

305 Lecture 3.6 - Rules for Truth Trees

Brian Weatherson

Plan

This lecture introduces the rules we use for building up truth trees.

Associated Reading

Boxes and Diamonds, sections 2.2-2.3.

What Rules Do

The rules tell you what new lines to write down given the lines you've already got.

- To some extent they simply have to be memorised.
- But hopefully they are all (except for the rules about \rightarrow) fairly intuitive.

Rules for \neg

1. $\mathbb{T} \neg A$
2. $\mathbb{F} A \quad \neg\mathbb{T}, 1$

1. $\mathbb{F} \neg A$
2. $\mathbb{T} A \quad \neg\mathbb{F}, 1$

Rules for \neg

1. $\mathbb{T} \neg A$

2. $\mathbb{F} A \quad \neg\mathbb{T}, 1$

1. $\mathbb{F} \neg A$

2. $\mathbb{T} A \quad \neg\mathbb{F}, 1$

- Note that the line numbers are just for illustration, and are arbitrary in two senses.
- First, you apply the rule wherever a sentence like $\neg A$ appears, not just at line 1.
- Second, you don't need to apply the rules immediately, so the successor line could come later than 2.

Rule for true \wedge sentence

1. $\mathbb{T} A \wedge B$
2. $\mathbb{T} A$ $\wedge \mathbb{T}, 1$
3. $\mathbb{T} B$ $\wedge \mathbb{T}, 1$

When you have a true \wedge sentence, you can write down that the sentences either side of it are true.

Rule for true \vee sentence



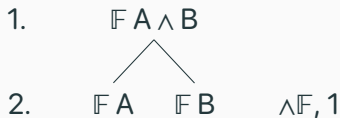
- When you have a true \vee sentence, you create two **branches**.
- The way to read the tree is that at least one of the branches must be all true.

Rule for true \vee sentence



- When you have a true \vee sentence, you create two **branches**.
- The way to read the tree is that at least one of the branches must be all true.
- The 'trunk' above the branching (in this case just line 1), is part of both branches.
- Branches are inclusive; you are saying that at least one branch is true, not that precisely one is.

Rule for false \wedge sentence



- If an \wedge sentence is false, then we know that one or other (or both) of the sides are false.
- So we create two branches, one where each side is false.

Rule for false \vee sentence

1. $\mathbb{F} A \vee B$
2. $\mathbb{F} A$ $\vee \mathbb{F}, 1$
3. $\mathbb{F} B$ $\vee \mathbb{F}, 1$

When you have a false \vee sentence, you know that each side is false, so you write down that they are both false.

Justifying the rule for false \vee sentences

Recall the truth table for \vee

A	B	A	\vee	B
T	T	T	T	T
T	F	T	T	F
F	T	F	T	T
F	F	F	F	F

- The only line where the whole sentence is **F** is line 4.
- So if a \vee sentence is **F**, we know that we're on line 4.
- And on line 4, both A and B are false.

Rule for false \rightarrow sentence

1. $\mathbb{F} A \rightarrow B$
2. $\mathbb{T} A \quad \rightarrow \mathbb{F}, 1$
3. $\mathbb{F} B \quad \rightarrow \mathbb{F}, 1$

When you have a false \rightarrow sentence, you know that the left side is true and the right side is false, so you write those things down.

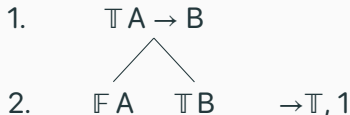
Justifying the rule for false \rightarrow sentences

Recall the truth table for \rightarrow

A	B	$A \rightarrow B$		
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- The only line where the whole sentence is F is line 2.
- So if a \rightarrow sentence is F, we know that we're on line 2.
- And on line 4, A is true and B are false.

Rule for true \rightarrow sentence



- When you have a true \rightarrow sentence, you create two **branches**.
- On the first, A is false. That covers lines 3 and 4 of the truth table.
- On the second, B is true. That covers lines 1 and 3 of the truth table.
- Between them, they cover lines 1, 3 and 4 of the truth table.
- And those are the lines where $A \rightarrow B$ is true.

Next Week

- We will look at some examples of truth trees.
- I find truth trees are much easier in practice than in theory, so if this was all a bit abstract, hopefully it will be more intelligible once we work through some examples.