305 Lecture 9.4 - Maximise Expected Utility

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Plan

- In this lecture we'll talk about the core rule of formal decision theory: Maximise Expected Utility.

Associated Reading

Odds and Ends, Chapter 12

The Rule

- The orthodox view in modern decision theory is that the right decision is the one that maximises the expected utility of your choice.
- The rational decision maximises not actual utility, but expected utility.

Airline Example (Several Variants)

It turns out that what to do turns on three factors.

1. How likely bad weather is.
2. How much you have to gain by flying the cheaper airline in good weather.
3. How much you have to lose by flying the cheaper airline in bad weather.

It is plausible, I think, that these three should matter.

Version One

Lots to gain, relatively little to lose, high probability of good weather.

	Good weather Pr=0.8	Bad Weather Pr=0.2
Fly Cheap Airline	10	0
Fly Good Airline	6	5

Utility Calculation

We can work out the expected utility of each action fairly easily.

$$\begin{aligned}
\text{Exp(Cheap Airline)} &= 0.8 \times 10 + 0.2 \times 0 \\
&= 8 + 0 \\
&= 8 \\
\text{Exp(Reliable Airline)} &= 0.8 \times 6 + 0.2 \times 5 \\
&= 4.8 + 1 \\
&= 5.8
\end{aligned}$$

- So the cheap airline has an expected utility of 8, the reliable airline has an expected utility of 5.8.
- The cheap airline has a higher expected utility, so it is what should be taken.

Other versions

- We'll now look at three changes to the example.
- Each change should intuitively change the correct decision, and we'll see that the maximise expected utility rule does change in each case.

More Downside if Bad Weather

	Good weather Pr=0.8	Bad Weather Pr=0.2
Fly Cheap Airline	10	-20
Fly Good Airline	6	5

Utility Calculations

Here are the new expected utility considerations.

$$\begin{aligned}
\text{Exp(Cheap Airline)} &= 0.8 \times 10 + 0.2 \times -20 \\
&= 8 + (-4) \\
&= 4 \\
\text{Exp(Reliable Airline)} &= 0.8 \times 6 + 0.2 \times 5 \\
&= 4.8 + 1 \\
&= 5.8
\end{aligned}$$

- Now the expected utility of catching the reliable airline is higher than the expected utility of catching the cheap airline.
- So it is better to catch the reliable airline.

Less to Gain by Cheaper Airline

	Good weather Pr=0.8	Bad Weather Pr=0.2
Fly Cheap Airline	10	0
Fly Good Airline	9	8

Utility Calculations

Here are the revised expected utility considerations.

$$\begin{aligned}
\text{Exp(Cheap Airline)} &= 0.8 \times 10 + 0.2 \times 0 \\
&= 8 + 0 \\
&= 8 \\
\text{Exp(Reliable Airline)} &= 0.8 \times 9 + 0.2 \times 8 \\
&= 7.2 + 1.6 \\
&= 8.8
\end{aligned}$$

And again this is enough to make the reliable airline the better choice.

Bad Weather More Likely

	Good weather Pr=0.3	Bad Weather Pr=0.7
Fly Cheap Airline	10	0
Fly Good Airline	6	5

Utility Calculations

We can work out the expected utility of each action fairly easily.

$$
\begin{aligned}
\text{Exp(Cheap Airline)} &= 0.3 \times 10 + 0.7 \times 0 \\
&= 3 + 0 \\
&= 3 \\
\text{Exp(Reliable Airline)} &= 0.3 \times 6 + 0.7 \times 5 \\
&= 1.8 + 3.5 \\
&= 5.3
\end{aligned}
$$

Summarising the Cases

We've looked at four versions of the same case. In each case the ordering of the outcomes, from best to worst, was:

1. Cheap airline and good weather
2. Reliable airline and good weather
3. Reliable airline and bad weather
4. Cheap airline and bad weather

But this doesn't settle what to do; these three factors all matter.

For Next Time

- We will look at the relationship between utility and money.