

# 305 Lecture 11.5 - Example Models

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# Plan

- To illustrate what we've done so far with some worked examples.

## Associated Reading

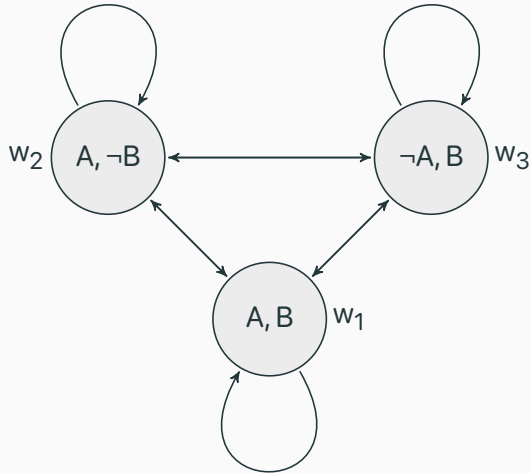
- Boxes and Diamonds, section 3.6 and 3.7.

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I'm going to go through the table on page 49 of the textbook, and show how there can be models where each of them is false. And I'll add a couple more in besides. Here is what we'll cover

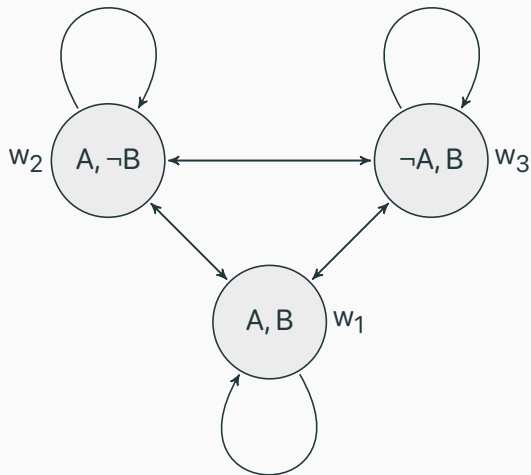
1.  $\Box(A \vee B) \rightarrow (\Box A \vee \Box B)$
2.  $(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$
3.  $A \rightarrow \Box A$
4.  $\Box A \rightarrow A$
5.  $\Box \Diamond A \rightarrow B$
6.  $\Box \Diamond A \rightarrow A$
7.  $\Box \Box A \rightarrow \Box A$
8.  $\Box A \rightarrow \Box \Box A$
9.  $\Box \Diamond A \rightarrow \Diamond \Box A$
10.  $\Box A \rightarrow \Diamond A$

$$\Box(A \vee B) \rightarrow (\Box A \vee \Box B)$$



At all points, either A  
or B is true, so  
 $\Box(A \vee B)$  is true.

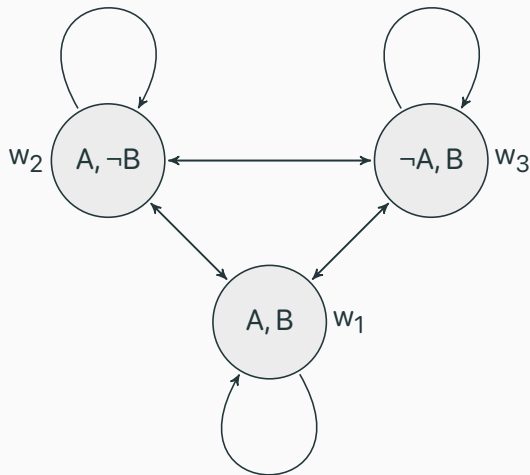
$$\Box(A \vee B) \rightarrow (\Box A \vee \Box B)$$



At all points, either  $A$  or  $B$  is true, so  $\Box(A \vee B)$  is true.

But  $\Box A$  and  $\Box B$  are false everywhere.

$$\Box(A \vee B) \rightarrow (\Box A \vee \Box B)$$

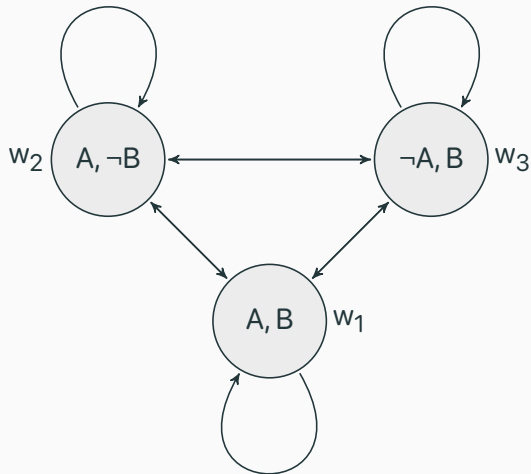


At all points, either  $A$  or  $B$  is true, so  $\Box(A \vee B)$  is true.

