Laboratory Manual

PHSC 12610 Black Holes

The University of Chicago

Winter 2020

Labs

Behavior of waves (the ripple tank)

1.1 Introduction

The Michelson interferometer, named after University of Chicago professor Albert A. Michelson (Nobel prize in Physics 1907), is an extremely sensitive instrument capable of measuring incredibly tiny displacements. A modern version of the Michelson interferometer has been developed by The Laser Interferometer Gravitational-Wave Observatory (LIGO) experiment to detect changes in distance of 10^{-19} m (much less than the size of the nucleus of an atom!). This displacement is sensed between mirrors separated by 4 km (see Figure 1.1). There are two sites for LIGO — one in Hanford, WA and the other in Livingston, LA. The LIGO interferometer has recently detected gravitational waves for the first time (September 15, 2015); the first announced gravitational wave detection fits, with remarkable precision, the expected signal from the merging of two black holes, 29 and 36 solar masses, located 410 Mpc away. The reported signal and the comparison to the fitted model are shown in Figure 1.2.

The working principle of the Michelson interferometer is the interference of light. In this lab, you will first explore the concepts of interference with sound waves, in a device known as a virtual ripple tank. In particular, in this first portion of the lab you will experimentally discover a relationship between wave frequency and wavelength, and then demonstrate constructive and destructive wave interference. You will then extend that understanding of interference to a wave geometry more appropriate to the second portion of the lab. The final measurement with the ripple tank will allow you to show that plane waves propagating through a slit behave as though the slit were a new source of waves, propagating radially (i.e. in a circular pattern).

Next week, you will measure interference phenomena with light, with a modern version of the famous double-slit experiment performed by Thomas Young in 1801. You will show that the interference properties of waves established in the first section of the lab apply to light as well, thus experimentally demonstrating that light behaves in a wavelike manner.

Having established the wavelike nature of light, you will then finally use a virtual Michelson interferometer to demonstrate how to measure changes in distances smaller than a human hair (not quite LIGO sensitivity, but still pretty impressive!).

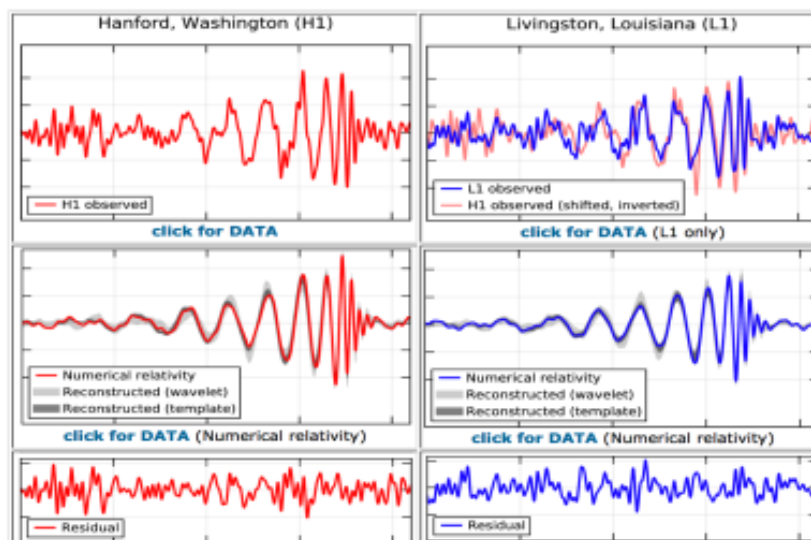Figure 1.1: An aerial view of the two LIGO sites.

Figure 1.2: The left panels show the LIGO signal at the Hanford site (top) and the best-fit model (middle) and the residual of the model minus the data (bottom). The residuals are consistent with noise. The right panels show the same for the Livingston site, with the Hanford signal plotted in red in the top panel to demonstrate the similarity of the two measurements (as expected in the event of a true gravitational wave signal). This first LIGO detection of a gravitational wave event marks a significant transformation in our collective ability to measure and understand black holes, and since that first detection, more black hole merger events have been detected and reported.

1.2 Learning Goals

- Learn how to conduct an observational experiment, including collecting data and analyzing the data to find and describe a pattern quantitatively.

- Discover the relationship between frequency and wavelength of waves.

- Learn how to conduct a testing experiment, including identifying a hypothesis, designing an experiment, making a prediction, and comparing it to an experimental outcome.

- Gain familiarity with wave interference.

1.3 Initial planning for your project and presentation.

Later in the course, you will write a project paper and make a presentation in lab section on the same topic. Project topic choices and presentation dates must be arranged with, and approved by your TA. Your choice of topic must be made in consultation with TAs and other students, so that no more than one person in any section will present on any given topic. A listing of "pre-approved" project topics is provided on Canvas. Other topics can also be accepted, with prior approval.

During the first week in lab section, **discuss the options with your TA and other students**. By the end of your second lab section meeting, mutually agree on plans for your course project topic and presentation date.

1.4 Group formation

1. Once you have a group, meet with each other and decide a) what tools you will use to communicate and collaborate, b) when you will meet, c) what you will do when you need to change an agreement, and d) what you will do when a member has a concern about how the group is functioning. **Record your agreements in your lab report.**

Team roles

2. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used

- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage

- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

Add members to Canvas lab report assignment group

3. On Canvas, navigate to the People section, then to the "Lab 1 Groups" tab. Find a group that is not yet used, and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

1.5 The Scientific Cycle[1]

One way of describing science is the process of incrementally improving a shared model of how our universe works. In different fields of science, different methods and cycles are used, so there is no "One True Scientific Method." One can still create a model for the process of science, and we describe here one such cycle (the hypothetico-deductive cycle), summarized in Figure 1.3.

In this cycle, there are three types of experiments, each one representing a different stage of the scientific effort. One stage, often started when encountering a novel phenomenon, is the **observational experiment**. This is an experiment that consists of deciding what to observe and how to observe it, collecting data, finding a pattern, and brainstorming possible explanations for what is observed (also called "hypotheses").

Once one has some trial explanations, one can test one or more of those with a **testing experiment**. Here, one designs a new experimental procedure and uses each hypothesis to predict what will happen. Then the prediction is compared to the procedure's outcome. If they are different, then the hypothesis is judged to be not a helpful explanation for that phenomenon. If they are the same, then it is still helpful. Throughout this stage, one may make various assumptions that would need to be validated, as they can effect the prediction or outcome.

Once a hypothesis has been tested enough for people to find it useful, then it can be applied to solve practical problems, or to determine properties of particular situations, in an "application experiment."
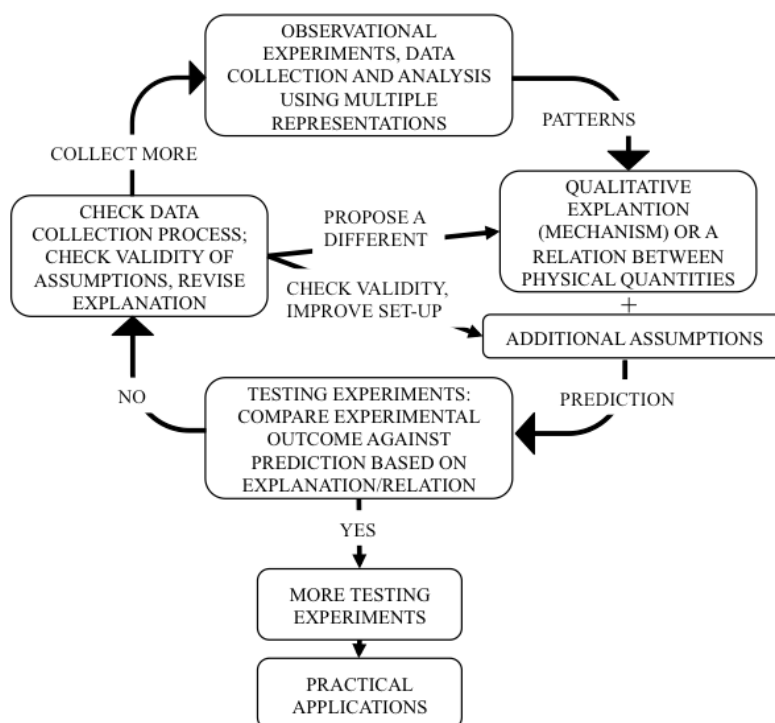
---

[1]adapted from [1]

Last updated May 3, 2021

Figure 1.3: A model of the process some scientists go through to create knowledge.[2]

1.6 Experiment 1: Observation of frequency and wavelength

Goal

Observe sound waves in a virtual ripple tank and determine a mathematical relationship between frequency and wavelength.

