305 Lecture 21 - Rules for Truth Trees

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Plan

This lecture introduces the rules we use for building up truth trees.

Associated Reading

Boxes and Diamonds, sections 2.2-2.3.

What Rules Do

The rules tell you what new lines to write down given the lines you've already got.

- To some extent they simply have to be memorised.
- But hopefully they are all (except for the rules about $\rightarrow$) fairly intuitive.

Rules for ¬

$$\frac{\mathsf{T}\ \neg A}{\mathsf{F}\ A}\ \neg\mathsf{T}$$

Rules for ¬

$$\frac{\mathsf{T}\ \neg A}{\mathsf{F}\ A}\ \neg\mathsf{T} \qquad \frac{\mathsf{F}\ \neg A}{\mathsf{T}\ A}\ \neg\mathsf{F}$$

Rules for ¬

$$\frac{\mathsf{T}\ \neg A}{\mathsf{F}\ A}\ \neg\mathsf{T} \qquad \frac{\mathsf{F}\ \neg A}{\mathsf{T}\ A}\ \neg\mathsf{F}$$

That is, if you have a negated sentence, write down the unnegated sentence with an opposite truth value.

Rules for $\wedge$

$$\frac{\mathsf{T}\ A \wedge B}{\begin{array}{c}\mathsf{T}\ A\\ \mathsf{T}\ B\end{array}}\ \wedge\mathsf{T}$$

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$$\frac{\mathsf{T}\ A \wedge B}{\begin{array}{c}\mathsf{T}\ A\\ \mathsf{T}\ B\end{array}}\ \wedge\mathsf{T} \qquad \frac{\mathsf{F}\ A \wedge B}{\mathsf{F}\ A \mid \mathsf{F}\ B}\ \wedge\mathsf{F}$$

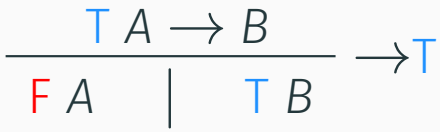
- If you have a true conjunction, write down each conjunct.
- If you have a false conjunction, create two branches, one for each way it can be false.

Rules for $\vee$

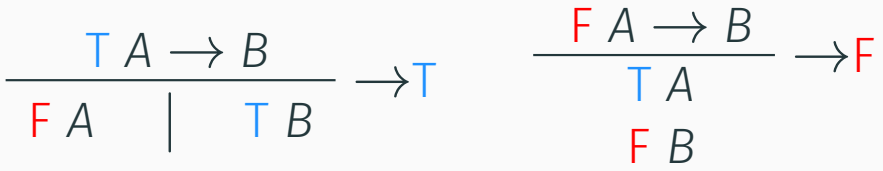
$$\frac{\mathsf{T}\ A \vee B}{\begin{array}{l|l}\mathsf{T}\ A & \mathsf{T}\ B\end{array}}\ \vee\mathsf{T} \qquad \frac{\mathsf{F}\ A \vee B}{\begin{array}{l}\mathsf{F}\ A\\ \mathsf{F}\ B\end{array}}\ \vee\mathsf{F}$$

- If you have a true disjunction, create two branches for the two disjuncts.
- If you have a false disjunction, write down that each disjunct is false.

Rules for $\rightarrow$



Rules for $\rightarrow$



Rules for $\rightarrow$

$$\frac{\mathsf{T}\ A \rightarrow B}{\mathsf{F}\ A \quad \mid \quad \mathsf{T}\ B}\ \rightarrow\mathsf{T} \qquad \frac{\mathsf{F}\ A \rightarrow B}{\begin{array}{c}\mathsf{T}\ A \\ \mathsf{F}\ B\end{array}}\ \rightarrow\mathsf{F}$$

- And this is the hard rule.
- When a conditional is true, we create two branches - one for the antecedent being false, the other for the consequent being true.
- When a conditional is false, we infer that the antecedent is true and the conclusion false.

For Next Time

We will look at some examples of truth trees.