305 Lecture 10.4 - Subjective Theories of Probability

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Plan

- We're going to look at versions of the subjective theory of probability

Associated Reading

Odds and Ends, chapter 16.

Subjective Theory

Probability is degree of confidence.

Two Questions

1. Whose confidence?

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2. Actual confidence or idealised confidence?

Actual

Question 2 seems easiest to answer.

- No matter how confident you or I am that the moon is made of green cheese, it is not probable that the moon is made of green cheese.
- If we're really confident that it is, then we're getting something very badly wrong.
- But if probability just was how confident we actually are in something, then it would be probable that the moon is made of green cheese.

Idealised

A better view is, as the textbook says, that

> Probability is ultimately about belief. It's about how certain you should be that something is true.

So not what you actually believe, but what you should believe. This gets out of the green cheese problem.

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We could do weeks on every one of these questions, but I’ll focus on 3/4.

Bayesianism

The main subjective theory is known as Bayesianism.

- The name comes from the Rev. Thomas Bayes, an 18th century mathematician.
- Just what is and is not a form of Bayesianism is a slightly contested, though mostly terminological question.

Conditionalisation

The core principle behind Bayesianism is that updating is by conditionalisation.

- That is, the new Pr(H) after getting evidence E is the old Pr(H|E).
- And Pr(H|E) is given by the formula.

$$\Pr(H|E) = \frac{\Pr(E|H)\,\Pr(H)}{\Pr(E)}$$

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- That tells you how to convert an old probability into a new probability given some evidence E.
- That is, it tells you how to generate a **posterior** probability.
- But it does not tell you where the **prior** probability comes from.
- And this question is one that Bayesians have never quite had a good answer to.

Green Cheese

To make the problem vivid, imagine that I start out with the following probabilities.

- The probability that the moon is made of green cheese is 0.99.
- The things that happen around here are probabilistically independent of hypotheses about the material composition of the moon.

The first of these is absurd, but the second isn't ridiculous I suppose.

Updating

Now some stuff happens, I get evidence E.

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- Since Pr(H) = 0.99, and H is independent of E, it follows that Pr(H|E) = 0.99.

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- Since Pr(H) = 0.99, and H is independent of E, it follows that Pr(H|E) = 0.99.
- So the Bayesians' favourite updating rule says that the new probability for H should be 0.99.
- But that's fairly absurd - what could it mean to say that I **should** have probability 0.99 in H?

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The very rough history of this is that 19th century folks were sympathetic to option 1, but in the 20th century, most of the focus was on option 2.

For Next Time

- We will take a short look at the ‘one true prior’ option.