305 Lecture 17 - Tautologies

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This lecture is about how we use truth tables to check for logical properties.

Carnap Book, Chapter 10, section "Truth Values, Truth Tables, Tautologies".
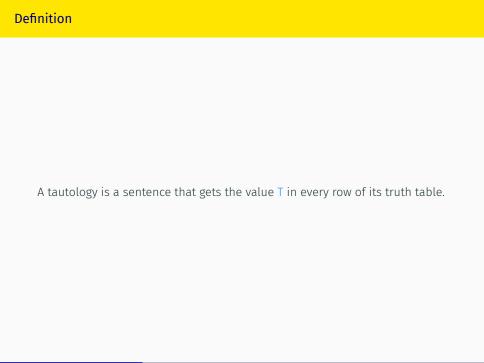
We are going to start with a particular kind of sentence, a **tautology**.

A tautology is a sentence that gets the value T in every row of its truth table.

What are some sentences that might fit the bill?

A	A ∨ ¬ A
T	T T F T
F	F T T F

A	¬ (A ∧ ¬ A)
T	T T F F T
F	T F F T F

A	A $\rightarrow$ A
T	T T T
F	F T F

A B	(A → B) ∨ (B → A)
T T	T T T T T T T
T F	T F F T F T T
F T	F T T T T F F
F F	F T F T F T F

- All tautologies are logical truths.
- But the converse isn't true - some logical truths are not tautologies.
- E.g., If Brian is necessarily a human, then Brian is a human.

We can also use truth tables to check for properties of arguments, and in particular to check for validity.

- An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are T, the conclusion is T as well.

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- Equivalently, an argument is invalid if there is a line where the premises are T and the conclusion F, and valid otherwise.

The argument A, therefore $A \wedge B$ is invalid because of the second line.

A B	A	A ∧ B
T T	T	T T T
T F	T	T F F
F T	F	F F T
F F	F	F F F

Another Invalidity Example

Note that there are several lines with T premises and conclusion. But the argument $A \rightarrow B$, so $A \rightarrow C$ is invalid because of line 2.

A B C	A $\rightarrow$ B	A $\rightarrow$ C
T T T	T T T	T T T
T T F	T T T	T F F
T F T	T F F	T T T
T F F	T F F	T F F
F T T	F T T	F T T
F T F	F T T	F T F
F F T	F T F	F T T
F F F	F T F	F T F

On the other hand the argument from $A \rightarrow B$ and $B \rightarrow C$ to $A \rightarrow C$ is valid.

A	B	C	A	$\rightarrow$	B	B	$\rightarrow$	C	A	$\rightarrow$	C
T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F	F	T	F	F
T	F	T	T	F	F	F	T	T	T	T	T
T	F	F	T	F	F	F	T	F	T	F	F
F	T	T	F	T	T	T	T	T	F	T	T
F	T	F	F	T	T	T	F	F	F	T	F
F	F	T	F	T	F	F	T	T	F	T	T
F	F	F	F	T	F	F	T	F	F	T	F

We'll talk about how to build more complicated truth tables.