305 Lecture 11.1 - Introduction to Modal Logic

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Plan

- Introducing modal logic.

Associated Reading

- Boxes and Diamonds, section 3.1 and 3.2.

What Modal Logic Is

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The logics of **must** and **might**.

- Why plural? Because we do not assume that these words have a single determinate meaning.

Examples of Must

1. If x = 2 + 2, then x must equal 4.
2. If something is a cat, then it must be a mammal.
3. If the gardener is innocent, then it must be the butler who did it.
4. You must drive under 70mph on I-94.
5. You must keep your promises.
6. If you set out a knife and fork, the fork must go on the left.

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To my ears, 1 is **logical** necessity, 2 is **metaphysical** necessity, 3 is **epistemic** necessity, 4 is **legal** necessity, 5 is **moral** (or **deontic**) necessity and 6 is **etiquette** necessity.

Examples of May/Might

1. If x is prime, then x might be even.
2. If x is a cat, then x might be male.
3. It might be the butler or the gardener that did it.
4. You may drive at any speed below 30mph on State Street.
5. You may lie to save a friend's life.
6. You may use white napkins or red napkins.

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To my ears, 1 is **logical** possibility, 2 is **metaphysical** possibility, 3 is **epistemic** possibility, 4 is **legal** possibility, 5 is **moral** (or **deontic**) possibility and 6 is **etiquette** possibility (though I'm not sure about any of these).

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- But it is very much not true on the legal, moral or etiquette interpretations.

So we want some logics where it is a logical truth, and some where it is not.

Language

We extend our language with two new operators: $\Box$ and $\Diamond$.

- If p is a sentence, so is $\Box$p and so is $\Diamond$p.
- These mean, respectively, that p must be true, and that p might be true.
- We interpret these somewhat similar to negations; they just bind what they are immediately next to.
- So $\Box p \to q$ means $(\Box p) \to q$, not $\Box(p \to q)$.

For Next Time

We'll start looking at what it takes for these sentences to be true.