Available equipment

- virtual ripple tank at www.falstad.com/ripple

> **Self-assessment:** To help you improve your scientific abilities, we provide you with self-assessment rubrics. A rubric is a scoring system. Self-assessment is determining how well you performed a particular task. So, these self-assessment rubrics are designed to help you evaluate your performance while you are designing and performing your experiment.
>
> The complete set of rubrics is available in Appendix B. In each lab, your report will be assessed using Rubric F, found in Table B.5, as well as 5 additional rubric rows listed in that lab. Each week, read through these and use them to evaluate your work as you design and perform the experiment. Your instructor will use the same rubrics to determine part of your grade for the lab.

Rubrics to focus on during this experiment

B7, B8, F1, F2. See Appendix B for details.

The virtual ripple tank

4. Open the virtual ripple tank by going to www.falstad.com/ripple in a web browser.

This is a simulation that demonstrates waves in two dimensions. The waves can represent water waves, sound, and light. When the simulation starts up, you will see a white square (called the "source") emitting circular waves. The light areas are positive and the dark areas are negative. So, if you prefer to think of the waves as sound waves, the light areas would be areas of high pressure, and the dark areas would be low pressure. The source might be a speaker of some sort. You can drag the source around wherever you want. Also you can create new waves (areas of high pressure) by clicking anywhere.

As you move the pointer around the tank, the position coordinates of the pointer, as well as the current time in the simulation, are displayed in the lower left corner. Note that the simulation is showing a slowed-down version of the actual sound waves, so the time in the simulation will pass much slower than the actual time.

The sliders to the right of the tank control various aspects of the simulation. For the frequency slider in particular, if the waves are set to represent sound, then the frequency of the emitted sound is equal to the number to the right of the slider times 54, with units of hertz (Hz, or cycles per second).

Suggestions for your experiment

5. Ensure that every group member knows what the terms frequency and wavelength mean, in relation to waves. Use whatever means at your disposal to do this.

6. This is an "observational experiment." Review Rubric B (Table B.2) and discuss any unclear expectations with your group and the instructor. Note that your lab report will be graded, in part, on demonstration of Abilities B7 and B8.

7. Brainstorm different methods you could use to determine the relationship between wavelength and frequency. Feel free to play with the simulation as you do so, seeing what the various sliders and checkboxes on the right do. Here are some things to consider:

   - Which variable will you control (and thus will be the independent variable) and which will you measure?

   - What is the range of the independent variable that you will use? How many different settings will you choose?

   - You will need to use several settings of the independent variable, and then plot the data in a graph, decide on what pattern you see, and give some justification for that pattern. You can use words like "proportional", "linear", "parabolic", "exponential", "logarithmic", and so on, if they fit. Ensure you use the mathematical definition of these.

   - How will you measure the wavelength? Is it a more precise measurement if you measure several of them at once and divide to get a single wavelength? You can use these coordinates to calculate the distance d between two points on the screen, (x_1, y_1) and (x_2, y_2), using the Pythagorean theorem
   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\,. \tag{1.1}$$

8. Decide on your measurement and analysis method and discuss it with an instructor before you begin. They will help increase the chances that your method will lead to successful results, or at least that the unhelpful path that you choose will take a short enough amount of time for you to change it when you discover it does not work. We want you to have productive failure that you have time to learn from.

9. Perform your experiment. Your lab report for this experiment should include:

- A labeled sketch or photo of the setup, and a description of the experimental procedure (see Rubric F1).

- A plot of wavelength vs. frequency (with the independent variable on the horizontal axis)

- A description of the pattern found. This can be done with a line (straight or curved) showing the pattern you see (either drawn manually or using the curve fitting function of the plotting program, e.g. LibreOffice Calc or Microsoft Excel) and with words describing what you found. (B7)

- An equation to represent the pattern. This can be taken from a curve fit or found by hand. Make sure there is some discussion of how well the equation agrees with the data, but you don't need to be very precise about it. (B8)

- A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2)

1.7 Experiment 2: Testing the conditions for destructive interference

Goal

A student from a different lab section came up with the idea that destructive interference between two waves occurs at positions where the distance from each source differs by an integer number of wavelengths, or

$$\Delta d = m\lambda, \tag{1.2}$$

where Δd is the "path length difference", λ is the wavelength, and m is any integer. The student has asked you to test this idea for them. Please do so and provide them feedback.

Available equipment

- Same as in the previous experiment

Setup

From the "Example" drop-down menu on the right, select "Example: Two Sources".

Rubric rows to be assessed in this experiment

C1, C4, C7, F1, F2. See Appendix B for details.

Testing this hypothesis

In general, one tests a hypothesis by using it to make a prediction about what will happen in a certain experimental procedure. With this hypothesis, it asserts a relationship between path length difference, wavelength, and destructive interference.

Destructive interference occurs whenever two waves overlap and attempt to disturb the medium in opposite directions, resulting in no disturbance. In this case, the color is the original color of the tank, neither lighter nor darker. In the 3-D view, the places of destructive interference are where the surface does not move up or down during the wave motion.

In this case, it is easier to start by finding those locations, picking a particular point, measuring the Δd, finding the wavelength for the given frequency using the relationship you found in Experiment 1, and solving for m. The hypothesis predicts that m should always be an integer. As a result, your experiment becomes this: find out how close the experimentally determined m's are to integers.

Brainstorm your experimental procedure, decide on it, discuss with your TA, then perform the experiment.

Your lab report for this experiment should include:

- A clear description of the hypothesis (see Rubric C1).

- A labeled sketch or photo of the setup, and a description of the experimental procedure (F1).

- A clear statement of the prediction that the hypothesis makes for this particular procedure (C4).

- A table of path lengths, path length differences, and measured m values.

- An analysis of how close the measured m values are to the prediction. Use some quantitative measure of this, but don't worry about being precise about uncertainties (C7).

- A judgment about the hypothesis. Is it supported, disproved, or undetermined? (C8, though not assessed this time)

- A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2).

1.8   Group functioning

10. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

1.9   Individual Homework: Observing plane waves encountering narrow gaps

These questions are to be answered individually and your answers should be submitted under the Lab 1 Homework assignment on Canvas.

This experiment does not clearly follow the model of the scientific cycle, but is closest to an observational experiment. In next week's lab, you will investigate the properties of light traveling through small slits. Ripples in a virtual tank are more obviously waves, so it is helpful to observe what happens here first.

1. Instead of point sources, select the "Example: Single Slit" and then set "Waves = Sound". This example is of a straight line wave, or, in two dimensions, a "plane wave", incident on a wall with a small opening. This is the same kind of wave and apparatus we will use next week with light. What wave pattern do you see? How does this pattern compare to the data you took using one source?

2. Is the wavelength of the pattern you observe consistent with the relationship between frequency and wavelength you measured with single point source? Include any measurements and calculations you make in answering this question in your homework response, and be quantitative.

3. Select "Example: Double Slit" and then set "Waves = Sound". What wave pattern do you see? How does this pattern compare to the data you took using two sources?

Behavior of electromagnetic waves in space (lasers and slits)

2.1 Introduction

In 1801, Thomas Young's "double-slit" experiment demonstrated the wave nature of light by showing that two coherent light sources produce interference patterns. You will perform a modern version of Young's experiment using a laser as light source. The laser illuminates two thin slits, each of width a separated by a distance d, which act as two coherent sources of light. This is analogous to what you have observed with water waves in the previous lab section, in which you saw a plane wave combined with an aperture (a slit) acting as a circular source of waves. An interference pattern appears on a viewing screen, placed at a distance L from the double slit, in the form of bright and dark regions corresponding to maxima and minima of interference. You will use the interference pattern to measure the wavelength λ of the laser, and show that the same framework of equations that is derived in the introduction to the previous lab holds for light too.

2.2 Topic selection for individual paper and presentation

By the end of lab today, ensure that you have chosen a topic for your individual project, and that it has been approved by your TA.

2.3 Team roles

1. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used

- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage

- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

2.4   Add members to Canvas lab report assignment group

2. On Canvas, navigate to the People section, then to the "Lab 2 Groups" tab. Find a group that is not yet used, and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

2.5   Experiment 1: Observing patterns made by 1 and by 2 slits

Goal

Describe the patterns made by a laser that is incident on 1 slit and on 2 slits, and the differences and similarities between them.

