

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \text{ (in K)}$$

Build a Tableau

To Check Whether it is Valid

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

1. 1, \mathbb{F} $(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$ Assumption

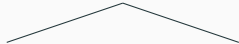
Start with it being false at 1.

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

- | | | |
|----|--|-----------------------------|
| 1. | $1, \mathbb{F} \quad (\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad \Diamond A \vee \Diamond B$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Diamond(A \vee B)$ | $\rightarrow \mathbb{F}, 1$ |

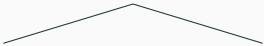
True antecedent, false consequent.

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

1.	$1, \mathbb{F} \quad (\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \vee \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond(A \vee B)$	$\rightarrow \mathbb{F}, 1$
		
4.	$1, \mathbb{T} \quad \Diamond A$ $1, \mathbb{T} \quad \Diamond B$	$\vee \mathbb{T}, 2$

Nothing to do but branch.

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

1.	1, \mathbb{F}	$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \checkmark$	Assumption
2.	1, \mathbb{T}	$\Diamond A \vee \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F}	$\Diamond(A \vee B)$	$\rightarrow \mathbb{F}, 1$
			
4.	1, \mathbb{T}	$\Diamond A \checkmark$	$\vee \mathbb{T}, 2$
5.	1.1, \mathbb{T}	A	$\Diamond \mathbb{T}, 4$
	1, \mathbb{T}	$\Diamond B \checkmark$	
	1.2, \mathbb{T}	B	

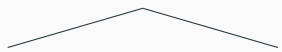
Diamond sentences have to be made true somehow.

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

1.	$1, \mathbb{F} \quad (\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \vee \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond(A \vee B)$	$\rightarrow \mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A \checkmark$	$\vee \mathbb{T}, 2$
5.	$1.1, \mathbb{T} \quad A$	$\Diamond \mathbb{T}, 4$
6.	$1.1, \mathbb{F} \quad A \vee B$	$\Diamond \mathbb{F}, 3$

False diamond sentences are false at all accessible worlds.

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

1.	1, \mathbb{F} $(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \checkmark$	Assumption
2.	1, \mathbb{T} $\Diamond A \vee \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Diamond(A \vee B)$	$\rightarrow \mathbb{F}, 1$
		
4.	1, \mathbb{T} $\Diamond A \checkmark$	$\vee \mathbb{T}, 2$
5.	1.1, \mathbb{T} A	$\Diamond \mathbb{T}, 4$
6.	1.1, \mathbb{F} $A \vee B \checkmark$	$\Diamond \mathbb{F}, 3$
7.	1.1, \mathbb{F} A	$\vee \mathbb{F}, 6$
	x	x

And false or sentences have each side of the or false.

$$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$$

1.	$1, \mathbb{F} \quad (\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B) \checkmark$	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \vee \Diamond B \checkmark$	$\rightarrow \mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond(A \vee B)$	$\rightarrow \mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A \checkmark$	$\vee \mathbb{T}, 2$
5.	$1.1, \mathbb{T} \quad A$	$\Diamond \mathbb{T}, 4$
6.	$1.1, \mathbb{F} \quad A \vee B \checkmark$	$\Diamond \mathbb{F}, 3$
7.	$1.1, \mathbb{F} \quad A$	$\vee \mathbb{F}, 6$
	x	x

I've cheated a bit here by just listing one justification for the lines after the branch. It's ok because the tree is completely symmetric; the same thing happens on each branch.