Laboratory Manual

PHSC 12620 The Big Bang

The University of Chicago

Spring 2022

Labs

LAB 1

Measuring distant objects with parallax

1.1 Introduction

Since it takes time for light to travel to us from objects in the universe, the further out an object is, the further back in time we see it. So for us to have an accurate picture of how the universe was in the past, we need to know how far away things are. For things that are nearby on Earth, we can travel there and see how far we went, or how long we took to get there. For things further away like the moon, we can use Kepler's laws, or we can bounce a beam of light off of it and see how long it takes to get back. For objects outside of our solar system, it would take too long, and the light would disperse too much, for us to use this last technique. For those objects that are still relatively nearby, we can use the parallax technique as the first rung on our distance ladder.

In this lab, you will visit the Hubble Lounge, at the north end of the 5th floor of Eckhardt Research Center (ERC). While there, you will use the parallax technique to find the distance to a nearby building.

1.2 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

1.3 Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the "L1 Parallax" tab. Find a group that is not yet used, and have each person in your group add themselves to that same lab group.

1.4 Worksheet

Complete the worksheet "The Parsec" on the following pages. You can draw the diagrams needed and include a picture of your diagrams, or use a drawing program to draw on them.

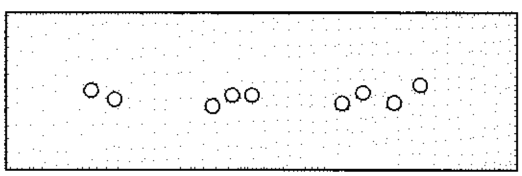
Part I: Stars in the Sky

Consider the diagram to the right.

1) Imagine that you are looking at the stars from Earth in January. Use a straightedge or a ruler to draw a straight line from Earth in January, through the Nearby Star (Star A), out to the Distant Stars. Which of the distant stars would appear closest to Star A in your night sky in January? Circle this distant star and label it "Jan."

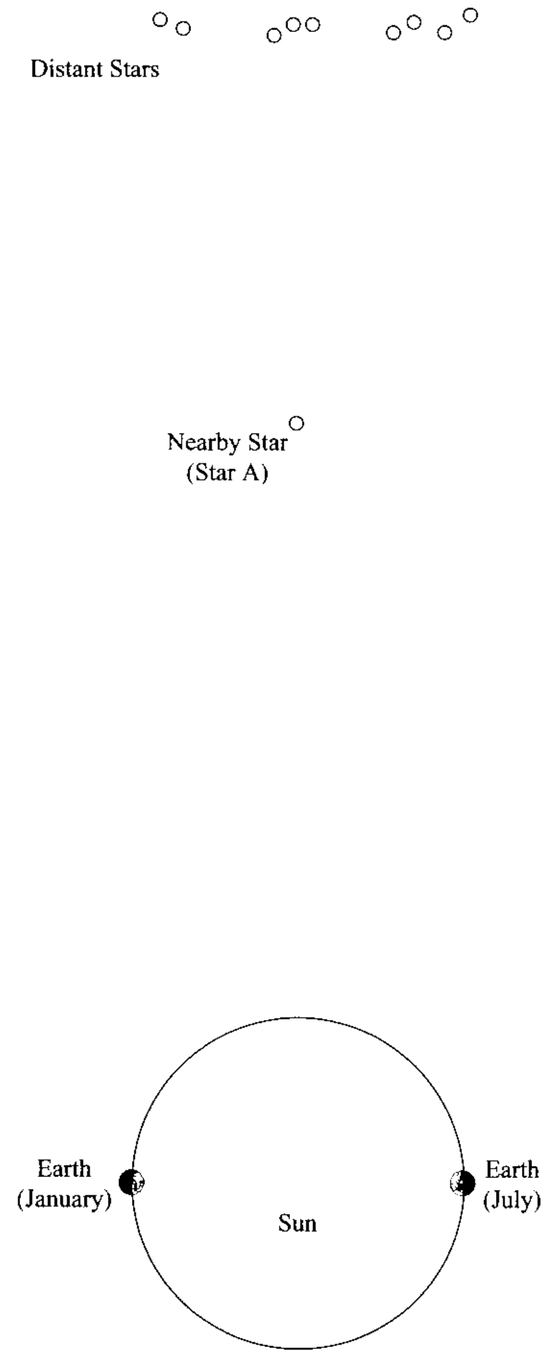
2) Repeat Question 1 for July and label the distant star "July."

3) In the box below, the same distant stars are shown as you would see them in the night sky. Draw a small × to indicate the position of Star A as seen in January and label it "Star A Jan."



4) In the same box, draw another × to indicate the position of Star A as seen in July and label it "Star A July."

5) Describe how Star A would appear to move among the distant stars as Earth orbits the Sun counterclockwise from January of one year, through July, to January of the following year.



The apparent motion of nearby objects relative to distant objects, which you just described, is called **parallax.**

6) Consider two stars (C and D) that both exhibit parallax. If Star C appears to move back and forth by a greater amount than Star D, which star do you think is actually closer to you? If you're not sure, just take a guess. We'll return to this question later in this activity.

Part II: What's a Parsec?

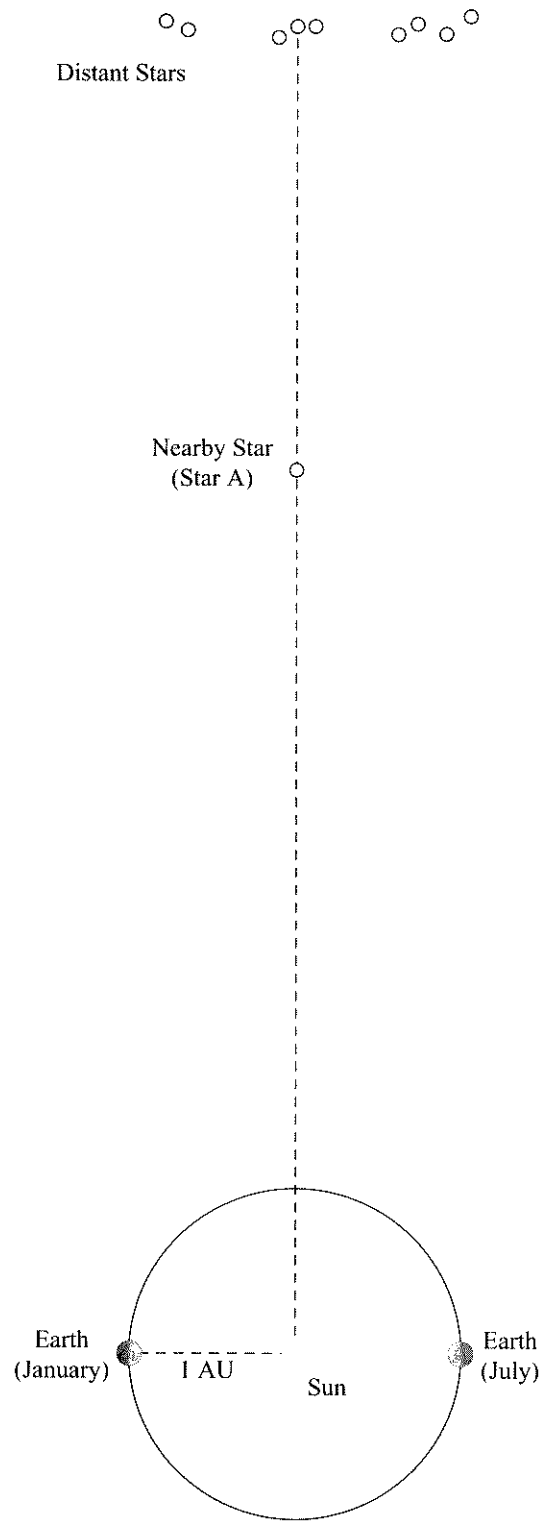
Consider the diagram to the right.

7) Starting from Earth in January, draw a line through Star A to the top of the page.

8) There is now a narrow triangle, created by the line you drew, the dotted line provided in the diagram, and the line connecting Earth and the Sun. The small angle, just below Star A, formed by the two longest sides of this triangle is called the **parallax angle** for Star A. Label this angle **"p_A."**

Knowing a star's parallax angle allows us to calculate the distance to the star. Since even the nearest stars are still very far away, parallax angles are extremely small. These parallax angles are measured in "arc-seconds" where an arcsecond is 1/3600 of 1 degree.

To describe the distances to stars, astronomers use a unit of length called the **parsec.** One parsec is defined as the distance to a star that has a **par**allax angle of exactly 1 arc**sec**ond. The distance from the Sun to a star 1 parsec away is 206,265 times the Earth–Sun distance or 206,265 AU. (Note that the diagram to the right is not drawn to scale.)



1.5 A quick measurement with hand tools

You will now use the parallax technique to measure the distance to an object in your environment without needed to travel to it. You'll first use crude measuring tools to compute the nearby star's distance with the parallax technique, and then use a more precise method. You'll use the little finger on your outstretched arm as a measurement of angular size — the width of this finger covers ("subtends") about 1 degree.

2. In the Hubble Lounge, look north towards the downtown skyline. Identify an object to be your "nearby star" and another as your "distant star", by analogy to the figure in the worksheet. An identifiable point on a white pillar rising from the Ratner gym serves well as the nearby star, while a corner of a building in the far distance can be your background star. See Fig. 1.1 for an example.

You'll first use crude measuring tools to compute the nearby star's distance with the parallax technique, and then use a more precise method. First, you'll use the pinky finger on your outstretched arm as a measurement of angular size — the width of this finger covers ("subtends") about 1 degree.

3. Identify an object to be your "nearby star" and another as your "distant star", by analogy to the figure in the worksheet. An identifiable point on a white pillar rising from the Ratner gym serves well as the nearby star, while a corner of a building in the distance can be your background star.

