305 Lecture 08 - Indirect Derivations

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July 8, 2020

Proving a Conditional

Intuitively, they way to show $A \longrightarrow B$ is to imagine/assume/suppose A is true, and show that then B will be true as well.

- That's what we'll do in Carnap.

Three New Tricks

1. Having the ‘Show’ be a conditional.
2. Starting with ‘AS’ not ‘PR’.
3. Ending with Conditional Derivation - CD.

Example

To prove: $P \rightarrow Q \vdash P \rightarrow \neg\neg Q$

```
1. Show: P -> ~~Q
2.      P           :AS
3.      P -> Q      :PR
4.      Q           :MP 3, 2
5.      ~~Q         :DNI 4
6. :CD 5
```

What goes in ‘Show’ is still the conclusion, but it isn’t what we end the proof with.

Example

To prove: $P \to Q \vdash P \to \neg\neg Q$

```
1. Show: P -> ~~Q
2.       P           :AS
3.       P -> Q      :PR
4.       Q           :MP 3, 2
5.       ~~Q         :DNI 4
6. :CD 5
```

We start with two kinds of underived lines.

1. Assumptions
2. Premises

Premises

To prove: $P \rightarrow Q \vdash P \rightarrow \neg\neg Q$

```
1. Show: P -> ~~Q
2.      P            :AS
3.      P -> Q       :PR
4.      Q            :MP 3, 2
5.      ~~Q          :DNI 4
6. :CD 5
```

The premises, in this case line 3, you are used to already.

Assumption

To prove: $P \to Q \vdash P \to \neg\neg Q$

```
1. Show: P -> ~~Q
2.      P           :AS
3.      P -> Q      :PR
4.      Q           :MP 3, 2
5.      ~~Q         :DNI 4
6. :CD 5
```

The assumption is the thing on the left of what you're trying to prove - the **antecedent** of the conditional.

Conditional Proof

To prove: $P \to Q \vdash P \to \neg\neg Q$

```
1. Show: P -> ~~Q
2.      P          :AS
3.      P -> Q     :PR
4.      Q          :MP 3, 2
5.      ~~Q        :DNI 4
6. :CD 5
```

Then you can use all the regular rules that you've used so far, with the same constraints.

Conditional Proof

To prove: $P \rightarrow Q \vdash P \rightarrow \neg\neg Q$

```
1. Show: P -> ~~Q
2.       P           :AS
3.       P -> Q      :PR
4.       Q           :MP 3, 2
5.       ~~Q         :DNI 4
6. :CD 5
```

The big difference is that you end with the **consequent** of what you're trying to show; the thing to the right of the $\rightarrow$.

Conditional Proof

To prove: $P \rightarrow Q \vdash P \rightarrow \neg\neg Q$

```
1. Show: P -> ~~Q
2.      P           :AS
3.      P -> Q      :PR
4.      Q           :MP 3, 2
5.      ~~Q         :DNI 4
6. :CD 5
```

And then (and this is a bit distinctive to Carnap), you write ‘CD’ for Conditional Derivation, not ‘DD’ for Direct Derivation.

For Next Time

- Read chapters 4 and 5.
- We will talk especially about what happens when these conditional derivations get **nested**.
- Finish the first assignment - the exercises from chapters 1 and 3.