

305 Lecture 22 - Examples of Truth Trees

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July 15, 2020

Plan

This lecture goes over some examples of truth trees.

Associated Reading

Boxes and Diamonds, section 2.4.


A Logical Truth

This is a tableau for showing $A \rightarrow (A \vee B)$ is a logical truth.

- | | | |
|----|---|---------------------------|
| 1. | $\text{F } A \rightarrow (A \vee B) \checkmark$ | Assumption |
| 2. | $\text{T } A$ | $\rightarrow \text{F}, 1$ |
| 3. | $\text{F } A \vee B \checkmark$ | $\rightarrow \text{F}, 1$ |
| 4. | $\text{F } A$ | $\vee \text{F}, 3$ |
| 5. | $\text{F } B$ | $\vee \text{F}, 3$ |
| | \otimes | |


An Argument

Here is a tableau for showing that $A \rightarrow B, \neg B \vdash \neg A$ is valid.

1.	$\text{T } A \rightarrow B \checkmark$	Assumption
2.	$\text{T } \neg B \checkmark$	Assumption
3.	$\text{F } \neg A \checkmark$	Assumption
4.	$\text{F } B$	$\neg\text{T}, 2$
5.	$\text{T } A$	$\neg\text{F}, 3$
		
6.	$\text{F } A \quad \text{T } B$	$\rightarrow\text{T}, 1$
	$\otimes \quad \otimes$	

An Open Tableau

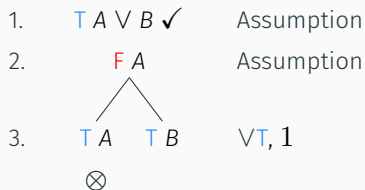
Here is a tableau for showing that $A \rightarrow B, \neg A \vdash \neg B$ is invalid.

- | | | |
|----|---|--------------------------|
| 1. | $\text{T } A \rightarrow B \checkmark$ | Assumption |
| 2. | $\text{T } \neg A \checkmark$ | Assumption |
| 3. | $\text{F } \neg B \checkmark$ | Assumption |
| 4. | $\text{F } A$ | $\neg\text{T}, 2$ |
| 5. | $\text{T } B$ | $\neg\text{F}, 3$ |
| |  | |
| 6. | $\text{F } A \quad \text{T } B$ | $\rightarrow\text{T}, 1$ |

This one is over the top - since both branches are open. You only need one open branch.

Another Open Tableau

Here's one for the argument $A \vee B \vdash A$



The right hand branch is open, so the whole tableau is open.

For Next Time

We will look at how we use truth trees to check for validity.