A → □ ◇ A (in S4)

Build a Tableau

To Check Whether it is Valid

A → □ ◇ A is a theorem of S4 = KT4.

- So we can use all the rules, plus the special rules for T and for 4.

1.   1, $\mathbb{F}$ A → □ ◇ A   Assumption

Start with it being false at 1.

1.	1, $\mathbb{F}$ A → □ ◇ A ✓	Assumption
2.	1, $\mathbb{T}$ A	→$\mathbb{F}$, 1
3.	1, $\mathbb{F}$ □ ◇A	→$\mathbb{F}$, 1

You know the drill – left hand side true, right hand side false.

A → □ ◇ A

1.	1, $\mathbb{F}$ A → □ ◇ A ✓	Assumption
2.	1, $\mathbb{T}$ A	→$\mathbb{F}$, 1
3.	1, $\mathbb{F}$ □ ◇A ✓	→$\mathbb{F}$, 1
4.	1.1, $\mathbb{F}$ ◇ A	□$\mathbb{F}$, 3

False □ sentences are false somewhere.

$A \rightarrow \Box \Diamond A$

1.   1, $\mathbb{F}$ $A \rightarrow \Box \Diamond A$ ✓   Assumption
2.   1, $\mathbb{T}$ A $\rightarrow\mathbb{F}$, 1
3.   1, $\mathbb{F}$ $\Box \Diamond A$ ✓ $\rightarrow\mathbb{F}$, 1
4.   1.1, $\mathbb{F}$ $\Diamond A$ $\Box\mathbb{F}$, 3
5.   1.1, $\mathbb{F}$ A $\mathbb{T}\Diamond$ 4

It's T, so false $\Diamond$ sentences have to actually be false.

1.	1, 𝔽 A → □ ◇ A ✓	Assumption
2.	1, 𝕋 A	→𝔽, 1
3.	1, 𝔽 □ ◇A ✓	→𝔽, 1
4.	1.1, 𝔽 ◇ A	□𝔽, 3
5.	1.1, 𝔽 A	T ◇ 4

- But that's it
- We've applied all the rules, and it doesn't close.
- So not a theorem.

- Two worlds, w_1, $w_{1.1}$.
- The accessibility relations are $w_1 R w_{1.1}$, $w_1 R w_1$ and $w_{1.1} R w_{1.1}$.
- The first is from the tree, the next two from reflexivity.
- In this case, transitivity doesn't require anything more. (Though double check this!)
- A is true at w_1 and and false at $w_{1.1}$.
- So $\Diamond$A will be false only at $w_{1.1}$.
- So $\Box \Diamond$ A will be false at w_1, while A is true, as required.