$\Box A \rightarrow \Box \Box A$ **(in KT)**

Build a Tableau

To Check Whether it is Valid

Hypothesis

$\Box A \rightarrow \Box \Box A$ is a theorem of KT.

- So we can use all the rules, plus the special rules for T.

$\Box A \to \Box \Box A$

1.	$1, \mathbb{F}$	$\Box A \to \Box \Box A$	Assumption

Start with it being false at 1.

$\Box A \to \Box \Box A$

1.	$1, \mathbb{F} \quad \Box A \to \Box \Box A$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Box A$	$\to\mathbb{F}, 1$

You know the drill - left hand side true, right hand side false.

$\Box A \to \Box \Box A$

1.	$1, \mathbb{F} \quad \Box A \to \Box \Box A$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Box A$	$\to\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad A$	$T\Box$ 2

It's T, so we have obligations when a $\Box$ sentence is true.

$\Box A \to \Box \Box A$

1.	$1, \mathbb{F} \quad \Box A \to \Box \Box A$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\to\mathbb{F}$, 1
3.	$1, \mathbb{F} \quad \Box \Box A$ ✓	$\to\mathbb{F}$, 1
4.	$1, \mathbb{T} \quad A$	T $\Box$ 2
5.	$1.1, \mathbb{F} \quad \Box A$	$\Box\mathbb{F}$, 3

Now we have to make the false $\Box$ sentence actually false.

$\Box A \to \Box \Box A$

1.	$1, \mathbb{F} \quad \Box A \to \Box \Box A$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box \Box A$ ✓	$\to\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad A$	T $\Box$ 2
5.	$1.1, \mathbb{F} \quad \Box A$	$\Box\mathbb{F}$, 3
6.	$1.1, \mathbb{T} \quad A$	$\Box\mathbb{T}$, 2

Carry down the true $\Box$ sentence.

□A → □ □ A

1.	1, 𝔽 □ A → □ □ A ✓	Assumption
2.	1, 𝕋 □ A	→𝔽, 1
3.	1, 𝔽 □ □A ✓	→𝔽, 1
4.	1, 𝕋 A	T □ 2
5.	1.1, 𝔽 □ A ✓	□𝔽, 3
6.	1.1, 𝕋 A	□𝕋, 2
7.	1.1.1, 𝔽 A	□𝔽, 5

- One more false □ sentence.

$\Box A \to \Box \Box A$

1.	$1, \mathbb{F} \quad \Box A \to \Box \Box A$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\to\mathbb{F}$, 1
3.	$1, \mathbb{F} \quad \Box \Box A$ ✓	$\to\mathbb{F}$, 1
4.	$1, \mathbb{T} \quad A$	T $\Box$ 2
5.	$1.1, \mathbb{F} \quad \Box A$ ✓	$\Box\mathbb{F}$, 3
6.	$1.1, \mathbb{T} \quad A$	$\Box\mathbb{T}$, 2
7.	$1.1.1, \mathbb{F} \quad A$	$\Box\mathbb{F}$, 5

- One more false $\Box$ sentence.
- And now we're done; all rules applied, and tree open. So it is not a theorem.

A Model

- Three worlds, w_1, $w_{1.1}$, $w_{1.1.1}$.
- The accessibility relations are $w_1Rw_{1.1}$, $w_{1.1}Rw_{1.1.1}$, w_1Rw_1, $w_{1.1}Rw_{1.1}$ and $w_{1.1.1}Rw_{1.1.1}$.
- The first two are from the tree, the next three from the restriction.
- A is true at w_1 and $w_{1.1}$ and false at $w_{1.1.1}$.
- So $\Box$A will be true only at w_1.
- So $\Box \Box$ A will be false at w_1, as required.