305 Lecture 3.2 - Tautologies

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Plan

This lecture is about how we use truth tables to check for whether something is a **tautology**.

Associated Reading

forall x, chapter 12, especially section 12.1.

Tautologies

We are going to start with a particular kind of sentence, a **tautology**.

Definition

A tautology is a sentence that gets the value $\mathbb{T}$ in every row of its truth table.

Examples of Tautologies

What are some sentences that might fit the bill?

The Law of Excluded Middle

A	$A \vee \neg A$
$\mathbb{T}$	$\mathbb{T}\ \mathbb{T}\ \mathbb{F}\ \mathbb{T}$
$\mathbb{F}$	$\mathbb{F}\ \mathbb{T}\ \mathbb{T}\ \mathbb{F}$

The Law of Non-Contradiction

A	$\neg (A \wedge \neg A)$
$\mathbb{T}$	$\mathbb{T}\ \mathbb{T}\ \mathbb{F}\ \mathbb{F}\ \mathbb{T}$
$\mathbb{F}$	$\mathbb{T}\ \mathbb{F}\ \mathbb{F}\ \mathbb{T}\ \mathbb{F}$

Reflexive Conditionals

A	$A \to A$	
$\mathbb{T}$	$\mathbb{T}$	$\mathbb{T}$ $\mathbb{T}$
$\mathbb{F}$	$\mathbb{F}$	$\mathbb{T}$ $\mathbb{F}$

A Surprising One

A	B	(A → B) ∨ (B → A)
𝕋	𝕋	𝕋 𝕋 𝕋 𝕋 𝕋 𝕋 𝕋
𝕋	𝔽	𝕋 𝔽 𝔽 𝕋 𝔽 𝕋 𝕋
𝔽	𝕋	𝔽 𝕋 𝕋 𝕋 𝕋 𝔽 𝔽
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Tautologies and Logical Truth

- All tautologies are logical truths.
- But the converse isn't true – some logical truths are not tautologies.
- E.g., If Brian is necessarily a human, then Brian is a human.

For Next Time

We will start using truth tables to look at whether arguments are valid.