in the data. When possible, the trend is described in words.
B8	**Is able to represent a pattern mathematically (if applicable)**	No attempt is made to represent a pattern mathematically.	The mathematical expression does not represent the trend.	No analysis of how well the expression agrees with the data is included, or some features of the pattern are missing.	The expression represents the trend completely and an analysis of how well it agrees with the data is included.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B9	**Is able to devise an explanation for an observed pattern**	No attempt is made to explain the observed pattern.	An explanation is vague, not testable, or contradicts the pattern.	An explanation contradicts previous knowledge or the reasoning is flawed.	A reasonable explanation is made. It is testable and it explains the observed pattern.

Table B.2: Rubric B: Ability to design and conduct an observational experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C1	**Is able to identify the hypothesis to be tested**	No mention is made of a hypothesis.	An attempt is made to identify the hypothesis to be tested but it is described in a confusing manner.	The hypothesis to be tested is described but there are minor omissions or vague details.	The hypothesis is clearly, specifically, and thoroughly stated.
C2	**Is able to design a reliable experiment that tests the hypothesis**	The experiment does not test the hypothesis.	The experiment tests the hypothesis, but due to the nature of the design it is likely the data will lead to an incorrect judgment.	The experiment tests the hypothesis, but due to the nature of the design there is a moderate chance the data will lead to an inconclusive judgment.	The experiment tests the hypothesis and has a high likelihood of producing data that will lead to a conclusive judgment.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C4	**Is able to make a reasonable prediction based on a hypothesis**	No prediction is made. The experiment is not treated as a testing experiment.	A prediction is made, but it is identical to the hypothesis, OR prediction is made based on a source unrelated to the hypothesis being tested, or is completely inconsistent with hypothesis being tested, OR prediction is unrelated to the context of the designed experiment.	Prediction follows from hypothesis but is flawed because relevant assumptions are not considered, OR prediction is incomplete or somewhat inconsistent with hypothesis, OR prediction is somewhat inconsistent with the experiment.	A prediction is made that follows from hypothesis, is distinct from the hypothesis, accurately describes the expected outcome of the experiment, and incorporates relevant assumptions if needed.
C5	**Is able to identify the assumptions made in making the prediction**	No attempt is made to identify assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or are confused with the hypothesis.	Relevant assumptions are identified but are not significant for making the prediction.	Sufficient assumptions are correctly identified, and are significant for the prediction that is made.
C6	**Is able to determine specifically the way in which assumptions might affect the prediction**	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.
C7	**Is able to decide whether the prediction and the outcome agree/disagree**	No mention of whether the prediction and outcome agree/disagree.	A decision about the agreement/disagreement is made but is not consistent with the results of the experiment.	A reasonable decision about the agreement/disagreement is made but experimental uncertainty is not taken into account.	A reasonable decision about the agreement/disagreement is made and experimental uncertainty is taken into account.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C8	**Is able to make a reasonable judgment about the hypothesis**	No judgment is made about the hypothesis.	A judgment is made but is not consistent with the outcome of the experiment.	A judgment is made, is consistent with the outcome of the experiment, but assumptions are not taken into account.	A judgment is made, is consistent with the outcome of the experiment, and assumptions are taken into account.

Table B.3: Rubric C: Ability to design and conduct a testing experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D1	**Is able to identify the problem to be solved**	No mention is made of the problem to be solved.	An attempt is made to identify the problem to be solved but it is described in a confusing manner.	The problem to be solved is described but there are minor omissions or vague details.	The problem to be solved is clearly stated.
D2	**Is able to design a reliable experiment that solves the problem.**	The experiment does not solve the problem.	The experiment attempts to solve the problem but due to the nature of the design the data will not lead to a reliable solution.	The experiment attempts to solve the problem but due to the nature of the design there is a moderate chance the data will not lead to a reliable solution.	The experiment solves the problem and has a high likelihood of producing data that will lead to a reliable solution.
D3	**Is able to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All of the chosen measurements can be made, but no details are given about how it is done.	All of the chosen measurements can be made, but the details about how they are done are vague or incomplete.	All of the chosen measurements can be made and all details about how they are done are provided and clear.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D4	**Is able to make a judgment about the results of the experiment**	No discussion is presented about the results of the experiment.	A judgment is made about the results, but it is not reasonable or coherent.	An acceptable judgment is made about the result, but the reasoning is incomplete, OR uncertainties are not taken into account, OR assumptions are not discussed, OR the result is written as a single number.	An acceptable judgment is made about the result, with clear reasoning. The effects of assumptions and experimental uncertainties are considered. The result is written as an interval.
D5	**Is able to evaluate the results by means of an independent method**	No attempt is made to evaluate the consistency of the result using an independent method.	A second independent method is used to evaluate the results. However there is little or no discussion about the differences in the results due to the two methods.	A second independent method is used to evaluate the results. The results of the two methods are compared correctly using experimental uncertainties. But there is little or no discussion of the possible reasons for the differences when the results are different.	A second independent method is used to evaluate the results and the evaluation is correctly done with the experimental uncertainties. The discrepancy between the results of the two methods, and possible reasons are discussed.
D7	**Is able to choose a productive mathematical procedure for solving the experimental problem**	Mathematical procedure is either missing, or the equations written down are irrelevant to the design.	A mathematical procedure is described, but is incorrect or incomplete, due to which the final answer cannot be calculated. Or units are inconsistent.	Correct and complete mathematical procedure is described but an error is made in the calculations. All units are consistent.	Mathematical procedure is fully consistent with the design. All quantities are calculated correctly with proper units. Final answer is meaningful.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D8	**Is able to identify the assumptions made in using the mathematical procedure**	No attempt is made to identify any assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevent or incorrect for the situation.	Relevant assumptions are identified but are not significant for solving the problem.	All relevant assumptions are correctly identified.
D9	**Is able to determine specifically the way in which assumptions might affect the results**	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.

