305 Lecture 4.2 - Examples of Truth Trees

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Plan

This lecture goes over some examples of truth trees.

Associated Reading

Boxes and Diamonds, section 2.4.

This is a tableau for showing $A \to (A \vee B)$ is a logical truth.

1.	$\mathbb{F}\ A \to (A \vee B)$ ✓	Assumption
2.	$\mathbb{T}\ A$	$\to\mathbb{F}$, 1
3.	$\mathbb{F}\ A \vee B$ ✓	$\to\mathbb{F}$, 1
4.	$\mathbb{F}\ A$	$\vee\mathbb{F}$, 3
5.	$\mathbb{F}\ B$	$\vee\mathbb{F}$, 3
	x	

Ticks and Crosses

- I've stared including ✓ on lines.
- That's just a reminder that I'm done with that line.
- It's not essential, but it's useful.

Ticks and Crosses

- It's useful in part because you don't always want to apply rules in order.
- In particular, you want to apply branching rules **after** non-branching rules.
- But if you do that, it's easy to forget which lines you still need to apply.
- So the tick marks help.

Ticks and Crosses

- The crosses at the end, saying which branches are closed, are essential.

Here is a tableau for showing that $A \to B, \neg B \vdash \neg A$ is valid.

1.	$\mathbb{T}\, A \to B$ ✓	Assumption
2.	$\mathbb{T}\, \neg B$ ✓	Assumption
3.	$\mathbb{F}\, \neg A$ ✓	Assumption
4.	$\mathbb{F}\, B$	$\neg\mathbb{T}$, 2
5.	$\mathbb{T}\, A$	$\neg\mathbb{F}$, 3
6.	$\mathbb{F}\, A$ $\quad$ $\mathbb{T}\, B$	$\to\mathbb{T}$, 1
	x $\quad$ x	

Here is a tableau for showing that $A \rightarrow B, \neg A \vdash \neg B$ is **invalid**.

1.	$\mathbb{T}\, A \rightarrow B$ ✓	Assumption
2.	$\mathbb{T}\, \neg A$ ✓	Assumption
3.	$\mathbb{F}\, \neg B$ ✓	Assumption
4.	$\mathbb{F}\, A$	$\neg\mathbb{T}$, 2
5.	$\mathbb{T}\, B$	$\neg\mathbb{F}$, 3
6.	$\mathbb{F}\, A$ | $\mathbb{T}\, B$	$\rightarrow\mathbb{T}$, 1

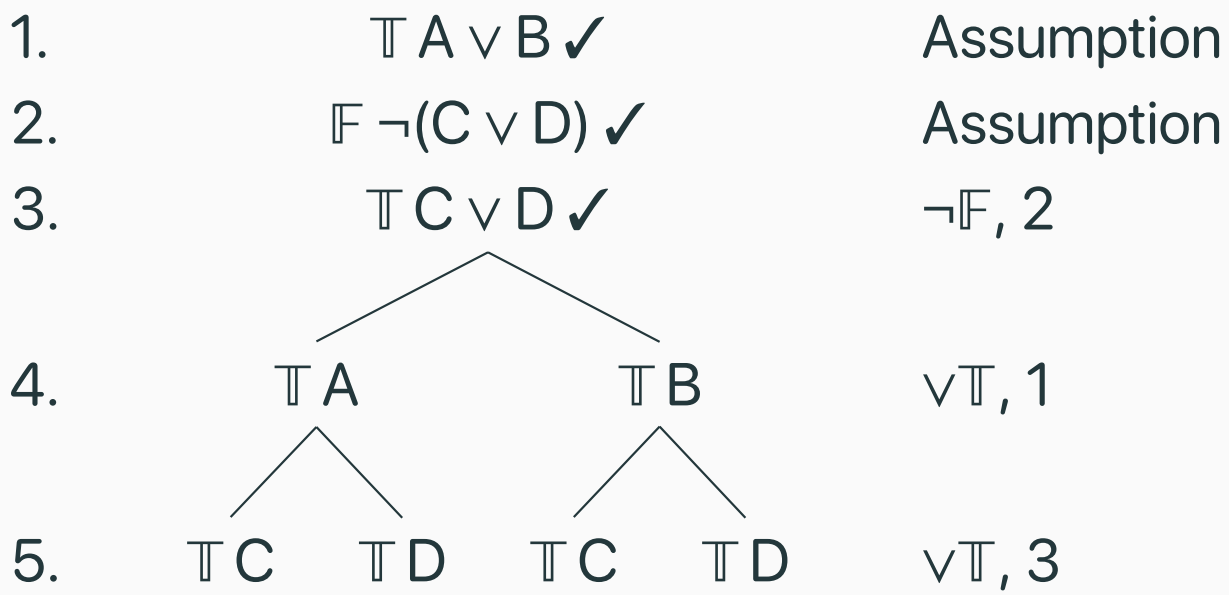
This one is over the top - since both branches are open. You only need one open branch.

Here's one for the argument $A \vee B \vdash A$

1.	$\mathbb{T}\, A \vee B$ ✓	Assumption
2.	$\mathbb{F}\, A$	Assumption
3.	$\mathbb{T}\, A$ | $\mathbb{T}\, B$	$\vee\mathbb{T}$, 1
	x	

The right hand branch is open, so the whole tableau is open.

Here's one for the obviously invalid $A \vee B \vdash \neg(C \vee D)$, to make a point about branching.



For Next Time

We will look at some more worked examples.