

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \text{ (in K)}$$

Build a Tableau

To Check Whether it is Valid

Hypothesis

$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$ is a theorem of K.

- So we can use the base modal rules.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1. 1, \mathbb{F} $(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$ Assumption

Start with it being false at 1.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

- | | | |
|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad \Diamond A \wedge \Diamond B$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Diamond(A \wedge B)$ | $\rightarrow \mathbb{F}, 1$ |

True antecedent, false consequent.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

- | | | |
|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad \Diamond A \wedge \Diamond B \checkmark$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Diamond(A \wedge B)$ | $\rightarrow \mathbb{F}, 1$ |
| 4. | $1, \mathbb{T} \quad \Diamond A$ | $\wedge \mathbb{T}, 2$ |
| 5. | $1, \mathbb{T} \quad \Diamond B$ | $\wedge \mathbb{T}, 2$ |

Break up the and.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1.	$1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \wedge \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A \checkmark$	$\wedge \mathbb{T}, 2$
5.	$1, \mathbb{T} \quad \Diamond B$	$\wedge \mathbb{T}, 2$
6.	$1.1, \mathbb{T} \quad A$	$\Diamond \mathbb{T}, 4$

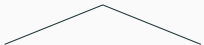
Make $\Diamond A$ true.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1.	$1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \wedge \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A \checkmark$	$\wedge \mathbb{T}, 2$
5.	$1, \mathbb{T} \quad \Diamond B$	$\wedge \mathbb{T}, 2$
6.	$1.1, \mathbb{T} \quad A$	$\Diamond \mathbb{T}, 4$
7.	$1.1, \mathbb{F} \quad A \wedge B$	$\Diamond \mathbb{F}, 3$

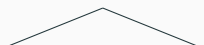
Since $A \wedge B$ is false everywhere accessible from 1, it is false at 1.1.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1.	1, \mathbb{F} $(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$	Assumption
2.	1, \mathbb{T} $\Diamond A \wedge \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Diamond(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	1, \mathbb{T} $\Diamond A \checkmark$	$\wedge \mathbb{T}, 2$
5.	1, \mathbb{T} $\Diamond B$	$\wedge \mathbb{T}, 2$
6.	1.1, \mathbb{T} A	$\Diamond \mathbb{T}, 4$
7.	1.1, \mathbb{F} $A \wedge B \checkmark$	$\Diamond \mathbb{F}, 3$
<div style="text-align: center;">  </div>		
8.	1.1, \mathbb{F} A 1.1, \mathbb{F} B	$\wedge \mathbb{F}, 7$
	x	

Two ways for $A \wedge B$ to fail; only one is possible.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1.	1, \mathbb{F} $(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$	Assumption
2.	1, \mathbb{T} $\Diamond A \wedge \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Diamond(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	1, \mathbb{T} $\Diamond A \checkmark$	$\wedge \mathbb{T}, 2$
5.	1, \mathbb{T} $\Diamond B \checkmark$	$\wedge \mathbb{T}, 2$
6.	1.1, \mathbb{T} A	$\Diamond \mathbb{T}, 4$
7.	1.1, \mathbb{F} $A \wedge B \checkmark$	$\Diamond \mathbb{F}, 3$
		
8.	1.1, \mathbb{F} A	1.1, \mathbb{F} B $\wedge \mathbb{F}, 7$
9.	x	1.2, \mathbb{T} B $\Diamond \mathbb{T}, 5$

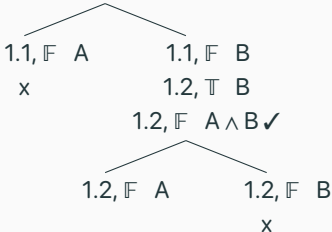
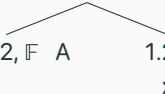
B has to be possible, so we'll say it is possible at 1.2 - using a name we haven't already used.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1.	1, \mathbb{F}	$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$	Assumption
2.	1, \mathbb{T}	$\Diamond A \wedge \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F}	$\Diamond(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	1, \mathbb{T}	$\Diamond A \checkmark$	$\wedge \mathbb{T}, 2$
5.	1, \mathbb{T}	$\Diamond B \checkmark$	$\wedge \mathbb{T}, 2$
6.	1.1, \mathbb{T}	A	$\Diamond \mathbb{T}, 4$
7.	1.1, \mathbb{F}	$A \wedge B \checkmark$	$\Diamond \mathbb{F}, 3$
8.	1.1, \mathbb{F}	A	$\wedge \mathbb{F}, 7$
9.	x	1.2, \mathbb{T}	$\Diamond \mathbb{T}, 5$
10.		1.2, \mathbb{F}	$\Diamond \mathbb{F}, 3$

$A \wedge B$ is still impossible.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

1.	1, \mathbb{F} $(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B) \checkmark$	Assumption
2.	1, \mathbb{T} $\Diamond A \wedge \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Diamond(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	1, \mathbb{T} $\Diamond A \checkmark$	$\wedge \mathbb{T}, 2$
5.	1, \mathbb{T} $\Diamond B \checkmark$	$\wedge \mathbb{T}, 2$
6.	1.1, \mathbb{T} A	$\Diamond \mathbb{T}, 4$
7.	1.1, \mathbb{F} $A \wedge B \checkmark$	$\Diamond \mathbb{F}, 3$
		
8.	1.1, \mathbb{F} A	$\wedge \mathbb{F}, 7$
9.	x	$\Diamond \mathbb{T}, 5$
10.	1.2, \mathbb{T} B	$\Diamond \mathbb{F}, 3$
		
11.	1.2, \mathbb{F} $A \wedge B \checkmark$	$\wedge \mathbb{F}, 7$
	1.2, \mathbb{F} A	x
	1.2, \mathbb{F} B	x

And we're done. All the rules have been applied - though you really need to check this - and the middle branch does not close.

A Model

When the tableau doesn't close, you should be able to build a model where the sentence is false at w_1 .

In fact, you should be able to read it off the tree.

A Model

- Three worlds, $w_1, w_{1.1}, w_{1.2}$.
- The only accessibility relations are $w_1 R w_{1.1}$ and $w_1 R w_{1.2}$.
- In general, the accessibility relations are the ones required by the restrictions (but there are none here), plus the ones required by the numbering, i.e., $w_x R w_{x.y}$.
- A is true at $w_{1.1}$ and false at $w_{1.2}$.
- B is true at $w_{1.2}$ and false at $w_{1.1}$.
- It doesn't matter what atomics are true at w_1 .
- But both $\Diamond A$ and $\Diamond B$ will be true at w_1 , while $\Diamond(A \wedge B)$ is false there.