305 Lecture 49 - Truth in a Model

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• To extend our discussion of truth at a world, to discussion of truth in a whole model.



 $\boldsymbol{\cdot}\:$ Boxes and Diamonds, section 3.5.

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- 3. A valuation function on those worlds, typically called $\it V$.

We'll write the models as $\langle W, R, V \rangle$.

Valuations

V is a function from atomic sentence letters to subsets of W.

- · It tells you when the atomic sentences are true.
- $\boldsymbol{\cdot}$ When an atomic sentence is not true, it is false.

Truth at a Point

The general theory of truth is built up in stages from the basic theory. Assume we have a model $\langle W, R, V \rangle$, and a point $w \in W$, and are asking whether an arbitrary sentence is true at w in $\langle W, R, V \rangle$.

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- p is true at w iff $w \in V(p)$.
- $\neg A$ is true at w iff A is not true at w.
- $A \wedge B$ is true at w iff A is true and w and B is true at w.
- A V B is true at w iff A is true and w or B is true at w.
- $A \rightarrow B$ is true at w iff A is false at w or B is true at w.

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This just leaves the modal formulae. I'll set out the rules, then do some worked examples.

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 metaphysical necessity, or an epistemic necessity, or a moral necessity,
 or anything else.
- And it is true at w just in case A is true at every world y such that wRy.
- · Necessary truth is truth at all accessible worlds.

Possible Truth at a Point

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· I'll read this as 'Diamond A'.

Possible Truth at a Point

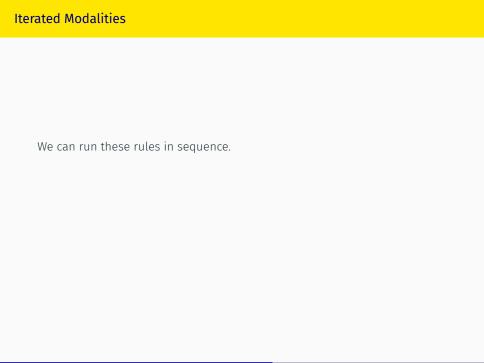
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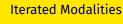
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 metaphysical necessity, or an epistemic necessity, or a moral necessity,
 or anything else.
- And it is true at w just in case A is true at some world y such that wRy.
- · Possible truth is truth at some accessible world.





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- It is for $\square A$ to be true at every y such that wRy.
- And that means that A has to be true at every world z such that yRz (for any y such that wRy).

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- If wRy, then you can 'step' from w to y.
- □A means that anywhere you can step to from w is a world where A is true.
- And □ □ A means that anywhere you can get to in two steps from w is a world where A is true.



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- What does it take for $\Diamond \Diamond A$ to be true at w?
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- What does it take for $\diamondsuit \diamondsuit A$ to be true at w?
- It is for $\Diamond A$ to be true at some y such that wRy.
- And that means that A has to be true at some world z such that yRz (for some y such that wRy).

We can run the rules in sequence.

- What does it take for $\Diamond \Diamond A$ to be true at w?
- It is for $\triangle A$ to be true at some y such that wRy.
- And that means that A has to be true at some world z such that yRz (for some y such that wRy).
- In the picturesque terms, you can get from w to an A-world in two steps.



What does it mean for $\Diamond \Box A$ to be true at w?



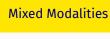
What does it mean for $\Diamond \Box A$ to be true at w?

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Mixed Modalities

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- There is some accessible world where $\Box A$ is true.
- That is, there is some accessible world such that everywhere you can go from there, A is true.



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What does it mean for $\square \diamondsuit A$ to be true at w?

- At all accessible worlds, $\Diamond A$ is true.
- That is, wherever you go, you can get to there is some accessible world such that everywhere you can go from there, A is true.

- It's for $p \lor (q \to \diamondsuit r)$ to be true everywhere you can get to (in one step) from w.
- That is, at every one of those worlds, either p is true or $q \to \Diamond r$ is true.

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- That is, at every one of those worlds, either p is true, or q is false, or $\Diamond r$ is true.
- That is, at every one of those worlds, either *p* is true, or *q* is false, or there is some world you can get to where *r* is true.

Box and connectives

The general rule is just to apply the rules for sentences inside the brackets at each world in W, and then apply the rule for \square or \diamondsuit . But there are three special cases worth thinking about.

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- $\cdot \Box (A \land B)$ means that all accessible worlds are A and B worlds.
- $\square(A \lor B)$ means that all accessible worlds make at least one of A and B true.
- $\cdot \Box (A \to B)$ means that all accessible A-worlds are B-worlds.

We'll use that last one a lot.

