305 Lecture 27 - Probability Trees

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Plan

- This lecture walks through a worked example of how to use probability trees to calculate a probability.

A Tree Example

Soccer Tournament

There is a big soccer tournament this weekend. The teams competing are

- Fireflies
- Penguins
- Huskies
- Bluebirds

Tournament Structure

There will be three games.

1. Fireflies vs Penguins
2. Huskies vs Bluebirds
3. Winner of Game 1 vs Winner of Game 2

Each game will have a winner one way or the other (maybe via penalty kicks or extra time).

Team Strength

The teams are not all equally good. They each have a ‘strength’. Here is their respective strengths

Team	Strength
Fireflies	5
Penguins	4
Huskies	3
Bluebirds	1

Win Probabilities

If a team with strength x plays a team with strength y, the team with strength x will win with probability

$$\frac{x}{x+y}$$

And the team with strength y will win with probability

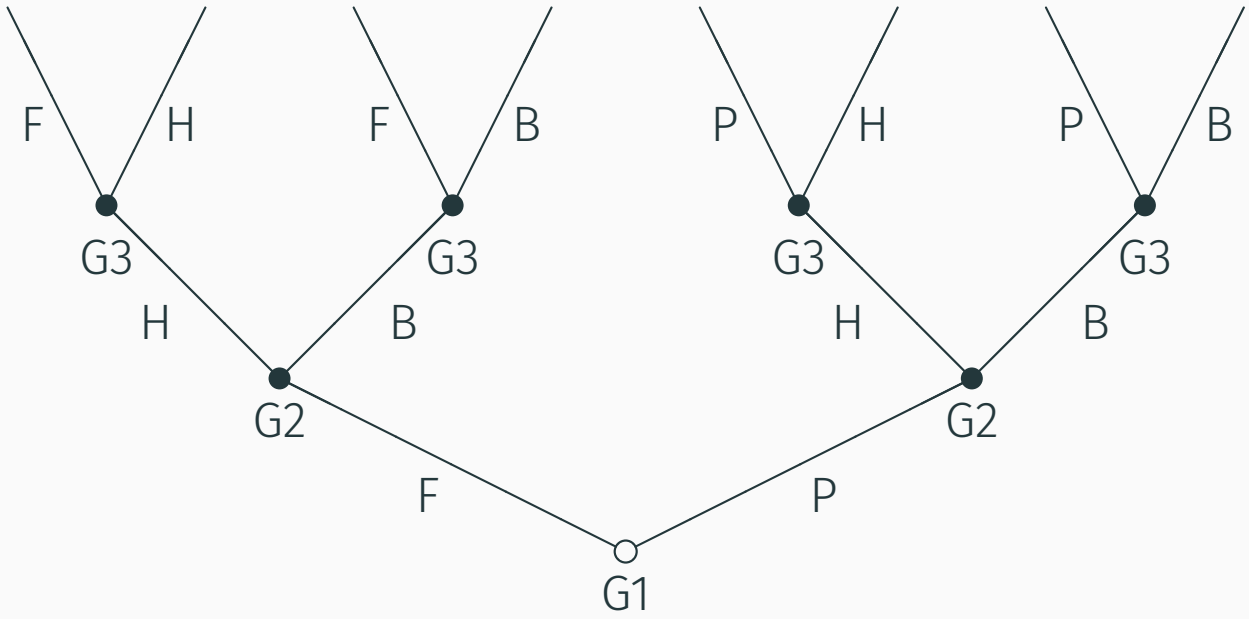
$$\frac{y}{x+y}$$

Question

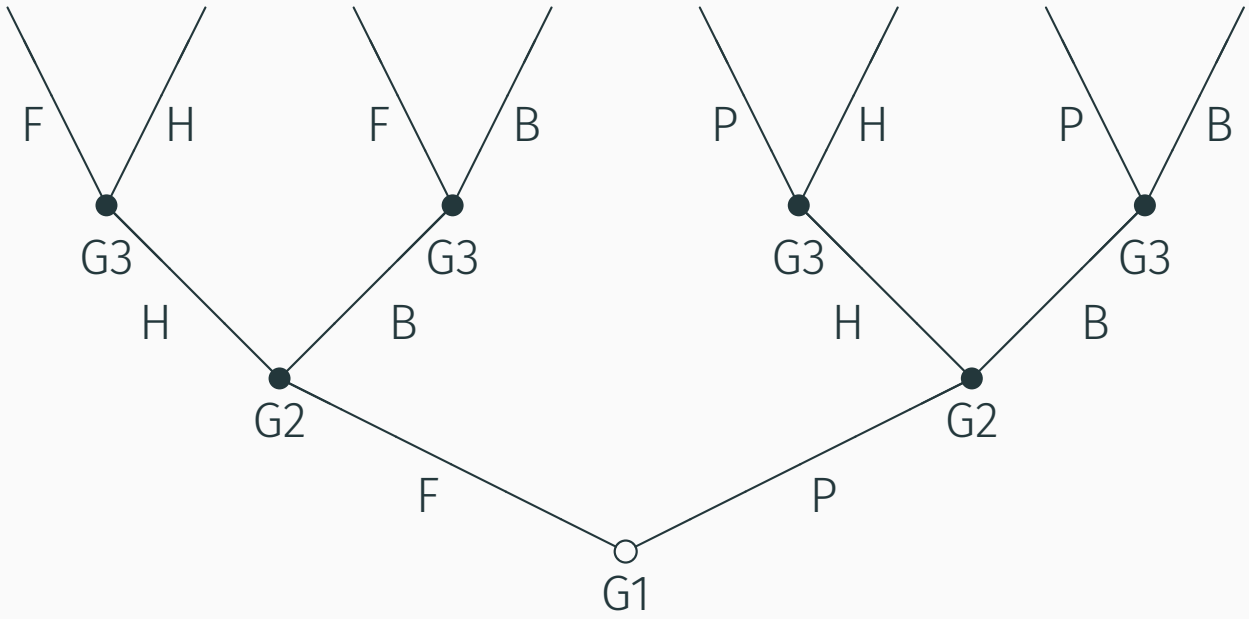
What is the probability that each team will win the tournament?

- We will answer this by doing a tree.

