

305 Lecture 17 - Tautologies

Brian Weatherson

July 13, 2020

Plan

This lecture is about how we use truth tables to check for logical properties.

Tautologies

We are going to start with a particular kind of sentence, a **tautology**.

Definition

A tautology is a sentence that gets the value **T** in every row of its truth table.

Examples of Tautologies

What are some sentences that might fit the bill?

The Law of Excluded Middle

| A | $A \vee \neg A$ | | | |
|---|-----------------|---|---|---|
| T | T | T | F | T |
| F | F | T | T | F |

The Law of Non-Contradiction

| A | $\neg (A \wedge \neg A)$ | | | | |
|---|--------------------------|---|---|---|---|
| T | T | T | F | F | T |
| F | T | F | F | T | F |

Reflexive Conditionals

| A | A \rightarrow A | | |
|---|-------------------|---|---|
| T | T | T | T |
| F | F | T | F |

A Surprising One

| A | B | $(A \rightarrow B) \vee (B \rightarrow A)$ | | | | | | |
|---|---|--|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T |
| T | F | T | F | F | T | F | T | T |
| F | T | F | T | T | T | T | F | F |
| F | F | F | T | F | T | F | T | F |

Tautologies and Logical Truth

- All tautologies are logical truths.
- But the converse isn't true - some logical truths are not tautologies.
- E.g., If Brian is necessarily a human, then Brian is a human.

We can also use truth tables to check for properties of arguments, and in particular to check for validity.

Truth Tables and Validity

- An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are T, the conclusion is T as well.

Truth Tables and Validity

- An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are **T**, the conclusion is **T** as well.
- Equivalently, an argument is invalid if there is a line where the premises are **T** and the conclusion **F**, and valid otherwise.

Example of Invalidity

The argument A , therefore $A \wedge B$ is invalid because of the second line.

| A | B | A | A | \wedge | B |
|---|---|---|---|----------|---|
| T | T | T | T | T | T |
| T | F | T | T | F | F |
| F | T | F | F | F | T |
| F | F | F | F | F | F |

Another Invalidity Example

Note that there are several lines with **T** premises and conclusion. But the argument $A \rightarrow B$, so $A \rightarrow C$ is invalid because of line 2.

| A B C | $A \rightarrow B$ | $A \rightarrow C$ |
|-------|-------------------|-------------------|
| T T T | T T T | T T T |
| T T F | T T T | T F F |
| T F T | T F F | T T T |
| T F F | T F F | T F F |
| F T T | F T T | F T T |
| F T F | F T T | F T F |
| F F T | F T F | F T T |
| F F F | F T F | F T F |

Hypothetical Syllogism

On the other hand the argument from $A \rightarrow B$ and $B \rightarrow C$ to $A \rightarrow C$ is valid.

| A B C | $A \rightarrow B$ | $B \rightarrow C$ | $A \rightarrow C$ |
|-------|-------------------|-------------------|-------------------|
| T T T | T T T | T T T | T T T |
| T T F | T T T | T F F | T F F |
| T F T | T F F | F T T | T T T |
| T F F | T F F | F T F | T F F |
| F T T | F T T | T T T | F T T |
| F T F | F T T | T F F | F T F |
| F F T | F T F | F T T | F T T |
| F F F | F T F | F T F | F T F |

For Next Time

We'll talk about how to build more complicated truth tables.