Table B.4: Rubric D: Ability to design and conduct an application experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
F1	**Is able to communicate the details of an experimental procedure clearly and completely**	Diagrams are missing and/or experimental procedure is missing or extremely vague.	Diagrams are present but unclear and/or experimental procedure is present but important details are missing. It takes a lot of effort to comprehend.	Diagrams and/or experimental procedure are present and clearly labeled but with minor omissions or vague details. The procedure takes some effort to comprehend.	Diagrams and/or experimental procedure are clear and complete. It takes no effort to comprehend.
F2	**Is able to communicate the point of the experiment clearly and completely**	No discussion of the point of the experiment is present.	The experiment and findings are discussed but vaguely. There is no reflection on the quality and importance of the findings.	The experiment and findings are communicated but the reflection on their importance and quality is not present.	The experiment and findings are discussed clearly. There is deep reflection on the quality and importance of the findings.

Table B.5: Rubric F: Ability to communicate scientific ideas [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G1	**Is able to identify sources of experimental uncertainty**	No attempt is made to identify experimental uncertainties.	An attempt is made to identify experimental uncertainties, but most are missing, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and instrumental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between instrumental and random uncertainty.
G2	**Is able to evaluate specifically how identified experimental uncertainties affect the data**	No attempt is made to evaluate experimental uncertainties.	An attempt is made to evaluate uncertainties, but most are missing, described vaguely, or incorrect. Or the final result does not take uncertainty into account.	The final result does take the identified uncertainties into account but is not correctly evaluated. Uncertainty propagation is not used or is used incorrectly.	The experimental uncertainty of the final result is correctly evaluated. Uncertainty propagation is used appropriately.
G3	**Is able to describe how to minimize experimental uncertainty and actually do it**	No attempt is made to describe how to minimize experimental uncertainty and no attempt to minimize is present.	A description of how to minimize experimental uncertainty is present, but there is no attempt to actually minimize it.	An attempt is made to minimize the uncertainty in the final result is made but the method is not very effective.	The uncertainty is minimized in an effective way.
G4	**Is able to record and represent data in a meaningful way**	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G5	**Is able to analyze data appropriately**	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains errors or omissions.	The analysis is appropriate, complete, and correct.

Table B.6: Rubric G: Ability to collect and analyze experimental data [3].

APPENDIX C

Lab Report Format

In a general sense, the labs should demonstrate Rubric Rows F1 and F2 (see Table B.5), in addition to the other rubric rows listed in the lab write-up.

C.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.
- If the rubric row refers to a particular part of your lab report, clearly label that part of the report with that rubric row. For example, you should label the section where you demonstrate uncertainty propagation with “G2” if that rubric row is being assessed in that lab.

C.2 Organizing the report

If the lab is clearly framed as an observational, testing, or application experiment, you can follow the corresponding rubric for the elements to include in the report (see, respectively, Rubrics B, C, and D in Appendix B).

In general, the report should include the following sections:

1. **Introduction.** A written description of what the lab is designed to investigate and a brief summary of the procedure used. This section should be at least a full paragraph long, and not more than 3 double-space pages.

   You don’t need to include too much detail here, but it should be a complete and concise description of the purpose and general method used in the lab. Imagine a classmate who hasn’t seen the lab writeup asked you, “what is this lab about? What do you do?” You should be able to hand them your introduction, and they’d be able to understand the purpose and general structure of the lab. But you need not mention every step and calculation here.

2. **Analysis and discussion.** For most labs that have more than one part, this should be broken up into parts and labeled in order. This section must include all of the following, in the same order in which these elements appear in the lab instructions.

- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.
- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$.
- Answers to any questions that appear in the lab handout.

3. **Conclusion.** This can be very short, and will generally only require one or two paragraphs. In your conclusion, you should summarize the point of the lab and what you learned, both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

C.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

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