

305 Lecture 3.3 - Validity

Brian Weatherson

Plan

This lecture is about how we use truth tables to check for whether an argument is valid.

Associated Reading

for all x , chapter 12, especially section 12.4.

We can also use truth tables to check for properties of arguments, and in particular to check for validity.

Truth Tables and Validity

- An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are \mathbb{T} , the conclusion is \mathbb{T} as well.

Truth Tables and Validity

- An argument is (truth-functionally) valid if (and only if) every line on the truth table where all the premises are \mathbb{T} , the conclusion is \mathbb{T} as well.
- Equivalently, an argument is invalid if there is a line where the premises are \mathbb{T} and the conclusion \mathbb{F} , and valid otherwise.

Example of Invalidity

The argument $A, \text{ therefore } A \wedge B$ is invalid because of the second line.

A B	A	A \wedge B
T T	T	T T T
T F	T	T F F
F T	F	F F T
F F	F	F F F

Another Invalidity Example

Note that there are several lines with \top premises and conclusion.
But the argument $A \rightarrow B$, so $A \rightarrow C$ is invalid because of line 2.

A B C	$A \rightarrow B$	$A \rightarrow C$
$\top \top \top$	$\top \top \top$	$\top \top \top$
$\top \top \text{F}$	$\top \top \top$	$\top \text{F} \text{F}$
$\top \text{F} \top$	$\top \text{F} \text{F}$	$\top \top \top$
$\top \text{F} \text{F}$	$\top \text{F} \text{F}$	$\top \text{F} \text{F}$
$\text{F} \top \top$	$\text{F} \top \top$	$\text{F} \top \top$
$\text{F} \top \text{F}$	$\text{F} \top \top$	$\text{F} \top \text{F}$
$\text{F} \text{F} \top$	$\text{F} \top \text{F}$	$\text{F} \top \top$
$\text{F} \text{F} \text{F}$	$\text{F} \top \text{F}$	$\text{F} \top \text{F}$

Hypothetical Syllogism

On the other hand the argument from $A \rightarrow B$ and $B \rightarrow C$ to $A \rightarrow C$ is valid.

A B C	$A \rightarrow B$	$B \rightarrow C$	$A \rightarrow C$
T T T	T T T	T T T	T T T
T T F	T T T	T F F	T F F
T F T	T F F	F T T	T T T
T F F	T F F	F T F	T F F
F T T	F T T	T T T	F T T
F T F	F T T	T F F	F T F
F F T	F T F	F T T	F T T
F F F	F T F	F T F	F T F

For Next Time

We'll finish up the discussion of truth tables and validity.