305 Lecture 43 - The One True Prior

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Plan

- We're going to look at a very objective version of the subjective theory, one that says there is a single true prior probability function.

Associated Reading

Odds and Ends, chapter 18.

Symmetry

There is a natural way to start looking for the one true prior.

- When we do probabilities involving games of chance, we naturally take the different possibilities as having equal probability.
- So when we give equal probability to heads and tails, or to each marble being drawn, or each side of the die coming up, that doesn't seem to be based on a careful analysis of coins, marbles or dice.
- Rather, it's just natural to divide probability evenly among the possibilities.

A Caveat

There is one way in which it is more complicated than that.

- We don't give any probability to the coin landing on its edge, or the die landing on a corner.
- I suspect this is because we've observed a lot about the world from a very young age, and noticed things like that just don't happen.
- This complicates the narrative that we're just using some natural principle of prior probability.

Logical Probability

The very rough idea is that there are logical symmetry principles, and the prior probability function is one that respects these symmetries.

- If p is a proposition that things are one side of one of these lines of symmetry in logical space, the prior probability of p is 0.5.

Logical Probability

The very rough idea is that there are logical symmetry principles, and the prior probability function is one that respects these symmetries.

- If p is a proposition that things are one side of one of these lines of symmetry in logical space, the prior probability of p is 0.5.
- Don't think about this too hard because it isn't actually going to work.
- The problem is that there are too many symmetries.

The Cube Factory

- Imagine that all you know about a factory is that it makes cubes with side lengths between 0 and 2cm.
- What is the probability that the next cube will have a side length less than or equal to 1cm?

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- What is the probability that the next cube will have a side length less than or equal to 1cm?
- Intuitively, it's 0.5, right?

The Cube Factory (reprise)

- Imagine that all you know about a factory is that it makes cubes with volumes between 0 and 8cm^3.
- What is the probability that the next cube will have a volume less than or equal to 1cm^3?

The Cube Factory (reprise)

- Imagine that all you know about a factory is that it makes cubes with volumes between 0 and 8cm^3.
- What is the probability that the next cube will have a volume less than or equal to 1cm^3?
- Intuitively, it's 1 in 8, right?

Problem

- These are the same question!
- To say the sides are between 0 and 2 just is to say the volume is between 0 and 8.
- And to say that the side length is at most 1 just is to say that the volume is at most 1.
- If we try to respect all intuitive symmetries, we are led into inconsistency.

Principle of Indifference

The intuitive rule we've been discussing here has a name, the Principle of Indifference.

- It says that given a partition of possibility space into n possibilities, and no reason to give higher probability to any one of them, give each of them probability $\frac{1}{n}$.
- But this is incoherent - since the possibility that the cubes are under 1 is both part of a 2-way partition (the partition by side lengths) and an 8-way partitio (the partition by volumes).

For Next Time

- We will take a short look at why some theorists thought it didn't matter if people start with very different priors.