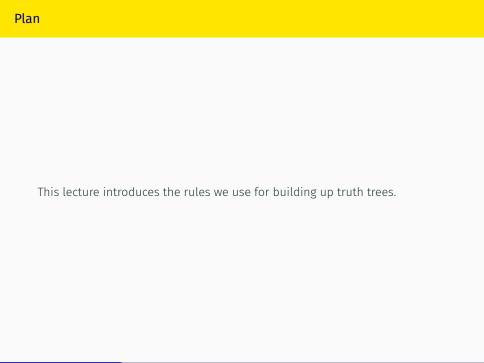
305 Lecture 21 - Rules for Truth Trees

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The rules tell you what new lines to write down given the lines you've already got.

- · To some extent they simply have to be memorised.
- · But hopefully they are all (except for the rules about ightarrow) fairly intuitive.

Rules for ¬



Rules for ¬

$$\frac{\mathsf{T} \neg A}{\mathsf{F} A} \neg \mathsf{T} \qquad \frac{\mathsf{F} \neg A}{\mathsf{T} A} \neg \mathsf{F}$$

Rules for ¬

$$\frac{\mathsf{T} \neg A}{\mathsf{F} A} \neg \mathsf{T} \qquad \frac{\mathsf{F} \neg A}{\mathsf{T} A} \neg \mathsf{F}$$

That is, if you have a negated sentence, write down the unnegated sentence with an opposite truth value.

Rules for \wedge

$$\begin{array}{c} \underline{ \quad T\ A\ \wedge\ B} \\ \underline{ \quad T\ A} \\ \underline{ \quad T\ B} \end{array} \wedge T$$

Rules for \wedge

$$\begin{array}{c|c} TA \land B \\ \hline TA \\ TB \end{array} \land T \qquad \begin{array}{c|c} FA \land B \\ \hline FA & FB \end{array} \land F$$

Rules for \wedge

$$\begin{array}{c|c}
 & TA \land B \\
\hline
TA \\
TB
\end{array} \land T$$

$$\begin{array}{c|c}
 & FA \land B \\
\hline
FA & FB
\end{array} \land F$$

- · If you have a true conjunction, write down each conjunct.
- If you have a false conjunction, create two branches, one for each way it can be false.

Rules for ∨

$$\begin{array}{c|c} TA \lor B \\ \hline TA & TB \end{array} \lor T \qquad \begin{array}{c|c} FA \lor B \\ \hline FA \\ \hline FB \end{array} \lor F$$

- If you have a true disjunction, create two branches for the two disjuncts.
- If you have a false disjunction, write down that each disjunct is false.

Rules for \rightarrow

$$\frac{TA \to B}{FA \mid TB} \to T$$

Rules for \rightarrow

$$\begin{array}{c|c} TA \to B \\ \hline FA & TB \end{array} \to T \qquad \begin{array}{c} FA \to B \\ TA \\ FB \end{array} \to F$$

Rules for \rightarrow

$$\frac{TA \to B}{FA \mid TB} \to T \qquad \frac{FA \to B}{TA} \to F$$

- · And this is the hard rule.
- When a conditional is true, we create two branches one for the antecedent being false, the other for the consequent being true.
- When a conditional is false, we infer that the antecedent is true and the conclusion false.