Available equipment

- In this remote environment, imagine you have a laser, slit card, and screen as shown in Figure 2.1.
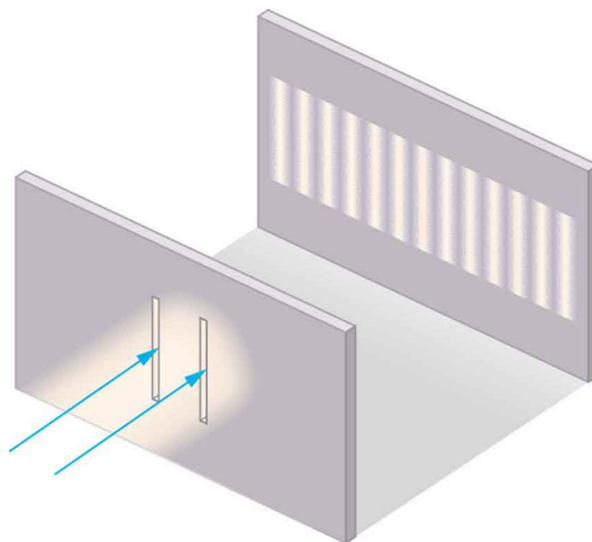

Figure 2.1: Setup for the double-slit experiment. Laser light of a single frequency is sent through two slits and a pattern forms on the screen. Figure from OpenStax. Access for free at https://openstax.org/books/college-physics/pages/27-3-youngs-double-slit-experiment
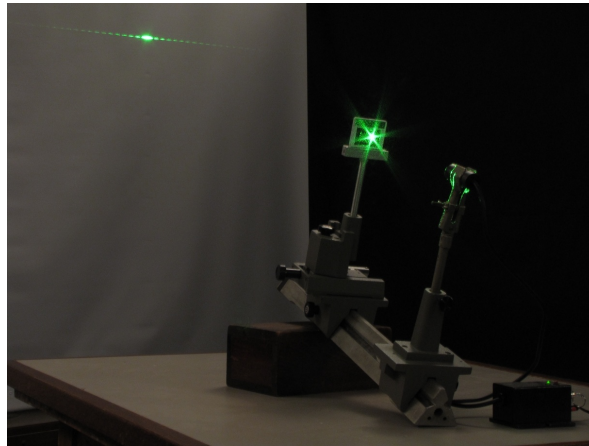
Figure 2.2: Setup for the double-slit experiment. Laser light of a single frequency originates from the laser on the right. The laser light strikes the card in the middle, some of it passing through two thin vertical slits. A pattern forms on the screen. Image from https://wiki.uchicago.edu/display/KER/Laser+Double+Slit+Interference
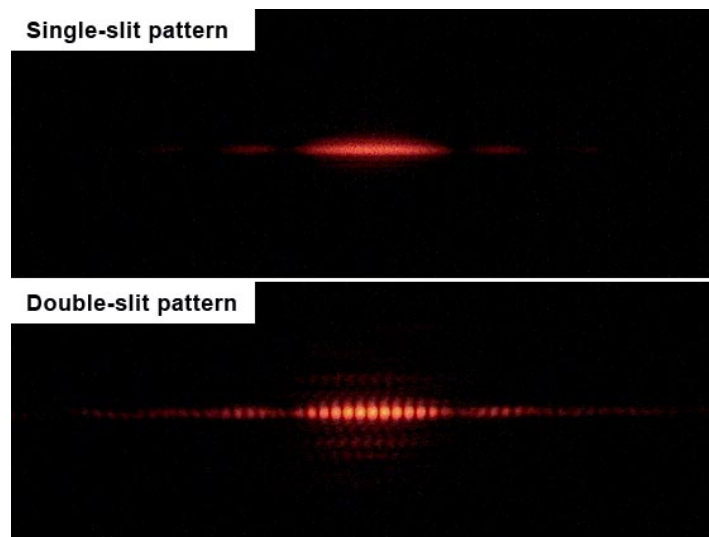

Figure 2.3: In a dark room, a laser is aimed at a card with a single or double slit in it. The light passes through the slit(s) and lights up a screen with the patterns shown. Image from https://en.wikipedia.org/wiki/Double-slit_experiment#/media/File:Single_slit_and_double_slit2.jpg

Rubrics to be assessed for this section

F1, F2. See Appendix B for details.

Steps

In our lab of the mind, we have a red laser sending a thin beam of light of a single wavelength through slits and onto a screen. A sketch of the situation is shown in Figure 2.1, and a photo of the setup is shown in Figure 2.2. We first use just a single slit, and then use both of them. With the room lights off, we take a picture of each case. The results are shown in Figure 2.3.

3. Discuss the patterns observed in Figure 2.3. Describe each in detail. How are they alike and different? **Record your description and comparison.**

2.6 Experiment 2: Testing the wave hypothesis

Goal

Determine whether light from a laser can be described as a plane wave.

Available equipment

- Simulation of laser light passing through slits, found at `https://physics.bu.edu/~duffy/HTML5/double_slit.html`

Rubrics to be assessed for this section

C4, C7, C8, G2, G4, F1, F2. See Appendix B for details.

Behavior of a plane wave incident on single and double slits

The following equation describes the location, y_m (measured relative to the center of the pattern), of the mth interference *minimum* (dark spot) seen on a screen when a plane wave is incident on a single slit.

$$y_m = \frac{m\lambda L}{a} \, , \tag{2.1}$$

where L is the distance from the slit to the screen, and a is the width of the slit.

For a double slit, the following equation describes the location y_n of the nth interference *maximum* (bright spot) seen on a screen when a plane wave is incident on a double slit.

$$y_n = \frac{n\lambda L}{d} \, , \tag{2.2}$$

where d is the distance between the two slits. These equations use the same principle of path length difference as used in the previous lab, but are derived for the case where the distance from slit to screen is much larger than the slit and spacing dimensions.

Suggestions for your experiment

- To get an uncertainty for your length measurements for the interference maxima and minima, you can have different teammates measure the same lengths and find the average and standard deviation.

- You can assume the values selected by the sliders are exact, with zero uncertainty.

- If you use a value with an uncertainty in a calculation, if you want to use that value for comparison, you must propagate the uncertainty through to the final value. See Appendix A.2.

- To compare your outcomes to your predictions, get a value with uncertainty for each, then compare them using the t' test, described in Section A.3.

Items to include in your report

Relevant rubric rows from Appendix B are listed in parentheses.

1. Statement of the hypothesis (C1).

2. Description of the experimental setup and procedure (C2, F1).

3. The quantitative prediction that the hypothesis makes about what will happen during the experimental procedure (C4). Ensure that uncertainty is handled correctly (G2).

4. A report of the experimental outcome (results), neatly organized (G4).

5. Determination of whether / how much the prediction agrees with the outcome, comparing using uncertainties (C7, G2).

6. Judgment about the hypothesis — based on this experiment, does it lead you to support the hypothesis more or less, about how much (qualitative)? (C8)

7. A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2).

2.7 Group functioning

4. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

2.8 Individual homework

In the simulation from Section 2.6, find the wavelengths of the red, green, and violet lasers. Given that this is a simulation, determine if these wavelengths are physically reasonable, given typical wavelengths for these colors.

Using light waves to measure small distance changes (Michelson interferometer)

3.1 Introduction

With the basic properties of waves and wave interference established (via the ripple tank) and the same behavior demonstrated in light (via the laser-based modern version of Young's double slit experiment) we are now finally ready to look at a Michelson interferometer. This technology is the basis of the LIGO experiment. You may want to refer back to the introduction of Lab 1 to remind yourself of some details. LIGO itself is a large experiment that has been constructed over several decades of work and technology development, and so is many orders of magnitude more precise and sensitive than what we can do in an hour on a lab bench. Nevertheless, the basic principles are the same.

3.2 Team roles

1. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used

- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage

- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

3.3   Add members to Canvas lab report assignment group

2. On Canvas, navigate to the People section, then to the "Lab 3 Groups" tab. Find a group that is not yet used, and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

3.4   Observation experiment: demonstration of the interferometer

Goal

Observe the flow of light in the Michelson interferometer simulation.

Available equipment

- A virtual Michelson interferometer found at https://www.geogebra.org/m/msfpudej

  - Note: in the sim, do not zoom in and out by scrolling — when this happens, the coordinate system appears to shrink and expand, leading to the mirrors seeming to move in space.

Rubrics to be assessed for this section

None.

3.5   Steps

3. Load the virtual apparatus at the address listed in the available equipment section.

4. Click the refresh button in the simulation, to the right of the Zeit slider. This fixes a bug where the green wave does not fully display.

5. Slide Zeit (German for "time") to the left to equal 0. Uncheck "Weg Spiegel 1" (wave path 1)

This is the Michelson interferometer, invented by Albert Michelson, a physicist whose family immigrated to the USA from Poland when he was 2 years old. He grew up in mining camps in California and Nevada and went on to found the physics department here at UChicago. Let's walk through the operation of the interferometer step by step. A schematic of the setup with parts labeled is found in Figure 3.1.

