$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$ **(in K)**

Build a Tableau

To Check Whether it is Valid

$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$

1.	$1, \mathbb{F}$	$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$	Assumption

Start with it being false at 1.

$(\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$

1.	$1, \mathbb{F}$	$(\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$ ✓	Assumption
2.	$1, \mathbb{T}$	$\Diamond A \vee \Diamond B$	$\to\mathbb{F}$, 1
3.	$1, \mathbb{F}$	$\Diamond (A \vee B)$	$\to\mathbb{F}$, 1

True antecedent, false consequent.

$(\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$

1.	$1, \mathbb{F} \quad (\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \vee \Diamond B$ ✓	$\to\mathbb{F}$, 1
3.	$1, \mathbb{F} \quad \Diamond (A \vee B)$	$\to\mathbb{F}$, 1
4.	$1, \mathbb{T} \quad \Diamond A$ $\qquad$ $1, \mathbb{T} \quad \Diamond B$	$\vee\mathbb{T}$, 2

Nothing to do but branch.

$(\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$

1.	$1, \mathbb{F}$ $(\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$ ✓		Assumption
2.	$1, \mathbb{T}$ $\Diamond A \vee \Diamond B$ ✓		$\to\mathbb{F}$, 1
3.	$1, \mathbb{F}$ $\Diamond (A \vee B)$		$\to\mathbb{F}$, 1
4.	$1, \mathbb{T}$ $\Diamond A$ ✓	$1, \mathbb{T}$ $\Diamond B$ ✓	$\vee\mathbb{T}$, 2
5.	$1.1, \mathbb{T}$ A	$1.2, \mathbb{T}$ B	$\Diamond\mathbb{T}$, 4

Diamond sentences have to be made true somehow.

$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$

1.	$1, \mathbb{F}$ $(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$ ✓		Assumption
2.	$1, \mathbb{T}$ $\Diamond A \vee \Diamond B$ ✓		$\rightarrow\mathbb{F}$, 1
3.	$1, \mathbb{F}$ $\Diamond (A \vee B)$		$\rightarrow\mathbb{F}$, 1
4.	$1, \mathbb{T}$ $\Diamond A$ ✓	$1, \mathbb{T}$ $\Diamond B$ ✓	$\vee\mathbb{T}$, 2
5.	$1.1, \mathbb{T}$ A	$1.2, \mathbb{T}$ B	$\Diamond\mathbb{T}$, 4
6.	$1.1, \mathbb{F}$ $A \vee B$	$1.2, \mathbb{F}$ $A \vee B$	$\Diamond\mathbb{F}$, 3

False diamond sentences are false at all accessible worlds.

$(\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$

1.	$1, \mathbb{F} \quad (\Diamond A \vee \Diamond B) \rightarrow \Diamond(A \vee B)$ ✓		Assumption
2.	$1, \mathbb{T} \quad \Diamond A \vee \Diamond B$ ✓		$\rightarrow\mathbb{F}$, 1
3.	$1, \mathbb{F} \quad \Diamond (A \vee B)$		$\rightarrow\mathbb{F}$, 1
4.	$1, \mathbb{T} \quad \Diamond A$ ✓	$1, \mathbb{T} \quad \Diamond B$ ✓	$\vee\mathbb{T}$, 2
5.	$1.1, \mathbb{T} \quad A$	$1.2, \mathbb{T} \quad B$	$\Diamond\mathbb{T}$, 4
6.	$1.1, \mathbb{F} \quad A \vee B$ ✓	$1.2, \mathbb{F} \quad A \vee B$ ✓	$\Diamond\mathbb{F}$, 3
7.	$1.1, \mathbb{F} \quad A$	$1.2, \mathbb{F} \quad B$	$\vee\mathbb{F}$, 6
	x	x	

And false or sentences have each side of the or false.

$(\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$

1.	$1, \mathbb{F}\quad (\Diamond A \vee \Diamond B) \to \Diamond(A \vee B)$ ✓		Assumption
2.	$1, \mathbb{T}\quad \Diamond A \vee \Diamond B$ ✓		$\to\mathbb{F}$, 1
3.	$1, \mathbb{F}\quad \Diamond (A \vee B)$		$\to\mathbb{F}$, 1
4.	$1, \mathbb{T}\quad \Diamond A$ ✓	$1, \mathbb{T}\quad \Diamond B$ ✓	$\vee\mathbb{T}$, 2
5.	$1.1, \mathbb{T}\quad A$	$1.2, \mathbb{T}\quad B$	$\Diamond\mathbb{T}$, 4
6.	$1.1, \mathbb{F}\quad A \vee B$ ✓	$1.2, \mathbb{F}\quad A \vee B$ ✓	$\Diamond\mathbb{F}$, 3
7.	$1.1, \mathbb{F}\quad A$	$1.2, \mathbb{F}\quad B$	$\vee\mathbb{F}$, 6
	x	x	

I've cheated a bit here by just listing one justification for the lines after the branch. It's ok because the tree is completely symmetric; the same thing happens on each branch.