305 Lecture 13.2 - Material Conditionals

Brian Weatherson

Plan

- To discuss the conditional from propositional logic, the material conditional.

Associated Reading

- Boxes and Diamonds, section 6.1-6.2

What is It

The material conditional is what the truth tables say is the conditional.

- We will write it as $A \supset B$
- This just means $\neg(A \wedge \neg B)$, i.e., $\neg A \vee B$.

One interesting hypothesis is that this is the right way to interpret English language conditionals.

What About Hamlet

Perhaps we should say that two interesting hypotheses are that the two kinds of English language conditionals are best represented as material conditionals.

- Hypothesis 1: If it were the case that A it would be the case that B = $A \supset B$.
- Hypothesis 2: If it is the case that A then it is the case that B = $A \supset B$.

The first of these is wildly implausible; the second is a bit more defensible.

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How can you complete this sentence so that it is true?

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How can you complete this sentence so that it is true?

- If the United States had entered World War II in 1939, then …

On the material conditional theory, any sentence whatsoever you put in place of the dots will make this true. That's not very plausible.

Material Indicatives

- But the material conditional theory is at least a little plausible for indicative conditionals.
- At least, some smart people have defended it

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4. Contraposition - $A \to B \vDash \neg B \to \neg A$
5. Antecedent Strengthening - $A \to B \vDash (A \wedge C) \to B$
6. Paradox 1 - $B \vDash A \to B$
7. Paradox 2 - $\neg A \vDash A \to B$
8. Strict Paradox - $\Box B \vDash A \to B$
9. Disjunction Paradox - $(A \to B) \vee (B \to A)$

Nursery Rhyme

For want of a nail the shoe was lost.

For want of a shoe the horse was lost.

For want of a horse the rider was lost.

For want of a rider the message was lost.

For want of a message the battle was lost.

For want of a battle the kingdom was lost.

And all for the want of a horseshoe nail.

As An Argument

1. If we hadn’t lost the nail, we wouldn’t have lost the shoe.
2. If we hadn’t lost the shoe, we wouldn’t have lost the horse.
3. If we hadn’t lost the horse, we wouldn’t have lost the rider
4. If we hadn’t lost the rider, we wouldn’t have lost the message
5. If we hadn’t lost the message, we wouldn’t have lost the battle
6. If we hadn’t lost the battle, we wouldn’t have lost the kingdom.

7. So, if we hadn’t lost the nail, we wouldn’t have lost the kingdom.

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Modus Ponens

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- And in this sport, Michigan is pretty good, while Ohio State is super lucky to make the last 16.

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- And in this sport, Michigan is pretty good, while Ohio State is super lucky to make the last 16.

Now consider the following two conditionals.

1. If a Big Ten team wins, then if it isn't Michigan, it will be Ohio State.
2. If Michigan doesn't win, Ohio State will win.

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- Conditional 1 is true, almost by definition.
- Conditional 2 is false.
- And those judgments stay correct even if it turns out that a Big Ten team does in fact win (especially if it is Michigan).
- In general, $A \to (B \to C)$ and A seem like they can be true but (especially when A is not known) $B \to C$ can seem false.

Agglomeration

- This is very hard to find intuitive counterexamples to.
- Take this to be a challenge.

Transitivity

- This often sound right if you run the examples in that order.
- But it can sound very bad if you don't.

For example:

1. If I win the lottery tonight, I'll be very happy.
2. If I get hit by a car on the way home and win the lottery tonight, then I'll win the lottery tonight.
3. So if I get hit by a car on the way home and win the lottery tonight, I'll be very happy.

Contraposition

Some of the examples to this can sound a little silly, but still they arguably do sound like English.

1. If we win this election, we won't win it by a lot.
2. So if we win by a lot, we won't win.

Antecedent Strengthening

The same cases that are problems for transitivity are problems here.

1. If I win the lottery tonight, I'll be very happy.
2. So if I get hit by a car on the way home and win the lottery tonight, I'll be vey happy

The Paradoxes

- These are fairly easy to generate intuitive counterexamples for.
- Note in particular that the disjunction one allows for $B = \neg A$.
- So we get the following: Either if we win, we'll lose, or if we lose, we'll win.

For Next Time

We’ll discuss the so called strict conditional.