305 Lecture 11.4 - Truth in a Model

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 To extend our discussion of truth at a world, to discussion of truth in a whole model.

Associated Reading

• Boxes and Diamonds, section 3.5.

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We'll write the models as $\langle W, R, V \rangle$.

Valuations

V is a function from atomic sentence letters to subsets of W.

- It tells you when the atomic sentences are true.
- When an atomic sentence is not true, it is false.

Truth at a Point

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- ¬A is true at w iff A is not true at w.
- A ∧ B is true at w iff A is true and w and B is true at w.
- A ∨ B is true at w iff A is true and w or B is true at w.
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This just leaves the modal formulae. I'll set out the rules, then do some worked examples.

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- And it is true at w just in case A is true at every world y such that wRy.
- Necessary truth is truth at all accessible worlds.

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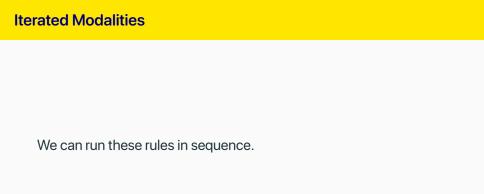
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- And it is true at w just in case A is true at some world y such that wRy.
- · Possible truth is truth at some accessible world.





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- And that means that A has to be true at every world z such that yRz (for any y such that wRy).

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- And that means that A has to be true at some world z such that yRz (for some y such that wRy).
- In the picturesque terms, you can get from w to an A-world in two steps.



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What does it mean for $\Diamond \Box A$ to be true at w?

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• At all accessible worlds, $\Diamond A$ is true.

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- That is, wherever you go, you can get to there is some accessible world such that everywhere you can go from there, A is true.

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- That is, at every one of those worlds, either p is true, or q is false, or there is some world you can get to where r is true.

Box and connectives

The general rule is just to apply the rules for sentences inside the brackets at each world in W, and then apply the rule for \Box or \Diamond . But there are three special cases worth thinking about.

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- □(A ∧ B) means that all accessible worlds are A and B worlds.
- □(A ∨ B) means that all accessible worlds make at least one of A and B true.
- □(A → B) means that all accessible A-worlds are B-worlds.

We'll use that last one a lot.



We'll discuss of examples of truth (and non-truth) in models to explain this material.