Tournament Tree

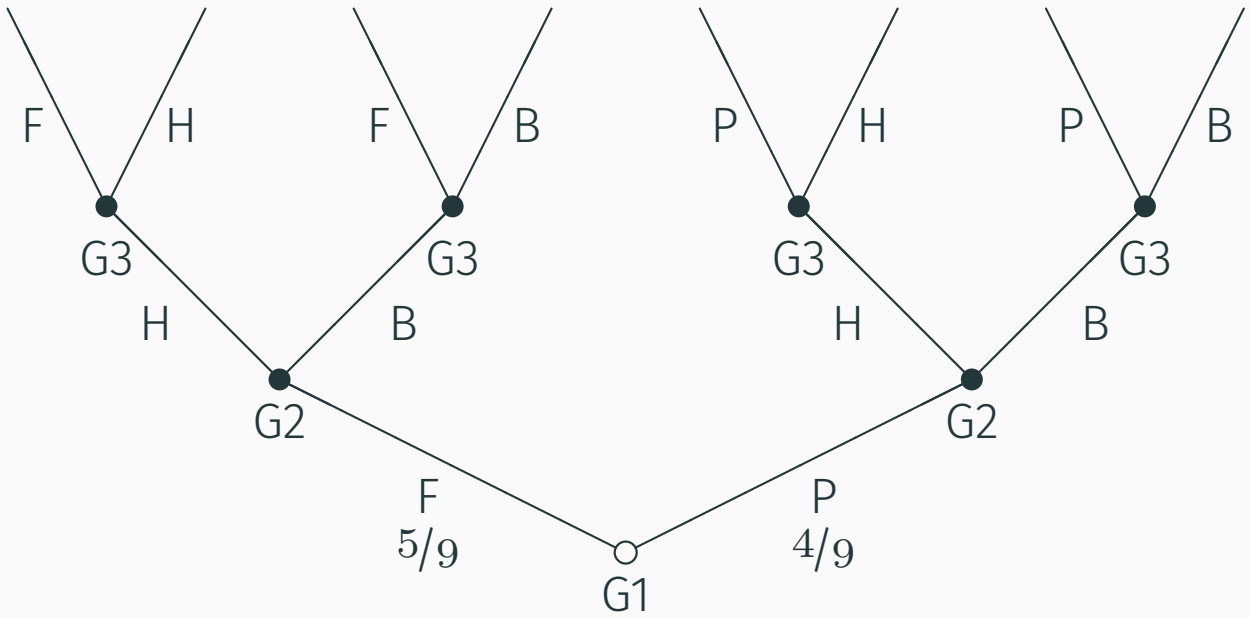


Tournament Tree



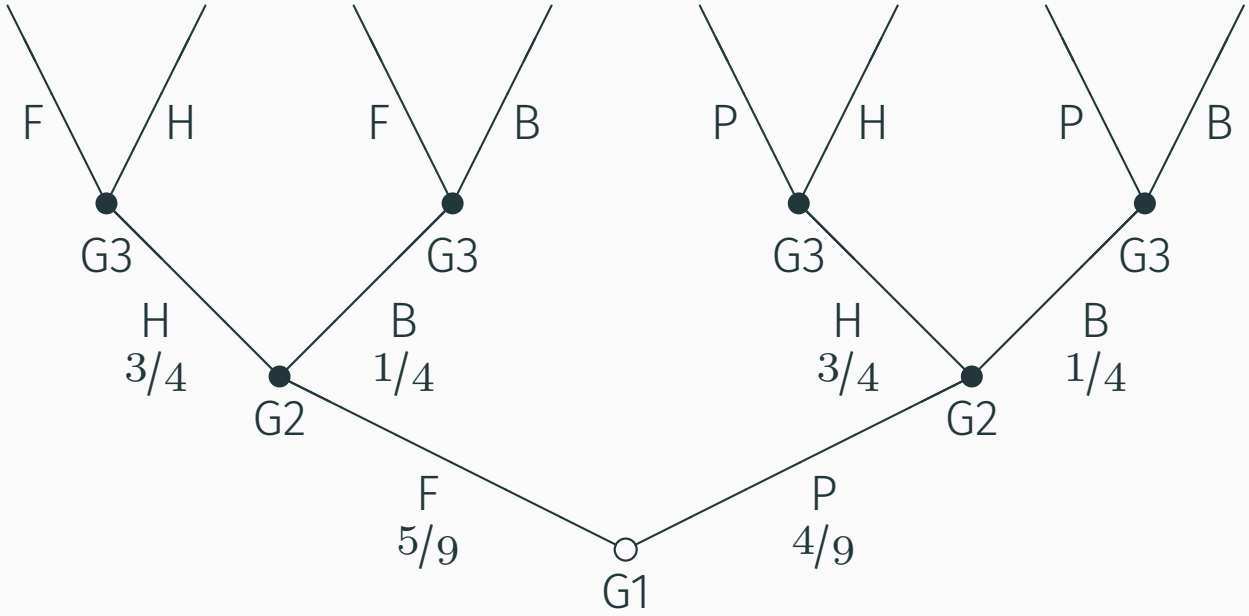
Now we have to add the probabilities to it.

