305 Lecture 15 - Truth Tables

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July 13, 2020

Plan for Today

Introducing truth tables.

Associated Reading

Carnap book, chapter 10, first half.

Conceptual Idea

- Think about all the combinations of truth values for the atomic sentences in a longer sentence, or in an argument.

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- Think about all the combinations of truth values for the atomic sentences in a longer sentence, or in an argument.
- For each possible combination, evaluate the truth of every part of every sentence in an argument.
- See if it is possible for the premises to be true and the conclusion false.

Truth Tables and Validity

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- If there is no such combination, mark the argument valid.
- If there is such a combination, tentatively mark the argument invalid.
- We'll come back to why 'tentatively'.

Structure

- We list each of the combinations in separate rows.
- In each column we list the truth value of the sentence such that the symbol at the top of that column is the main connective.
- That's I think a lot easier to understand in practice than in theory, so let's start with some examples.

A Truth Table

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

One Sentence

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

This is a truth table for a single sentence, not an argument. We'll get to arguments in a bit.

Understanding

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

We will also get (even sooner) to how to build these monsters. What I first want to talk about is how to read them.

Four Rows

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	**F**	**T**	**F**	**F**	**T**	**F**	**T**	**T**
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

Each of the four rows represent a way things could be. For instance, the second row (bolded here) represents how things are if P is true and Q is false.

Four Rows

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

There are four rows because there are 2 sentence letters - P and Q - each of which could take 2 values, so there are $2 \times 2 = 4$ combinations of values.

More Rows!

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

- If there had been three sentence letters, there would be eight rows.
- Four sentence letters would mean 16 rows, etc.

The Columns

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

- The columns under the letters reflect the value of the atomic sentences in each row.
- As you can see, they are just cut-and-paste from the left hand side.

Atomic Columns

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	**T**	T	T	T	T	T	**T**
T	F	**T**	F	F	T	F	T	**T**
F	T	**F**	T	T	T	T	F	**F**
F	F	**F**	T	F	T	F	T	**F**

I've bolded all the truth values for *P*, which as you can see were just copied and pasted from the columns on the far left.

Intermediate Sentences

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	**T**	T	T	T	T	T
T	F	T	**F**	F	T	F	T	T
F	T	F	**T**	T	T	T	F	F
F	F	F	**T**	F	T	F	T	F

The surprising thing (or at least the thing that surprised me as a student) was what we mean by the column under the →, which I've bolded.

Intermediate Sentences

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	**T**	T	T	T	T	T
T	F	T	**F**	F	T	F	T	T
F	T	F	**T**	T	T	T	F	F
F	F	F	**T**	F	T	F	T	F

Each letter here is giving the truth value of the sentence that has that first → as its main connective. That is, $P \rightarrow Q$.

Intermediate Sentences

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	**T**	T
T	F	T	F	F	T	F	**T**	T
F	T	F	T	T	T	T	**F**	F
F	F	F	T	F	T	F	**T**	F

And this column gives the truth values for $Q \longrightarrow P$.

Conditionals

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

Don't worry for now about why we write those letters down; we'll get to that in the next lecture. For now I just want to go over how to read these tables.

The Big Red Column

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

- And the column that I've put in red gives the truth value of the sentence whose main connective is ∨.
- That is, in this case, the whole sentence.

The Big Red Column

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

Ultimately the red column is all we really care about - the others are essentially scaffolding.

Logical Truth

P	Q	(P	→	Q)	∨	(Q	→	P)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

- There is something distinctive about this table - the red column is all T.
- That means the sentence is a logical truth.
- We'll have more to say about this presently in future lectures.

For Next Time

We'll talk about how to build truth tables.