305 Lecture 14 - Explosion

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Plan for Today

- We're going to talk two interesting consequences of the rules we've produced so far.
- The first is that the rules validate explosion, the idea that everything follows from a contradiction.
- The second is that they validate all four of what are called DeMorgan's Laws.

Associated Reading

No associated reading, but again, this will be relevant to our later work.

Explosion

What It is

$P, \neg P \vdash Q$

One Weird Thing About it

It violates **relevance** - the constraint that any valid argument should have at least some term in common between premise and conclusion.

Informal Argument

- An argument is valid if it is impossible for the premises to be true and the conclusion false.

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- In **Explosion**, it is impossible for the premises to be true.
- So in **Explosion**, it is impossible for the premises to be true and the conclusion false.

Informal Argument

- An argument is valid if it is impossible for the premises to be true and the conclusion false.
- In **Explosion**, it is impossible for the premises to be true.
- So in **Explosion**, it is impossible for the premises to be true and the conclusion false.
- So **Explosion** is valid.

Formal Argument

Prove **Explosion** in Carnap.

First Attempt

P, ¬P ⊢ Q

```
1. P          :PR             +
2. ~P         :PR             +
3. Show: ~~Q                  +
4.|    ~Q     :AS             +
5. :ID 1| 2
6. Q          :DNE 3          +
```

Expand

1.	P	PR
2.	¬P	PR
3.	Show: ¬¬Q	
4.	¬Q	AS
5.		ID 1, 2
6.	Q	DNE 3

Figure 1: $P, \neg P \vdash Q$

Second Attempt

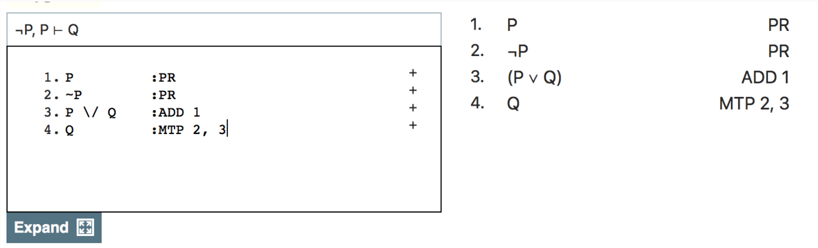

Figure 2: $P, \neg P \vdash Q$

Choice Point

1. Reject Addition
2. Reject Disjunctive Syllogism
3. Accept Explosion

- Most people who reject explosion also reject disjunctive syllogism.

Why Reject Explosion

1. Truth in a Database

Why Reject Explosion

1. Truth in a Database
2. Truth in a Story

Why Reject Explosion

1. Truth in a Database
2. Truth in a Story
3. There are actual true contradictions, but some things are not true.

Explosion is fine - we just have to be careful with databases and stories.

DeMorgan Laws

First Equivalence

1. $\neg(P \vee Q)$
2. $\neg P \wedge \neg Q$

Prove these are equivalent.

First Direction

¬(P ∨ Q) ⊢ (¬P ∧ ¬Q)

```
 1. Show: ~P /\ ~Q                                  +
 2.|     ~(P \/ Q)      :PR                         +
 3.|     Show: ~P                                   +
 4.|     |    P         :AS                         +
 5.|     |    P \/ Q    :ADD 4                      +
 6.|     :ID 2 5
 7.|     Show: ~Q                                   +
 8.|     |    Q         :AS                         +
 9.|     |    P \/ Q    :ADD 8                      +
10.|     :ID 2 9
11.|     ~P /\ ~Q       :ADJ 3, 7                   +
12. :DD 11
```

Expand

1.	Show: (¬P ∧ ¬Q)	
2.	¬(P ∨ Q)	PR
3.	Show: ¬P	
4.	P	AS
5.	(P ∨ Q)	ADD 4
6.		ID 2, 5
7.	Show: ¬Q	
8.	Q	AS
9.	(P ∨ Q)	ADD 8
10.		ID 2, 9
11.	(¬P ∧ ¬Q)	ADJ 3, 7
12.		DD 11

