

305 Lecture 2.2 - Forms in Propositional Logic

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Plan for This Lecture

- We're going to look at how to translate English sentences into a form where we can use the tools of propositional logic on them.

forall x, chapter 5, "Connectives"

- Note that the book goes into more detail than I'm going into here about the range of English sentences that can be translated (or at least symbolized) using the tools of propositional logic.

Sentence Letters

- We use letters, usually starting with p and then continuing through the alphabet, for 'atomic' sentences.
- for all x uses capital letters, others use lowercase. This is a matter of taste.

Symbols

- "and" is represented by \wedge
- "or" is represented by \vee
- "not" is represented by \neg
- "if" is represented by \rightarrow
- "if and only if" is represented by \leftrightarrow

An Easy Example

It will rain on Monday or it will rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

$$M \vee T$$

Another Easy Example

It will rain on Monday and it will rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

$$M \wedge T$$

Only Slightly Harder Example

If it rains on Monday, it will rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

$$M \rightarrow T$$

And A Bit Harder Again

It will rain on Monday, but it won't rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

$$M \wedge \neg T$$

Parentheses

- As in math, these tell you what order to compute things in.
- They let you avoid the ambiguity in the following sentence.

It will rain on Monday and it will rain on Tuesday or it will rain on Wednesday.

Two Translations

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday
- W = It will rain on Wednesday

Translations

1. $(M \wedge T) \vee W$
2. $M \wedge (T \vee W)$

What's the Difference?

1. $(M \wedge T) \vee W$

2. $M \wedge (T \vee W)$

What is a situation where one of these is true but not the other?

- If it only rains on Wednesday, then 1 is true but 2 is false.

What's the Difference?

1. $(M \wedge T) \vee W$

2. $M \wedge (T \vee W)$

What is a situation where one of these is true but not the other?

- If it only rains on Wednesday, then 1 is true but 2 is false.
- Are there any others?

For Next Time

We'll look at how the processes I've described in this lecture can be iterated.