6. Select "Animation" and watch as the wave leaves the laser source and select it again to pause as the wave encounters the beam splitter.

The beam splitter is a device that reflects some of the beam and transmits the rest. This one is a 50/50 beam splitter, so it splits the beam into equal parts.

7. Select "Animation" again to watch the reflected part of the beam travel until it reaches the output/viewing screen. You'll watch the transmitted part of the beam at a later step. For now it is not displayed. *If the reflected beam does not appear, refresh the animation using the circular arrow button to the right of the Zeit slider.*

This reflected beam travels to the stationary mirror a distance d_2, reflects from that mirror, travels directly through the beam splitter on the way back, and exits the device to be seen on the viewing screen. (we ignore the part that is reflected when it strikes the beam splitter on the way back). The arrow that appears at the output represents the phase of the wave that is passing that point. For example, when the wave is cresting, the arrow points up, and when the wave is at a trough, the arrow points down.
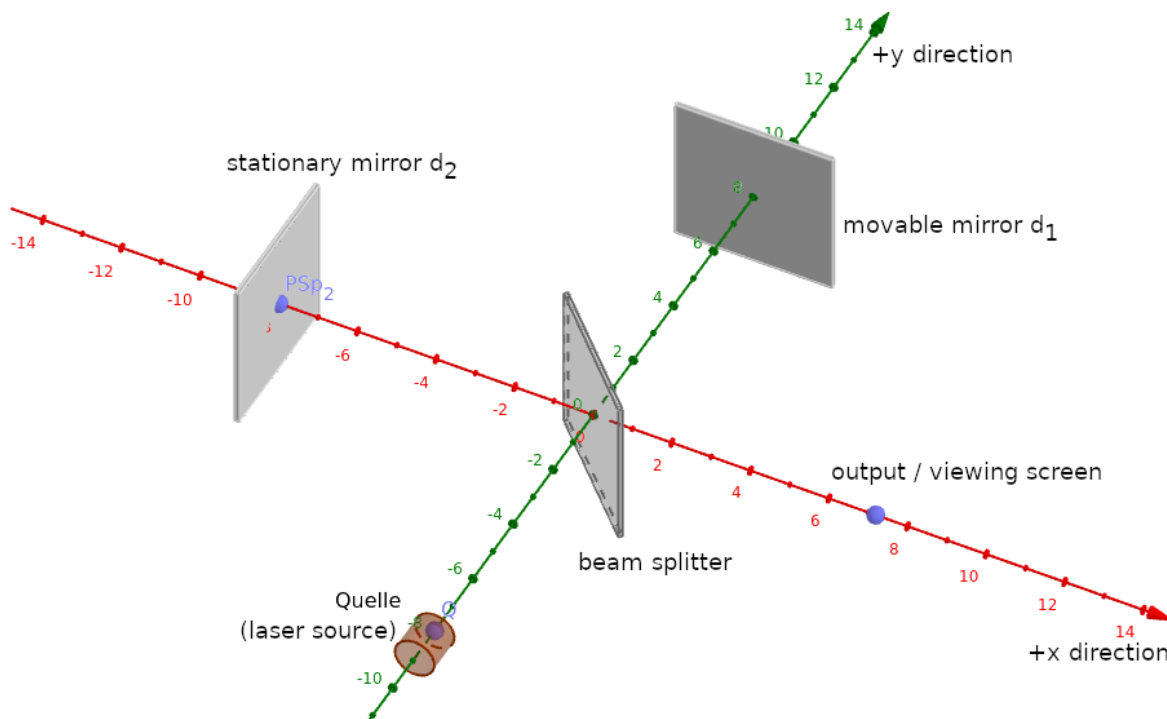
Figure 3.1: Diagram of the Michelson interferometer simulation with parts labeled.

8. Move Zeit back to zero. Check the box for Weg Spiegel 1 and uncheck Weg Spiegel 2. Watch the animation again to follow the path of the beam that is initially transmitted through the beam splitter.

This beam strikes the mirror at d_1, which is movable using the slider on the left or by typing in the desired distance. The typing field accepts numbers with precision up to 4 decimal places.

9. Move Zeit back to zero, check both boxes, and watch the beams overlap with each other at the output.

In this way the original beam of light splits, and portions of the resulting beams are brought back together to interfere with each other. The black arrow represents the sum of the two waves. The direction is the phase and the length is proportional to the amplitude of the resulting wave.

3.6 Testing experiment: testing the theory of wave interference

Goal

Test if the theory of wave interference and path length difference predicts the correct behavior in the situation of the Michelson interferometer. This theory states that for two different sources of waves that are of the same, single wavelength, and which started out in phase, then constructive interference occurs at a location when the difference in travel distances (path lengths) between the two sources is an integer number of wavelengths. Destructive interference occurs when the path length difference is a half-integer, for example 0.5, 1.5, or 2.5 wavelengths.

Available equipment

- A virtual Michelson interferometer as in the previous experiment.

17

Rubrics to be assessed for this section

- **C4:** Is able to make a reasonable prediction based on a hypothesis

- **C7:** Is able to decide whether the prediction and the outcome agree/disagree

- **F1:** Is able to communicate the details of an experimental procedure clearly and completely

- **F2:** Is able to communicate the point of the experiment clearly and completely

See Appendix B for details.

Suggestions for your experiment

- Ensure that all group members are familiar with how path length difference results in constructive and destructive interference.

- Examine the geometry of the setup to determine the path length of the beam that travels through each arm of the interferometer.

- Develop a specific prediction of what setting combinations of wavelength and mirror distance will result in different observable interference effects.

- Note that in the simulation, you can type in a mirror distance d_1 with a precision of 4 decimal places. Press enter after typing it in to save it and have the result displayed.

Items to include in your report

Relevant rubric rows from Appendix B are listed in parentheses.

1. Clear statement of the hypothesis you are testing (C1, not assessed)

2. Prediction that follows from the hypothesis, along with justification (C4)

3. Description of experimental procedure to produce outcome (F1)

4. Determination of how much the prediction agrees with the outcome (C7)

5. A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2)

3.7   Application experiment: measuring length changes with the interferometer

Goal

The Michelson interferometer is used for LIGO since gravitational waves create very small vibrations in spacetime as they pass. These vibrations literally alter the distance between the beam splitter and mirror along one axis. This MI simulates this with a movable mirror. Using this virtual MI, **develop a procedure** to determine the change in mirror distance d_1 if given the image of the black arrow as displayed on the screen (the arrow's length is proportional to the brightness at that location and moment). **Demonstrate this procedure** with an example initial and final mirror position.

   Determine how small a change in the mirror distance you can measure using your procedure (the *resolution* of the instrument. **Identify methods** to improve this resolution.

Available equipment

- A virtual Michelson interferometer as in the previous experiment.

Rubrics to be assessed for this section

- **D2:** Is able to design a reliable experiment that solves the problem

- **G2:** Is able to evaluate specifically how identified experimental uncertainties affect the data

- **G3:** Is able to describe how to minimize experimental uncertainty and actually do it

- **F1:** Is able to communicate the details of an experimental procedure clearly and completely

- **F2:** Is able to communicate the point of the experiment clearly and completely

See Appendix B for details.

Suggestions for your experiment

- Remember that the laser wavelength λ and the mirror distances are given in micrometers (μm). This makes it an infrared laser.

- Decide how you will measure the black arrow's length. This can many methods, for example holding up a ruler to the screen or taking a screenshot and measuring the pixel length in an image analysis program like ImageJ.

- Note that in the simulation, you can type in a mirror distance d_1 with a precision of 4 decimal places. Press enter after typing it in to save it and have the result displayed.

Items to include in your report

Relevant rubric rows from Appendix B are listed in parentheses.

1. Description of procedure to use image of black arrow to determine the change in mirror distance d_1 (D2, F1)

2. Demonstration of this procedure

3. Determination of measurement resolution (G2)

4. Identification of ways to improve this resolution (G3)

5. A discussion of the findings of the experiment and why it's helpful (for you and/or for science) (F2)

3.8 Group functioning

10. A 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what do you want to continue doing or do differently?

3.9 Individual homework

1. If you were to add a mirror between the beam splitter and the movable mirror, such that the light struck the movable mirror, then our new mirror, then back to the movable mirror, then back to the beam splitter, how would this change the resolution of your measuring device from the application experiment, if at all?

2. Given the resolution of your instrument that you found in the application experiment, what are examples of objects at that length scale that you would be able to use this to measure with 1% accuracy? You can use this website as a starting point: https://htwins.net/scale2/

Black Hole at the Galactic Center?