4. Find a place where you can stand and move a meter or two side to side and still see both "stars". The movement will simulate the Earth moving from its January to its July position. When picking the stars and your Earth positions, check to see that from each Earth position, the distant star does not appear to move much, compared to the nearby star, and your guessed distance to the nearby star is at least 10 times greater than the distance between your two Earth positions.

5. Looking from just one eye, move so that the two stars appear to be directly overlapping. Mark your current position as Earth (January). Hold up your smallest finger at arm's length and move to your left or right until your finger fits just in between the two stars. This means that they are 1 degree away from each other in angular separation. Mark this position as Earth (July).

6. Draw a diagram, similar to the second figure in the worksheet, and find your own Earth-Sun distance (half the distance between your Earth positions). Calculate the parallax angle, which is half the 1 degree you measured with your finger, and convert it to radians.

Now the distance to the nearby star can be found using the triangle formed by the line segments Sun-Earth, Earth-Star, and Star-Sun (see Figure 1.2). Trigonometry relates these lengths to each other according to

$$\tan p = \frac{a}{d}\,, \tag{1.1}$$

where p is the parallax angle in radians, d is the distance to the star, and a is half of the distance between the two measurement positions. Since the length d is much greater than a, the angle p is very small, and so we can use the small angle approximation $\tan u \approx u$, and therefore

$$p = \frac{a}{d}\,. \tag{1.2}$$

7. Use Equation 1.2 to calculate the distance to the nearby star.

8. Conduct the data collection twice more to get two more Earth-Star distances. You can use this to calculate the *measurement uncertainty* — the uncertainty is half the range of values, and the distance is the average of the 3 distances you found.

Figure 1.1: Example images. My foreground "star" was the point defined by the left intersection of the lower cable and the white pillar on top of the gym on campus. One of my reference "stars" was the top right corner of the building in the background. Note that the images produced by this telescope are upside down.

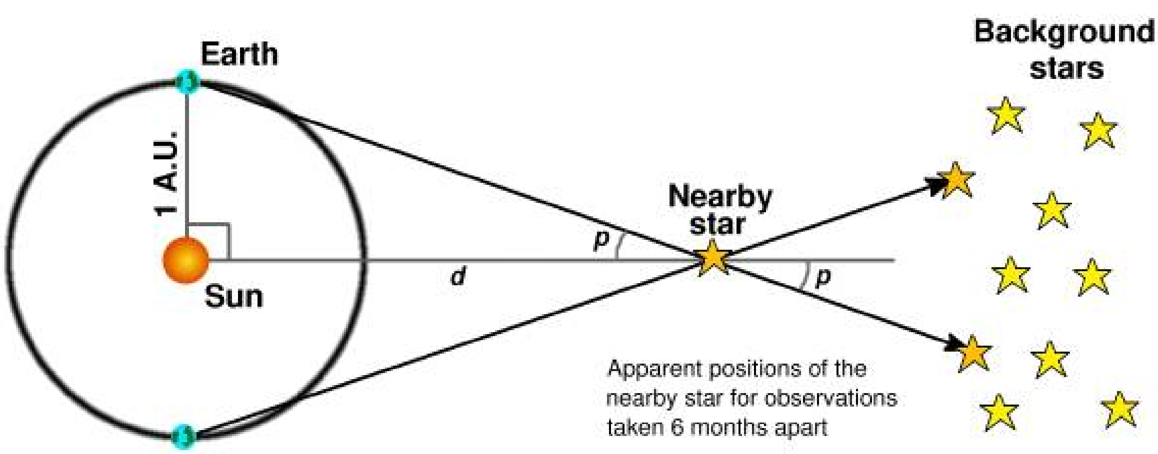


Figure 1.2: Illustration of the geometry involved in a parallax measurement to determine d, the distance to a nearby star.

9. Find the distance to the nearby star in a more direct way — measure the distance with a measuring tape or pieces of paper, or for more distant objects, find them on Google Maps. For this value, also measure this distance 3 times and find the value and uncertainty as in the previous step.

> **Notation for uncertain values**
> To be precise about how imprecise our measurements are, scientists often express the quantities as "$\langle$value$\rangle \pm \langle$uncertainty$\rangle$", followed by the units. For example, if you measured your average distance to be 3.3 meters, and your uncertainty of that distance to be 0.4 meters, then to express in a succinct way, you can write the distance as "3.3 ± 0.4 meters".

One way of describing how different two values are, without considering the uncertainty of those values, is to calculate the percent difference:

$$\text{percent difference} = \frac{d_1 - d_2}{\left(\frac{d_1+d_2}{2}\right)} \times 100\% \tag{1.3}$$

10. How close is your parallax distance measurement to the direct measurement? Report the percent difference.

In the next section, you will compare these values with each other using their uncertainties as well.

1.6 Measuring with more precise equipment

Using a finger for measuring angular separation is not very precise. Here you'll use the parallax technique to determine the same distance to the nearby star, but using a telescope, camera and analysis software instead.

You will now use the telescope apparatus to measure the parallax of a "foreground star" (actually a nearby building), with the Chicago skyline serving as "background stars."

11. Aim the telescopes at the same "nearby star" that you measured with the finger method. The white pillars look similar to each other, so be careful to aim at the same one. Ensure that the telescopes are at the same height, so we don't have to factor in vertical displacements.

12. Pick a point in the background to be your "background star". This can be different from the background star you used in the first method, and should be visible in both telescopes.

13. Take a picture from a digital camera (likely your phone camera) through the telescopes from the vantage point of each Earth position used above. Be careful to keep the camera still while taking the picture, and use the level to keep the phone level, also to ensure we don't need to consider vertical displacement.

14. Upload your photographs to a computer. The simplest way to do this may be to email the images to yourself from your phone. Give each file a descriptive name (e.g. "`parallax_left_telescope`").

To find the parallax angle using these images, we need to measure the angular separations in the images. A length in an image corresponds to an "angular distance" in degrees or radians, rather than a physical distance like meters. This is perfect for us, since we need to measure the angle. To measure these lengths, we will first find the pixel scale of the image — how much angular separation each pixel subtends.

Finding the pixel scale

Notice that with the finger, I told you that your little finger, outstretched, is about 1 degree wide. This was a conversion between the linear size of your finger, for example 1 cm, to an angular size, 1 degree. With the camera, we need to find out the similar conversion — how many pixels in an image corresponds to what angular size, also known as the *pixel scale* of the image. To do this, you can take a known angular size and measure its length in pixels.

In this case, the telescope that we are using has a known field of view (FOV) of 1.5°. This means that the width of the view (the diameter of the visible circle) subtends 1.5°. You will use an image analysis program to measure the FOV in pixels and find the pixel scale. These instructions are for using ImageJ, a free software tool, but you can use your own if you have a different preference.

15. Open ImageJ on a lab computer, or install it on your own computer (`https://imagej.nih.gov/ij/download.html`) and open it there.
16. Open your image using the File > Open... menu.
17. Select menu option Analyze > Measure to open the Results menu, which will give the results of your length measurements.
18. Select the line tool by clicking the 5th button from the left.
19. Click and drag on the image to draw a line that you want to measure.
20. Press "m" on your keyboard to take a measurement of that line. The length in pixels will be among the numbers given in the Results window.
21. Use this length to calculate the pixel scale in radians per pixel.

Finding the parallax angle

Now that you have the pixel scale of the image, you can use that to measure the parallax angle of the your nearby star and thus find the distance to that object like you did with the little finger method.

22. Open the first parallax image. Measure the pixel length from the nearby star to the background star (this length is zero if the two stars are completely overlapping). Convert the length to angular separation in radians using the pixel scale you found earlier. Repeat these measurements for the second parallax image using the same background star.
23. Find the total angular distance the star moved between images by subtracting the two separations. However, if the stars switched orientations, for example the background star switched from being on the left to being on the right of the nearby star, then you should add the two separations.
24. Divide this by two to get the parallax angle.
25. Use Equation 1.2 to find your new calculation for the distance to the nearby star.
26. Calculate the percent difference between this and the direct measurement like you did in the previous section.
27. Use the t' statistic described in Appendix B.3 to compare the two values and interpret the result — do these two ways of distance measurement really measure the same thing?

1.7 Report checklist and grading

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. Your group's agreements about communication.
2. The completed worksheet "The Parsec".
3. Work and final answer for your distance measurements using your finger, with uncertainty.
4. Work and final answer for direct distance measurement, along with percent difference between the two measurements.
5. A figure with your two parallax images.
6. The angular separations from background star to nearby star.
7. Final determined value of the distance and comparison with the direct distance using percent difference and t' statistic. Show your work (see Appendix A).
8. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

LAB 2

Galactic distances and the Hubble diagram

2.1 Introduction

In 1929, Edwin Hubble measured that distant galaxies were systematically redshifted relative to galaxies that were closer. From this data, Hubble inferred that the universe was expanding, an idea initially worked out by Georges Lemaitre using Einstein's theory of gravity.

In this lab, you will conduct a measurement similar to Hubble's and will produce your own version of his famous Hubble diagram shown below.

2.2 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

1. On Canvas, navigate to the People section, then to the "L2 Hubble" tab. Find a group that is not yet used, and have each person in your group add themselves to that same lab group.

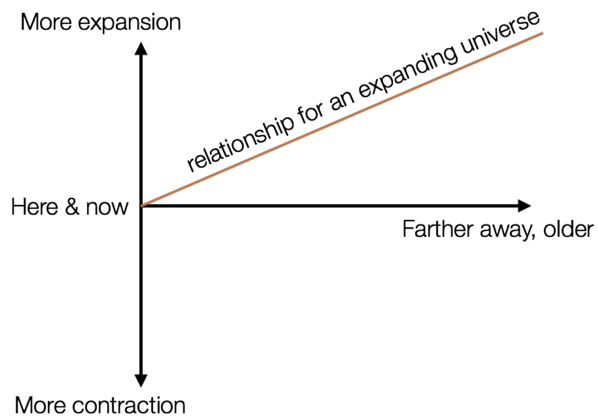


Figure 2.1: Schematic of a Hubble diagram plot. It illustrates the relationship between the expansion experienced by a photon and the distance of its emitter.