Tournament Tree



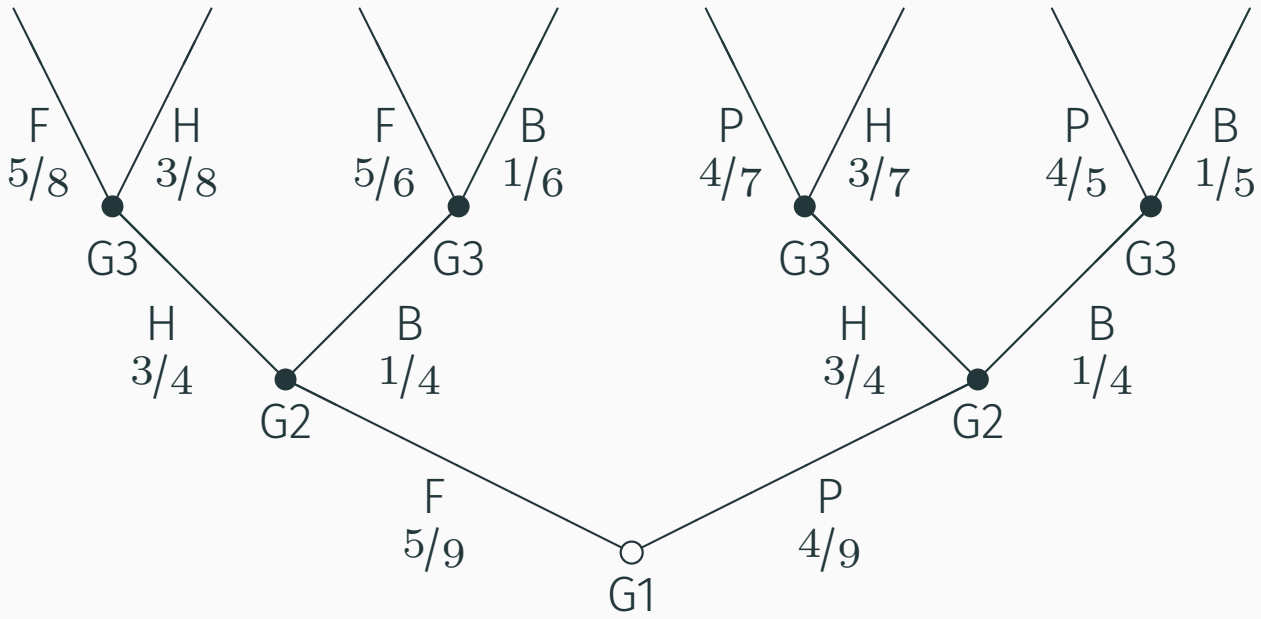
The first game is strength 5 vs strength 4, so the win probability for the stronger team is $5/5+4$, i.e., $5/9$.

