305 Lecture 32 - The Crashing Websites

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 This lecture will go over exercise 8.8 in the book, as an illustration of how to invert conditional probabilities in a slightly more complicated case.



Odds and Ends, Chapter 8

Odds and Ends 8.8

A company makes websites, always powered by one of three server platforms: Bulldozer, Kumquat, or Penguin. Bulldozer crashes 1 out of every 10 visits, Kumquat crashes 1 in 50 visits, and Penguin only crashes 1 out of every 200 visits.

Half of the websites are run on Bulldozer, 30% are run on Kumquat, and 20% are run on Penguin.

You visit one of their sites for the first time and it crashes. What is the probability it was run on Penguin?

Start with a Table

	Crash	No Crash
Bulldozer		
Kumquat		
Penguin		

We will start by filling in the table - though really it is the left hand column that matters here

	Crash	No Crash
Bulldozer	$0.5 \times 0.1 = 0.05$	
Kumquat		
Penguin		

The formular for Bulldozer-and-Crash is

$$\Pr(\text{Bulldozer}) \times \Pr(\text{Crash}|\text{Bulldozer}) = 0.5 \times 0.1 = 0.05$$

	Crash	No Crash
Bulldozer	$0.5 \times 0.1 = 0.05$	
Kumquat	$0.3 \times 0.02 = 0.006$	
Penguin		

The formula for Kumquat-and-Crash is

$$\Pr(\text{Kumquat}) \times \Pr(\text{Crash}|\text{Kumquat}) = 0.3 \times 0.02 = 0.006$$

	Crash	No Crash
Bulldozer	$0.5 \times 0.1 = 0.05$	
Kumquat	$0.3 \times 0.02 = 0.006$	
Penguin	$0.2 \times 0.005 = 0.001$	

The formula for Penguin-and-Crash is

$$\Pr(\text{Penguin}) \times \Pr(\text{Crash}|\text{Penguin}) = 0.2 \times 0.005 = 0.001$$

ash No Crash
05
006
001

Let's rewrite it without the workings.

	Crash	No Crash
Bulldozer	0.05	0.45
Kumquat	0.006	0.294
Penguin	0.001	0.199

We can fill in the right-hand column by noting that the rows have to add up to 0.5, 0.3 and 0.2 respectively; since those are the probabilities of the three server types.

Crash!

	Crash	No Crash
Bulldozer	0.05	0.45
Kumquat	0.006	0.294
Penguin	0.001	0.199

So the probability of a crash is

$$0.05 + 0.006 + 0.001 = 0.057$$

Penguin given Crash

So the probability of Penguin given Crash is

$$\frac{\Pr(\text{Penguin-and-Crash})}{\Pr(\text{Crash})} = \frac{0.001}{0.057} \approx 0.0175$$

Penguin given Crash

So the probability of Penguin given Crash is

$$\frac{\Pr(\text{Penguin-and-Crash})}{\Pr(\text{Crash})} = \frac{0.001}{0.057} \approx 0.0175$$

That's really low, because Penguin sites don't crash.

For Next Time			

• We will look at a formula that some people find helpful for solving these kinds of problems.