But  $\Box A$  and  $\Box B$  are false everywhere. So the conditional is false everywhere.

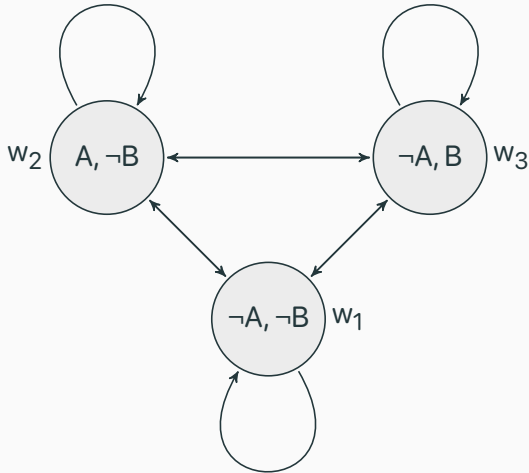


$$\Box(A \vee B) \rightarrow (\Box A \vee \Box B)$$



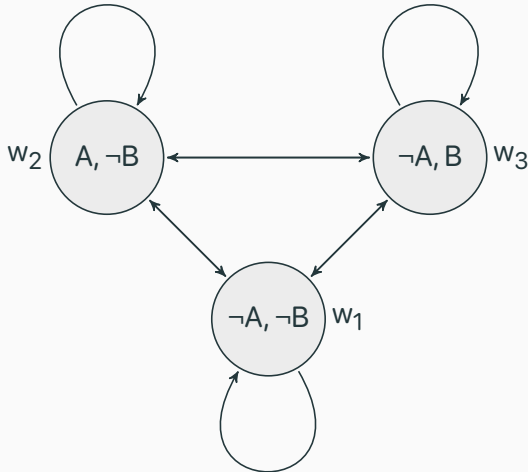
Note that this is overkill. We just need to show that the formula can be false somewhere in order to show that it is not a theorem.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$



At  $w_1$ , we have  
 $\Diamond A \wedge \Diamond B$  true.

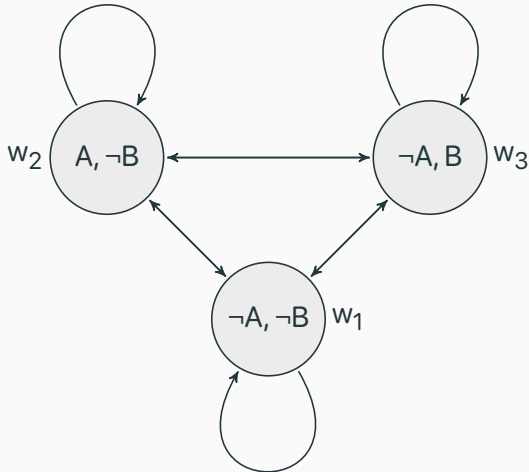
$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$



At  $w_1$ , we have  
 $\Diamond A \wedge \Diamond B$  true.

But nowhere is  $A \wedge B$   
true, so  $\Diamond(A \wedge B)$  is  
false at  $w_1$ . So the  
conditional is false.

$$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$$

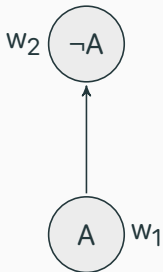


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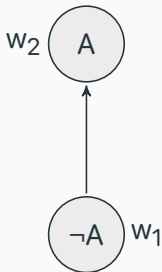
Again, this is overkill.

$$A \rightarrow \Box A$$



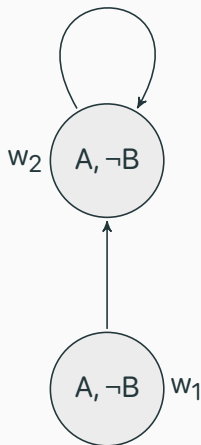
- At  $w_1$   $A$  is true.
- But  $\Box A$  is false, since  $w_1$  can access  $w_2$ , and  $A$  is false there.
- So  $A \rightarrow \Box A$  is false.

$$\Box A \rightarrow A$$



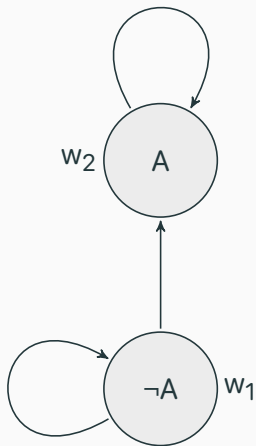
- At  $w_1$   $\Box A$  is true. The only accessible world is  $w_2$ , and  $A$  is true there.
- But  $A$  is false there.
- So  $\Box A \rightarrow A$  is false.

