305 Lecture 2.5 - Basic Truth Tables

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Plan

This lecture is the truth tables for the basic connectives.

Associated Reading

- We’re still working through forall x chapters 9-11.
- This is primarily about chapter 9.
- We’re not going to cover biconditionals here (or elsewhere in this course).

Four Main Connectives

- Building truth tables requires, unfortunately, a small amount of memorization.
- In particular, you just have to memorize the truth tables for each of the connectives.
- Equally unfortunately, justifying yourself using truth tables requires justifying these basic tables.
- And as we'll see, that's not trivial.
- But that's for much down the line - let's learn how to use these first, then we'll get to justifying them.

Negation Table

A	$\neg$	A
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$

You should read it as saying that if A is $\mathbb{T}$ then $\neg A$ is $\mathbb{F}$, and if A is $\mathbb{F}$, then $\neg A$ is $\mathbb{T}$.

The Conjunction Table

A	B	A	$\wedge$	B
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$

Conjunction in Words

- A conjunction is $\mathbb{T}$ if both conjuncts are $\mathbb{T}$, and is $\mathbb{F}$ otherwise.

The Disjunction Table

A	B	A	$\vee$	B
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$

Disjunction in Words

- A disjunction is $\mathbb{T}$ if either disjunct is $\mathbb{T}$, and is $\mathbb{F}$ otherwise.

The Conditional Table

A	B	A	$\to$	B
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$

Material Implication

Note that these three sentences have exactly the same table.

A	B	A	$\rightarrow$	B	$\neg$	A	$\vee$	B	$\neg$	$($	A	$\wedge$	$\neg$	B	$)$
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$		$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	
$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$		$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	
$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$		$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	$\mathbb{T}$		$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$	$\mathbb{F}$	

This conditional is sometimes called **material implication**.

Oddities

It is certainly an odd interpretation of ‘if’ that makes these sentences turn out true.

- If I am 200 years old, then Michigan is part of Canada.
- If I am in Los Angeles, then I am in Ann Arbor.

But they are both true on this table.

Arguments

- It turns out that interpreting the conditional this way makes the most sense of the role of conditionals in certain arguments, in particular to do with disjunctive syllogism.
- There is an allusion to this at the end of chapter 1 of Boxes and Diamonds.

Arguments

The big advantage of thinking of ‘if’ this way is that it guarantees that for any value of A, B, C, these two arguments agree on validity - that is, they are either both valid or both invalid.

$$A, B \vdash C$$

$$A \vdash B \to C$$

And plausibly those should be the same. A suffices for $B \to C$ just in case A and B together suffice for C.

For Next Time

We'll talk about how to use these basic truth tables to build larger truth tables.