305 Lecture 12.4 - S5

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Plan

- The simple rules for the logic S5.

Associated Reading

- Boxes and Diamonds, section 5.8.

Universal relations and Equivalence Relations

Strictly speaking, S5 is the logic of frames where R is an **equivalence relation**. That is, it is a relation that is:

- Transitive
- Reflexive
- Symmetric

But we can simplify things by just focussing on the case where R is a **universal** relation - that is, a relation that holds between any two points.

Universal relations and Equivalence Relations

Not all equivalence relations are universal relations:

- The relation Has the same birthday as is an equivalence relation, but not universal.

But anything that's true on all models for universal R will be true on all models for equivalence relation R.

- That's because from the perspective of any point, R may as well be universal.
- All that matters to truth at any point is what happens at points you can reach in a finite number of steps.
- So R may as well be a universal relation.

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Drop the rules you've already got for $\Box$ and $\Diamond$. And drop the decimal points; all worlds just get referred to by a single number.

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- If □A is true at x, then for any y on the branch, A is true at y.
- If ◇A is false at x, then for any y on the branch, A is false at y

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- If □A is true at x, then for any y on the branch, A is true at y.
- If ◇A is false at x, then for any y on the branch, A is false at y.
- If □A is false at x, then add a new world y with A false at y.
- If ◇A is true at x then add a new world y with A true at y.

For Next Time

On Wednesday we'll look at using trees to prove invalidity, and go over a bunch of examples.