305 Lecture 8.2 - Probability Revision

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Plan

- This is a short lecture just for revising some of the basic principles about probability.

Associated Reading

None - this is just revision

The Negation Rule

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4. So, from 1, 2, 3, we get $Pr(A) + Pr(\neg A) = 1$.
5. So, from 4, we get $Pr(\neg A) = 1 - Pr(A)$.

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Multiplying both sides by Pr(B) gives us

$$\Pr(A \wedge B) = \Pr(A|B)\Pr(B)$$

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6. Also by the multiplication rule, $Pr(\neg A \wedge B) = Pr(B|\neg A)\,Pr(\neg A)$.

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1. B is logically equivalent to $(A \wedge B) \vee (\neg A \wedge B)$.
2. So, $\Pr(B) = \Pr((A \wedge B) \vee (\neg A \wedge B))$.
3. $(A \wedge B)$ and $(\neg A \wedge B)$ are exclusive.
4. So $\Pr((A \wedge B) \vee (\neg A \wedge B)) = \Pr(A \wedge B) + \Pr(\neg A \wedge B)$.
5. By the multiplication rule, $\Pr(A \wedge B) = \Pr(B|A)\Pr(A)$.
6. Also by the multiplication rule, $\Pr(\neg A \wedge B) = \Pr(B|\neg A)\Pr(\neg A)$.
7. Putting all these together, we get

$$\Pr(B) = \Pr(B|A)\Pr(A) + \Pr(B|\neg A)\Pr(\neg A)$$

Another Conditional Probability Rule

Putting that formula for Pr(B) into the definition of conditional probability, we get

$$\Pr(A|B) = \frac{\Pr(A \wedge B)}{\Pr(B|A)\Pr(A) + \Pr(B|\neg A)\Pr(\neg A)}$$

Yet Another Conditional Probability Rule

$$
\begin{aligned}
\Pr(B|A) \times \frac{\Pr(A)}{\Pr(B)} &= \frac{\Pr(B \wedge A)}{\Pr(A)} \times \frac{\Pr(A)}{\Pr(B)} \\
&= \frac{\Pr(A \wedge B)}{\Pr(A)} \times \frac{\Pr(A)}{\Pr(B)} \\
&= \frac{\Pr(A \wedge B)}{\Pr(B)} \\
&= \Pr(A|B)
\end{aligned}
$$

Yet Another Conditional Probability Rule

Or, as it is usually written

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$$

For Next Time

- We will look at a more complicated example of inverting conditional probabilities.