305 Lecture 31 - Probability Revision

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Plan

- This is a short lecture just for revising some of the basic principles about probability.

Associated Reading

None - this is just revision

The Negation Rule

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4. So, from 1, 2, 3, we get $\Pr(A) + \Pr(\neg A) = 1$.
5. So, from 4, we get $\Pr(\neg A) = 1 - \Pr(A)$.

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2. Multiplying both sides by $\Pr(B)$ gives us $\Pr(A \wedge B) = \Pr(A|B)\Pr(B)$.

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6. Also by the multiplication rule, $\Pr(\neg A \wedge B) = \Pr(B|\neg A)\Pr(\neg A)$.

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2. So, $\Pr(B) = \Pr((A \wedge B) \vee (\neg A \wedge B))$.
3. $(A \wedge B)$ and $(\neg A \wedge B)$ are exclusive.
4. So $\Pr((A \wedge B) \vee (\neg A \wedge B)) = \Pr(A \wedge B) + \Pr(\neg A \wedge B)$.
5. By the multiplication rule, $\Pr(A \wedge B) = \Pr(B|A)\Pr(A)$.
6. Also by the multiplication rule, $\Pr(\neg A \wedge B) = \Pr(B|\neg A)\Pr(\neg A)$.
7. Putting all these together, we get

$$\Pr(B) = \Pr(B|A)\Pr(A) + \Pr(B|\neg A)\Pr(\neg A)$$

Another Conditional Probability Rule

Putting that formula for $\Pr(B)$ into the definition of conditional probability, we get

$$Pr(A|B) = \frac{\Pr(A \wedge B)}{\Pr(B|A)\Pr(A) + \Pr(B|\neg A)\Pr(\neg A)}$$

Yet Another Conditional Probability Rule

$$
\begin{aligned}
\Pr(B|A) \times \frac{\Pr(A)}{\Pr(B)} &= \frac{\Pr(B \wedge A)}{\Pr(A)} \times \frac{\Pr(A)}{\Pr(B)} \\
&= \frac{\Pr(A \wedge B)}{\Pr(A)} \times \frac{\Pr(A)}{\Pr(B)} \\
&= \frac{\Pr(A \wedge B)}{\Pr(B)} \\
&= \Pr(A|B)
\end{aligned}
$$

Yet Another Conditional Probability Rule

Or, as it is usually written

$$Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$$

For Next Time

- We will look at a more complicated example of inverting conditional probabilities.