

$\Box A \rightarrow \Box \Box A$ (in KTB)

Build a Tableau

To Check Whether it is Valid

Hypothesis

$\Box A \rightarrow \Box \Box A$ is a theorem of KTB.

- So we can use all the rules, plus the special rules for B.

$$\Box A \rightarrow \Box \Box A$$

1. $1, \mathbb{F} \quad \Box A \rightarrow \Box \Box A$ Assumption

Start with it being false at 1.

$$\Box A \rightarrow \Box \Box A$$

- | | | |
|----|---|-----------------------------|
| 1. | $1, \mathbb{F} \quad \Box A \rightarrow \Box \Box A \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad \Box A$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Box \Box A$ | $\rightarrow \mathbb{F}, 1$ |

You know the drill - left hand side true, right hand side false.

$$\Box A \rightarrow \Box \Box A$$

1.	$1, \mathbb{F} \quad \Box A \rightarrow \Box \Box A \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Box A$	$\rightarrow \mathbb{F}, 1$
4.	$1, \mathbb{T} \quad A$	$\mathbb{T} \Box 2$

It's \mathbb{T} , so true box sentences are true.

$$\Box A \rightarrow \Box \Box A$$

1.	$1, \mathbb{F} \quad \Box A \rightarrow \Box \Box A \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Box A \checkmark$	$\rightarrow \mathbb{F}, 1$
4.	$1, \mathbb{T} \quad A$	$\mathbb{T} \Box 2$
5.	$1.1, \mathbb{F} \quad \Box A$	$\Box \mathbb{F}, 3$

False \Box sentences have to be false somewhere.

$$\Box A \rightarrow \Box \Box A$$

1.	$1, \text{F } \Box A \rightarrow \Box \Box A \checkmark$	Assumption
2.	$1, \text{T } \Box A$	$\rightarrow \text{F}, 1$
3.	$1, \text{F } \Box \Box A \checkmark$	$\rightarrow \text{F}, 1$
4.	$1, \text{T } A$	$\text{T } \Box 2$
5.	$1.1, \text{F } \Box A$	$\Box \text{F}, 3$
6.	$1.1, \text{T } A$	$\Box \text{T}, 2$

True \Box sentences have to be true everywhere accessible.

$$\Box A \rightarrow \Box \Box A$$

1.	1, \mathbb{F} $\Box A \rightarrow \Box \Box A$ ✓	Assumption
2.	1, \mathbb{T} $\Box A$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Box \Box A$ ✓	$\rightarrow \mathbb{F}, 1$
4.	1, \mathbb{T} A	$\mathbb{T} \Box 2$
5.	1.1, \mathbb{F} $\Box A$ ✓	$\Box \mathbb{F}, 3$
6.	1.1, \mathbb{T} A	$\Box \mathbb{T}, 2$
7.	1.1.1, \mathbb{F} A	$\Box \mathbb{F}, 5$

Another false \Box sentence that needs to be made false.

$$\Box A \rightarrow \Box \Box A$$

1.	1, \mathbb{F} $\Box A \rightarrow \Box \Box A$ ✓	Assumption
2.	1, \mathbb{T} $\Box A$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Box \Box A$ ✓	$\rightarrow \mathbb{F}, 1$
4.	1, \mathbb{T} A	$\mathbb{T} \Box 2$
5.	1.1, \mathbb{F} $\Box A$ ✓	$\Box \mathbb{F}, 3$
6.	1.1, \mathbb{T} A	$\Box \mathbb{T}, 2$
7.	1.1.1, \mathbb{F} A	$\Box \mathbb{F}, 5$

And that's it. We've applied all the rules. The extra rules for T and B don't change much here from the basic tableau in K.

A Model

- Three worlds, $w_1, w_{1.1}, w_{1.1.1}$.
- The accessibility relations are
 $w_1 R w_{1.1}, w_{1.1} R w_{1.1.1}, w_1 R w_1, w_{1.1} R w_{1.1}, w_{1.1.1} R w_{1.1.1}, w_{1.1} R w_1$
and $w_{1.1.1} R w_{1.1}$
- The first two are from the tree, the next three from T and the last two from B.
- A is true at w_1 and $w_{1.1}$ and false at $w_{1.1.1}$.
- So $\Box A$ will be true only at w_1 .
- So $\Box \Box A$ will be false at w_1 , as required.