305 Lecture 11.1 - Introduction to Modal Logic

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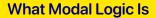
• Introducing modal logic.

Associated Reading

• Boxes and Diamonds, section 3.1 and 3.2.



The logics of **must** and **might**.



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 Why plural? Because we do not assume that these words have a single determinate meaning.

Examples of Must

- 1. If x = 2 + 2, then x must equal 4.
- 2. If something is a cat, then it must be a mammal.
- 3. If the gardener is innocent, then it must be the butler who did it.
- 4. You must drive under 70mph on I-94.
- 5. You must keep your promises.
- 6. If you set out a knife and fork, the fork must go on the left.

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To my ears, 1 is **logical** necessity, 2 is **metaphysical** necessity, 3 is **epistemic** necessity, 4 is **legal** necessity, 5 is **moral** (or **deontic**) necessity and 6 is **etiquette** necessity.

Examples of May/Might

- 1. If x is prime, then x might be even.
- 2. If x is a cat, then x might be male.
- 3. It might be the butler or the gardener that did it.
- 4. You may drive at any speed below 30mph on State Street.
- 5. You may lie to save a friend's life.
- 6. You may use white napkins or red napkins.

Examples of May/Might

- 1. If x is prime, then x might be even.
- 2. If x is a cat, then x might be male.
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- 5. You may lie to save a friend's life.
- 6. You may use white napkins or red napkins.

To my ears, 1 is **logical** possibility, 2 is **metaphysical** possibility, 3 is **epistemic** possibility, 4 is **legal** possibility, 5 is **moral** (or **deontic**) possibility and 6 is **etiquette** possibility (though I'm not sure about any of these).

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If something must be true, then it is true.

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- But it is very much not true on the legal, moral or etiquette interpretations.

So we want some logics where it is a logical truth, and some where it is not.

Language

We extend our language with two new operators: \Box and \Diamond .

- If p is a sentence, so is \Box p and so is \Diamond p.
- These mean, respectively, that p must be true, and that p might be true.
- We interpret these somewhat similar to negations; they just bind what they are immediately next to.
- So $\Box p \rightarrow q$ means $(\Box p) \rightarrow q$, not $\Box (p \rightarrow q)$.

