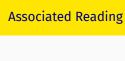
## 305 Lecture 43 - The One True Prior

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 We're going to look at a very objective version of the subjective theory, one that says there is a single true prior probability function.



Odds and Ends, chapter 18.

### **Symmetry**

There is a natural way to start looking for the one true prior.

- When we do probabilities involving games of chance, we naturally take the different possibilities as having equal probability.
- So when we give equal probability to heads and tails, or to each marble being drawn, or each side of the die coming up, that doesn't seem to be based on a careful analysis of coins, marbles or dice.
- Rather, it's just natural to divide probability evenly among the possibilities.

#### A Caveat

There is one way in which it is more complicated than that.

- We don't give any probability to the coin landing on its edge, or the die landing on a corner.
- I suspect this is because we've observed a lot about the world from a very young age, and noticed things like that just don't happen.
- This complicates the narrative that we're just using some natural principle of prior probability.

# **Logical Probability**

The very rough idea is that there are logical symmetry principles, and the prior probability function is one that respects these symmetries.

 If p is a proposition that things are one side of one of these lines of symmetry in logical space, the prior probability of p is 0.5.

## **Logical Probability**

The very rough idea is that there are logical symmetry principles, and the prior probability function is one that respects these symmetries.

- If p is a proposition that things are one side of one of these lines of symmetry in logical space, the prior probability of p is 0.5.
- Don't think about this too hard because it isn't actually going to work.
- The problem is that there are too many symmetries.

#### The Cube Factory

- Imagine that all you know about a factory is that it makes cubes with side lengths between 0 and 2cm.
- What is the probability that the next cube will have a side length less than or equal to 1cm?

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- What is the probability that the next cube will have a side length less than or equal to 1cm?
- Intuitively, it's 0.5, right?

### The Cube Factory (reprise)

- Imagine that all you know about a factory is that it makes cubes with volumes between 0 and  $8\,\mathrm{cm}^3$ .
- What is the probability that the next cube will have a volume less than or equal to 1cm<sup>3</sup>?

### The Cube Factory (reprise)

- Imagine that all you know about a factory is that it makes cubes with volumes between 0 and  $8 \text{cm}^3$ .
- $\cdot$  What is the probability that the next cube will have a volume less than or equal to 1cm  $^3?$
- Intuitively, it's 1 in 8, right?

#### **Problem**

- · These are the same question!
- To say the sides are between 0 and 2 just is to say the volume is between 0 and 8.
- And to say that the side length is at most 1 just is to say that the volume is at most 1.
- If we try to respect all intuitive symmetries, we are led into inconsistency.

### **Principle of Indifference**

The intuitive rule we've been discussing here has a name, the Principle of Indifference.

- It says that given a partition of possibility space into n possibilities, and no reason to give higher probability to any one of them, give each of them probability  $\frac{1}{n}$ .
- But this is incoherent since the possibility that the cubes are under 1
  is both part of a 2-way partition (the partition by side lengths) and an
  8-way partitio (the partition by volumes).

For Next Time
<ul> <li>We will take a short look at why some theorists thought it didn't matter if people start with very different priors.</li> </ul>