305 Lecture 15 - Truth Tables

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Plan for Today

Introducing truth tables.

Carnap book, chapter 10, first half.

Conceptual Idea

- Think about all the combinations of truth values for the atomic sentences in a longer sentence, or in an argument.

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- For each possible combination, evaluate the truth of every part of every sentence in an argument.
- See if it is possible for the premises to be true and the conclusion false.

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- If there is such a combination, tentatively mark the argument invalid.
- We'll come back to why 'tentatively'.

- We list each of the combinations in separate rows.
- In each column we list the truth value of the sentence such that the symbol at the top of that column is the main connective.
- That's I think a lot easier to understand in practice than in theory, so let's start with some examples.

A Truth Table

P	Q	(	P	→	Q	)	∨	(	Q	→	P	)
T	T		T	T	T		T		T	T	T	
T	F		T	F	F		T		F	T	T	
F	T		F	T	T		T		T	F	F	
F	F		F	T	F		T		F	T	F	

P	Q	(P	→	Q	)	∨	(Q	→	P	)
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$		
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$		
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$		
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$		

This is a truth table for a single sentence, not an argument. We'll get to arguments in a bit.

P	Q	(P	→	Q	)	∨	(Q	→	P	)
𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	
𝕋	𝔽	𝕋	𝔽	𝔽	𝕋	𝔽	𝕋	𝕋		
𝔽	𝕋	𝔽	𝕋	𝕋	𝕋	𝕋	𝔽	𝔽		
𝔽	𝔽	𝔽	𝕋	𝔽	𝕋	𝔽	𝕋	𝔽		

We will also get (even sooner) to how to build these monsters. What I first want to talk about is how to read them.

P Q	(P → Q) ∨ (Q → P)
T T	T T T T T T T
T F	T F F T F T T
F T	F T T T T F F
F F	F T F T F T F

Each of the four rows represent a way things could be. For instance, the second row (in blue here) represents how things are if *P* is true and *Q* is false.

P	Q	(P	→	Q	) ∨ (	Q	→	P	)
T	T	T	T	T	T	T	T	T	
T	F	T	F	F	T	F	T	T	
F	T	F	T	T	T	T	F	F	
F	F	F	T	F	T	F	T	F	

There are four rows because there are 2 sentence letters - *P* and *Q* - each of which could take 2 values, so there are $2 \times 2 = 4$ combinations of values.
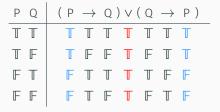
P Q	(P $\rightarrow$ Q) $\vee$ (Q $\rightarrow$ P)
$\mathbb{T}$ $\mathbb{T}$	$\mathbb{T}$ $\mathbb{T}$ $\mathbb{T}$ $\color{red}{\mathbb{T}}$ $\mathbb{T}$ $\mathbb{T}$ $\mathbb{T}$
$\mathbb{T}$ $\mathbb{F}$	$\mathbb{T}$ $\mathbb{F}$ $\mathbb{F}$ $\color{red}{\mathbb{T}}$ $\mathbb{F}$ $\mathbb{T}$ $\mathbb{T}$
$\mathbb{F}$ $\mathbb{T}$	$\mathbb{F}$ $\mathbb{T}$ $\mathbb{T}$ $\color{red}{\mathbb{T}}$ $\mathbb{T}$ $\mathbb{F}$ $\mathbb{F}$
$\mathbb{F}$ $\mathbb{F}$	$\mathbb{F}$ $\mathbb{T}$ $\mathbb{F}$ $\color{red}{\mathbb{T}}$ $\mathbb{F}$ $\mathbb{T}$ $\mathbb{F}$

· If there had been three sentence letters, there would be eight rows.

· Four sentence letters would mean 16 rows, etc.

P	Q	(P	→	Q	)	∨	(Q	→	P	)
T	T	T	T	T	T	T	T	T		
T	F	T	F	F	T	F	T	T		
F	T	F	T	T	T	T	F	F		
F	F	F	T	F	T	F	T	F		

- The columns under the letters reflect the value of the atomic sentences in each row.
- As you can see, they are just cut-and-paste from the left hand side.

P	Q	(P	→	Q	) ∨ (	Q	→	P	)
T	T	T	T	T	T	T	T	T	
T	F	T	F	F	T	F	T	T	
F	T	F	T	T	T	T	F	F	
F	F	F	T	F	T	F	T	F	

I've put in blue all the truth values for *P*, which as you can see were just copied and pasted from the columns on the far left.

P	Q	(P	→	Q	) ∨ (	Q	→	P	)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

The surprising thing (or at least the thing that surprised me as a student) was what we mean by the column under the →, which I've put in blue.

P	Q	(P	→	Q	) ∨ (Q	→	P	)
T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

Each letter here is giving the truth value of the sentence that has that first → as its main connective. That is, $P \rightarrow Q$.

P	Q	(P	→	Q	)	∨	(Q	→	P	)
T	T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	F	T	F

And this column gives the truth values for $Q \rightarrow P$.

P	Q	(P	→	Q	)	∨	(Q	→	P	)
T	T	T	T	T	T	T	T	T	T	
T	F	T	F	F	T	F	T	T	T	
F	T	F	T	T	T	T	T	F	F	
F	F	F	T	F	T	F	T	T	F	

Don't worry for now about why we write those letters down; we'll get to that in the next lecture. For now I just want to go over how to read these tables.

P Q	(P → Q) ∨ (Q → P)
𝕋 𝕋	𝕋 𝕋 𝕋 𝕋 𝕋 𝕋 𝕋
𝕋 𝔽	𝕋 𝔽 𝔽 𝕋 𝔽 𝕋 𝕋
𝔽 𝕋	𝔽 𝕋 𝕋 𝕋 𝕋 𝔽 𝔽
𝔽 𝔽	𝔽 𝕋 𝔽 𝕋 𝔽 𝕋 𝔽

- And the column that I've put in red gives the truth value of the sentence whose main connective is ∨.
- That is, in this case, the whole sentence.

P Q	(P	→	Q)	∨	(Q	→	P)
𝕋 𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	𝕋	𝕋
𝕋 𝔽	𝕋	𝔽	𝔽	𝕋	𝔽	𝕋	𝕋
𝔽 𝕋	𝔽	𝕋	𝕋	𝕋	𝕋	𝔽	𝔽
𝔽 𝔽	𝔽	𝕋	𝔽	𝕋	𝔽	𝕋	𝔽

Ultimately the red column is all we really care about - the others are essentially scaffolding.

P Q	(P → Q) ∨ (Q → P)
T T	T T T **T** T T T
T F	T F F **T** F T T
F T	F T T **T** T F F
F F	F T F **T** F T F

- There is something distinctive about this table - the red column is all T.
- That means the sentence is a logical truth.
- We'll have more to say about this presently in future lectures.

We'll talk about how to build truth tables.