

$\Box A \rightarrow (A \wedge \Box \Box A)$  (in S4)

---

Build a Tableau

To Check Whether it is Valid

# Hypothesis

$\Box A \rightarrow (A \wedge \Box \Box A)$  is a theorem of  $S4 = KT4$ .

- So we can use all the rules, plus the special rules for T and for 4.

$$\Box A \rightarrow (A \wedge \Box \Box A)$$

1.                      1,  $\mathbb{F}$     $\Box A \rightarrow (A \wedge \Box \Box A)$       Assumption

Start with it being false at 1.

$$\Box A \rightarrow \Box \Box A$$

- |    |  |                             |
|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad \Box A \rightarrow (A \wedge \Box \Box A) \checkmark$ | Assumption                  |
| 2. | $1, \mathbb{T} \quad \Box A$   | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad A \wedge \Box \Box A$                                 | $\rightarrow \mathbb{F}, 1$ |

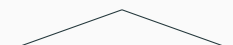
You know the drill - left hand side true, right hand side false.

$$\Box A \rightarrow \Box \Box A$$

- |    |  |                             |
|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad \Box A \rightarrow (A \wedge \Box \Box A) \checkmark$ | Assumption                  |
| 2. | $1, \mathbb{T} \quad \Box A$   | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad A \wedge \Box \Box A$                                 | $\rightarrow \mathbb{F}, 1$ |
| 4. | $1, \mathbb{T} \quad A$  | $\mathbb{T} \Box 2$         |

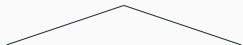
We have  $\mathbb{T}$ , so necessary truths are true.

$$\Box A \rightarrow \Box \Box A$$

1.	$1, \text{F } \Box A \rightarrow (A \wedge \Box \Box A) \checkmark$	Assumption
2.	$1, \text{T } \Box A$	$\rightarrow \text{F}, 1$
3.	$1, \text{F } A \wedge \Box \Box A \checkmark$	$\rightarrow \text{F}, 1$
4.	$1, \text{T } A$	$\text{T } \Box 2$
		
5.	$1, \text{F } A$ $\times$	$1, \text{F } \Box \Box A$ $\wedge \text{F}, 3$

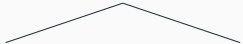
False  $\wedge$  means branching, but happily one branch closes immediately.

$$\Box A \rightarrow \Box \Box A$$

1.	$1, \mathbb{F} \quad \Box A \rightarrow (A \wedge \Box \Box A) \checkmark$	Assumption	
2.	$1, \mathbb{T} \quad \Box A$	$\rightarrow \mathbb{F}, 1$	
3.	$1, \mathbb{F} \quad A \wedge \Box \Box A \checkmark$	$\rightarrow \mathbb{F}, 1$	
4.	$1, \mathbb{T} \quad A$	$\mathbb{T} \Box 2$	
			
5.	$1, \mathbb{F} \quad A$	$1, \mathbb{F} \quad \Box \Box A \checkmark$	$\wedge \mathbb{F}, 3$
6.	x	$1.1, \mathbb{F} \quad \Box A$	$\Box \mathbb{F}, 5$

False  $\Box$  sentences require new worlds where the unboxed thing is false.

$$\Box A \rightarrow \Box \Box A$$

1.	$1, \text{F } \Box A \rightarrow (A \wedge \Box \Box A) \checkmark$	Assumption
2.	$1, \text{T } \Box A$	$\rightarrow \text{F}, 1$
3.	$1, \text{F } A \wedge \Box \Box A \checkmark$	$\rightarrow \text{F}, 1$
4.	$1, \text{T } A$	$\text{T } \Box 2$
		
5.	$1, \text{F } A$	$1, \text{F } \Box \Box A \checkmark$ $\wedge \text{F}, 3$
6.	x	$1.1, \text{F } \Box A$ $\Box \text{F}, 5$
7.		$1.1, \text{T } \Box A$ $4 \Box 2$
		x

But 4 requires we carry down  $\Box$  sentences, and now we're done.  
Both branches close, so this is a logical truth.