

# 305 Lecture 12.1 - Modal Tableau

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# Plan

- To introduce tableau for proving things in modal logic.

## Associated Reading

- Boxes and Diamonds, section 5.1

One big difference:

- On each line, we include reference to a world.
- The line says that a particular proposition is true or false at a world.
- The tableau only close if the tableau says the same proposition is both true and false at the same world.

## Referring to Worlds

We refer to a world with a string of numbers, such as 1.2.1.3.

- The string tells you something (but not everything) about R relations.
- World  $x$  can always access world  $x.y$ .
- So there is an R-relation from 1.2.1 to 1.2.1.3, and indeed to 1.2.1. $x$  for any  $x$ .
- These don't exhaust the R-relations; perhaps there is also an R-relation from 1.2.1.3 back to 1.2.1.
- But the relation from  $x$  to  $x.y$  is guaranteed.
- Note that worlds can be picked out by multiple strings - we do not assume that 1.1 and 1.2 are different, though we don't assume they are the same either.

# Rules

The rules for the old connectives stay as they are.

- The only difference is that you have to note which world you are in.
- So if you have that  $A \wedge B$  is true in 1.4.7, then you have to write down that  $A$  is true in 1.4.7, and that  $B$  is true in 1.4.7.
- And if  $A \vee B$  is true in 1.6.8 you have to have two branches, one where  $A$  is true in 1.6.8, and the other where  $B$  is true in 1.6.8.
- But otherwise things are as they were before.

## Rules for $\Box$

If  $\Box A$  is true at  $x$ , then for any  $x.y$  that is already on the tree, we can infer that  $A$  is true at  $x.y$ .

- Note: If there is no  $x.y$  on the tree, we can't assume that there is one.
- $\Box A$  can be true at a world because that world can't access any other world.

## Rules for $\Box$ (cont)

If  $\Box A$  is false at  $x$ , then we have to add a **new**  $x.y$ , and make  $A$  false at  $x.y$ .

- It is very important that  $x.y$  be new.
- We know that  $A$  is false at some accessible world, but we don't know which one.
- For any given world,  $A$  might be true there, as long as it is false somewhere.
- That's why we use a new number.
- Remember that it might be that multiple strings refer to the same world.



## Rules for $\Diamond$

If  $\Diamond A$  is true at  $x$ , then we have to add a **new**  $x.y$ , and make  $A$  true at  $x.y$ .

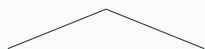
- It is very important that  $x.y$  be new.
- We know that  $A$  is true at some accessible world, but we don't know which one.
- For any given world,  $A$  might be false there, as long as it is false somewhere.
- That's why we use a new number.
- Remember that it might be that multiple strings refer to the same world.

## Rules for $\Diamond$ (cont)

If  $\Diamond A$  is false at  $x$ , then for any  $x.y$  that is already on the tree, we can infer that  $A$  is false at  $x.y$ .

- Note: If there is no  $x.y$  on the tree, we can't assume that there is one.
- $\Diamond A$  can be false at a world because that world can't access any other world.

$$(\Box A \wedge \Box B) \rightarrow \Box(A \wedge B)$$

1.	1, $\mathbb{F}$ $(\Box A \wedge \Box B) \rightarrow \Box(A \wedge B)$	Assumption
2.	1, $\mathbb{T}$ $\Box A \wedge \Box B$	$\rightarrow \mathbb{F}, 1$
3.	1, $\mathbb{F}$ $\Box(A \wedge B)$	$\rightarrow \mathbb{F}, 1$
4.	1, $\mathbb{T}$ $\Box A$	$\wedge \mathbb{T}, 2$
5.	1, $\mathbb{T}$ $\Box B$	$\wedge \mathbb{T}, 2$
6.	1.1, $\mathbb{F}$ $A \wedge B$	$\Box \mathbb{F}, 3$
		
7.	1.1, $\mathbb{F}$ $A$	$\wedge \mathbb{F}, 6$
	1.1, $\mathbb{F}$ $B$	
8.	1.1, $\mathbb{T}$ $A$	$\Box \mathbb{T}, 4; \Box \mathbb{T}, 5$
	1.1, $\mathbb{T}$ $B$	
	x	x

$$\Diamond(A \vee B) \rightarrow (\Diamond A \vee \Diamond B)$$

1.	1, $\mathbb{F}$	$\Diamond(A \vee B) \rightarrow (\Diamond A \vee \Diamond B)$	Assumption
2.	1, $\mathbb{T}$	$\Diamond(A \vee B)$	$\rightarrow \mathbb{F}, 1$
3.	1, $\mathbb{F}$	$\Diamond A \vee \Diamond B$	$\rightarrow \mathbb{F}, 1$
4.	1, $\mathbb{F}$	$\Diamond A$	$\vee \mathbb{F}, 3$
5.	1, $\mathbb{F}$	$\Diamond B$	$\vee \mathbb{F}, 3$
6.	1.1, $\mathbb{T}$	$A \vee B$	$\Diamond \mathbb{T}, 2$
7.	1.1, $\mathbb{T}$	$A$	$\vee \mathbb{T}, 6$
8.	1.1, $\mathbb{F}$	$A$	$\Diamond \mathbb{F}, 4; \Diamond \mathbb{F}, 5$
	x		
		x	

## For Next Time

We'll look at how to extend this approach to other logics.