$\Box \Diamond A \rightarrow B$



- At  $w_1$   $\Box \Diamond A$  is true. The only accessible world is  $w_2$ , and  $\Diamond A$  is true there. (Why?)
- But  $B$  is false at  $w_1$ .
- So  $\Box \Diamond A \rightarrow B$  is false.

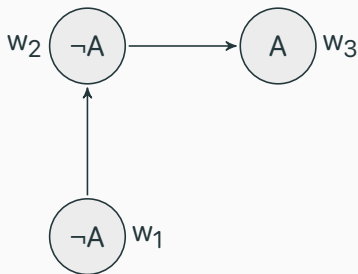
$\Box \Diamond A \rightarrow A$



- At  $w_1$   $\Box \Diamond A$  is true. At every world,  $w_2$  is accessible, and  $A$  is true there.
- But  $A$  is false at  $w_1$ .
- So  $\Box \Diamond A \rightarrow A$  is false at  $w_1$ .

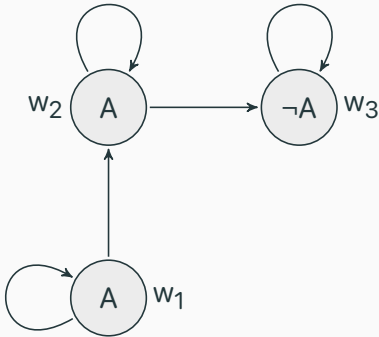


$$\Box\Box A \rightarrow \Box A$$



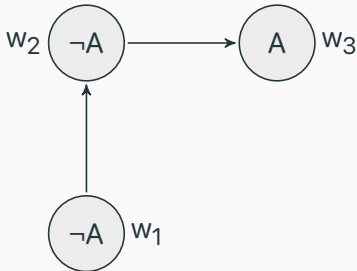
- The only world  $w_2$  can access is  $w_3$ , and  $A$  is true there, so  $\Box A$  is true at  $w_2$ .
- The only world  $w_1$  can access is  $w_2$ , and  $\Box A$  is true there, so  $\Box\Box A$  is true at  $w_1$ .
- But  $\Box A$  is false at  $w_1$ .
- So  $\Box\Box A \rightarrow \Box A$  is false at  $w_1$ .

$$\Box A \rightarrow \Box \Box A$$



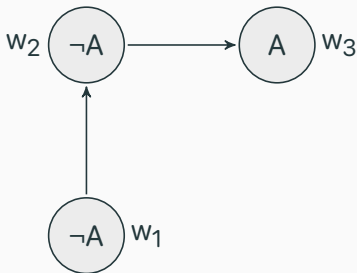
- Since  $A$  is false at  $w_3$ , and  $w_2$  can access  $w_3$ ,  $\Box A$  is false at  $w_2$ .
- Since  $\Box A$  is false at  $w_2$ , and  $w_1$  can access  $w_2$ ,  $\Box \Box A$  is false at  $w_1$ .
- But  $\Box A$  is true at  $w_1$ .
- So  $\Box A \rightarrow \Box \Box A$  is false at  $w_1$ .

$$\Box A \rightarrow \Diamond A$$



- Focus on  $w_3$ .
- There is no accessible world where  $A$  is false, so  $\Box A$  is true there.
- But there is no accessible world where  $A$  is true, so  $\Diamond A$  is false there.
- So  $\Box A \rightarrow \Diamond A$  is false there.

$$\Box A \rightarrow \Diamond A$$



Whenever there are no accessible worlds, the following two weird things happen.

1. All  $\Box$ -sentences (i.e., sentences that start with a  $\Box$  that takes scope over the whole sentence) are true.
2. All  $\Diamond$ -sentences (i.e., sentences that start with a  $\Diamond$  that takes scope over the whole sentence) are false.

## For Next Time

We will start looking at frames - what you get when you remove the valuation from a model.