Figure 3: $\neg(P \vee Q) \vdash \neg P \wedge \neg Q$

Second Direction

(¬P ∧ ¬Q) ⊢ ¬(P ∨ Q)

```
1. Show: ~(P \/ Q)                  +
2.|     P \/ Q      :AS            +
3.|     ~P /\ ~Q    :PR            +
4.|     ~P          :S 3           +
5.|     ~Q          :S 3           +
6.|     Q           :MTP 2, 4      +
7. :ID 5 6
```

Expand

1.	Show: ¬(P ∨ Q)	
2.	(P ∨ Q)	AS
3.	(¬P ∧ ¬Q)	PR
4.	¬P	S 3
5.	¬Q	S 3
6.	Q	MTP 2, 4
7.		ID 5, 6

Figure 4: $\neg P \wedge \neg Q \vdash \neg(P \vee Q)$

Second Equivalence

1. $\neg P \vee \neg Q$
2. $\neg(P \wedge Q)$

First Direction

(¬P ∨ ¬Q) ⊢ ¬(P ∧ Q)

```
1. Show: ~(P /\ Q)            +
2.|     P /\ Q     :AS        +
3.|     ~P \/ ~Q   :PR        +
4.|     P          :S 2       +
5.|     Q          :S 2       +
6.|     ~~Q        :DNI 5     +
7.|     ~P         :MTP 3, 6  +
8. :ID 4 7
```

Expand

1.	Show: ¬(P ∧ Q)	
2.	(P ∧ Q)	AS
3.	(¬P ∨ ¬Q)	PR
4.	P	S 2
5.	Q	S 2
6.	¬¬Q	DNI 5
7.	¬P	MTP 3, 6
8.		ID 4, 7

Figure 5: $\neg(P \wedge Q) \vdash \neg P \vee \neg Q$

Second Direction

¬(P ∧ Q) ⊢ (¬P ∨ ¬Q)

```
 1. Show: ~P \/ ~Q                              +
 2.|    Show: ~~(~P \/ ~Q)                      +
 3.|    |    ~(~P \/ ~Q)        :AS             +
 4.|    |    ~(P /\ Q)          :PR             +
 5.|    |    Show: ~~P                          +
 6.|    |    |    ~P          :AS               +
 7.|    |    |    ~P \/ ~Q    :ADD 6            +
 8.|    |    :ID 3 7
 9.|    |    P               :DNE 5             +
10.|    |    Show: ~~Q                          +
11.|    |    |    ~Q          :AS               +
12.|    |    |    ~P \/ ~Q    :ADD 11           +
13.|    |    :ID 3 12
14.|    |    Q               :DNE 10            +
15.|    |    P /\ Q          :ADJ 9, 14         +
16.|    :ID 4 15
17.|    ~P \/ ~Q             :DNE 2             +
18. :DD 17
```

Expand

1.	Show: (¬P ∨ ¬Q)	
2.	Show: ¬¬(¬P ∨ ¬Q)	
3.	¬(¬P ∨ ¬Q)	AS
4.	¬(P ∧ Q)	PR
5.	Show: ¬¬P	
6.	¬P	AS
7.	(¬P ∨ ¬Q)	ADD 6
8.		ID 3, 7
9.	P	DNE 5
10.	Show: ¬¬Q	
11.	¬Q	AS
12.	(¬P ∨ ¬Q)	ADD 11
13.		ID 3, 12
14.	Q	DNE 10
15.	(P ∧ Q)	ADJ 9, 14
16.		ID 4, 15
17.	(¬P ∨ ¬Q)	DNE 2
18.		DD 17

Figure 6: $\neg(P \wedge Q) \vdash \neg P \vee \neg Q$

General Principle

To move a negation from outside the parentheses to inside it, you do two things.

1. Negate each of the parts.
2. Flip the conjunction/disjunction symbol upside down.

This process is reversible.

For Next Time

Do the weekly assignment, and we'll start talking about truth tables.