305 Lecture 9.1 - Expected Utility

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Plan

- Today we’re going to talk about the role of probability in decision making.
- And to do this, we need to introduce a new concept, **Expected Value**.

Associated Reading

Odds and Ends, chapter 11; though like with chapter 8 I’m going to take a different path through the material to the book.

Random Variables

- A **random variable** is simply a variable that takes different numerical values in different states.
- In other words, it is a function from possibilities to numbers.
- It need not be ‘random’ in any familiar sense.
- The function from possible situations to the value of 2 + 2 in that situation is a random variable, albeit a constant one.
- It’s just a slightly confusing term for any variable that takes different, numerical, values in different situations.

Labels

- Typically, random variables are denoted by capital letters.
- So we might have a random variable X whose value is the age of the next President of the United States, and his or her inauguration.
- Or we might have a random variable Y that is the number of children you will have in your lifetime.
- Basically any mapping from possibilities to numbers can be a random variable.

An Example

- You’ve asked each of your friends who will win the Lakers v Clippers game.
- 12 said the Lakers will win.
- 7 said the Clippers will win.

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- You've asked each of your friends who will win the Lakers v Clippers game.
- 12 said the Lakers will win.
- 7 said the Clippers will win.
- Then we can let X be a random variable measuring the number of your friends who correctly predicted the result of the game.

$$X = \begin{cases} 12, & \text{if Lakers win,} \\ 7, & \text{if Clippers win.} \end{cases}$$

Expected Value

- Given a random variable X and a probability function Pr, we can work out the **expected value** of that random variable with respect to that probability function.
- Intuitively, the expected value of X is a weighted average of the possible values of X, where the weights are given by the probability (according to Pr) of each value coming about.

Calculating Expected Value

- More formally, we work out the expected value of X this way.
- For each possibility, we multiply the value of X in that case by the probability of the possibility obtaining.
- Then we sum the numbers we've got, and the result is the expected value of X.
- We'll write the expected value of X as Exp(X).

Back to the Example

- So if the probability that the Lakers win is 0.7, and the probability that the Clippers win is 0.3, then

$$\begin{aligned} \mathrm{Exp}(X) &= 12 \times 0.7 + 7 \times 0.3 \\ &= 8.4 + 2.1 \\ &= 10.5 \end{aligned}$$

Notes

1. The expected value of X isn't in any sense the value that we expect X to take. It's more like an average.
2. If this kind of situation recurs a lot, you would expect the long run average value X takes to be roundabout the expected value.
3. That's a better way of conceptualising what expected values are.

For Next Time

- We will look at how to formally model a decision problem.