4.1 Mystery at the Center of the Milky Way

1. Take a moment to watch the video found in the following link www.astro.ucla.edu/~ghezgroup/gc/animations.html under the heading **3D Movie of Stellar Orbits in the Central Parsec**.

At first glance the video might not seem all too surprising as having learned about the solar system you likely expect orbiting planets to be a mundane fixture of the universe. However, what if you were to learn that the objects were not planets, but in fact stars and that what you see in the video spans a distance of 3 light years? For comparison, Pluto is only about 0.0006 ly from the sun. In fact, the video you just saw is a visualization of a phenomenon in the center of our galaxy which puzzled astronomers for a long time. As you might have learned all objects exert a gravitational force which is proportional to the mass of the object. For this reason, smaller objects tend to be "pulled in" by larger objects, forming the orbital relationships we see in our daily lives: the moon orbiting the Earth, the Earth orbiting the Sun, and so on. Some of the most massive objects in the universe are stars which is why they tend to form the center of orbital systems. However, given that all the objects in the video were stars, this meant that there had to be a much, much more massive object in the center of our galaxy attracting them, one which seemed to be invisible, save for radio signals coming from the location of the object. There were many theories as to what the object, whose signal is dubbed Sagittarius A*, could be. Some believed it to be a collection of massive objects such as stars or small black holes. The most compelling theory, however, was that thee source responsible for the signal was, in fact, a Supermassive Black Hole (SMBH). Black holes are some of the most extreme objects in the universe which were first theorized to exist as a result of Einstein's theory of general relativity. In the most basic terms, a black hole is an extremely massive and dense object whose gravitational pull is so strong, that not even light can escape. This fact that light cannot escape from a black hole, however, makes them incredibly difficult to observe directly. That said, due to the strength of their gravitational pull, black holes can often be detected indirectly based on their influence over nearby objects.

In this lab you will examine the gravitational system you saw in the video and see what judgment you can make about whether the object in the center of the Milky Way is, in fact, a black hole.

4.2 Learning Goals

- Understand Kepler's laws of planetary motion and be able to use them to extract information about orbital systems.

- Be able to gather data using a variety of tools and understand the limitations of experimental data.

- Be able to make inferences about physical properties of objects which cannot be directly measured.

- Identify assumptions made during analysis and their effects on calculations.

4.3 Team roles

2. **Decide on roles** for each group member.

The available roles are:

- Facilitator: ensures time and group focus are efficiently used

- Scribe: ensures work is recorded

- Technician: oversees apparatus assembly, usage

- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

4.4 Add members to Canvas lab report assignment group

3. On Canvas, navigate to the People section, then to the "Lab 4 Groups" tab. Find a group that is not yet used, and have each person in your group add themselves to that same lab group.

This enables group grading of your lab report. Only one person will submit the group report, and all members of the group will receive the grade and have access to view the graded assignment.

4.5 Installing "ImageJ"

For some parts of this lab you will be using a image analysis software called "ImageJ" to gather numerical data from images.

4. Go to the following link to install the imageJ software https://imagej.nih.gov/ij/download.html

5. Click on the link to the software version for your OS. This will download a .zip file.

6. Once the file is downloaded, right click on the folder and choose "extract all". Once it is finished go into the folder you extracted the files to and click on the icon labeled "Imagej"

4.6 General Relativity and Schwarzschild Radii

While Newtonian dynamics is useful for describing most orbital systems, extreme systems or objects such as black holes cannot be fully described without also incorporating general relativity. In particular for this lab we will be using a particular description of the universe in which gravity, rather than being an "attraction" between objects, is actually the result of curved "spacetime". To visualize this, imagine space-time as sheet of stretched out fabric. Normally, if you were to try to roll light objects across the sheet they would travel in a straight line. However, if you were to place a large weight in the center, the fabric would droop inwards and any object you tried to roll would instead fall inwards towards the depressed region (the following video demonstrates this analogy: https://youtu.be/MTY1Kje0yLg). This is analogous to the effect that gravity has on spacetime. The key to this description is that anything traveling through spacetime will follow this curvature, even if it has no mass such as light. This means that, theoretically, an object can exist which bends spacetime so much that not even light can climb back out and escape (it would orbit the object or fall back inward). Using the principles of gravitation developed by Newton, we can estimate how massive and small such an object would be.

In Newtonian dynamics, the minimum speed an object needs to escape the gravitational pull of an object is given by

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \,. \tag{4.1}$$

where r is the distance from its center of mass M.

7. First, manipulate this equation in order to get an expression for r in terms of the other variables.

If you now plug in the speed of light $c = 2.998 \times 10^8$ m/s as the escape velocity into the equation you just derived, you get an expression for what is known as the Schwarszchild radius. The Schwarzschild radius is an estimate of the upper limit of the radius of a black hole with a given mass. That is, according to this model, if a chunk of matter of mass M is squeezed into a radius as small or smaller than the radius r, then light cannot escape, and it is a black hole.

8. To gain some intuition about how compact black holes would need to be, calculate the Schwarzchild radius for each of the following objects, and compare them to their actual radii.

    a) One of your group members.
    b) the Earth.
    c) the Sun.
    d) the Solar System.
    e) The Milky Way Galaxy.

To determine whether the object at the center of our galaxy is a black hole, we need to measure its mass, calculate its Schwarzchild radius based on that, then measure its radius and compare them.

4.7 Understanding orbits

In general, an *orbit* is what we call the path an object follows when under the gravitational influence of a larger mass. The interactions between two gravitationally bound objects can be approximated using Newtonian mechanics, with the force of gravity between two objects given by

$$F_{\text{grav}} = G\frac{m_1 \, m_2}{r^2} \,, \tag{4.2}$$

where m_1 and m_2 are the masses of the two objects, r is the distance between them, and G is the Newtonian constant of gravitation, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. This equation, coupled with Newton's Second Law of Motion $F = ma$ or force F equals mass m times acceleration a, tells us
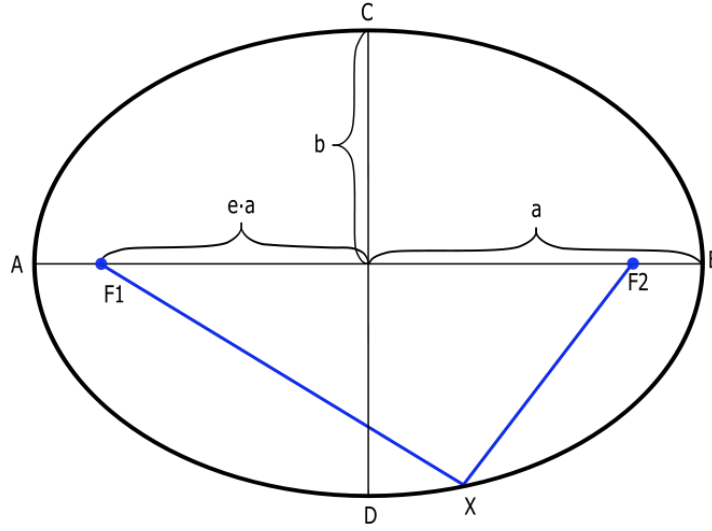
Figure 4.1: The geometry of an ellipse: a is the semi-major axis of the ellipse, F1 and F2 are each a focus of the ellipse, b is the semi-minor axis, and e is the eccentricity. The eccentricity describes the extent to which the ellipse is oblong: an ellipse with $e = 0$ is just a circle. The foci are defined such that the distance from F1 to X, added to the distance from F2 to X, is the same no matter where X is located on the ellipse. For a circle, the foci coincide at the center. The Newtonian generalization of Kepler's First Law tells us that a small mass will orbit a much larger mass on an ellipse, and the larger mass will be located at one of the foci.

that the stronger the force of gravity, the greater the acceleration due to gravity. With these two fundamental principles, it is possible to derive many of the properties of orbital mechanics. Moreover, they allow us to understand the physics behind the mathematical description of orbits formulated by Johannes Kepler nearly a century earlier.

Orbits, ellipses, and Kepler's First Law

Kepler's First Law states that *a planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse.* This is true generally for any mass orbiting a much more massive object. An *ellipse* (commonly referred to as an oval) is a generalization of a circle, allowing for the circle to be stretched along a certain direction. See Figure 4.1 for details.

Kepler's Third Law

Kepler's Third law addresses the path of an object m as it orbits a much more massive object of mass M. Specifically, it relates the orbital period P (the time it takes for one complete orbit to occur) to the semi-major axis a of the ellipse according to

$$P^2 = \frac{4\pi^2}{GM}a^3 \, , \tag{4.3}$$

where other objects are also considered to be not affecting the orbit of m.