2.3 Building intuition

The graph in Figure 2.1 illustrates the impact of an expanding universe of photons emitted from distant objects. Because the speed of light is constant, photons that we measure today were emitted in the past, with photons originating from objects that are further away being emitted earlier in time. This means that photons from objects that are further away are older, and thus, those photons have experienced more expansion by the universe.

2. Sketch the plot from Figure 2.1, then sketch on the plot two more lines corresponding to the relationship for 1) a contracting universe, and 2) a static universe.

From this relationship, we can determine whether the universe is expanding, contracting, or static by looking at a number of galaxies and measuring their distance (corresponding to the horizontal axis) and the expansion experienced by their photons (corresponding to the vertical axis).

For this lab, we will use galaxy images and spectra listed in an online table.

2.4 Measuring distance

We will use geometry to measure the distance of our galaxies. Galaxies that are closer will look bigger and will subtend a larger angle whereas galaxies that are further will looks smaller and will subtend a smaller angle. This relationship between the angular size of the galaxy and its distance is illustrated in Figure 2.2.

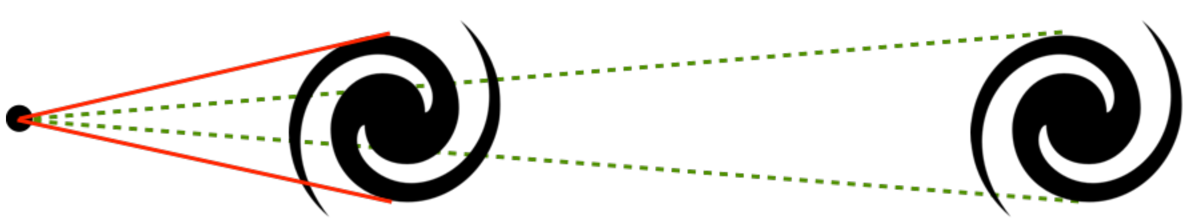


Figure 2.2: Looking from the dot on the left, there are two galaxies that are the same size, one further away than the other. The more distant galaxy subtends a smaller angle (dashed green lines) than the closer galaxy (solid red lines).

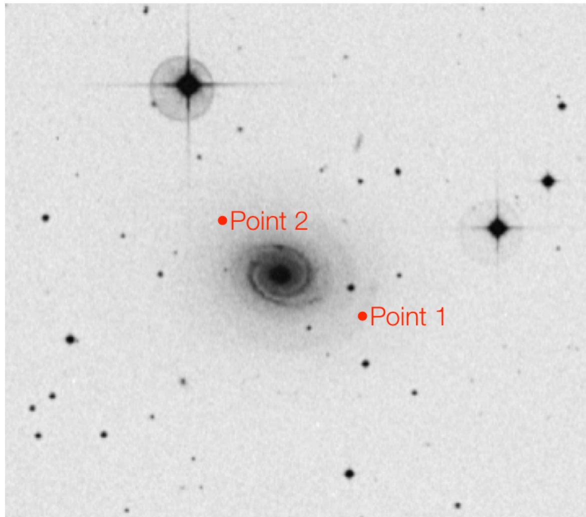


Figure 2.3: Example of galaxy image. Colors are inverted here, and Points 1 and 2 mark the furthest extent of the galaxy

Our galaxies will all be nearly the same size (22 kpc). Using the geometry shown in the illustration, we can arrive at the following relationship:

$$\text{angular size} = \frac{22\text{ kpc}}{\text{distance}}\,. \tag{2.1}$$

So, by measuring the angular size of our galaxy images, we can use the above equation to determine the distance to the galaxy.

From the Modules > Lab section of the Canvas site, download and extract to a folder `HubbleDataWebpage.zip`. In that folder, open `HubbleDataPage.html`. You will find a list of galaxy names. Click on the "Image" link for the first galaxy, NGC 1357. In the new tab, you will see an image of galaxy NGC 1357 similar to Figure 2.3.

3. What kind of galaxy is it (spiral, elliptical, unclear)? Note this in your spreadsheet.

4. Are there any noteworthy features in the image? Use your spreadsheet to record your answers.

We want to measure the angular size of NGC 1357, which you can do by measuring the angular separation between two appropriate points spanning the entire galaxy. In the lower left hand corner of google skymaps, google shows you the coordinates of your cursor. Record the coordinates (RA and DEC) for two points spanning the galaxy. Then use an online calculator (e.g. `http://cads.iiap.res.in/tools/angularSeparation`) to calculate the angular separation between the points. Be careful and make sure that you don't choose points that are too far outside or inside the galaxy image.

5. Enter the value for the angular size (in radians, not degrees) in a spreadsheet.

6. Divide up the remaining galaxy images between your groupmates and repeat Steps 3–5 for each of the galaxies recording your notes and measurements in the spreadsheet.

7. Once you have measured the angular size of all the galaxies, use Equation 2.1 to estimate the distance for each galaxy and record the values in a "distance" column.

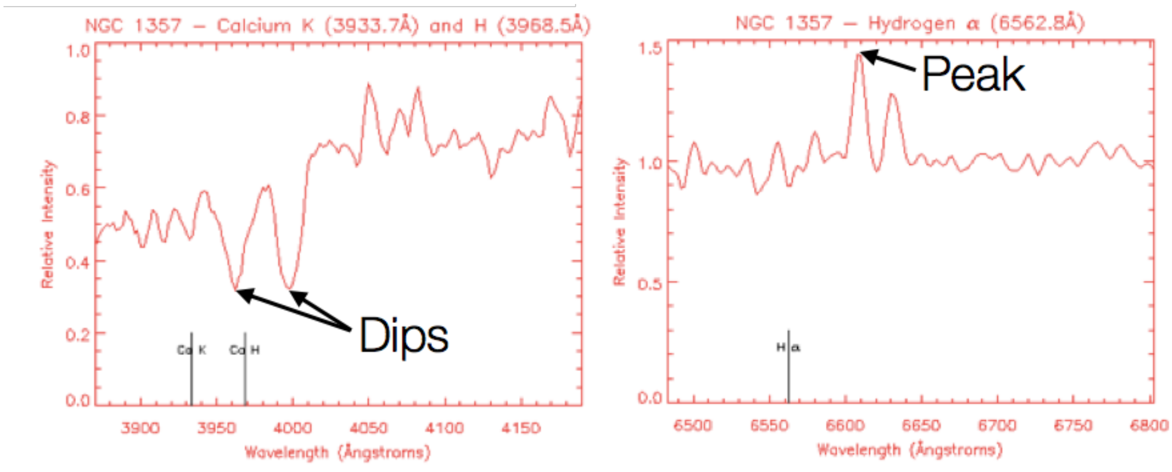


Figure 2.4: Spectrum of light detected from NGC 1357. The dips and peak that you will use to identify redshift are identified.

2.5 Measuring expansion

The wavelength of light changes as the universe expands, an effect known as cosmological redshifting. If the universe expands, the wavelength is stretched, becoming longer and redder. For a contracting universe, the wavelength will be compressed becoming bluer. We define the redshift, z, as

$$z = \frac{\lambda_{\text{measured}} - \lambda_{\text{original}}}{\lambda_{\text{original}}}, \tag{2.2}$$

where λ represents the wavelength. The redshift is a measure of how much the wavelength has been stretched or compressed.

We can measure the redshift by examining spectra (the energy emitted in different wavelengths) of the same galaxies we just measured. For NGC 1357, click on the link "Ca Spectra." The link will show spectra associated with Ca absorption which produces dips at wavelengths of 3933.7 angstroms and 3968.5 angstroms. You should see two prominent dips in the data. See Figure 2.4 for example spectra.

Go back to the galaxy list and click on the link "H-alpha spectrum" to bring up the spectra associated with Hydrogen alpha emission of light with wavelength 6562.8 Angstroms. You should see a clear peak in the data. The H-alpha peak is the leftmost of the distribution. Since we know the original wavelengths for these processes, we can compare the measured wavelength of these features with their original wavelength to determine how much the light has stretched.

8. For each of the spectra estimate the value for the two Ca dips and the H-alpha peak. Record the values for the two Ca absorption lines and the H-alpha emission line in the excel worksheet.

9. Use Equation 2.2 to calculate the redshift for each of the lines and take the average to estimate the redshift for the galaxy. Record the redshift in an "average redshift" column in the spreadsheet.

10. Repeat this process for each of your galaxies.

2.6 The Hubble diagram

11. Using the measurements in your worksheet, make a plot of redshift versus distance for your galaxies.

12. Is there a trend in your data? Is the trend clear?

13. Compare your plot with the sketches from Step 2. Does your data indicate that the universe is expanding? Contracting? Static? Why?

Hubble's constant H_0 gives a relation between the recessional speed v of an object and its distance D, according to the equation

$$v = H_0 D\,. \tag{2.3}$$

You will determine Hubble's constant from your data. You have the distances of the galaxies already. To find the velocities, multiply the redshift by the speed of light c,

$$v = zc\,. \tag{2.4}$$

This equation is valid for low values of redshift.

14. Make a Hubble diagram by plotting velocity (in km/s) vs. distance (in Mpc).

15. Use this plot and Equation 2.3 to fit a line to the data and find Hubble's constant, which should be the slope of that line. If you are doing a linear fit, you need to force the y-intercept of the line to be zero. You may also need to specify that the fit equation be displayed on the plot.

