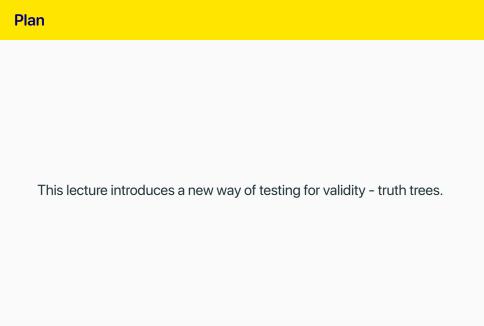
305 Lecture 3.5 - Introducing Truth Trees

Brian Weatherson





Boxes and Diamonds, section 2.1

What Tableaux Are

- A way for determining whether some combinations are logically possible.
- That can be used for determining whether some arguments are valid - if the truth of the premises and the falsity of the conclusion is not logically possible; then the argument is valid.

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- Each time one of those things implies that some other things must be the case, write those down too.
- For example, if you write down that A ∧ B is true, also write down that A is true and that B is true.
- Each time there are multiple ways to make something you've written true, create multiple branches for those ways.
- For example, if you write down that A ∨ B is true, create a branch where A is true, and a branch where B is true.

Closing

- A branch of a tableau is closed if it contains that some particular claim has incompatible truth values.
- For now, this means that one sentence is both true and false.

Closing

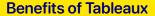
- A branch of a tableau is closed if it contains that some particular claim has incompatible truth values.
- For now, this means that one sentence is both true and false.
- The whole tableau is closed if every branch is closed.

What Closure Means

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- If you are evaluating an argument, this means that the argument is valid.
- If you start the tableau by just saying that one sentence is false, the closure of the tableau means that that sentence is a logical truth.



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Benefits of Tableaux

Tableaux have two big benefits over truth tables.

- 1. They don't grow exponentially when you increase the number of variables.
- 2. They can be generalised to things beyond propositional logic.

We are introducing them here because of point 2.

Open Tableaux

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Open Tableaux

Here is something the book doesn't make a big deal of, but is kind of important.

- · A closed tableau can show that an argument is valid.
- An open and completed tableau can show that an argument is invalid.
- The trick here is that it's hard to tell when a tableau is completed in the relevant sense.
- This will be easier to illustrate in practice than in theory, so let's start building tableau up.

Signs

The system we are using is what is called a **signed tableau** system.

- That means that every line consists of two parts.
- The bigger, second, part is a sentence.
- The first part is a sign, which for now is a truth value.
- That is, it is either \mathbb{T} or \mathbb{F} .

What Lines Mean

So each line either says that a particular sentence is true, or says that it is false.

- The book for some reason includes the word 'might' here.
- That's misleading; what they should say is that each line says what is true given (a) the starting assumptions and (b) the assumptions we made for branching purposes.

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Both of these are somewhat idiosyncratic, though not abnormal. Unlike the truth tables, there just aren't well defined conventions for how to write these things out.

