305 Lecture 06 - Recursive Composition Rules

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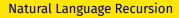
 We're going to look at how and why we can iterate the translation procedures we've been investigating.

Recursion

The language of propositional logic has some fairly simple composition rules.

- · It says what the basic sentences are.
- It has some rules saying that if some things are sentences, so are some other things.

The effect is that there are an infinity of possible sentences.



As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

· It will rain.

- · It will rain.
- · Alex thinks that it will rain.

- · It will rain.
- · Alex thinks that it will rain.
- · Kim thinks that Alex thinks that it will rain.

- · It will rain.
- · Alex thinks that it will rain.
- · Kim thinks that Alex thinks that it will rain.
- · Alex thinks that Kim thinks that Alex thinks that it will rain.

- · It will rain.
- · Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain
- · Alex thinks that Kim thinks that Alex thinks that it will rain.
- · And so on, to infinity, without adding any more words.

Recursive Rule

- If S is a sentence, and N is a name, then N thinks that S is a sentence.
- Note that the output of this rule can be the input to a new instance of it.

Formal Language Recursion

- The letters P, Q, R... are sentences.
- If S and T are sentences, then so are:
- ¬S
- 2. S V T
- 3. $S \wedge T$
- 4. $S \rightarrow T$
- 5. $S \leftrightarrow T$

Multiple Steps

So these are all sentences. (Note that I'm playing fast and loose with parentheses here.)

- 1. P
- 2. Q
- 3. $P \wedge Q$
- 4. $Q \rightarrow (P \land Q)$
- 5. ¬*P*
- 6. O ∨ ¬P
- 7. $(Q \rightarrow (P \land Q)) \leftrightarrow (Q \lor \neg P)$

The last one follows from the fact that 4 and 6 are sentences.

Main Connective

For any sentence you can make, there will be a 'last step' in the demonstration that it is a sentence

- That last step will involve copying down 1 or 2 other sentences, and adding a connective.
- On the previous slide, you copy down 4 and 6, and put a \leftrightarrow between them.
- That connective you add is the main connective of the sentence.
- · It covers all the material in the sentence.

Main Connective

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

$$P \wedge (Q \rightarrow R)$$

The Λ is the main connective because either side of it are

- . P
- $\cdot (Q \rightarrow R)$

And they are both sentences.

Main Connective

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

$$P \wedge (Q \rightarrow R)$$

The ightarrow is not the main connective because either side of it are

- $\cdot P \wedge (Q)$
- · R)

And they are not both sentences.



Figure 1: Step 1



Figure 2: Step 2

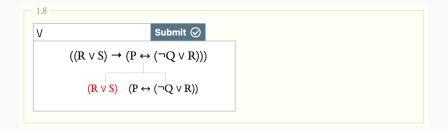


Figure 3: Step 3

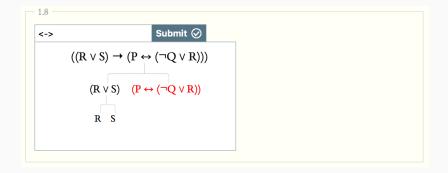


Figure 4: Step 4

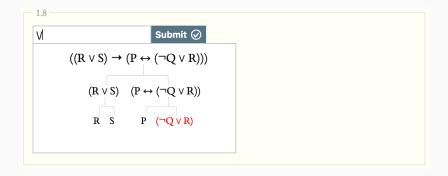


Figure 5: Step 5

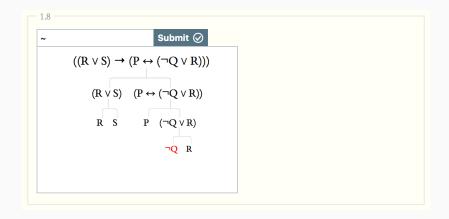


Figure 6: Step 6

For Next Time

- Before next lecture, read Chapter 3 of The Carnap Book
- $\boldsymbol{\cdot}$ We're going to start applying these tools to analysing arguments.