305 Lecture 11 - The Conditional Theorem

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Plan for Today

We're going to go over an important general fact about conditional in Carnap - the conditional theorem. This is going to be very short.

Associated Reading

This isn’t covered in Carnap, but it’s relevant to our work later in the course, so I’m introducing it now.

The Conditional Theorem

These two things are equivalent in Carnap for any A, B, C.

1. $A, B \vdash C$
2. $A \vdash B \to C$

This is somewhat intuitive, though it's worth going over why it is so.

Argument for 1 to 2 (Informal)

- Assume we have a proof from A, B to C
- Now imagine a proof that starts with A, then has 'Show: $B \longrightarrow C$' followed by an assumption of B.
- Whatever you did in the first proof to get to C, you can repeat in the second proof.
- So eventually, you'll get C.
- So you can now use 'Conditional Derivation' to get $B \longrightarrow C$.

Argument for 2 to 1

- Assume that from A we have a proof of $B \longrightarrow C$
- Continue that proof for one more line with premise B.
- Then use MPP on the last two lines to get C.
- And now you've got a proof with premises A, B and conclusion C.

Metalogic

- The last two slides were super-duper informal.
- But the basic arguments were sound.
- And this is how you prove results in **metalogic**.
- You aren't trying to prove that some particular things are true, you're trying to prove general things about the nature of what can be proven.
- We're not going to do much metalogic in this course, but it's worth knowing about.

For Next Time

We're going to start on some material from chapter 8, in particular, we'll talk about how to prove things involving 'and'.