An important qualification to be made about Kepler's Laws is that they apply only to two-body systems. Kepler's third law breaks down when you have more than 2 orbiting objects in a system. However, they are nonetheless a very good approximation for the orbits of small masses around a much larger mass, in which case the gravitational force of the massive object dominates over the intra-small-object interactions, and thus each smaller body approximately behaves independently from the other

small objects. And so the motion of each small object, to a good approximation, can be modeled by Kepler's laws.

4.8 Developing Orbital Dynamics Skills

Goal

Use Kepler's 3rd Law to analyze a test system.

Available equipment

- Elliptical Orbits and Kepler's Laws simulator: https://ophysics.com/f6.html

Steps

9. Open the link provided in the Available Equipment section above. This will open an orbit simulator.

10. Make sure the simulation is paused. Now manipulate the initial distance from the sun, the initial speed of the planet, and the mass of the sun by moving the slider over the bars on the left-hand side on the screen. For now, don't pay particular attention to the different variables, in this step you just need to focus on the orbit itself. How does the orbit change shape as you manipulate the initial conditions? How does this align with what Kepler's First Law predicts? Are there exceptions? **Record your answers.**

11. Reset the simulation by clicking the arrow symbol to the right of the "zoom out" button. This time, you will carry out a similar process as the previous step, however, this time, you will only be manipulating one variable at a time. How does the shape of the orbit change as you change each variable? Does this change in shape align with predictions from Kepler's third law? How does the value of the semi-major axis a change? How is the change in period reflected in the orbital path? **Record your answers.**

12. Use Equation 4.3 to find the mass of the Sun, given the period of the Earth's orbit (converted to seconds) and its semi-major axis (approximately the radius, since Earth has a nearly circular orbit). Check your answer with standard references to ensure that you have used this method correctly.

13. To test that you have an effective technique for using Kepler's third law with a visual orbit, reset the simulation to the default initial conditions and use Kepler's third law to predict the period of this orbit. Ensure that you get about 3.3 seconds.

4.9 Determining the mass of Sag A*

Now that you have an understanding of orbits, you will analyze orbital data gathered by UCLA and use these to calculate the mass of the object located at Sag A*.

Goal

Use measurements from two stars orbiting the object located at Sag A* to estimate its mass.

Available Equipment

- ImageJ: https://imagej.nih.gov/ij/download.html. This is an image processing program which you will use this to extract numerical data from the video.

- Stars Orbiting Galactic Center: https://youtu.be/7vcSKbXnLJA. This is the video you will be analyzing.

Steps

14. First, watch the video several times and take note of the different stars and their paths around the central star symbol, which represents Sag A*. What are your initial impressions? How could this video be used to estimate the mass of Sag A*? **Record your answers.**

15. Determine two stars for which you can find and measure the semi-major axis a and the period P. Note that the timestamp in the upper-left corner is in decimal years. **Record which stars you picked and why you picked them.**

16. Set the video to full screen and take a screenshot near the end of the video, so that you can carefully measure the semi-major axis of both stars.

Gathering data with ImageJ

This section will guide you through the process of taking length measurements using ImageJ.
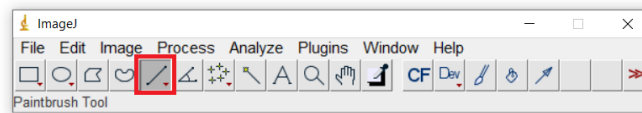
Figure 4.2: Top panel of ImageJ software with straight line tool highlighted.

17. First, note the white arrow located on the left-hand side of the image. This indicates the angular scale of the image. In the top menu bar, click on the straight line icon (see figure 4.2). Now click one end of the arrow and drag the line to the other end.

18. Normally, the straight line tool measures pixel count, however, you can change the scale so that it gives you the the actual measurements according to the scale of the image. To do this, click on "Analyze" above the icons and select the "set scale" option. In the "known distance" box

Figure 4.3: Table generated by ImageJ when measuring.

Last updated May 3, 2021

enter "0.1". This allows you to measure the angular separation d of the objects in the image in arcseconds.

19. Once the scale is set, use the straight line tool (the same you used to set the scale) to draw a line along the length you want to measure and press "m" on the keyboard. This will generate a window with different measurements which updates each time you hit "m" (see figure 4.3 for an example table). You will only be using the "length" (in arcseconds) measurements.

20. Measure the semi-major axis of both stars that you chose. Make an estimate of the uncertainty of these measurements as well. **Record the star labels and their axes measurements with uncertainties.**

21. Currently your measurement is an angular separation in units of arcseconds. Kepler's Third Law deals with the physical distance, so you will need to convert the angular separation to physical distance. Convert the angular separation from arcseconds to radians. Then multiply by the distance from Earth to Sag A*, $2.47 \pm 0.05 \times 10^{20}$ m. This gives you the distance s in meters. Propagate uncertainty according to Appendix A.2. **Record your work and results.**

22. Use the video to measure the period of each star's orbit. If the star you picked does not complete an entire period, develop a way to estimate the entire period. Note that the time is given in decimal years (so 2020.3 means 3/10 of the year past the beginning of 2020). **Record your work and results.**

23. Use Kepler's Third Law to calculate the mass of Sag A* from the orbital properties of both stars that you analyzed. Your measurement should be on the order of millions of solar masses. **Record the two calculated masses and their uncertainties.**

24. Use the t' statistic to determine if the calculated masses from both stars are plausibly the same value ($t' < 1$). If they are not, reconsider your methods and uncertainty analysis. **Record your work and results.**

25. Make a final determination of your measurement of the mass (with uncertainty), based on your two calculated masses.

4.10 Finding the size of Sag A* and testing for black hole plausibility

26. Using the last frame of the image, use the orbital path traced in the video which came closest to Sag A* to place an upper limit on its radius. Use the straight line tool to measure this limit in arcseconds and convert to distance in meters as in Step 21. **Record your work and results.**

27. Now, using the estimate you found for the mass of Sag A*, calculate its Schwarzschild radius. How does this compare to the upper limit you estimated for its radius? **Record your work and results.**

28. Based on this alone, how likely do you think it is that Sag A* is a black hole? What additional evidence would you need in order to conclude this? What sources of error could be affecting your estimates? What assumptions were made in this analysis that might be incorrect? **Record your discussion and determination.**

4.11 Checking an assumption about perspective

29. Go back and watch the video you watched at the start of the lab. This time, focus on one or two orbits and not how they change as the camera moves. How does the observed 2D shape of the orbit change? **Record your observations in the lab report**

30. Once you have a good idea of how the change in perspective affects our observation of the different orbits, discuss in your group how this might lead to errors when estimating orbital parameters. Think back to the different calculations you made throughout the lab, what errors could have arisen from assuming that you were always observing orbits directly from above? If we tried using this assumption to calculate the mass of the central object in an orbit, and that assumption was false, would the mass be over- or under-estimated? **Record your answer with justification.**

4.12   Group functioning

31. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what do you want to continue doing or do differently?

Items to include in your report

1. Expression for Schwarzchild radius in terms of mass, gravitational constant, and speed of light Step 7 {0.25 pt}

2. Schwarzchild radii for various objects (Step 8) {0.25 pt}

3. Observations and comparing with Kepler's first law (Step 10) {0.25 pt}

4. Observations and comparing with Kepler's third law (Step 11) {0.25 pt}

5. Initial impressions of video and identification of stars to measure (Steps 14–15) {0.25 pt}

6. Measurement of semi-major axis of each star and conversion to distance (Steps 20-21) {0.25 pt}

7. Measurement of period of each star's orbit (Step 22) {0.25 pt}

8. Calculation of Sag A* mass using each star's orbit (Step 23) [Rubric Row G2] {1 pt}

9. Comparison of two masses, discussion, and final judgment of Sag A*'s mass (Steps 24-25) [Rubric Row D4] {1 pt}.

10. Measurement of upper limit on Sag A* radius and calculation of its Schwarzchild radius (Steps 26-27) {0.25 pt}

11. Judgment on the hypothesis that Sag A* is a black hole with discussion of errors and assumptions (Step 28) [Rubric Rows C8, D8] {2 pt}

12. Analysis of effects of angular orientation assumption (Step 29–30) [Rubric Row D9] {1 pt}

13. A discussion of the findings of the experiment and why it's helpful (for you and/or for science) [Rubric Row F2] {1 pt}

