305 Lecture 4.1 - Using Truth Trees

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Plan

Showing how we use truth trees to check for logical truth and validity.

Reading

Boxes and Diamonds, section 2.4.

1.	$\mathbb{T}\,\neg A$	
2.	$\mathbb{F}\,A$	$\neg\mathbb{T}, 1$

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2.	$\mathbb{T}\,A$	$\neg\mathbb{F}, 1$

1.	$\mathbb{T}\,A \wedge B$	
2.	$\mathbb{T}\,A$	$\wedge\mathbb{T}, 1$
3.	$\mathbb{T}\,B$	$\wedge\mathbb{T}, 1$

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2.	$\mathbb{F}\,A$	$\vee\mathbb{F}, 1$
3.	$\mathbb{F}\,B$	$\vee\mathbb{F}, 1$

1.	$\mathbb{T}\,A \to B$		
2.	$\mathbb{F}\,A$	$\mathbb{T}\,B$	$\to\mathbb{T}, 1$

1.	$\mathbb{F}\,A \to B$	
2.	$\mathbb{T}\,A$	$\to\mathbb{F}, 1$
3.	$\mathbb{F}\,B$	$\to\mathbb{F}, 1$

Closure

- A tree without branches closes if there is a sentence on the tree that is marked as both true and false.
- Here is an example. The first line is stipulated, the rest are derived.

1.	$\mathbb{T}\, A \wedge \neg A$	
2.	$\mathbb{T}\, A$	$\wedge\mathbb{T}$, 1
3.	$\mathbb{T}\, \neg A$	$\wedge\mathbb{T}$, 1
4.	$\mathbb{F}\, A$	$\neg\mathbb{T}$, 3
	x	

Interpreting Closure

- If a tree is closed, it means the initial assumptions can't be true.
- So this tree means that the initial assumption $A \wedge \neg A$ can't be true.

1.	$\mathbb{T}\, A \wedge \neg A$	
2.	$\mathbb{T}\, A$	$\wedge\mathbb{T}$, 1
3.	$\mathbb{T}\, \neg A$	$\wedge\mathbb{T}$, 1
4.	$\mathbb{F}\, A$	$\neg\mathbb{T}$, 3
	x	

Closure with Branches

- A branching tree closes if every branch closes.
- The next slide has an example, with in this case the top two lines stipulated.

1.	$\mathbb{T}\,\neg A \wedge \neg B$		
2.	$\mathbb{T}\,A \vee B$		
3.	$\mathbb{T}\,\neg A$		$\wedge\mathbb{T}, 1$
4.	$\mathbb{T}\,\neg B$		$\wedge\mathbb{T}, 1$
5.	$\mathbb{F}\,A$		$\neg\mathbb{T}, 3$
6.	$\mathbb{F}\,B$		$\neg\mathbb{T}, 4$
7.	$\mathbb{T}\,A$	$\mathbb{T}\,B$	$\vee\mathbb{T}, 2$
	x	x	

Closure with Branches

- A tree with any open branches is open, i.e., not closed.
- If a tree has some open branches and some closed branches, it is **open**.
- All that matters is if all branches are closed.
- The next slide is an example of an open tree.

1.	$\mathbb{T}\,\neg A \wedge B$		
2.	$\mathbb{T}\,A \vee B$		
3.	$\mathbb{T}\,\neg A$		$\wedge\mathbb{T}, 1$
4.	$\mathbb{T}\,B$		$\wedge\mathbb{T}, 1$
5.	$\mathbb{F}\,A$		$\neg\mathbb{T}, 3$
6.	$\mathbb{T}\,A$	$\mathbb{T}\,B$	$\vee\mathbb{T}, 2$
	x		

Logical Truth

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3. If the tree does not close, it is not a logical truth (of propositional logic).

Looking for Counterexamples (Tables)

- Truth tables didn’t just tell us that something failed to be a logical truth (of propositional logic).
- It told us where the failure was.
- You didn’t just know that there was an $\mathbb{F}$ in the relevant column, you knew which row it was on.
- And that told you where to look for counterexamples.

Looking for Counterexamples (Trees)

- The same thing happens with trees.
- By reading off the open branch, you can see where the sentence fails.
- The trick is to focus on the **atomic** sentences on the branch.
- These are the ones with no connectives at all.

1.	$\mathbb{F}\, A \to (A \land B)$	
2.	$\mathbb{T}\, A$	$\to\mathbb{T}$, 1
3.	$\mathbb{F}\, A \land B$	$\to\mathbb{T}$, 1
4.	$\mathbb{F}\, A$ $\quad$ $\mathbb{F}\, B$	$\land\mathbb{F}$, 3
	x	

- The right hand branch doesn't close.
- The atomics on that branch are that A is $\mathbb{T}$ and B is $\mathbb{F}$.
- So that's the line on the truth table where $A \to (A \land B)$ is $\mathbb{F}$.

Validity

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3. If the tree closes, the argument is valid.
4. If the tree does not close, the argumnt is not valid (in propositional logic).

For Next Time

We will illustrate this algorithm.