305 Lecture 11.4 - Truth in a Model

Brian Weatherson

Plan

- To extend our discussion of truth at a world, to discussion of truth in a whole model.

Associated Reading

- Boxes and Diamonds, section 3.5.

Models

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We'll write the models as $\langle W, R, V \rangle$.

Valuations

V is a function from atomic sentence letters to subsets of W.

- It tells you when the atomic sentences are true.
- When an atomic sentence is not true, it is false.

Truth at a Point

The general theory of truth is built up in stages from the basic theory. Assume we have a model $\langle W, R, V \rangle$, and a point $w \in W$, and are asking whether an arbitrary sentence is true at w in $\langle W, R, V \rangle$.

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- p is true at w iff $w \in V(p)$.
- $\neg A$ is true at w iff A is not true at w.
- $A \wedge B$ is true at w iff A is true and w and B is true at w.
- $A \vee B$ is true at w iff A is true and w or B is true at w.
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This just leaves the modal formulae. I'll set out the rules, then do some worked examples.

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- Intuitively, it means **It must be that A**, where **must** could be a metaphysical necessity, or an epistemic necessity, or a moral necessity, or anything else.
- And it is true at w just in case A is true at every world y such that wRy.
- Necessary truth is truth at all accessible worlds.

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- And it is true at w just in case A is true at some world y such that wRy.
- Possible truth is truth at some accessible world.

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- And that means that A has to be true at every world z such that yRz (for any y such that wRy).

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- $\Box A$ means that anywhere you can step to from w is a world where A is true.
- And $\Box\Box A$ means that anywhere you can get to in two steps from w is a world where A is true.

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- And that means that A has to be true at some world z such that yRz (for some y such that wRy).
- In the picturesque terms, you can get from w to an A-world in two steps.

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- There is some accessible world where $\Box A$ is true.
- That is, there is some accessible world such that everywhere you can go from there, A is true.

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What does it mean for $\Box\Diamond A$ to be true at w?

- At all accessible worlds, $\Diamond A$ is true.
- That is, wherever you go, you can get to there is some accessible world such that everywhere you can go from there, A is true.

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What does it mean for $\Box(p \vee (q \to \Diamond r))$ to be true at w?

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- That is, at every one of those worlds, either p is true, or q is false, or $\Diamond r$ is true.
- That is, at every one of those worlds, either p is true, or q is false, or there is some world you can get to where r is true.

Box and connectives

The general rule is just to apply the rules for sentences inside the brackets at each world in W, and then apply the rule for $\Box$ or $\Diamond$. But there are three special cases worth thinking about.

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- $\Box(A \wedge B)$ means that all accessible worlds are A and B worlds.
- $\Box(A \vee B)$ means that all accessible worlds make at least one of A and B true.
- $\Box(A \rightarrow B)$ means that all accessible A-worlds are B-worlds.

We'll use that last one a lot.

For Next Time

We’ll discuss of examples of truth (and non-truth) in models to explain this material.