14. Analysis of group functioning (Step 31) {0.5 pt}

4.13   Individual Homework

Throughout the lab, you attempted to determine whether the object located at Sag A* was a black hole by constraining its size and mass through its gravitational influence on nearby stars. It is possible to place further constraints on its size by measuring the time between fluctuations in the electromagnetic signal emitted by the object. Although Sag A* does not emit any significant light in the optical range of the spectrum, it does emit strong X-ray and radio frequencies. Moreover, the signals in these frequencies flare up on average once a day for short periods of time. However, since the speed of light
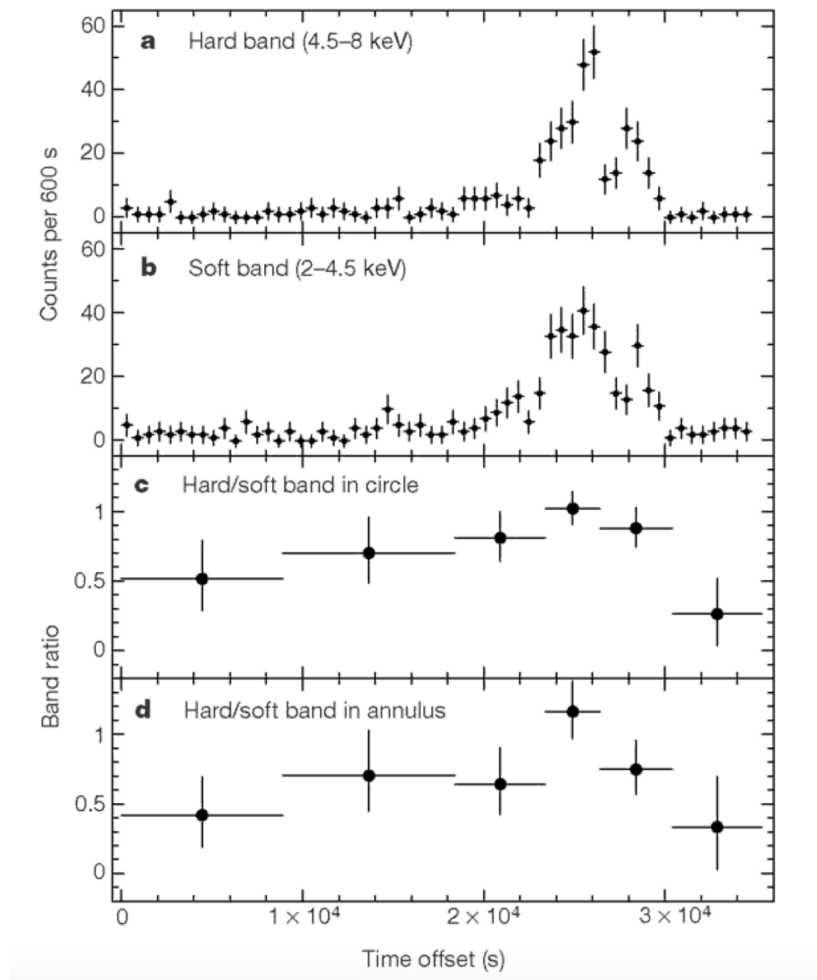
Figure 4.4: Sag A* light curve, showing significant flaring on the timescale of a day. (from https://heasarc.gsfc.nasa.gov/docs/objects/galaxies/sag-astar.html)

is finite, light from one side of the object facing away from us will take more time to reach us than light from the side facing us. We can take advantage of this time difference to estimate how big an object is.

1. Use figure 4.4 to find an upper bound on the size of Sag A* in this way. How does this compare to the size you calculated during lab?

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "5 ± 1 m" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

A.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

---

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet, or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}} \,, \tag{A.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}} \,. \tag{A.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as $52 \pm 3 \, \text{m}$.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

A.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{A.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{A.4}$$

$$\text{For } c = a^n, \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{A.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table A.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table A.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the 'e' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

A.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a - b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{A.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

---

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.

APPENDIX B

Rubrics

The scientific abilities rubrics are found on the following pages.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
A11	Graph	No graph is present.	A graph is present but the axes are not labeled. There is no scale on the axes.	The graph is present and axes are correctly labeled, but the axes do not correspond to the independent and dependent variables, or the scale is not accurate.	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate.

Table B.1: Rubric A: Ability to represent information in multiple ways

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B1	Is able to identify the phenomenon to be investigated	No phenomenon is mentioned	The description of the phenomenon to be investigated is confusing, or it is not the phenomenon of interest.	The description of the phenomenon is vague or incomplete.	The phenomenon to be investigated is clearly stated.
B2	Is able to design a reliable experiment that investigates the phenomenon	The experiment does not investigate the phenomenon.	The experiment may not yield any interesting patterns.	Some important aspects of the phenomenon will not be observable.	The experiment might yield interesting patterns relevant to the investigation of the phenomenon.
B3	Is able to decide what physical quantities are to be measured and identify independent and dependent variables	The physical quantities are irrelevant.	Only some of physical quantities are relevant.	The physical quantities are relevant. However, independent and dependent variables are not identified.	The physical quantities are relevant and independent and dependent variables are identified.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B4	**Is able to describe how to use available equipment to make measurements**	At least one of the chosen measurements cannot be made with the available equipment.	All chosen measurements can be made, but no details are given about how it is done.	All chosen measurements can be made, but the details of how it is done are vague or incomplete.	All chosen measurements can be made and all details of how it is done are clearly provided.
B5	**Is able to describe what is observed without trying to explain, both in words and by means of a picture of the experimental setup.**	No description is mentioned.	A description is incomplete. No labeled sketch is present. Or, observations are adjusted to fit expectations.	A description is complete, but mixed up with explanations or pattern. Or the sketch is present but is difficult to understand.	Clearly describes what happens in the experiments both verbally and with a sketch. Provides other representations when necessary (tables and graphs).
B6	**Is able to identify the shortcomings in an experiment and suggest improvements**	No attempt is made to identify any shortcomings of the experiment.	The shortcomings are described vaguely and no suggestions for improvement are made.	Not all aspects of the design are considered in terms of shortcomings or improvements.	All major shortcomings of the experiment are identified and reasonable suggestions for improvement are made.
B7	**Is able to identify a pattern in the data**	No attempt is made to search for a pattern.	The pattern described is irrelevant or inconsistent with the data.	The pattern has minor errors or omissions. Terms like "proportional" used without clarity, e.g. is the proportionality linear, quadratic, etc.	The pattern represents the relevant trend in the data. When possible, the trend is described in words.
B8	**Is able to represent a pattern mathematically (if applicable)**	No attempt is made to represent a pattern mathematically.	The mathematical expression does not represent the trend.	No analysis of how well the expression agrees with the data is included, or some features of the pattern are missing.	The expression represents the trend completely and an analysis of how well it agrees with the data is included.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
B9	Is able to devise an explanation for an observed pattern	No attempt is made to explain the observed pattern.	An explanation is vague, not testable, or contradicts the pattern.	An explanation contradicts previous knowledge or the reasoning is flawed.	A reasonable explanation is made. It is testable and it explains the observed pattern.

Table B.2: Rubric B: Ability to design and conduct an observational experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C1	Is able to identify the hypothesis to be tested	No mention is made of a hypothesis.	An attempt is made to identify the hypothesis to be tested but it is described in a confusing manner.	The hypothesis to be tested is described but there are minor omissions or vague details.	The hypothesis is clearly, specifically, and thoroughly stated.
C2	Is able to design a reliable experiment that tests the hypothesis	The experiment does not test the hypothesis.	The experiment tests the hypothesis, but due to the nature of the design it is likely the data will lead to an incorrect judgment.	The experiment tests the hypothesis, but due to the nature of the design there is a moderate chance the data will lead to an inconclusive judgment.	The experiment tests the hypothesis and has a high likelihood of producing data that will lead to a conclusive judgment.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C4	**Is able to make a reasonable prediction based on a hypothesis**	No prediction is made. The experiment is not treated as a testing experiment.	A prediction is made, but it is identical to the hypothesis, OR prediction is made based on a source unrelated to the hypothesis being tested, or is completely inconsistent with hypothesis being tested, OR prediction is unrelated to the context of the designed experiment.	Prediction follows from hypothesis but is flawed because relevant assumptions are not considered, OR prediction is incomplete or somewhat inconsistent with hypothesis, OR prediction is somewhat inconsistent with the experiment.	A prediction is made that follows from hypothesis, is distinct from the hypothesis, accurately describes the expected outcome of the experiment, and incorporates relevant assumptions if needed.
C5	**Is able to identify the assumptions made in making the prediction**	No attempt is made to identify assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or are confused with the hypothesis.	Relevant assumptions are identified but are not significant for making the prediction.	Sufficient assumptions are correctly identified, and are significant for the prediction that is made.
C6	**Is able to determine specifically the way in which assumptions might affect the prediction**	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.
C7	**Is able to decide whether the prediction and the outcome agree/disagree**	No mention of whether the prediction and outcome agree/disagree.	A decision about the agreement/disagreement is made but is not consistent with the results of the experiment.	A reasonable decision about the agreement/disagreement is made but experimental uncertainty is not taken into account.	A reasonable decision about the agreement/disagreement is made and experimental uncertainty is taken into account.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
C8	Is able to make a reasonable judgment about the hypothesis	No judgment is made about the hypothesis.	A judgment is made but is not consistent with the outcome of the experiment.	A judgment is made, is consistent with the outcome of the experiment, but assumptions are not taken into account.	A judgment is made, is consistent with the outcome of the experiment, and assumptions are taken into account.

