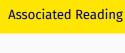
305 Lecture 31 - Probability Revision

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July 22, 2020



• This is a short lecture just for revising some of the basic principles about probability.



None - this is just revision

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- 4. So, from 1, 2, 3, we get $Pr(A) + Pr(\neg A) = 1$.
- 5. So, from 4, we get $Pr(\neg A) = 1 Pr(A)$.

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- 2. Multiplying both sides by Pr(B) gives us $Pr(A \land B) = Pr(A|B) Pr(B)$.

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- 6. Also by the multiplication rule, $\Pr(\neg A \land B) = \Pr(B | \neg A) \Pr(\neg A)$.

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- 2. So, $Pr(B) = Pr((A \land B) \lor (\neg A \land B))$.
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- 4. So $Pr((A \land B) \lor (\neg A \land B)) = Pr(A \land B) + Pr(\neg A \land B)$.
- 5. By the multiplication rule, $Pr(A \land B) = Pr(B|A) Pr(A)$.
- 6. Also by the multiplication rule, $Pr(\neg A \land B) = Pr(B|\neg A) Pr(\neg A)$.
- 7. Putting all these together, we get

$$Pr(B) = Pr(B|A) Pr(A) + Pr(B|\neg A) Pr(\neg A)$$

Another Conditional Probability Rule

Putting that formula for Pr(B) into the definition of conditional probability, we get

$$Pr(A|B) = \frac{Pr(A \land B)}{Pr(B|A) Pr(A) + Pr(B|\neg A) Pr(\neg A)}$$

Yet Another Conditional Probability Rule

$$Pr(B|A) \times \frac{Pr(A)}{Pr(B)} = \frac{Pr(B \land A)}{Pr(A)} \times \frac{Pr(A)}{Pr(B)}$$
$$= \frac{Pr(A \land B)}{Pr(A)} \times \frac{Pr(A)}{Pr(B)}$$
$$= \frac{Pr(A \land B)}{Pr(B)}$$
$$= Pr(A|B)$$

Yet Another Conditional Probability Rule

Or, as it is usually written

$$Pr(A|B) = \frac{Pr(B|A) Pr(A)}{Pr(B)}$$

For Next Time
 We will look at a more complicated example of inverting conditional probabilities.