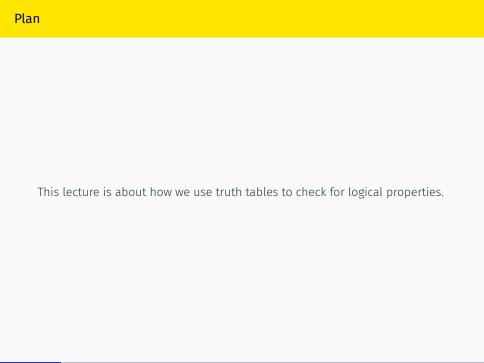
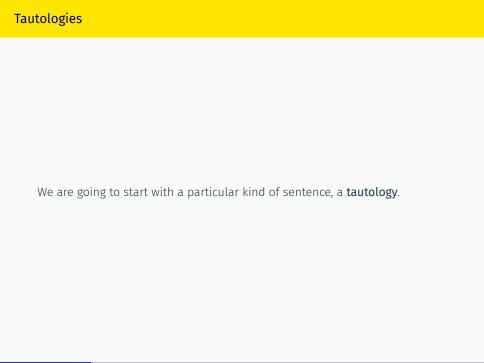
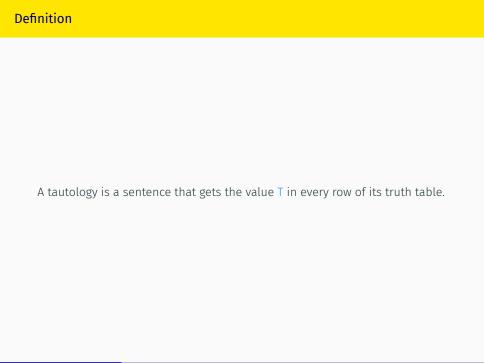
305 Lecture 17 - Tautologies

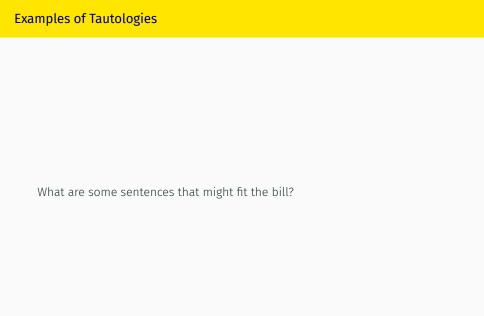
Brian Weatherson

July 13, 2020









The Law of Excluded Middle

The Law of Non-Contradiction

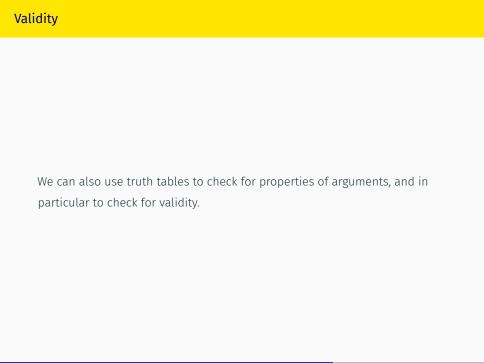
Reflexive Conditionals

$$\begin{array}{c|cccc}
A & A \rightarrow A \\
\hline
T & T & T \\
F & F & T & F
\end{array}$$

A Surprising One

Tautologies and Logical Truth

- · All tautologies are logical truths.
- But the converse isn't true some logical truths are not tautologies.
- $\boldsymbol{\cdot}\;$ E.g., If Brian is necessarily a human, then Brian is a human.



Truth Tables and Validity	

• An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are T, the conclusion is T as well.

Truth Tables and Validity

- An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are T, the conclusion is T as well.
- Equivalently, an argument is invalid if there is a line where the premises are T and the conclusion F, and valid otherwise.

Example of Invalidity

The argument A, therefore $A \wedge B$ is invalid because of the second line.

АВ	А	А∧В
ТТ	Т	TTT
T F	Т	TFF
FΤ	F	FFT
F F	F	FFF

Another Invalidity Example

Note that there are several lines with T premises and conclusion. But the argument $A \to B$, so $A \to C$ is invalid because of line 2.

АВС	$A\rightarrowB$	$A\rightarrowC$
ТТТ	ттт	ттт
TTF	TTT	T F F
TFT	T F F	ттт
TFF	T F F	T F F
FTT	FTT	FTT
FTF	FTT	FTF
FFT	F T F	FTT
F F F	F T F	F T F

Hypothetical Syllogism

On the other hand the argument from $A \to B$ and $B \to C$ to $A \to C$ is valid.

АВС	$A\rightarrowB$	${ t B} o { t C}$	A o C
ТТТ	TTT	ТТТ	TTT
T T F	ттт	TFF	TFF
TFT	TFF	FTT	ттт
T F F	TFF	FTF	TFF
FTT	FTT	ттт	FTT
FTF	FTT	TFF	FTF
FFT	FTF	FTT	FTT
F F F	FTF	FTF	FTF