Table B.3: Rubric C: Ability to design and conduct a testing experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D1	Is able to identify the problem to be solved	No mention is made of the problem to be solved.	An attempt is made to identify the problem to be solved but it is described in a confusing manner.	The problem to be solved is described but there are minor omissions or vague details.	The problem to be solved is clearly stated.
D2	Is able to design a reliable experiment that solves the problem.	The experiment does not solve the problem.	The experiment attempts to solve the problem but due to the nature of the design the data will not lead to a reliable solution.	The experiment attempts to solve the problem but due to the nature of the design there is a moderate chance the data will not lead to a reliable solution.	The experiment solves the problem and has a high likelihood of producing data that will lead to a reliable solution.
D3	Is able to use available equipment to make measurements	At least one of the chosen measurements cannot be made with the available equipment.	All of the chosen measurements can be made, but no details are given about how it is done.	All of the chosen measurements can be made, but the details about how they are done are vague or incomplete.	All of the chosen measurements can be made and all details about how they are done are provided and clear.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D4	**Is able to make a judgment about the results of the experiment**	No discussion is presented about the results of the experiment.	A judgment is made about the results, but it is not reasonable or coherent.	An acceptable judgment is made about the result, but the reasoning is incomplete, OR uncertainties are not taken into account, OR assumptions are not discussed, OR the result is written as a single number.	An acceptable judgment is made about the result, with clear reasoning. The effects of assumptions and experimental uncertainties are considered. The result is written as an interval.
D5	**Is able to evaluate the results by means of an independent method**	No attempt is made to evaluate the consistency of the result using an independent method.	A second independent method is used to evaluate the results. However there is little or no discussion about the differences in the results due to the two methods.	A second independent method is used to evaluate the results. The results of the two methods are compared correctly using experimental uncertainties. But there is little or no discussion of the possible reasons for the differences when the results are different.	A second independent method is used to evaluate the results and the evaluation is correctly done with the experimental uncertainties. The discrepancy between the results of the two methods, and possible reasons are discussed.
D7	**Is able to choose a productive mathematical procedure for solving the experimental problem**	Mathematical procedure is either missing, or the equations written down are irrelevant to the design.	A mathematical procedure is described, but is incorrect or incomplete, due to which the final answer cannot be calculated. Or units are inconsistent.	Correct and complete mathematical procedure is described but an error is made in the calculations. All units are consistent.	Mathematical procedure is fully consistent with the design. All quantities are calculated correctly with proper units. Final answer is meaningful.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
D8	Is able to identify the assumptions made in using the mathematical procedure	No attempt is made to identify any assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevent or incorrect for the situation.	Relevant assumptions are identified but are not significant for solving the problem.	All relevant assumptions are correctly identified.
D9	Is able to determine specifically the way in which assumptions might affect the results	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of assumptions are determined and the assumptions are validated.

Table B.4: Rubric D: Ability to design and conduct an application experiment [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
F1	Is able to communicate the details of an experimental procedure clearly and completely	Diagrams are missing and/or experimental procedure is missing or extremely vague.	Diagrams are present but unclear and/or experimental procedure is present but important details are missing. It takes a lot of effort to comprehend.	Diagrams and/or experimental procedure are present and clearly labeled but with minor omissions or vague details. The procedure takes some effort to comprehend.	Diagrams and/or experimental procedure are clear and complete. It takes no effort to comprehend.
F2	Is able to communicate the point of the experiment clearly and completely	No discussion of the point of the experiment is present.	The experiment and findings are discussed but vaguely. There is no reflection on the quality and importance of the findings.	The experiment and findings are communicated but the reflection on their importance and quality is not present.	The experiment and findings are discussed clearly. There is deep reflection on the quality and importance of the findings.

Table B.5: Rubric F: Ability to communicate scientific ideas [3].

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G1	**Is able to identify sources of experimental uncertainty**	No attempt is made to identify experimental uncertainties.	An attempt is made to identify experimental uncertainties, but most are missing, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and instrumental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between instrumental and random uncertainty.
G2	**Is able to evaluate specifically how identified experimental uncertainties affect the data**	No attempt is made to evaluate experimental uncertainties.	An attempt is made to evaluate uncertainties, but most are missing, described vaguely, or incorrect. Or the final result does not take uncertainty into account.	The final result does take the identified uncertainties into account but is not correctly evaluated. Uncertainty propagation is not used or is used incorrectly.	The experimental uncertainty of the final result is correctly evaluated. Uncertainty propagation is used appropriately.
G3	**Is able to describe how to minimize experimental uncertainty and actually do it**	No attempt is made to describe how to minimize experimental uncertainty and no attempt to minimize is present.	A description of how to minimize experimental uncertainty is present, but there is no attempt to actually minimize it.	An attempt is made to minimize the uncertainty in the final result is made but the method is not very effective.	The uncertainty is minimized in an effective way.
G4	**Is able to record and represent data in a meaningful way**	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	Scientific Ability	Missing	Inadequate	Needs Improvement	Adequate
G5	**Is able to analyze data appropriately**	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains errors or omissions.	The analysis is appropriate, complete, and correct.

Table B.6: Rubric G: Ability to collect and analyze experimental data [3].

Lab Report Format

In a general sense, the labs should demonstrate Rubric Rows F1 and F2 (see Table B.5), in addition to the other rubric rows listed in the lab write-up.

C.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.

- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

- If the rubric row refers to a particular part of your lab report, clearly label that part of the report with that rubric row. For example, you should label the section where you demonstrate uncertainty propagation with "G2" if that rubric row is being assessed in that lab.

C.2 Organizing the report

If the lab is clearly framed as an observational, testing, or application experiment, you can follow the corresponding rubric for the elements to include in the report (see, respectively, Rubrics B, C, and D in Appendix B).

In general, the report should include the following sections:

1. **Introduction.** A written description of what the lab is designed to investigate and a brief summary of the procedure used. This section should be at least a full paragraph long, and not more than 3 double-space pages.

   You don't need to include too much detail here, but it should be a complete and concise description of the purpose and general method used in the lab. Imagine a classmate who hasn't seen the lab writeup asked you, "what is this lab about? What do you do?" You should be able to hand them your introduction, and they'd be able to understand the purpose and general structure of the lab. But you need not mention every step and calculation here.

2. **Analysis and discussion.** For most labs that have more than one part, this should be broken up into parts and labeled in order. This section must include all of the following, in the same order in which these elements appear in the lab instructions.

- Any data that you've collected: tables, figures, measured values, sketches. Whenever possible, include an estimate of the uncertainty of measured values.

- Any calculations that you perform using your data, and the final results of your calculation. Note that you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.

- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92 \, \text{m/s}^2)t + 0.2 \, \text{m/s}$.

- Answers to any questions that appear in the lab handout.

3. **Conclusion.** This can be very short, and will generally only require one or two paragraphs. In your conclusion, you should summarize the point of the lab and what you learned, both in the frame of a scientist conducting the experiment ("What did the experiment tell us about the world?") and in the frame of a student ("What skills or mindsets did I learn?").

C.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.

- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".

- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

Bibliography

[1] E. Etkina, G. Planinsic, and A. Van Heuvelen, *College physics: explore and apply*, 2nd (Pearson, New York, 2014), 981 pp.

[2] E. Etkina, "Millikan award lecture: students of physics—listeners, observers, or collaborative participants in physics scientific practices?", American Journal of Physics **83**, 669 (2015).

[3] E. Etkina, A. Van Heuvelen, S. White-Brahmia, D. T. Brookes, M. Gentile, S. Murthy, D. Rosengrant, and A. Warren, "Scientific abilities and their assessment", Physical Review Special Topics - Physics Education Research **2**, 10.1103/PhysRevSTPER.2.020103 (2006).