16. Do some research on the history and background of Hubble's measurement. Write a one paragraph summary of this lab and discuss the following:

    a) What was the historical context of Hubble's measurement?
    b) Why was it important?
    c) What did you do in this lab and how do your measurements and conclusions compare with Hubble's?

2.7 Report checklist and grading

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. Data table

2. Sketch of predicted relations for expanding, contracting, and static universes

3. Your Hubble diagram (velocity vs. distance)

4. Your Hubble constant

5. Answers to Questions 12, 13, and 16.

6. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

LAB 3

Is the universe getting... faster?

3.1 Introduction

In 1929, Edwin Hubble discovered that the universe was expanding. At the end of the 20 th century, astronomers made another stunning discovery associated with the expansion of the universe. In 1998 two independent projects obtained results showing that the expansion of the universe was accelerating, a result that eventually led to Nobel prizes for Perlmutter, Riess, and Schmidt in 2011. The two teams used the same technique: measuring the distance and redshift of Type Ia supernova. In this lab, you will work through some supernova data looking for evidence of the accelerated expansion of the universe.

A supernova is essentially an exploding star. This "explosion" produces a tremendous amount of light which then slowly fades over a period of weeks to months. Measuring the light associated with the supernova over time produces what is known as the supernova light curve. You can use the following link to build some intuition about supernovae and their light curves: https://youtu.be/TY6Y5_7xQ8o. An example of a supernova light curve is shown in Figure 3.1. The vertical axis is in "magnitudes," the standard astronomical measure of brightness where a larger magnitude corresponds to a fainter object.

Supernova are used to measure distance in a manner similar to how we measured distance in the Hubble lab. In the Hubble lab, we used the fact that objects that are father away look smaller, that is, they subtend a smaller angle on the sky. So, if we know the physical size of the object, we can measure the object's angular size and from that determine its distance. For supernova, astronomers utilize the fact that objects that are closer, look brighter, and objects that are farther away, look dimmer. So, if we know the intrinsic brightness of an object (called its absolute luminosity), we can then compare that to our measured brightness (called its apparent brightness) to determine the object's distance. This concept for measuring distance is illustrated in Figure 3.2 showing how an object of a known brightness (or size) looks fainter (smaller) when it is farther away.

The 1998 supernova teams studied Type Ia supernova because it is possible to use the shape of the supernova light curve to deduce the intrinsic brightness of the supernova. By comparing our inferred intrinsic brightness with our measured apparent brightness, we can then use the supernova to measure the distance.

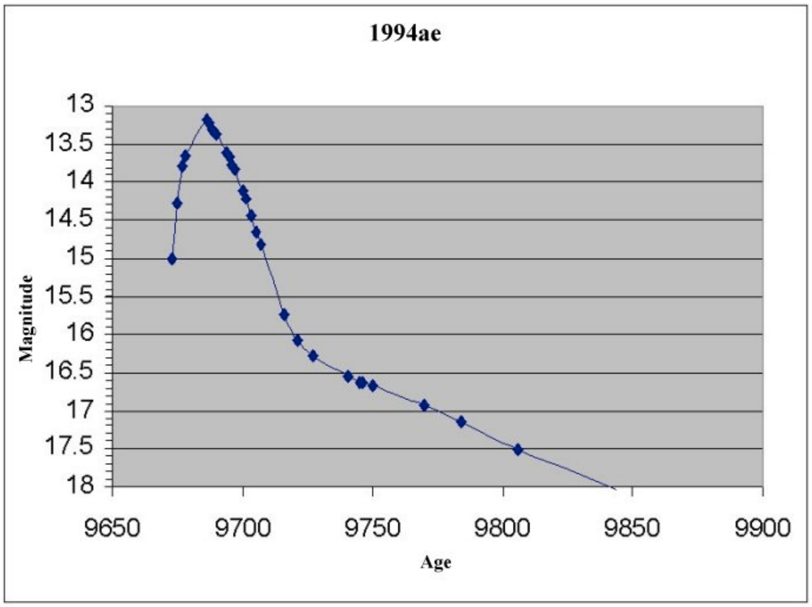


Figure 3.1: Light curve for supernova 1994ae. The age is given in days.

Figure 3.2: A method to determine the distance of an object. For objects with the same absolute luminosity, those are are further away will appear dimmer.

3.2 Team roles

Decide on roles for each group member. The available roles are:

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Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

3.3 Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L3 Faster [number]" that isn't used and have each person in your group add themselves to that same lab group.

3.4 Making a supernova Hubble diagram

The first activity for this lab is to make a Hubble diagram, like in the Hubble lab, using Type Ia supernovae. Then you will analyze your Hubble diagram to qualitatively measure cosmic acceleration. Your data is in the Excel spreadsheet (available on Canvas in the Lab module in the compressed file DarkEnergyLab.zip) named SNe_Reiss_2004, and consists of distance moduli and redshifts for 186 supernova from Reiss, et al., 2004.

2. Download DarkEnergyLab.zip from the location mentioned above and extract its contents into a directory on the computer.

As discussed above, the apparent magnitude of the supernova can be used as a measurement of distance if we know the intrinsic brightness (also called absolute brightness) of the supernova. In astronomy, we call the difference between apparent and absolute brightness the "distance modulus," μ (greek letter pronounced "myu" as in "music"), which is defined as

$$\mu = m - M \,, \tag{3.1}$$

where m is the apparent brightness (in magnitudes), and M is the absolute brightness (in magnitudes). The distance to the object is related to the distance modulus by

$$\text{distance} = 10^{(\mu-25)/5}\,\text{Mpc}\,. \tag{3.2}$$

3. In the spreadsheet, in a new column, calculate the distance for each supernova from the supernova's distance modulus.

After determining distances for objects, the next part of the Hubble diagram involves making a measurement of the expansion of space. As with the Hubble lab, we will estimate the expansion using the redshift distortion of an object's spectrum. Recall that we measured the redshift by looking for a specific spectral feature (either a dip or a peak) which occurs at a known wavelength. A larger redshift indicates a faster recession velocity.

A constant expansion rate for the universe means that the amount of expansion is proportional to the amount of time. Mathematically, we can write this as

$$\text{expansion} \propto \text{time}\,. \tag{3.3}$$

As expected, light emitted by more distant (and therefore older) objects will undergo more expansion. This expansion leads to the redshifting we discussed above. So, we can write that for a constant expansion rate, the relationship between redshift and distance is

$$\text{redshift} = \frac{H}{c} \times \text{distance}\,, \tag{3.4}$$

where c is the speed of light (300,000 km/s) and H is called the Hubble constant. This relation is known as "Hubble's Law." It describes a linear relationship between redshift and distance, the two axes of the Hubble diagram.

We can measure the expansion rate of the universe by using our Hubble diagrams and fitting the data to Hubble's law. The slope of the line that fits the data will give a measurement of the Hubble constant, H, which parameterizes the expansion rate of the universe.

4. In the spreadsheet, sort data from closest to furthest. Then separately fit linear relationships to the 40 nearest and 40 furthest supernovae to obtain two Hubble constant measurements (Note that in Equation 3.4, the y-intercept is set to zero, so be sure to do this in your fit as well). Plot both linear relationships, as well as the data being fit, on a single graph. In order to see both sets of data more easily, set both axes to log scale to make a log-log plot.

5. What value of the Hubble constant best fits the data for the 40 closest supernova?

6. What value of the Hubble constant best fits the data for the 40 farthest supernova?

7. How do these two results show that the expansion rate is accelerating?

8. If the expansion rate is accelerating, what do you predict you should see if you fit data for 40 supernovae at intermediate distances?

9. What value of the Hubble constant best fits the data for 40 intermediate distance supernovae? Add this to your plot.

3.5 Finding and measuring supernovae

Supernova are random events. In order to use supernova for cosmological studies, astronomers must find them first. In this final section of the lab, you will look at real data from the Dark Energy Survey. You will search for a supernova and measure its light curve. Dr. Daniel Scolnic, a former KICP fellow at the University of Chicago, has graciously provided the images for this section.

10. In the file you downloaded earlier, find the folder "SNe_search" and open the two image files SNe1_search.jpeg and SNe2_search.jpeg. Compare these two images. These images correspond to pictures taken of the same patch of sky on two different nights. A supernova is in one of the images. Can you see it?

In general, it is difficult to find supernova in a raw image of the sky because of all the other objects in the image. Take a moment to look at the different objects. Almost all of them correspond to stars and galaxies.

Since the majority of objects in an image of the sky are not supernovae, astronomers can try to remove them by generating a template image for that patch of sky and then subtracting it from the image. Once these non-supernova objects are removed (or mostly removed), it becomes easier to search for supernovae.

11. Open the file SNe1_template.jpeg and compare it to SNe1_search.jpeg.

SNe1_template.jpeg is the template file and it should look very similar (though not exactly the same) to SNe1_search.jpeg. Subtracting the two yields a "difference" image which is file SNe1_diff.jpeg.

12. Open SNe1_diff.jpeg.

You should notice two things, 1) most of the features are now gone, and 2) the subtraction is imperfect. The imperfect subtraction introduces some artifacts in the differenced image such as the spiderweb-like patterns and perfect geometric shapes (like uniformly dark or bright squares and circles). You will note that most of the artifacts from the imperfect subtraction are in locations where there were very bright objects in the original image.

A supernova in the difference images will look like a round solid blob that is very bright. Because a supernova is transient, the brightness of the blob will not be the same in all images. In fact, there should be a few images where there is no blob at all.

13. Open the two difference images SNe1_diff.jpeg and SNe2_diff.jpeg. Compare these two difference images and identify the supernova. Remember, the supernova will look like a bright solid round blob in one of the images.

Once you have found the supernova, the next step is to measure the supernova light curve. We will do this using the DS9 software on your computer or the lab computer.

