# 305 Lecture 5.3 - Rules for Not

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This lecture discusses the rules for negation.

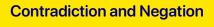
## **Associated Reading**

forall x, section 16.8.

## A New Symbol

• 1

Read this as 'contradiction', or 'the false'. It is a sentence that can't be true.



• How do we know that a contradiction has obtained?

### **Contradiction and Negation**

- How do we know that a contradiction has obtained?
- By proving some sentence and the negation of that very sentence.

### **Contradiction and Negation**

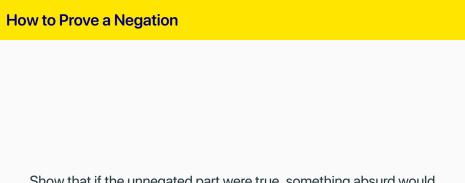
- How do we know that a contradiction has obtained?
- By proving some sentence and the negation of that very sentence.
- This is going to be our rule for proving things from a negation.

# **Neg-Elimination**

```
egin{array}{c|c} m & \neg \mathcal{A} \\ n & \mathcal{A} \\ & \bot & \neg \to m, \ n \end{array}
```

Neg-Elimination

From contradictory sentences, infer  $\bot$ .



Show that if the unnegated part were true, something absurd would follow.

# **Absurdity**

 In engineering, a perpetual motion machine, or some other kind of free energy.

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- In engineering, a perpetual motion machine, or some other kind of free energy.
- In finance, a risk-free way to make a guaranteed profit.
- In logic, a sentence and its negation.

# **Neg-Introduction**

$$egin{array}{c|c} i & & \mathcal{A} \\ j & & \perp \\ \neg \mathcal{A} & \neg \mathrm{I} \ i-j \end{array}$$

Neg-Introduction

- If A implies a contradiction, infer  $\neg A$ .

## **Indirect Proof**

$$egin{array}{c|c} i & & \neg \mathcal{A} \\ j & & \bot \\ & & \mathcal{A} & & \mathrm{IP} \; i{-}j \end{array}$$

**Indirect Proof** 

• If ¬A implies a contradiction, infer A.

### **Explosion**

$$egin{array}{c|c} m & \perp & & \\ \mathscr{A} & \mathbf{X} & m & & \\ \end{array}$$

#### **Explosion**

- · A contradiction implies anything.
- Note that this rule is redundant; we can replicate it using Indirect Proof.
- I think they've added it because it is an interesting rule if you don't like Indirect Proof.

#### **For Next Time**

- That's a lot of rules we've set out.
- We will start looking at how they work in practice.