305 Lecture 9.6 - Allais Paradox

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Plan

- In this lecture we'll talk about a famous puzzle for the story I've been telling you about the relationship between utility and money: the Allais paradox.

Associated Reading

Odds and Ends, section 13.1.

Sure Thing Principle

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Then the Sure Thing Principle says the following.

- Changing what that return is won’t change your preference between A and B.

Intuitive Example

Assume that B is a **conditional bet** - a bet on q that only takes place if something is true. So if you take the bet, the following things happen.

- So if the condition, and q is true, you win, let's say, $10.
- If the condition, and q is false, you lose $10.
- But if the condition is not met, then the bet is called off.

E.g., you might bet a friend that if the Rose Bowl is played this year, Michigan will win it. The bet is simply called off if the Rose Bowl doesn't happen. Let A be the action of simply not taking this bet, and staying at the status quo. And let p be that the Rose Bowl doesn't happen. So A and B have the same payoff if p, since then the bet is called off.

A Change

You find out, apparently because you've been doing more gambling in your spare time than is good for you, that you have another bet that wins 5 dollars if p, i.e., if the Rose Bowl doesn't happen, and returns nothing otherwise.

- Could this change your preferences over A vs B?

The Argument for No

Either p is true or it isn’t.

- If it is, then whether you would have got $5 if it were not doesn’t make a difference to whether you prefer A or B.
- If it is not, then you should still be indifferent between A and B.
- And this doesn’t look like it just applies to this case.
- It looks like a perfectly general weak dominance argument.

Expected Utility Theory and Constraints on Choice

- Orthodox expected utility theory, the theory that says you should maximise expected utility, puts very few constraints on individual choices.
- But it puts quite striking constraints on sets of choices.
- It says you can't prefer A to B, and B to C, and C to A, for example.
- And it says that the Sure Thing Principle, a principle about what changes in payouts licence a change of preferences, holds.

Allais

Maurice Allais (1911-2010) developed the most famous objection to the Sure Thing Principle.

- It is a pair of two-way choices, and an intuitively rational pair of preferences among them.
- Expected utility can make sense of either one of the pair, but not both.

Allais - First Part

You have a choice between:

A. A 10% chance of $5,000,000.
B. An 11% chance of $1,000,000.

What do you choose?

Allais - Second Part

That was a hypothetical. Now for real you have a choice between:

C. A 10% chance of $5,000,000, an 89% chance of $1,000,000, and a 1% chance of nothing.
D. $1,000,000.

What do you choose?

Allais's Argument

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- It is rational to prefer A to B, and D to C.
- Expected utility theory says that you can't prefer both A to B, and D to C.
- So expected utility theory is false.

Allais Table

Imagine you have 10 blue marbles in an urn, 89 maize marbles, and 1 scarlet marble.

	Blue	Maize	Scarlet
A	5M	0	0
B	1M	0	1M
C	5M	1M	0
D	1M	1M	1M

All that changed from AB to CD was that we changed how much the payout was if Maize, without changing the fact that it was equal.

Why Expected Utility Theory Can't Handle This

Let $u(5M) = x$ and $u(1M) = y$.

- If A is preferred to B, then $0.1x > 0.11y$, since those are the expected utilities of A and B.

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- If A is preferred to B, then $0.1x > 0.11y$, since those are the expected utilities of A and B.
- So adding $0.89y$ to both sides, we get $0.1x + 0.89y > y$.
- But those just are the expected utilities of C and D.
- So if A is preferred to B, then C is preferred to D.

An argument against Allais

Assume you'll find out, both in the AB case and the CD case, whether the marble was maize, or not-maize before you are told its color.

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- At that point, in the AB case, what will you wish you'd chosen?
- If B, or you are indifferent, then you shouldn't have preferred A in the first place.
- If A, then do the same thing in the CD case.
- If you find out the marble is maize, you don't care.
- If you find out it's not maize, then you're back in the exact same puzzle, so you should prefer C to D.
- So by weak dominance, you should prefer C to D overall.

Decision Theory for Allais Agents

- This was originally developed by the Australian economist John Quiggin, and recently expanded by the American philosopher Lara Buchak.
- Very roughly, you replace the Pr in expected utility theory with some function $f(Pr)$ where f measures the agent's attitude to risk.
- If $f(x) < x$, the agent is risk averse, if $f(x) > x$ they are risk seeking.
- If you let $f(x) = x^2$, it's easy to model the Allais preferences.

For Next Time

- We will move on to thinking about how to use probability in learning about the world.