Installing DS9 on your computer (optional)

You can download and install DS9 from https://sites.google.com/cfa.harvard.edu/saoimageds9/download. SAOImage DS9, or DS9 for short, is an image viewer, analyzer, and processor written and used by astronomers for working with astronomical images.

If you click the link to download, it might say "redirecting" while never actually redirecting. In this case, copy the link into the address bar directly.

For MacOS, unless you know otherwise, choose from the top set of choices (Aqua, rather than X11). To find your version, from the Apple menu in the corner of the screen, choose "About This Mac". If it displays a warning and prevents you from installed from an unidentified developer, follow the instructions at the following link to create an exception: https://support.apple.com/guide/mac-help/open-a-mac-app-from-an-unidentified-developer-mh40616/mac

14. Open DS9 and press the "File" button to bring up the file menu. Press "open" and open the file 1-25-2014.diff.fits. This is the original file for the image with the supernova above.

In DS9, you will need to press the "scale" button followed by the "zscale" button so that the image will appear properly. You can drag the box to look more closely at different parts of the image.

15. Play around with the settings associated with the zoom, scale, and color buttons and examine different parts of the image.

16. Find the supernova in this file (it is in the same location as in the earlier jpeg images). As you move the mouse around on top of the supernova, the "value" field will show you the value of the pixel underneath the mouse. Find the location on the supernova where this value is maximal and record the (X,Y) coordinates (the numbers for Image X and Y) below. We will use these (X, Y) coordinates as the coordinates for the supernova. Also record the pixel value.

17. To measure the supernova's light curve, load up each of the other *.fits files. Move the mouse to the (X, Y) coordinates you determined for the supernova and record the date and pixel value in your spreadsheet. Remember, the supernova is a transient object and it may not be present in all images. The filename tells you the date the image was taken.

18. In the spreadsheet software, make a plot of the pixel value versus time. Make sure you use the date of the image to put your data in chronological order and to determine the time between each of the images.

3.6 Report checklist and grading

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. Your Hubble diagram with all three sets of 40 supernovae and the three best-fit lines with their equations.

2. Your three determinations of the Hubble constant (near, far, and mid), with work showing how you found them.

3. Answers to questions 7 and 8.

4. The table and plot of your experimental supernova light curve.

5. Write one paragraph summarizing this lab. What is the significance and historical context of the discovery of cosmic acceleration (also called Dark Energy)? Why was it important? How does your analysis of the data in this lab demonstrate that the expansion rate of the universe is accelerating?

6. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

LAB 4

Taking the temperature of the universe

Introduction

In this experiment we will perform a measurement first carried out in the mid 1960s by Arno Penzias and Robert Wilson, for which they shared the 1978 Physics Nobel Prize for the discovery of the Cosmic Microwave Background Radiation.[1] We will measure the temperature of the Cosmic Microwave Background Radiation by comparing the power of this emission directly with the emission from thermal sources in the lab.

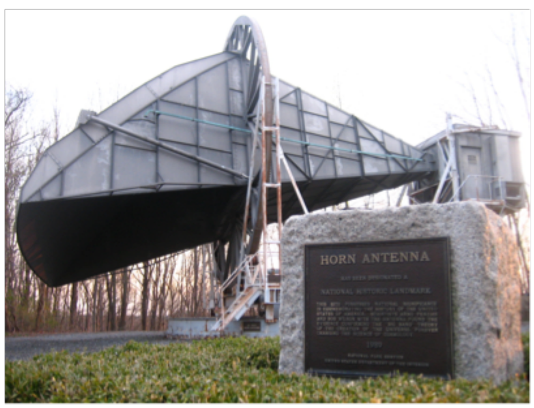

Figure 4.1: A photo of the horn antenna with which Arno Penzias and Robert Wilson discovered the Cosmic Microwave Background radiation. This lab uses a smaller horn working at a much higher frequency (shorter wavelength), so that the "beam patterns" are similar (recall the Small Radio Telescope lab and beam measurements to see why).

[1] A note on the Penzias and Wilson Nobel-prize winning measurement can be found at: http://www.bell-labs.com/project/feature/archives/cosmology/.

4.1 Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L4 Temp [number]" that isn't used and have each person in your group add themselves to that same lab group.

4.2 Measuring Temperatures with Thermal Radiation

We use the basic fact that at sufficiently long wavelengths, the power emitted by a perfect thermal emitter is proportional to the temperature of the emitter:

$$P = \alpha T \tag{4.1}$$

where P is the power emitted, α is a constant that depends on area, wavelength, and other factors, and T is the physical temperature of the emitter in Kelvin.

To measure the power we use an instrument that can measure the incoming radiation power and it has input optics arranged so that it "views" a small range of angles in front of the instrument. This is the basis of a radiometer. You can imagine this as a reverse flashlight. In the case of the flashlight, the filament of the light bulb is hot and emits through an arrangement of mirrors to form a narrow beam. For the radiometer, the "heat radiation" from a narrow range of angles funnels into the instrument and the power is detected and the amount of power read out on a meter.

In practice, the receiver used to measure radiation is not perfect and generates some 'noise' power of its own. So even if we pointed the receiver at an object with $T = 0$, it would still measure some power. This extra power is denoted in terms of the temperature, $T_{\rm rx}$, you would measure if you looked at a zero temperature emitter and did not correct for this fact:

$$P = \alpha(T + T_{\rm rx}). \tag{4.2}$$

To compute temperature of an emitter, T, from the power, P, measured by receiver, we need to know α and $T_{\rm rx}$. It would be hard to calculate the factor α from first principles, especially because in order to be useful the calculation must include all of the details of the measuring equipment. However, it is straightforward to obtain an accurate measurement of α experimentally by measuring the power for different temperatures.

We will determine the factor α by placing first an emitter of one known temperature in front of the radiometer, noting the power reading on the meter, then placing a different emitter with a different known temperature in front of the radiometer and noting the new power level. According to equations 4.1 and 4.2, these two measurements can be used to calculate α:

$$\alpha = \frac{P_1 - P_2}{T_1 - T_2} \tag{4.3}$$

Once α is calculated, we can solve for $T_{\rm rx}$ from measurement of one of the known temperature emitters:

$$T_{\rm rx} = \frac{P_1}{\alpha} - T_1 \tag{4.4}$$

With α and $T_{\rm rx}$ measured, we can "point" the radiometer anywhere and remotely measure the temperature of any emitter corresponding to the received power P by solving eq. 4.2 for T,

$$T = \frac{P}{\alpha} - T_{\rm rx} \tag{4.5}$$

Note that when you will be measuring the temperature of the "hot" load you will need to convert the temperature in degrees Celsius or Fahrenheit it reports into Kelvins.

4.3 The receiver and experimental set up

Figure 4.2 shows the experimental set up you will be using in this lab. The 30 GHz receiver is enclosed in a cryostat enclosure that keeps it very cold to reduce its noise. It is mounted on a cart in a contraption that allows to change its pointing for different elevations. Radiation measured by the receiver enters through receiver window at the top of the enclosure. The power measured by the receiver is displayed in digital form on the power readout screen. Pointing of the receiver can be changed using the elevation control lever and current elevation can be read on the electronic angle meter mounted on the enclose (this meter should be calibrated at the zero angle at the beginning of the lab with the help of your TA).

Figure 4.3 shows details of the receiver when it is taken out of the cryostat enclosure.

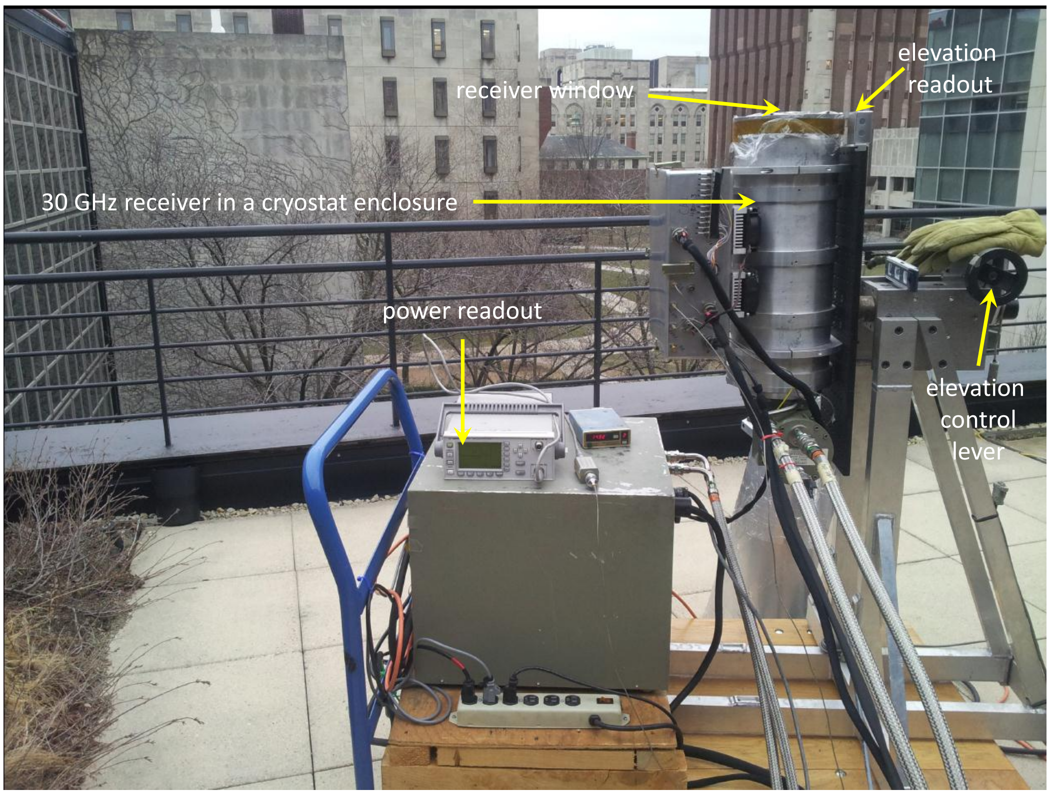

Figure 4.2: Photo of the 30 GHz receiver enclosed in a cryostat enclosure mounted on the cart. This set up on a patio of KPTC 312 will be used in this lab for measurements of temperature of the sky.

