

305 Lecture 13.3 - Strict Conditionals

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Plan

- To discuss the strict conditional.

Associated Reading

- Boxes and Diamonds, section 6.3

Definition of Strict Conditional

- If A, B means Necessarily, A materially implies B.
- In symbols, $A \rightarrow B = \Box(A \supset B)$

What Box

Two main options

- Universal - $\Box(A \supset B)$ means that every A world is a B world, i.e., $A \wedge \neg B$ is impossible.
- Epistemic $\Box(A \supset B)$ means that every A world that could be actual for all we know is a B world, i.e., $A \wedge \neg B$ is known to be false.

And there are other complications you could try. We'll mostly stay out of these.

The Nine Puzzle Cases

1. Modus Ponens - $A, A \rightarrow B \models B$
2. Agglomeration - $A \rightarrow B, A \rightarrow C \models A \rightarrow (B \wedge C)$
3. Transitive - $A \rightarrow B, B \rightarrow C \models A \rightarrow C$
4. Contraposition - $A \rightarrow B \models \neg B \rightarrow \neg A$
5. Antecedent Strengthening - $A \rightarrow B \models (A \wedge C) \rightarrow B$
6. Paradox 1 - $B \models A \rightarrow B$
7. Paradox 2 - $\neg A \models A \rightarrow B$
8. Strict Paradox - $\Box B \models A \rightarrow B$
9. Disjunction Paradox - $(A \rightarrow B) \vee (B \rightarrow A)$

Modus Ponens

Still valid.

- Assume A, and that every A world is a B world.
- Then this is an A world, so it is a B world.

Agglomeration.

Still valid.

- Assume every A world is a B world, and that every A world is a C world.
- Then every A world will be a $B \wedge C$ world.
- So $A \rightarrow (B \wedge C)$

Transitivity

Still valid.

- Assume every A world is a B world, and every B world is a C world.
- Then every A world is a C world.
- So $A \rightarrow C$

Contraposition

Still valid.

- Assume every A world is a B world.
- Consider an arbitrary $\neg B$ world.
- It can't be an A world, because then some A world would not be a B world.
- So it's a $\neg A$ world.
- So all $\neg B$ worlds are $\neg A$ worlds
- So $\neg B \rightarrow \neg A$.

Antecedent Strengthening

Still valid.

- Assume every A world is a B world.
- Consider an $A \wedge C$ world.
- It's an A world, by the rule for true \wedge .
- So it's a B world.
- So $(A \wedge C) \rightarrow B$.

Paradox 1

Not valid!

- Even if B is true, as long as $A \wedge \neg B$ is possible, $A \rightarrow B$ will be false.

Paradox 2

Not valid!

- Even if A is not actually true, as long as some world make $A \wedge \neg B$ true, then $A \rightarrow B$ will be false.

Strict Paradox

Still valid.

- Assume every world is a B world.
- Then every A world is a world, and hence a B world.
- So $A \rightarrow B$.

Disjunction Paradox

Not valid.

- Just let A , B be two independent propositions, so all four binary possibilities are realised.
- Then neither disjunction will be true.

Summary

This is better than the material conditional theory, but not perfect.
And we haven't said anything about the subjunctive conditionals.

For Next Time

We'll talk a bit more about one of these rules, antecedent strengthening.