

$A \rightarrow \Box \Diamond A$ (in KTB)

Build a Tableau

To Check Whether it is Valid

Hypothesis

$A \rightarrow \Box \Diamond A$ is a theorem of KTB.

- So we can use all the rules, plus the special rules for B and T.

$$A \rightarrow \Box \Diamond A$$

1. 1, \mathbb{F} $A \rightarrow \Box \Diamond A$ Assumption

Start with it being false at 1.

$$A \rightarrow \Box \Diamond A$$

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|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad A \rightarrow \Box \Diamond A \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad A$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Box \Diamond A$ | $\rightarrow \mathbb{F}, 1$ |

You know the drill - left hand side true, right hand side false.

$$A \rightarrow \Box \Diamond A$$

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|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad A \rightarrow \Box \Diamond A \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad A$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Box \Diamond A \checkmark$ | $\rightarrow \mathbb{F}, 1$ |
| 4. | $1.1, \mathbb{F} \quad \Diamond A$ | $\Box \mathbb{F}, 3$ |

False \Box sentences are false somewhere.

$$A \rightarrow \Box \Diamond A$$

1.	$1, \mathbb{F} \quad A \rightarrow \Box \Diamond A \checkmark$	Assumption
2.	$1, \mathbb{T} \quad A$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Diamond A \checkmark$	$\rightarrow \mathbb{F}, 1$
4.	$1.1, \mathbb{F} \quad \Diamond A$	$\Box \mathbb{F}, 3$
5.	$1, \mathbb{F} \quad A$	$B \Diamond 4$

In B, false \Diamond sentences have to be false one step backwards as well as one step forwards.

$$A \rightarrow \Box \Diamond A$$

1.	$1, \mathbb{F} \quad A \rightarrow \Box \Diamond A \checkmark$	Assumption
2.	$1, \mathbb{T} \quad A$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Diamond A \checkmark$	$\rightarrow \mathbb{F}, 1$
4.	$1.1, \mathbb{F} \quad \Diamond A$	$\Box \mathbb{F}, 3$
5.	$1, \mathbb{F} \quad A$	$B \Diamond 4$
	x	

And that closes the tree, so it is a theorem.