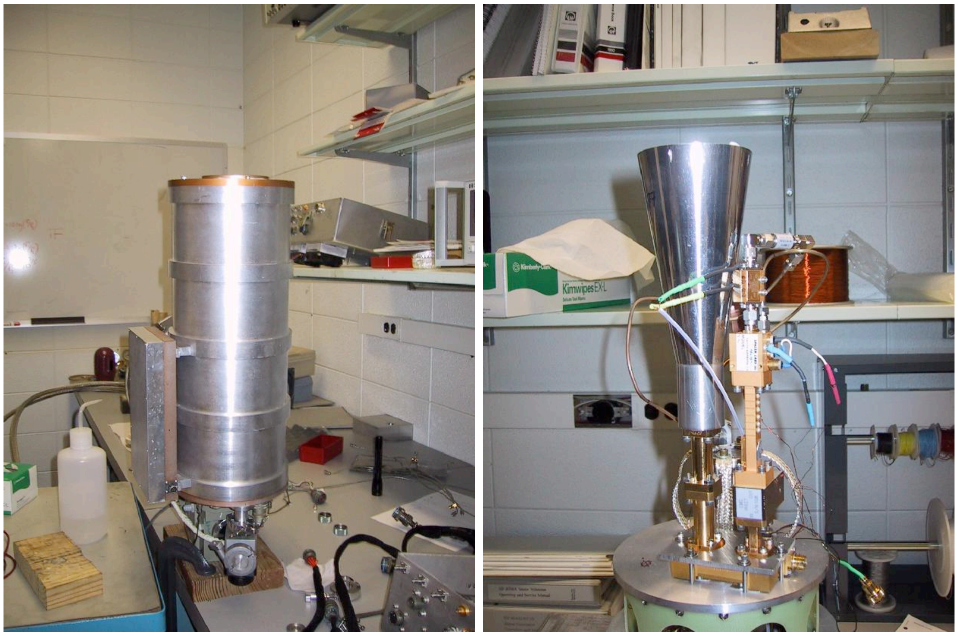

Figure 4.3: *Left:* The CMB lab 30 GHz receiver closed in the lab. *Right:* A photo of the inside of the cryostat showing the horn and electronics of the receiver. The outer shell of the cryostat provides the vacuum seal for the interior components that are cooled to roughly 15 K (-258 C) inside of thermal radiation shields (not shown) to lower the noise generated by the electronics, and even the noise due to the radiation from the horn. The electronics amplify the signal by roughly a factor of 10,000. By cooling and using a specially designed amplifier for this research, the receiver only adds a small amount of noise, roughly equivalent to that which would result by placing 10 K at the input of an idealized noiseless receiver (if only such things existed!). This receiver was built here at the university and is also used to at the CARMA radio astronomy observatory, see `http://www.mmarray.org/`, for which The University of Chicago is a partner. It is one of a handful of the most sensitive 30 GHz receivers in the world.

4.4 Measuring the Temperature of Items in the Lab

We will first measure the temperature of several items using the radiometer.

Some things you could measure:

1. The wall or ceiling
2. The sky through the door of the lab
3. Your cupped hands above the receiver or your face

For each item, you should make three or more measurements quickly in succession of the cold load, the hot load, and the source for which you want to measure temperature. The hot load is a piece of material that is a good emitter and the temperature is near room temperature and is measured with a a digital thermometer located at the top of the load (see left panel of Fig. 4.4). The cold load is a similar material that is first immersed completely in liquid Nitrogen (which has temperature $T_{\mathrm{cold}} = 77.4$ K),[2] as shown in the middle panel of Fig. 4.4. When you are ready to make a measurement with the cold load, take it out of the dewar, let the liquid Nitrogen drip for $\sim$5 seconds and place it in front of the

[2]It is important that the entire cone is immersed in the liquid Nitrogen or else the effective temperature of the load will be poorly determined and this will lead to incorrect measurement of the source temperature.

receiver window. Be careful to fully cover the window, but do not rest the cold load on the cryostat. Then comes the source of unknown temperature. For each of these objects you will need to read the power reading on the power meter attached to the receiver (see Fig. 1).

Find the unknown temperature using equation 4.2 to find α, equation 4.4 to find T_{rx}, with T_1 and P_1 being the temperature and power for the hot load, T_2 and P_2 being the temperature and power for the cold load. Then using equation 4.5, find the unknown temperature using the power measured while the unknown was in front of the receiver.

Figure 4.4: *Left:* The "hot" load with digital thermometer at the top. *Center:* "cold" load submerged in a dewar with liquid Nitrogen. Make sure that the copper part of the load is fully submerged with the liquid surface at a wooden part of the load. *Right:* ground shield which will be used to minimize radiation from nearby buildings and the ground when measuring radiation from the sky.

4.5 Measuring the Temperature of the CMB

Measuring the temperature of the CMB is conceptually the same as measuring the temperature of the items in the lab. We will compare the power we get from the sky to the hot and cold loads.

Because the CMB temperature is very low (just a few degrees K) we have to take a more careful account of a few things than we did for the previous section. First, we will have to get T_{rx} pretty accurately because the CMB temperature, T_{CMB} is less than T_{rx}. Second, there are hot sources (compared to the CMB) all around the experiment. You can imagine that even a small bit of one of these hot sources in part of the beam would change the oncoming power a lot and cause a big change in what we infer for the CMB temperature.

We will also have to worry about two other sources. One is the hot ground and buildings around us. We will guess about these sources by placing a ground shield on the receiver, which keeps the ground emission out of the input (see right panel of Fig. 4.4 to see how the ground shield looks like). We will check how much difference this makes and subtract that source if it seems to be an important factor.

The second, and more difficult extraneous source to remove is emission from our own atmosphere. While the atmosphere is *almost* transparent (and therefore almost non-emissive), it is not perfect. What's more, the main emitter is water-vapor which varies a lot. There is not much water vapor on those clear, crisp days when the sky is deep blue. On the other hand, when it's wet and cloudy, there is a lot of water vapor, and consequently, if we point our radiometer up at the sky we will get a hotter temperature on humid or wet days.

As you also know water vapor is not well mixed in our atmosphere (e.g. clouds), so on a poor day you will see the output power of the radiometer changing quickly as the winds blows different blobs of water vapor in front of the radiometer. Try comparing the stability of the radiometer when staring at the warm load compared to staring at the sky. On a good day the power when staring at the sky is as stable as that when staring at the load.

How can we estimate the amount of power coming from the atmosphere when we point the radiometer at the sky? We need to do this to get a good estimate of T_{CMB}. We do this by measuring the effective temperature as a variety of angles from the vertical. We can calculate how much atmosphere we are going through for each angle and from that, extrapolate the temperature we would get if were looking through zero atmosphere. The atmosphere contribution approximately follows

$$T_{\rm atm}(z) = T_0 A(z) \tag{4.6}$$

where $A(z)$ is the number of airmasses you are looking through. $A(z) = \sec(z)$, where $\sec(z) = 1/\cos(z)$ is the secant function – reciprocal of the cosine function and is equal to 1 when looking straight up ($z = 0$). T_0 is the atmosphere temperature at the zenith and z is zenith angle (the angle between straight up and where the radiometer is looking). The total temperature we will measure when looking at the sky (after subtracting $T_{\rm rx}$ according to equation 4.5) will be

$$T_{\rm total} = T_{\rm CMB} + T_{\rm atm}(z) \tag{4.7}$$

To get an estimate of $T_{\rm CMB}$, we will measure the $T_{\rm total}$ at a variety of zenith angles. We then plot $T_{\rm total}$ vs $A(z)$ to get a straight line for $A(z)$ going from 1 to about 2 (z going from 0 to 60°). Then we can extrapolate the straight line to $A = 0$ and read off $T_{\rm CMB}$.

A flow chart of the measurement strategy is shown in Figure 4.5. It is important to understand the quality of the data as you take it – your TA will help you with this. There are many reasons the data could be corrupted, e.g., poor cold load temperature if not well immersed in liquid Nitrogen, the radiometer not being leveled, not centering the loads on the window or aligning the reflecting shield along the beam, buildings in the way, poor and unstable atmospheric transmission, etc. Good zenith angles to give well sampled air mass measurements are: 0^o, 15^o, 25^o, 35^o, 40^o, 45^o, 48^o, 52^o, 54^o, 56^o, 58^o, 60^o, although you may find that the buildings do not allow good results at the higher zenith angles, even after correcting for the sidelobe response (see below). Be sure to measure T_{total} at each of the zenith angles with the hot load – cold load – hot load method as shown in the flow chart.

Because the receiver has some sensitivity to emission from angles far outside of its beam referred to as the beam sidelobe response (and at extreme angles as the far sidelobe response), it is important to account for the excess power picked up by the sidelobe response from the warm buildings, the ground and also the Sun. We do this by making measurements at each zenith angle with the "ground shield" (see right panel of Fig. 4.4). Once the sky and load measurements are obtained at a given zenith angle, take several measurements in quick succession of the power on the sky with the ground shield off and placed along the beam axis. If the shield is well aligned (use two spotters to make sure it is), then the receiver output power will be lower when the shield is in place, especially at lower elevations (higher zenith angles) due to emission from the warm buildings. Determine the percentage change to the sky power for each zenith angle. You then correct the power in Eq. 4.5 before calculating the T_{total} for Eq. 4.7.

