305 Lecture 2.3 - Symbolization and Recursion

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Plan for This Lecture

- We’re going to look at how and why we can iterate the translation procedures we’ve been investigating.

Associated Reading

forall x, chapter 6, “Sentences of TFL”

Recursion

The language of propositional logic has some fairly simple composition rules.

- It says what the basic sentences are.
- It has some rules saying that if some things are sentences, so are some other things.

The effect is that there are an infinity of possible sentences.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.
- Alex thinks that Kim thinks that Alex thinks that it will rain.

Natural Language Recursion

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.
- Alex thinks that Kim thinks that Alex thinks that it will rain.
- And so on, to infinity, without adding any more words.

Recursive Rule

- If S is a sentence, and N is a name, then N thinks that S is a sentence.
- Note that the output of this rule can be the input to a new instance of it.

Formal Language Recursion

- The letters P, Q, R... are sentences.
- If S and T are sentences, then so are:

1. $\neg S$
2. $S \vee T$
3. $S \wedge T$
4. $S \rightarrow T$
5. $S \leftrightarrow T$

Multiple Steps

So these are all sentences. (Note that I'm playing fast and loose with parentheses here.)

1. P
2. Q
3. $P \wedge Q$
4. $Q \rightarrow (P \wedge Q)$
5. $\neg P$
6. $Q \vee \neg P$
7. $(Q \rightarrow (P \wedge Q)) \leftrightarrow (Q \vee \neg P)$

The last one follows from the fact that 4 and 6 are sentences.

Main Connective

For any sentence you can make, there will be a 'last step' in the demonstration that it is a sentence.

- That last step will involve copying down 1 or 2 other sentences, and adding a connective.
- On the previous slide, you copy down 4 and 6, and put a $\leftrightarrow$ between them.
- That connective you add is the **main connective** of the sentence.
- It covers all the material in the sentence.

Main Connective

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

$$P \land (Q \rightarrow R)$$

The $\land$ is the main connective because either side of it are

- P
- $(Q \rightarrow R)$

And they are both sentences.

Main Connective

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

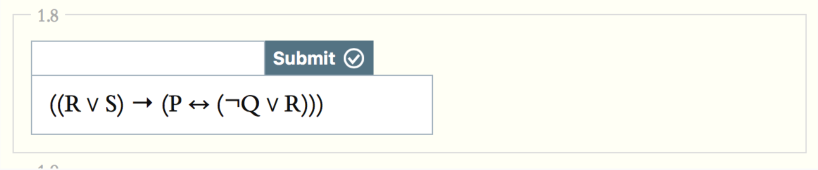
$$P \land (Q \to R)$$

The $\to$ is not the main connective because either side of it are

- $P \land (Q$
- $R)$

And they are not both sentences.

A Worked Example



Step 1

A Worked Example

1.8

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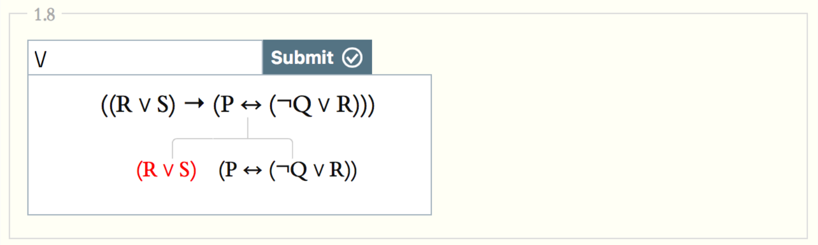
Submit

$((R \vee S) \rightarrow (P \leftrightarrow (\neg Q \vee R)))$

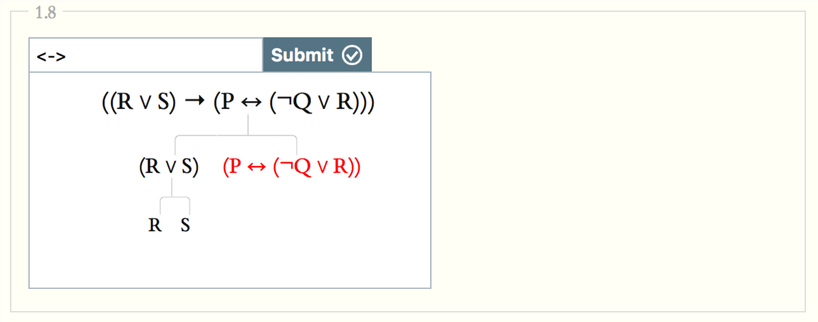
Step 2

A Worked Example



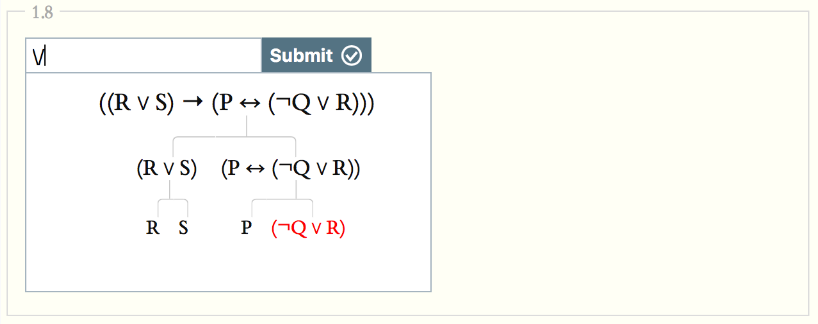
Step 3

A Worked Example



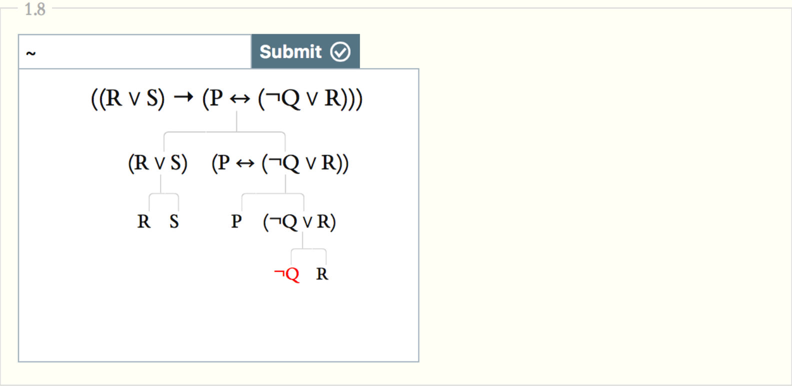
Step 4

A Worked Example



Step 5

A Worked Example



Step 6

For Next Time

- We are going to start on how these connectives help us figure out the truth value of longer sentences.
- And we're going to use that to analyse arguments.