305 Lecture 8.9 - More Sampling without Replacement

Brian Weatherson

Plan

- To go over a more complicated example of sampling without replacement.

Associated Reading

Odds and Ends, Chapter 9

Last (Difficult) Example

- There are four urns in the room, three of type X, one of type Y.
- The type X urns have 4 blue marbles and 2 yellow marbles.
- The type Y urn has 5 blue marbles and 3 yellow marbles.
- One of the four urns was selected at random.
- Then two marbles were selected **without replacement** from the randomly selected urn.
- The first was blue, the second was yellow.
- A third marble is about to be selected.
- What is the probability that it is blue?

Urn	Blue-then-Yellow
Type X	$\frac{3}{4} \times \frac{4}{6} \times \frac{2}{5} = \frac{1}{5}$
Type Y	
Total	

$$Pr(X \wedge Blue_1 \wedge Yellow_2) = Pr(X) \times Pr(Blue_1 | X) \times Pr(Yellow_2 | X \wedge Blue_1)$$

Urn	Blue-then-Yellow
Type X	$\frac{3}{4} \times \frac{4}{6} \times \frac{2}{5} = \frac{1}{5}$
Type Y	$\frac{1}{4} \times \frac{5}{8} \times \frac{3}{7} = \frac{15}{224}$
Total	

$$Pr(Y \wedge Blue_1 \wedge Yellow_2) = Pr(Y) \times Pr(Blue_1 | Y) \times Pr(Yellow_2 | Y \wedge Blue_1)$$

Urn	Blue-then-Yellow
Type X	1/5
Type Y	15/224
Total	299/1120

You should double check this, but I think

$$\frac{1}{5} + \frac{15}{224} = \frac{299}{1120}$$

So that's $\Pr(\text{Blue}_1 \wedge \text{Yellow}_2)$

Conditional Probabilities

$$\Pr(X|\mathrm{Blue}_1 \wedge \mathrm{Yellow}_2) = \frac{\Pr(X \wedge \mathrm{Blue}_1 \wedge \mathrm{Yellow}_2)}{\Pr(\mathrm{Blue}_1 \wedge \mathrm{Yellow}_2)} = \frac{\frac{1}{5}}{\frac{299}{1120}} = \frac{224}{299}$$

$$\Pr(Y|\mathrm{Blue}_1 \wedge \mathrm{Yellow}_2) = \frac{\Pr(Y \wedge \mathrm{Blue}_1 \wedge \mathrm{Yellow}_2)}{\Pr(\mathrm{Blue}_1 \wedge \mathrm{Yellow}_2)} = \frac{\frac{15}{224}}{\frac{299}{1120}} = \frac{75}{299}$$

The probability of Y is ever so fractionally higher than when we started.

Next Marble

- If X (and Blue-followed-by-Yellow), the probability of next marble being blue is $\frac{3}{4}$.
- If Y (and Blue-followed-by-Yellow), the probability of next marble being blue is $\frac{2}{3}$.

Next Marble

- If X (and Blue-followed-by-Yellow), the probability of next marble being blue is $\frac{3}{4}$.
- If Y (and Blue-followed-by-Yellow), the probability of next marble being blue is $\frac{2}{3}$.
- So overall probability of next marble being blue is

$$\frac{224}{299} \times \frac{3}{4} + \frac{75}{299} \times \frac{2}{3} = \frac{218}{299} \approx 0.729$$

General Strategy of Last Slide

- If there are two hypotheses X and Y, and you want to know the probability of some event E, it will be given by

$$\Pr(E) = \Pr(X)\Pr(E|X) + \Pr(Y)\Pr(E|Y)$$

And that generalises to the case where there are multiple hypotheses $H_1, \dots H_n$

$$\Pr(E) = \Pr(H_1)\Pr(E|H_1) + \cdots + \Pr(H_n)\Pr(E|H_n)$$

For Next Time

Next week we will look at the use of probability in decision making, and in science.