Note, because of the sidelobe response, it is important that everyone is far from beam of the receiver when you are taking measurements. It is best for everyone to stay behind the plane defined by the front of the receiver.

During the measurements, different students in the lab will take on a particular role to handle hot or cold load, read off power, put on ground shield, record the data.

2. Your measurements should be recorded in a table, which has elevation, power for the cold load, power for the hot load, temperature of the hot load read off at each measurement, power of the sky without shield, power of the sky with shield. You will take the measurements jointly but will analyze them to measure $T_{\rm CMB}$ on your own.

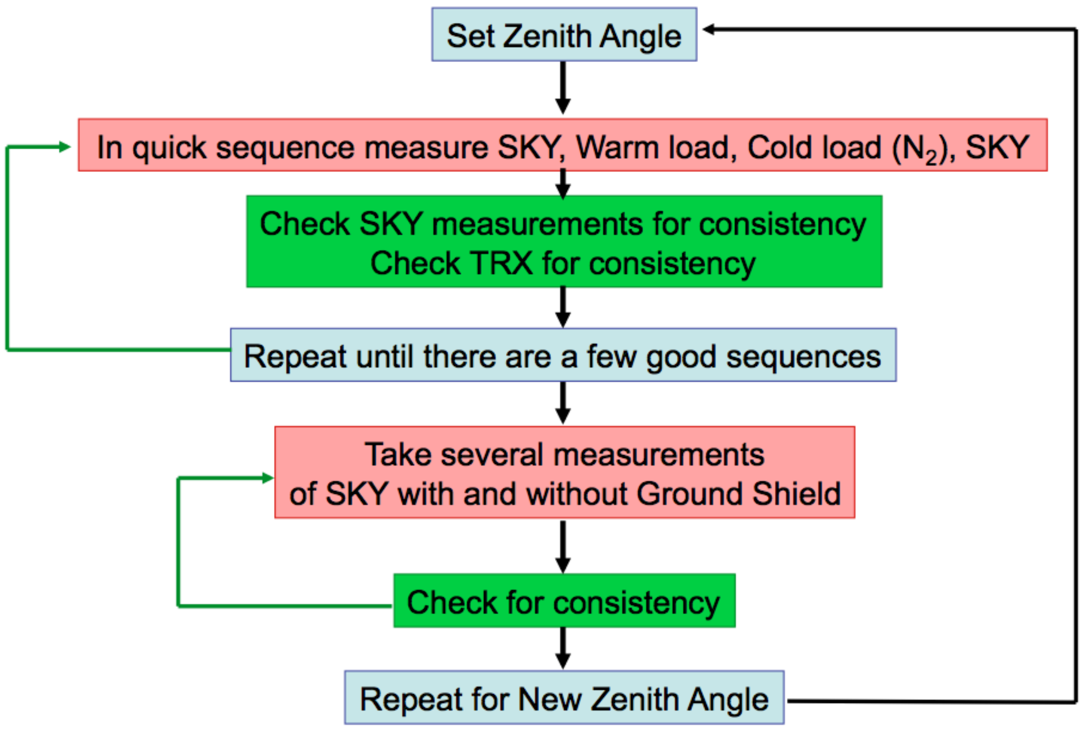

Figure 4.5: Flow chart for taking sky data. It is important to make sure you obtain consistent results for each elevation angle. The ratio of the power on ambient to liquid nitrogen load should be very stable, since the scale factor α in Eq. 4.1 cancels in the ratio. If the ratio is not stable there is a problem with the instrument (or liquid nitrogen load was not full immersed) and you will not get reliable results for Cosmic Microwave Background results. The ratio of the output power for the ambient load to the nitrogen cooled load should be about 3.4 to 3.6, depending on the temperature of the ambient load. The typical receiver temperature should be around 11 K. On a cloudy day the sky measurements may not be stable enough to achieve reliable results. Why is that? *Hint, light at cm wavelengths is also known as microwave radiation. What is it that your microwave oven is heating up? And, if it absorbs the microwave radiation, then it must also emit it.*

4.6 Analysis

3. Plot your data, T_{Sky} vs. airmass, with airmass on the horizontal axis. Airmass is defined as

$$A(z) = \sec(z), \tag{4.8}$$

   where z is the elevation angle.

4. Fit your data to a line and record the slope and intercept for the line. *Tip: double check your units. For elevation angle, we usually measure angles in degrees, but most calculators expect units of radians. Also, we will measure all temperatures in Kelvin.*

4.7 Report checklist and grading

Each item below is worth 10 points.

1. Data and determination of temperature of two items inside the lab.
2. Data table from Step 2.
3. The plot and fit parameters from Steps 3 and 4.
4. Does your data agree with the model where the emission is dominated by the atmosphere? What value to get for any residual temperature (T_{CMB}) and how do you interpret it?
5. What are some of the uncertainties associated with your measurement? What steps did you take to measure or mitigate these uncertainties?
6. Write a paragraph discussing the quality of your data and the historical significance of the Penzias and Wilson measurement. Include a discussion of the following: What was the context of the Penzias and Wilson measurement? Why was this measurement important? What were some of the competing cosmological theories at that time and how did the Penzias and Wilson measurement fit in to that context?
7. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

LAB 5

What is the universe made of?

5.1 Introduction

In this lab, you will analyze the fluctuations of the cosmic microwave background (CMB) to determine the proportion of normal matter, dark matter, and dark energy in the universe, as well as finding its age and curvature.

For introductory background on the CMB and fluctuations in it, see this online book chapter: `https://openstax.org/books/astronomy/pages/29-4-the-cosmic-microwave-background`

5.2 Team roles

Decide on roles for each group member. The available roles are:

- Facilitator: ensures time and group focus are efficiently used
- Scribe: ensures work is recorded
- Technician: oversees apparatus assembly, usage
- Skeptic: ensures group is questioning itself

These roles can rotate each lab, and you will report at the end of the lab report on how it went for each role. If you have fewer than 4 people in your group, then some members will be holding more than one role. For example, you could have the skeptic double with another role. Consider taking on a role you are less comfortable with, to gain experience and more comfort in that role.

Additionally, if you are finding the lab roles more restrictive than helpful, you can decide to co-hold some or all roles, or think of them more like functions that every team needs to carry out, and then reflecting on how the team executed each function.

5.3 Add members to Canvas lab report assignment group

1. On Canvas, navigate to the People section, then to the "Groups" tab. Scroll to a group called "L5 Made [number]" that isn't used and have each person in your group add themselves to that same lab group.

5.4 Steps

2. Open the CMB Simulator at https://chrisnorth.github.io/planckapps/Simulator/
3. Click on the info button at the top right (gray circle with an 'i') and read this to get an understanding of what you are looking at.

Displayed on the right is an image of the CMB fluctuations as produced by a simulation. In the upper right, the simulation matches the properties of the experimentally determined CMB. On the lower right is the simulation run with the parameters set by the sliders on the left. Below the image are other properties of your simulated universe.

4. Using the image, design a procedure to measure the fundamental scale — that is, quantitatively answer the question: "generally how big are the blobs", in degrees. In the scale of the image, the circle in the lower right corner of the image is 1° in diameter. **Record your procedure.**
5. For three different slider configurations (click the refresh button in the upper right to generate random configurations), use your procedure to determine the fundamental scale. **Record screenshots of the images and sufficient data to follow along with your work.**
6. Determine how close your result is to the scale listed below the image by calculating the percent difference and **record your result**.
7. Click the power spectrum button (upper right corner of page, second button from the right) to display the power spectrum for your simulated universe. **Describe what feature of the plot corresponds to the fundamental scale you found.**
8. Determine how moving each slider affects the power spectrum and image. **Record your findings of how the plot and image change.** *For example, does it shift any parts of the spectrum in different directions or expand any parts? Do parts of the image get brighter or dimmer or fuzzier?*
9. The apparent need for dark matter was not widely accepted by astronomers before 1980 or so. Set dark matter (Ω_{C}) and dark energy (Ω_{Λ}) to zero. Assuming only normal matter exists and adjusting Ω_{b}, how close can you make your simulated universe match the real one (what percentage)? **Record your finding.**
10. Now, assuming normal and dark matter can exist, but not yet dark energy, how close can you make your simulated universe match the real one (what percentage)? **Record your finding.**
11. The first direct evidence for dark energy was found in 1998. Now using all three sliders, how close can you make your simulated universe match the real one (what percentage)? **Record your finding.**
12. Based on your simulation studies here, what is the age and curvature of the universe?

5.5 Testing the Big Bang Theory

Here is a hypothetical observation that is *not* predicted by the Big Bang theory:[1]

> "evidence for an increase in the cosmic microwave background temperature with time"

13. Imagine what would happen if it were actually observed — whether it could be explained with the existing Big Bang theory, could be explained with a revision to the Big Bang theory, or would force us to abandon the Big Bang theory. **Write down your team's reasoning.**

[1] This discussion is from Bennett, Donahue, Schneider, Voit, The Cosmic Perspective, 9th ed. (2020)

5.6 Report checklist and grading

Include the following in your lab report. See Appendix A for formatting details. Each item below is worth 10 points.

1. Procedure for measuring the fundamental scale (Step 4)
2. Screenshots and determination of fundamental scale for three different universes, with percent differences compared to the displayed scale (Step 5–6)
3. Description of how the parameters affect the power spectrum (Step 8)
4. Matching the real universe using different parameter constraints and reporting your determination of the age and curvature of the universe (Steps 9–12)
5. Conclusion and reasoning for effects of hypothetical observation on Big Bang theory (Step 13)
6. Write a 100–200 word reflection on group dynamics. Address the following topics: who did what in the lab, how did you work together, how group roles functioned, what successes and challenges in group functioning did you have, and what might you want to do differently next time?