Tournament Tree



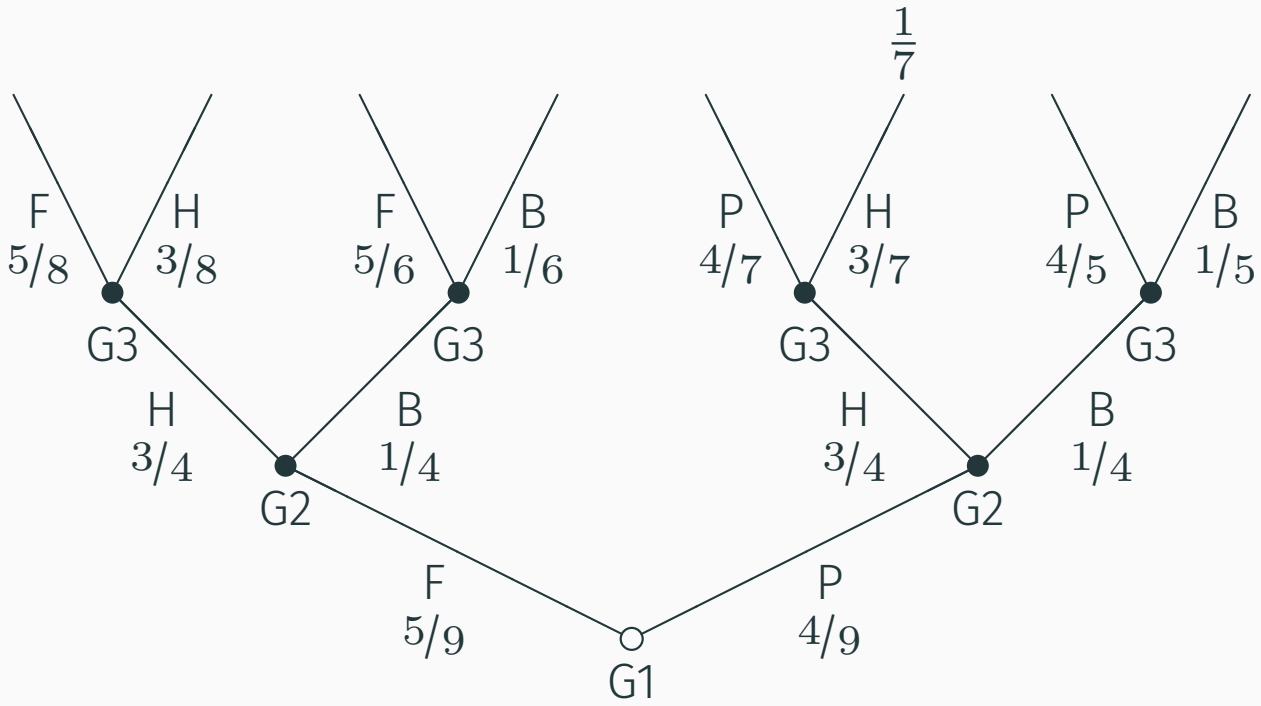
The second game is strength 3 vs strength 1, so the win probability for the stronger team is $3/3+1$, i.e., $3/4$. And it doesn't matter how the first game went - that's the probability for the second game.

