305 Lecture 8.3 - Sampling with Replacement

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- To illustrate a special kind of updating on two data points: Sampling with Replacement

Odds and Ends, Chapter 9

An Example

There are two urns in front of us.

- One of them - urn A - has 4 red marbles, 3 green marbles, and 3 blue marbles.
- The other - urn B- has 8 red marbles, 1 green marbles and 1 blue marbles.

An Example

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- One of them - urn A - has 4 red marbles, 3 green marbles, and 3 blue marbles.
- The other - urn B- has 8 red marbles, 1 green marbles and 1 blue marbles.

One of the urns will be selected at random, and then a marble drawn from it.

- If the marble is red, what is the probability that Urn A was selected?

I'll just do the column for red marble selected.

	Red
Urn A	$0.5 \times 0.4 = 0.2$
Urn B	$0.5 \times 0.8 = 0.4$
Total	$0.2 + 0.4 = 0.6$

$$\Pr(A|Red) = \frac{\Pr(A \wedge Red)}{\Pr(Red)} = \frac{0.2}{0.6} = \frac{1}{3}$$

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Another Example

There are two urns in front of us.

- One of them - urn A - has 4 red marbles, 3 green marbles, and 3 blue marbles.
- The other - urn B- has 8 red marbles, 1 green marbles and 1 blue marbles.

One of the urns will be selected at random, and then two marbles drawn from it **with replacement**.

- If both draws are red, what is the probability that Urn A was selected?

	Red–Red
Urn A	$0.5 \times 0.4^2 = 0.08$
Urn B	$0.5 \times 0.8^2 = 0.32$
Total	$0.08 + 0.32 = 0.4$

$$\Pr(A|Red-Red) = \frac{\Pr(A \wedge Red-Red)}{\Pr(Red-Red)} = \frac{0.08}{0.4} = \frac{1}{5}$$

The probability of Urn A fell by a lot.

Yet Another Example

There are two urns in front of us.

- One of them – urn A – has 4 red marbles, 3 green marbles, and 3 blue marbles.
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Yet Another Example

There are two urns in front of us.

- One of them - urn A - has 4 red marbles, 3 green marbles, and 3 blue marbles.
- The other - urn B- has 8 red marbles, 1 green marbles and 1 blue marbles.

One of the urns will be selected at random, and then two marbles drawn from it **with replacement**.

- If the first draw is red and the second green, what is the probability that Urn A was selected?

	Red–Green
Urn A	$0.5 \times 0.4 \times 0.3 = 0.06$
Urn B	$0.5 \times 0.8 \times 0.1 = 0.04$
Total	$0.06 + 0.04 = 0.1$

$$\Pr(A|\text{Red} - \text{Green}) = \frac{\Pr(A \wedge \text{Red} - \text{Green})}{\Pr(\text{Red} - \text{Green})} = \frac{0.06}{0.1} = \frac{3}{5}$$

The probability of Urn A rose by a lot.

We'll look at what happens when the first data point changes the probability of the later data points.