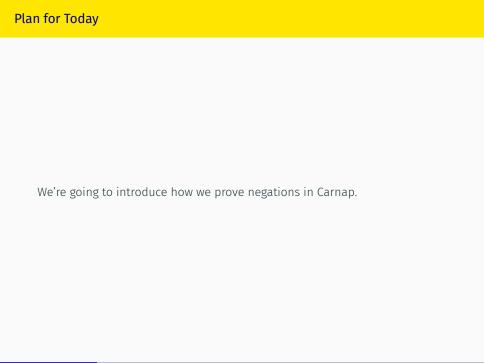
305 Lecture 10 - Indirect Proof

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Carnap book, chapter 6.



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- · In logic, a sentence and its negation.

Structure of Proof

- 1. Say that we will show $\neg X$
- 2. Assume X
- 3. Derive some pair of sentences, Y and $\neg Y$
- 4. Close the indirect proof by listing the lines where Y and $\neg Y$ appear.
- 5. Say that $\neg X$ has been shown at the 'show' line, and not citing any of the lines between it and the 'Indirect Proof' line.

Example

To prove:
$$P \rightarrow Q, P \rightarrow \neg Q \vdash \neg P$$

- 1. Show: ~P
- 2. P :AS
- 3. $P \rightarrow Q :PR$
- 4. P -> ~Q :PR
- 5. Q :MP 2, 3
- 6. ~Q :MP 2, 4
- 7. :ID 5 6

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- · Oddly, we don't put a comma between the lines cited at the end.
- · Once you end the sub-proof, you can't cite lines from in it again.
- But in the sub-proof, you can cite lines from earlier; they could even be one of the pair of lines that make up the absurdity.

One More Example

To prove:
$$P \rightarrow Q, R \rightarrow \neg Q \vdash R \rightarrow \neg P$$

$$3. \quad P \rightarrow Q : PR$$

4.
$$R \rightarrow Q$$
:PR