Tournament Tree



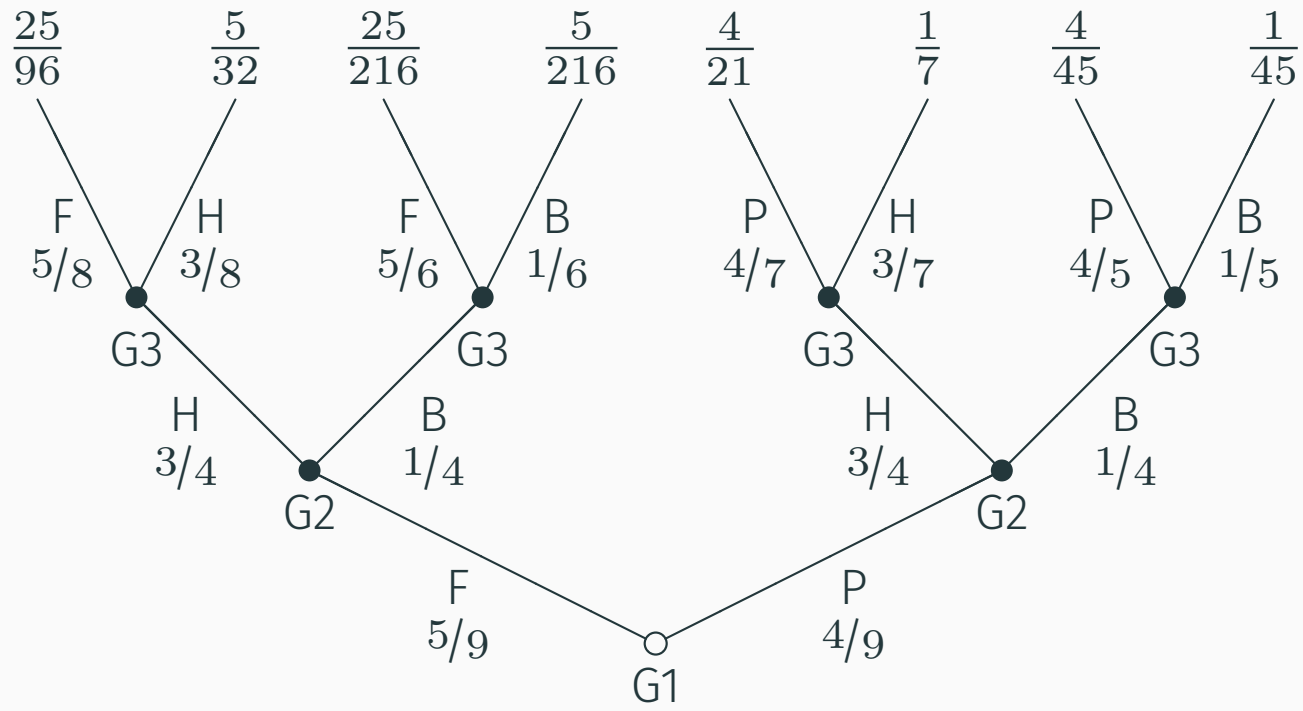
And now for each possible match up in game 3, we apply the formula to get the win probability for each team.

Tournament Tree



- The probability of each completed branch is the product of each of the smaller branches.
- So the one I've marked is $\frac{4}{9} \times \frac{3}{4} \times \frac{3}{7} = \frac{1}{7}$.

Tournament Tree



I've included all the others - they usually don't cancel as nicely as that one.

Tournament Table

It might be easier to see the results in a table

Winner	Runner-Up	Probability	Approx
Fireflies	Huskies	$\frac{25}{96}$	0.260
Huskies	Fireflies	$\frac{5}{32}$	0.156
Fireflies	Bluebirds	$\frac{25}{216}$	0.116
Bluebirds	Fireflies	$\frac{5}{216}$	0.023
Penguins	Huskies	$\frac{4}{21}$	0.190
Huskies	Penguins	$\frac{1}{7}$	0.143
Penguins	Bluebirds	$\frac{4}{45}$	0.089
Bluebirds	Penguins	$\frac{1}{45}$	0.022

Tournament Table

And we can rearrange that so the rows where each team wins are adjacent.

Winner	Runner-Up	Probability	Approx
Fireflies	Huskies	$\frac{25}{96}$	0.260
Fireflies	Bluebirds	$\frac{25}{216}$	0.116
Huskies	Fireflies	$\frac{5}{32}$	0.156
Huskies	Penguins	$\frac{1}{7}$	0.143
Penguins	Huskies	$\frac{4}{21}$	0.190
Penguins	Bluebirds	$\frac{4}{45}$	0.089
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Tournament Table

And then just adding up the probabilities for the two ways each team can win, we get the actual probabilities of each win. (I'm just doing the decimals now.)

Winner	Approx Probability
Fireflies	0.376
Huskies	0.299
Penguins	0.279
Bluebirds	0.045

(Those numbers don't sum to 1 precisely because of rounding.)

For Next Time

- We will talk about a well known piece of fallacious reasoning: the gamblers' fallacy.