305 Lecture 21 - Rules for Truth Trees

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This lecture introduces the rules we use for building up truth trees.

The rules tell you what new lines to write down given the lines you've already got.

- To some extent they simply have to be memorised.
- But hopefully they are all (except for the rules about $\rightarrow$) fairly intuitive.

$$\frac{\text{T } \neg A}{\text{F } A} \; \neg\text{T}$$

$$\frac{\text{T } \neg A}{\text{F } A} \; \neg\text{T} \qquad \frac{\text{F } \neg A}{\text{T } A} \; \neg\text{F}$$

$$\frac{\text{T } \neg A}{\text{F } A} \ \neg\text{T} \qquad \frac{\text{F } \neg A}{\text{T } A} \ \neg\text{F}$$

That is, if you have a negated sentence, write down the unnegated sentence with an opposite truth value.

Rules for $\wedge$

$$\frac{\mathsf{T}\ A \wedge B}{\begin{array}{c}\mathsf{T}\ A \\ \mathsf{T}\ B\end{array}}\ \wedge\mathsf{T}$$

Rules for ∧

$$\frac{\text{T } A \wedge B}{\begin{array}{c}\text{T } A \\ \text{T } B\end{array}} \wedge\text{T} \qquad \frac{\text{F } A \wedge B}{\text{F } A \quad \big| \quad \text{F } B} \wedge\text{F}$$

$$\frac{\text{T } A \wedge B}{\begin{array}{c}\text{T } A \\ \text{T } B\end{array}} \wedge\text{T} \qquad \frac{\text{F } A \wedge B}{\text{F } A \quad | \quad \text{F } B} \wedge\text{F}$$

- If you have a true conjunction, write down each conjunct.
- If you have a false conjunction, create two branches, one for each way it can be false.

$$\frac{\text{T } A \lor B}{\text{T } A \quad | \quad \text{T } B} \lor \text{T} \qquad \frac{\text{F } A \lor B}{\text{F } A \atop \text{F } B} \lor \text{F}$$

- If you have a true disjunction, create two branches for the two disjuncts.
- If you have a false disjunction, write down that each disjunct is false.

$$\frac{\mathsf{T}\ A \rightarrow B}{\mathsf{F}\ A \quad | \quad \mathsf{T}\ B}\ \rightarrow\mathsf{T}$$

$$\frac{\mathsf{T}\ A \rightarrow B}{\mathsf{F}\ A \quad | \quad \mathsf{T}\ B} \rightarrow\mathsf{T} \qquad \frac{\mathsf{F}\ A \rightarrow B}{\begin{array}{c}\mathsf{T}\ A \\ \mathsf{F}\ B\end{array}} \rightarrow\mathsf{F}$$

$$\frac{\text{T } A \rightarrow B}{\text{F } A \quad | \quad \text{T } B} \rightarrow \text{T}$$

$$\frac{\text{F } A \rightarrow B}{\begin{array}{c} \text{T } A \\ \text{F } B \end{array}} \rightarrow \text{F}$$

- And this is the hard rule.
- When a conditional is true, we create two branches - one for the antecedent being false, the other for the consequent being true.
- When a conditional is false, we infer that the antecedent is true and the conclusion false.

We will look at some examples of truth trees.