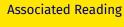
# 305 Lecture 33 - Bayes's Theorem

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• This lecture will go over one of the most famous theorems in probability: Bayes's Theorem.



Still chapter 8 of Odds and Ends

## One Important Equivalence

$$Pr(B|A) Pr(A) = Pr(A|B) Pr(B)$$

We know that because these are two different ways of expressing  $\Pr(A \land B)$ .

## One Important Equivalence

Divide both sides by  $\Pr(B)$  (and flip sides around) and you get

$$Pr(A|B) = \frac{Pr(B|A) Pr(A)}{Pr(B)}$$

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This is the formula that's written in neon in the textbook.

### **Another Important Equivalence**

Start again with our canonical formula for conditional probability.

$$Pr(A|B) = \frac{Pr(A \land B)}{Pr(B)}$$

### **Another Important Equivalence**

Replace  $\Pr(A \land B)$  with its definition in terms of conditional probability.

$$Pr(A|B) = \frac{Pr(B|A) Pr(A)}{Pr(B)}$$

### **Another Important Equivalence**

Now replace Pr(B) with the formula we derived for it in an earlier lecture.

$$Pr(A|B) = \frac{Pr(B|A) Pr(A)}{Pr(B|A) Pr(A) + Pr(B|\neg A) Pr(\neg A)}$$

And this is what is sometimes called Bayes's Theorem.

#### **General Version**

- Let  $X_1, \ldots, X_n$  be a partition of possibility space.
- Then B is equivalent to  $(B \wedge X_1) \vee \cdots \vee (B \wedge X_n)$ .
- · So  $Pr(B) = Pr(B \land X_1) + \cdots + Pr(B \land X_n)$
- $\boldsymbol{\cdot}$  And we can use that to get the very general form of Bayes's theorem

#### **General Version**

$$Pr(X_i|B) = \frac{Pr(B|X_i) Pr(X_i)}{\sum_{k=1}^{n} Pr(B|X_k) Pr(X_k)}$$

So if you know the prior probability of each cell in the partition, and the probability of *B* conditional on each cell, you can work out the probability of being in a particular cell given *B*.

#### **General Version**

- · A lot of people make a big deal about this formula.
- I rarely find myself in situations where it is easier to use than something like the trees or tables.
- But the fact that so many people fuss so much about it suggests that for a lot of applications it is very helpful.
- For the assignment questions this week, it's totally up to you whether to use the formula, or trees, or tables.