APPENDIX A

Lab Report Format

A.1 General

- The report should be typed for ease of reading. Text should be double-spaced, and the page margins (including headers and footers) should be approximately 2.5 cm, for ease of marking by the grader. Each page should be numbered.
- The first page should include the title of the lab; lab section day, time, and number; and the names of the members of your lab team.

A.2 Organizing the report

The report should follow the sequence of the report checklist. Answers to questions and inclusion of tables and figures should appear in the order they are referenced in the manual. In general, include the following:

- For any calculations that you perform using your data, and the final results of your calculation, you must show your work in order to demonstrate to the grader that you have actually done it. Even if you're just plugging numbers into an equation, you should write down the equation and all the values that go into it. This includes calculating uncertainty and propagation of uncertainty.
- If you are using software to perform a calculation, you should explicitly record what you've done. For example, "Using Excel we fit a straight line to the velocity vs. time graph. The resulting equation is $v = (0.92\,\mathrm{m/s^2})t + 0.2\,\mathrm{m/s}$."
- Answers to any questions that appear in the lab handout. Each answer requires providing justification for your answer.

A.3 Graphs, Tables, and Figures

Any graph, table, or figure (a figure is any graphic, for example a sketch) should include a caption describing what it is about and what features are important, or any helpful orientation to it. The reader should be able to understand the basics of what a graph, table, or figure is saying and why it is important without referring to the text. For more examples, see any such element in this lab manual.

Each of these elements has some particular conventions.

Tables

A table is a way to represent tabular data in a quantitative, precise form. Each column in the table should have a heading that describes the quantity name and the unit abbreviation in parentheses. For example, if you are reporting distance in parsecs, then the column heading should be something like "distance (pc)". This way, when reporting the distance itself in the column, you do not need to list the unit with every number.

Graphs

A graph is a visual way of representing data. It is helpful for communicating a visual summary of the data and any patterns that are found.

The following are necessary elements of a graph of two-dimensional data (for example, distance vs. time, or current vs. voltage) presented in a scatter plot.

- **Proper axes.** The conventional way of reading a graph is to see how the variable on the vertical axis changes when the variable on the horizontal axis changes. If there are independent and dependent variables, then the independent variable should be along the horizontal axis.
- **Axis labels.** The axes should each be labeled with the quantity name and the unit abbreviation in parentheses. For example, if you are plotting distance in parsecs, then the axis label should be something like "distance (pc)".
- **Uncertainty bars.** If any quantities have an uncertainty, then these should be represented with so-called "error bars", along both axes if present. If the uncertainties are smaller than the symbol used for the data points, then this should be explained in the caption.

APPENDIX B

Analysis of Uncertainty

A physical quantity consists of a value, unit, and uncertainty. For example, "$5 \pm 1\,\mathrm{m}$" means that the writer believes the true value of the quantity to most likely lie within 4 and 6 meters[1]. Without knowing the uncertainty of a value, the quantity is next to useless. For example, in our daily lives, we use an implied uncertainty. If I say that we should meet at around 5:00 pm, and I arrive at 5:05 pm, you will probably consider that within the range that you would expect. Perhaps your implied uncertainty is plus or minus 15 minutes. On the other hand, if I said that we would meet at 5:07 pm, then if I arrive at 5:10 pm, you might be confused, since the implied uncertainty of that time value is more like 1 minute.

Scientists use the mathematics of probability and statistics, along with some intuition, to be precise and clear when talking about uncertainty, and it is vital to understand and report the uncertainty of quantitative results that we present.

B.1 Types of measurement uncertainty

For simplicity, we limit ourselves to the consideration of two types of uncertainty in this lab course, instrumental and random uncertainty.

Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the precision of the instrument. Usually this value is taken as a half of the smallest increment of the instrument's scale. For example, 0.5 mm is the precision of a standard metric ruler; 0.5 s is the precision of a watch, etc. For electronic digital displays, the equipment's manual often gives the instrument's resolution, which may be larger than that given by the rule above.

Instrumental uncertainties are the easiest ones to estimate, but they are not the only source of the uncertainty in your measured value. You must be a skillful experimentalist to get rid of all other sources of uncertainty so that all that is left is instrumental uncertainty.

[1] The phrase "most likely" can mean different things depending on who is writing. If a physicist gives the value and does not given a further explanation, we can assume that they mean that the measurements are randomly distributed according to a normal distribution around the value given, with a standard deviation of the uncertainty given. So if one were to make the same measurement again, the author believes it has a 68% chance of falling within the range given. Disciplines other than physics may intend the uncertainty to be 2 standard deviations.

Random uncertainties

Very often when you measure the same physical quantity multiple times, you can get different results each time you measure it. That happens because different uncontrollable factors affect your results randomly. This type of uncertainty, random uncertainty, can be estimated only by repeating the same measurement several times. For example if you measure the distance from a cannon to the place where the fired cannonball hits the ground, you could get different distances every time you repeat the same experiment.

For example, say you took three measurements and obtained 55.7, 49.0, 52.5, 42.4, and 60.2 meters. We can quantify the variation in these measurements by finding their standard deviation using a calculator, spreadsheet (like Microsoft Excel, LibreOffice Calc, or Google Sheets), or the formula (assuming the data distributed according to a normal distribution)

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}}\,, \tag{B.1}$$

where $\{x_1, x_2, \ldots, x_N\}$ are the measured values, $\bar{x}$ is the mean of those values, and N is the number of measurements. For our example, the resulting standard deviation is 6.8 meters. Generally we are interested not in the variation of the measurements themselves, but how uncertain we are of the average of the measurements. The uncertainty of this mean value is given, for a normal distribution, by the so-called "standard deviation of the mean", which can be found by dividing the standard deviation by the square root of the number of measurements,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}\,. \tag{B.2}$$

So, in this example, the uncertainty of the mean is 3.0 meters. We can thus report the length as 52 ± 3 m.

Note that if we take more measurements, the standard deviation of those measurements will not generally change, since the variability of our measurements shouldn't change over time. However, the standard deviation of the mean, and thus the uncertainty, will decrease.

B.2 Propagation of uncertainty

When we use an uncertain quantity in a calculation, the result is also uncertain. To determine by how much, we give some simple rules for basic calculations, and then a more general rule for use with any calculation which requires knowledge of calculus. Note that these rules are strictly valid only for values that are normally distributed, though for the purpose of this course, we will use these formulas regardless of the underlying distributions, unless otherwise stated, for simplicity.

If the measurements are completely independent of each other, then for quantities $a \pm \delta a$ and $b \pm \delta b$, we can use the following formulas:

$$\text{For } c = a + b \text{ (or for subtraction), } \delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{B.3}$$

$$\text{For } c = ab \text{ (or for division), } \frac{\delta c}{c} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \tag{B.4}$$

$$\text{For } c = a^n,\ \frac{\delta c}{c} = n\frac{\delta a}{a} \tag{B.5}$$

For other calculations, there is a more general formula not discussed here.

Expression	Implied uncertainty
12	0.5
12.0	0.05
120	5
120.	0.5

Table B.1: Expression of numbers and their implied uncertainty.

What if there is no reported uncertainty?

Sometimes you'll be calculating with numbers that have no uncertainty given. In some cases, the number is exact. For example, the circumference C of a circle is given by $C = 2\pi r$. Here, the coefficient, 2π, is an exact quantity and you can treat its uncertainty as zero. If you find a value that you think is uncertain, but the uncertainty is not given, a good rule of thumb is to assume that the uncertainty is half the right-most significant digit. So if you are given a measured length of 1400 m, then you might assume that the uncertainty is 50 m. This is an assumption, however, and should be described as such in your lab report. For more examples, see Table B.1.

How many digits to report?

After even a single calculation, a calculator will often give ten or more digits in an answer. For example, if I travel 11.3 ± 0.1 km in 350 ± 10 s, then my average speed will be the distance divided by the duration. Entering this into my calculator, I get the resulting value "`0.0322857142857143`". Perhaps it is obvious that my distance and duration measurements were not precise enough for all of those digits to be useful information. We can use the propagated uncertainty to decide how many decimals to include. Using the formulas above, I find that the uncertainty in the speed is given by my calculator as "`9.65683578099600e-04`", where the '`e`' stands for "times ten to the". I definitely do not know my uncertainty to 14 decimal places. For reporting uncertainties, it general suffices to use just the 1 or 2 left-most significant digits, unless you have a more sophisticated method of quantifying your uncertainties. So here, I would round this to 1 significant digit, resulting in an uncertainty of 0.001 km/s. Now I have a guide for how many digits to report in my value. Any decimal places to the right of the one given in the uncertainty are distinctly unhelpful, so I report my average speed as "0.032 ± 0.001 km/s". You may also see the equivalent, more succinct notation "0.032(1) km/s".

B.3 Comparing two values

If we compare two quantities and want to find out how different they are from each other, we can use a measure we call a t' value (pronounced "tee prime"). This measure is not a standard statistical measure, but it is simple and its meaning is clear for us.

Operationally, for two quantities having the same unit, $a \pm \delta a$ and $b \pm \delta b$, the measure is defined as[2]

$$t' = \frac{|a-b|}{\sqrt{(\delta a)^2 + (\delta b)^2}} \tag{B.6}$$

If $t' \lesssim 1$, then the values are so close to each other that they are indistinguishable. It is either that they represent the same true value, or that the measurement should be improved to reduce the uncertainty.

If $1 \lesssim t' \lesssim 3$, then the result is inconclusive. One should improve the experiment to reduce the uncertainty.

If $t' \gtrsim 3$, then the true values are very probably different from each other.

[2]Statistically, if δa and δb are uncorrelated, random uncertainties, then t' represents how many standard deviations the difference $a - b$ is away from zero.