Assignment Week 4

Philosophy 305

Due July 31, 2020

Note that all these questions are from *Odds and Ends*, occasionally with minor changes to the wording.

Expected Value Questions

Questions 1 to 3

1. What is the expected monetary value of playing a slot machine that costs $\$100$ to play, and has a $1/25$ chance of paying out $\$500$? (The rest of the time it pays nothing.)
2. Suppose a slot machine pays off $\$25$ a fiftieth of the time and costs a $\$1$ to play, and a video poker machine pays off $\$10$ a twentieth of the time and costs $\$2$ to play. Which machine is the better bet in terms of expected monetary value?
3. You're considering downloading a new game for your phone. The game costs $\$0.99$. But as a promotion, the first $50,000$ downloaders are being entered in a fair lottery with a $\$10,000$ cash prize. If you know you'll be one of the first $50,000$ downloaders, what is the expected monetary value of downloading the game?

Question 4

A local casino offers a game which costs $\$2$ to play. A fair coin is flipped up to three times, and the payouts work as follows:

- If the coin lands heads on the first toss, you win $\$2$ and the game is over.
- If the coin lands heads on the second toss, you win $\$4$ and the game is over.
- If the coin lands heads on the third toss, you win $\$8$ and the game is over.
- If the coin lands tails all three times, you win $\$0$.

4. What is the expected monetary value of this game?

Question 5

5. Suppose you can bet on either of two dogs: Santa's Little Helper or She's the Fastest. If you bet on Santa's Little Helper and he wins, you get $\$5$. If he loses you pay $\$2$. If you bet on She's the Fastest and she loses, you pay $\$10$. The two dogs have the same chance of winning. How much would a winning bet on She's the Fastest have to pay for the bets to have the same value?

Questions 6 to 9

Suppose Michigan is deciding whether to enact a new tax. If the tax is enacted, it will bring in $700 million in revenue. But it could also hurt the economy. The chance of harm to the economy is small, just $1/5$. But it would cost the country $1,200 million in lost earnings. (The $700 million in revenue would still be gained, partially offsetting this loss.) Treat gains as positive and losses as negative.

6. What is the expected monetary value of enacting the new tax?

The government also has the option of conducting a study before deciding whether to enact the new tax. If the study's findings are bad news, that means the chance of harm to the economy is actually double what they thought. If its findings are good news, then the chance of harm to the economy is actually half of what they thought.

7. Suppose the government conducts the study and its findings are good news. What will the expected monetary value of enacting the tax be then?
8. Suppose the government conducts the study and its findings are bad news. What will the expected monetary value of enacting the tax be then?
9. If the expected monetary value of the tax is what you said in question 6, and the study is bound to deliver good news or bad news, what is the probability of it delivering good news?

Question 10

Consider the following game: I'm going to flip a fair coin up to three times. If it comes up heads on the first toss, the game is over and you win $2. If it comes up heads for the first time on the second toss, you win $40 and the game is over. If the first heads comes up on the third toss, you win $800 and the game is over. If it comes up tails every time, you have to pay me $x.

10. What does x have to be to make the game fair?

Question 11

Some workplaces hold a weekly lottery. Suppose there are 30 people in your workplace lottery, and each person pays in $5 every Monday. A finalist is chosen at random every Friday, for three weeks. Then, on the fourth Friday, one of the three finalists from the previous three weeks is chosen at random. That person gets all the prize money.

11. What is the expected value of being in this lottery?

Utility Questions

Question 12

Sonia has tickets to see The Weeknd tomorrow night. Her friend has tickets to see Beyoncé, and also tickets to Katy Perry. Beyoncé is Sonia's favourite performer, in fact she would rather see Beyoncé than The Weeknd. Sonia's friend offers a gamble in exchange for her tickets to The Weeknd. The gamble has a $9/10$ chance of winning, in which case Sonia gets the Beyoncé tickets (utility 1). Otherwise she gets the Katy Perry tickets (utility 0).

12. If Sonia declines the gamble, what can we conclude?

a. For Sonia, the utility of seeing The Weeknd is $9/10$.
b. For Sonia, the utility of seeing The Weeknd is greater than $9/10$.
c. For Sonia, the utility of seeing The Weeknd is less than $9/10$.
d. For Sonia, the utility of seeing The Weeknd is $1/10$.

Questions 13 to 14

After giving her calculus midterm, Professor X always offers her students a chance to improve their grade by trying to solve an optional "challenge" problem. If they get it right, their grade is increased by one letter grade: F changes to D, D changes to C, etc. But if they get it wrong, their grade goes down by one letter-grade: A changes to B, B changes to C, etc.

Hui got a C on his midterm. He asks the professor how often students get the challenge problem right and she says they get it right half the time. Hui decides to stick with his C. But he would be willing to try the challenge problem if the chances of getting it right were higher: $2/3$ or more. Suppose getting a D has utility $5/10$ for Hui, while a B has utility $8/10$.

13. What is the expected utility for Hui of trying the challenge problem?
14. How much utility does a C have for Hui?

Questions 15 to 17 (2 points each)

Eleanor wants to get a job at Google so she's going to university to study computer science. She has to decide between Wayne State and Michigan State. Suppose $1/100$ of Wayne State's computer science students get jobs at Google and the rest get jobs at Facebook. For Eleanor, a job at Google has utility 200 while a job at Facebook has utility 50.

15. What is the expected utility of going to Wayne State for Eleanor?

Suppose Michigan State students have better odds of getting a job at Google: $5/400$. And $360/400$ students go to work at Amazon, which Eleanor would prefer to Facebook. On the other hand, the remaining $35/400$ of them don't get a job at all, which has utility zero for Eleanor. After thinking about it, she can't decide: Wayne State and Michigan State seem like equally good options to her.

16. How much utility does working at Amazon have for Eleanor?

Suppose Eleanor ends up going to Wayne State, and now she's about to graduate. Unfortunately, Google isn't hiring any more. The only jobs available are at Amazon and Facebook. She would have to take a special summer training program to qualify for a job at Amazon, though. And that would mean she can't get a job at Facebook. Facebook is offering her a job, but she has to take it now or never. So, she has to either take the guaranteed job at Facebook right now, or gamble on the summer program. The summer program could get her a job at Amazon, or it could leave her unemployed.

17. How high would the probability of getting a job at Amazon have to be for Eleanor to be indifferent between taking and not taking the special summer program?