305 Lecture 12.5 - Proving Invalidity

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Plan

- Discuss how to use tableau to show invalidity.

Associated Reading

- Not in book.

Two Uses of Trees

We tell that an argument is valid or that a sentence is a theorem by drawing a closed tree.

- In principle, we can also use trees to show that an argument is invalid, or that something is not a theorem.
- In practice, it's a little tricky.

Case One: Draw an Open Tree

Imagine we're working in KT, and we want to show that $\Box A \rightarrow \Box\Box A$ is not a theorem. Then we really can draw a tree.

$\Box A \to \Box\Box A$

1.	$1, \mathbb{F} \quad \Box A \to \Box\Box A$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box A$	$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Box\Box A$ ✓	$\to\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad A$	$T\Box 2$
5.	$1.1, \mathbb{F} \quad \Box A$ ✓	$\Box\mathbb{F}, 3$
6.	$1.1, \mathbb{T} \quad A$	$\Box\mathbb{T}, 2$
7.	$1.1.1, \mathbb{F} \quad A$	$\Box\mathbb{F}, 5$

A Model

- Three worlds, w_1, $w_{1.1}$, $w_{1.1.1}$.
- The accessibility relations are $w_1 R w_{1.1}$, $w_{1.1} R w_{1.1.1}$, $w_1 R w_1$, $w_{1.1} R w_{1.1}$ and $w_{1.1.1} R w_{1.1.1}$.
- The first two are from the tree, the next three from the restriction.
- A is true at w_1 and $w_{1.1}$ and false at $w_{1.1.1}$.
- So $\Box A$ will be true only at w_1.
- So $\Box\Box A$ will be false at w_1, as required.

First Problem

We typically can't just tick off the sentences as we apply the rules for them.

- Lots of the rules, especially for true $\Box$ sentences and false $\Diamond$ sentences, are open.
- So to check the tree is finished, you have to go back and look at each of these sentences, and be sure that you really really have applied all the rules.

Second Problem

Sometimes the tree never ends.

- Imagine we're working in KT4.
- And we're trying to work out whether this is a theorem
- $\Box \Diamond A \rightarrow \Diamond \Box B$
- At one level, it's obvious that it isn't a theorem.
- But the tree is a mess.

1.	$1, \mathbb{F}\ \Box\Diamond A \to \Diamond\Box B$ ✓	Assumption
2.	$1, \mathbb{T}\ \Box\Diamond A$	$\to\mathbb{F}$, 1
3.	$1, \mathbb{F}\ \Diamond\Box B$	$\to\mathbb{F}$, 1
4.	$1, \mathbb{T}\ \Diamond A$ ✓	$T\Box$, 2
5.	$1, \mathbb{F}\ \Box B$ ✓	$T\Diamond$, 2
6.	$1.1, \mathbb{T}\ A$	$\Diamond\mathbb{T}$, 4
7.	$1.1, \mathbb{T}\ \Diamond A$	$\mathbb{T}\Box$, 2
8.	$1.1, \mathbb{F}\ \Box B$	$\Diamond\mathbb{F}$, 3
9.	$1.1, \mathbb{T}\ \Box\Diamond A$	$4\Box$, 2

What Went Wrong

We can sort of kind of see the problem.

- The tree just repeats.
- Maybe we can turn that into a model.

The General Recipe

What shows something is not a theorem is a model where it is false at a world.

- Take the open tree.
- Each number on the tree is a world.
- World x is always related to world x.y.
- Add R-relations that are required by the relevant restrictions.
- Read truth for atomic sentences off what the tree says. It should be reasonably specific, though often it will leave gaps.

The Harder Case

What to do if the tree doesn’t close.

1. Describe (but obviously don’t draw) the infinite model.
2. Draw a model where one world stands in for many different strings of numbers.

For Up/Down Verdicts

If you can see that the tree will cycle and never complete, that’s sort of good enough.

For Next Time

We'll go on to examples.