

305 Lecture 10 - Indirect Proof

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Plan for Today

We're going to introduce how we prove negations in Carnap.

Associated Reading

Carnap book, chapter 6.

How to Prove a Negation

Show that if it were true, something absurd would follow.

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- In finance, a risk-free way to make a guaranteed profit.
- In logic, a sentence and its negation.

Structure of Proof

1. Say that we will show $\neg X$
2. Assume X
3. Derive some pair of sentences, Y and $\neg Y$
4. Close the indirect proof by listing the lines where Y and $\neg Y$ appear.
5. Say that $\neg X$ has been shown at the 'show' line, and not citing any of the lines between it and the 'Indirect Proof' line.

Example

To prove: $P \rightarrow Q, P \rightarrow \neg Q \vdash \neg P$

1. Show: $\sim P$

2. P :AS

3. $P \rightarrow Q$:PR

4. $P \rightarrow \sim Q$:PR

5. Q :MP 2, 3

6. $\sim Q$:MP 2, 4

7. :ID 5 6

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- Oddly, we don't put a comma between the lines cited at the end.
- Once you end the sub-proof, you can't cite lines from in it again.
- But in the sub-proof, you can cite lines from earlier; they could even be one of the pair of lines that make up the absurdity.

One More Example

To prove: $P \rightarrow Q, R \rightarrow \neg Q \vdash R \rightarrow \neg P$

1. Show: $R \rightarrow \sim P$

2. R :AS

3. $P \rightarrow Q$:PR

4. $R \rightarrow \sim Q$:PR

5. $\sim Q$:MP 2, 4

6. Show: $\sim P$

7. P :AS

8. Q :MP 3,7

9. :ID 5 8

10. :CD 6

For Next Time

We will talk about an interesting general fact about what you can prove in Carnap.