305 Lecture 8.6 - Updating on Multiple Data Points

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Plan

- To look at what happens when we learn two facts about some environment.

Associated Reading

Odds and Ends, Chapter 9

Conditional Independence

In a lot of cases, the two data points we get will not be probabilistically independent, but they will be **conditionally independent**.

That is, if B_1 and B_2 are the data points, and X is an arbitrary hypothesis (like A, ¬A), we will have

$$Pr(B_1|X)\,Pr(B_2|X) = Pr(B_1 \wedge B_2|X)$$

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- The results of two flips of the same coin are not indepdendent.
- If one flip lands heads, that is evidence of a bias towards heads, and hence it increases the probability of heads on the next flip.
- But conditional on a hypothesis about the bias of the coin, the flips are independent.

Skilled Activity

A perhaps more real-life case of this is skilled action, like shooting free throws.

- The success of one attempt is not independent of the success of the previous.
- But conditional on the skill of the actor, the attempts are (probably, more or less) independent.

Sampling With Replacement

Drawing from a selection **with replacement** produces conditional independence.

- If I don't know how many black marbles are in an urn, then drawing a black marble **and replacing it** will be evidence that the next marble is black.
- But conditional on a hypothesis about the nature of the urn, the draws with replacement will be independent.

Yesterday, Today, Tomorrow

This is a little off topic, but a lot of real world phenomena satisfy (roughly) the following condition.

- How things were yesterday is a good (probabilistic) guide to how things will be tomorrow.
- So how things will be tomorrow is not independent of how things were yesterday.

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- How things were yesterday is a good (probabilistic) guide to how things will be tomorrow.
- So how things will be tomorrow is not independent of how things were yesterday.
- But, conditional on how things are today, how things were yesterday and will be tomorrow are independent.
- Knowing how things were yesterday doesn’t tell you any more about how things will be tomorrow once you know how things are today.

Markov Chains

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- Lots of real world processes are (more or less) Markov Chains.
- Weather systems, for instance, are probably more or less Markov Chains.
- And lots of ecological models assume that animal populations are Markov Chains.
- And the core idea is just conditional independence.

Conditional Independence

In cases where the data points B_1 and B_2 are independent, we have an easy story about how to work out the probabilities.

$$Pr(B_1 \wedge B_2|X) = Pr(B_1|X)\,Pr(B_2|X)$$

Same Event

There is an even simpler formula where B_1 and B_2 are the 'same' event, like the coin landing heads both time, or the same color marble being drawn.

$$Pr(B_1 \wedge B_2|X) = Pr(B_1|X)^2$$

For Next Time

We’ll start illustrating this with some worked examples.