305 Lecture 06 - Recursive Composition Rules

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Plan for This Lecture

- We’re going to look at how and why we can iterate the translation procedures we’ve been investigating.

Recursion

The language of propositional logic has some fairly simple composition rules.

- It says what the basic sentences are.
- It has some rules saying that if some things are sentences, so are some other things.

The effect is that there are an infinity of possible sentences.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.
- Alex thinks that Kim thinks that Alex thinks that it will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.
- Alex thinks that Kim thinks that Alex thinks that it will rain.
- And so on, to infinity, without adding any more words.

Recursive Rule

- If S is a sentence, and N is a name, then N *thinks that* S is a sentence.
- Note that the output of this rule can be the inpute to a new instance of it.

Formal Language Recursion

- The letters P, Q, R... are sentences.
- If S and T are sentences, then so are:

1. $\neg S$
2. $S \vee T$
3. $S \wedge T$
4. $S \longrightarrow T$
5. $S \leftrightarrow T$

Multiple Steps

So these are all sentences. (Note that I'm playing fast and loose with parentheses here.)

1. P
2. Q
3. $P \wedge Q$
4. $Q \rightarrow (P \wedge Q)$
5. $\neg P$
6. $Q \vee \neg P$
7. $(Q \rightarrow (P \wedge Q)) \leftrightarrow (Q \vee \neg P)$

The last one follows from the fact that 4 and 6 are sentences.

Main Connective

For any sentence you can make, there will be a 'last step' in the demonstration that it is a sentence.

- That last step will involve copying down 1 or 2 other sentences, and adding a connective.
- On the previous slide, you copy down 4 and 6, and put a $\leftrightarrow$ between them.
- That connective you add is the **main connective** of the sentence.
- It covers all the material in the sentence.

Main Connective

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

$$P \land (Q \rightarrow R)$$

The $\land$ is the main connective because either side of it are

- P
- $(Q \rightarrow R)$

And they are both sentences.

Main Connective

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

$$P \wedge (Q \rightarrow R)$$

The $\rightarrow$ is the main connective because either side of it are

- $P \wedge (Q$
- $R)$

And they are not both sentences.

A Worked Example

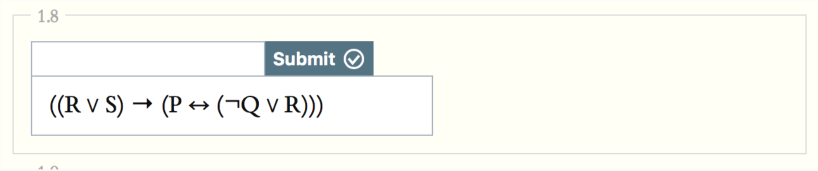

Figure 1: Step 1

A Worked Example

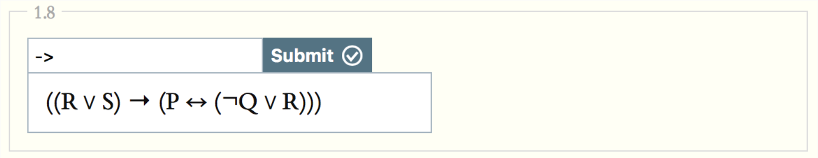

Figure 2: Step 2

A Worked Example

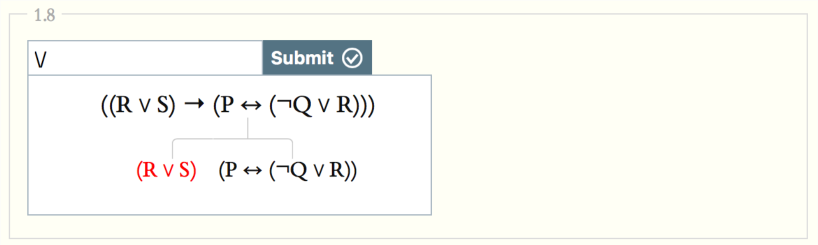

Figure 3: Step 3

A Worked Example

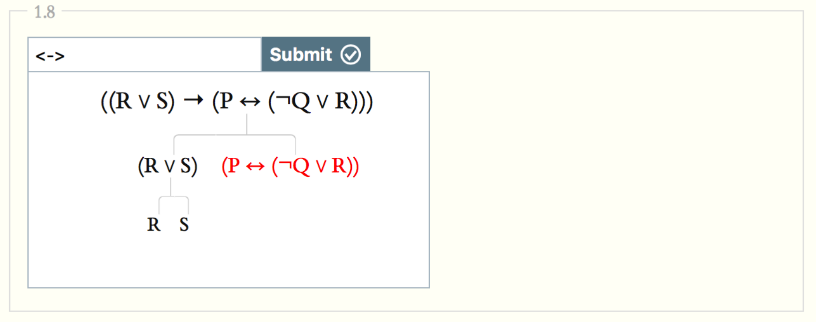

Figure 4: Step 4

A Worked Example

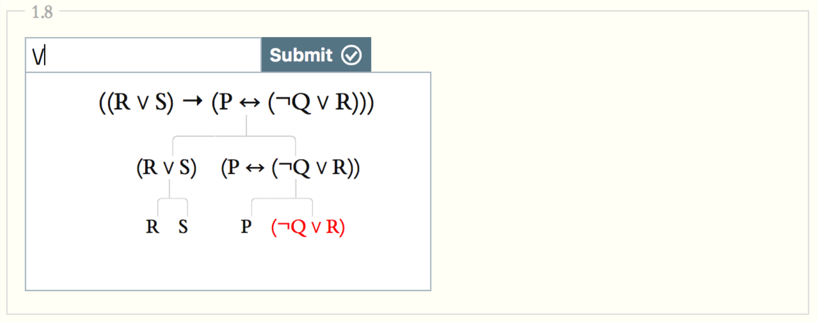

Figure 5: Step 5

A Worked Example

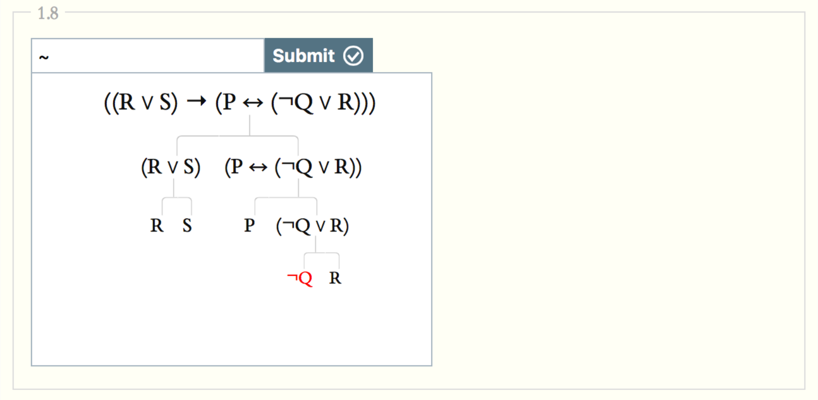

Figure 6: Step 6

For Next Time

- That's all for today's lectures
- For next time, read Chapter 3 of The Carnap Book
- We're going to start applying these tools to analysing arguments.