$(\Diamond A \land \Diamond B) \to \Diamond(A \land B)$ **(in K)**

Build a Tableau

To Check Whether it is Valid

Hypothesis

$(\Diamond A \land \Diamond B) \rightarrow \Diamond(A \land B)$ is a theorem of K.

- So we can use the base modal rules.

$(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$

1.	$1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$	Assumption

Start with it being false at 1.

$(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$

1.	$1, \mathbb{F}$	$(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$ ✓	Assumption
2.	$1, \mathbb{T}$	$\Diamond A \wedge \Diamond B$	$\to\mathbb{F}$, 1
3.	$1, \mathbb{F}$	$\Diamond (A \wedge B)$	$\to\mathbb{F}$, 1

True antecedent, false consequent.

$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$

1.	$1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \wedge \Diamond B$ ✓	$\rightarrow\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond (A \wedge B)$	$\rightarrow\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A$	$\wedge\mathbb{T}, 2$
5.	$1, \mathbb{T} \quad \Diamond B$	$\wedge\mathbb{T}, 2$

Break up the and.

$(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$

1.	$1, \mathbb{F}$	$(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$ ✓	Assumption
2.	$1, \mathbb{T}$	$\Diamond A \wedge \Diamond B$ ✓	$\to\mathbb{F}, 1$
3.	$1, \mathbb{F}$	$\Diamond (A \wedge B)$	$\to\mathbb{F}, 1$
4.	$1, \mathbb{T}$	$\Diamond A$ ✓	$\wedge\mathbb{T}, 2$
5.	$1, \mathbb{T}$	$\Diamond B$	$\wedge\mathbb{T}, 2$
6.	$1.1, \mathbb{T}$	A	$\Diamond\mathbb{T}, 4$

Make $\Diamond A$ true.

$(\Diamond A \land \Diamond B) \to \Diamond(A \land B)$

1.	$1, \mathbb{F} \quad (\Diamond A \land \Diamond B) \to \Diamond(A \land B)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Diamond A \land \Diamond B$ ✓	$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond (A \land B)$	$\to\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A$ ✓	$\land\mathbb{T}, 2$
5.	$1, \mathbb{T} \quad \Diamond B$	$\land\mathbb{T}, 2$
6.	$1.1, \mathbb{T} \quad A$	$\Diamond\mathbb{T}, 4$
7.	$1.1, \mathbb{F} \quad A \land B$	$\Diamond\mathbb{F}, 3$

Since $A \land B$ is false everywhere accessible from 1, it is false at 1.1.

$(\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$

1.	$1, \mathbb{F} \quad (\Diamond A \wedge \Diamond B) \to \Diamond(A \wedge B)$ ✓		Assumption
2.	$1, \mathbb{T} \quad \Diamond A \wedge \Diamond B$ ✓		$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond (A \wedge B)$		$\to\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A$ ✓		$\wedge\mathbb{T}, 2$
5.	$1, \mathbb{T} \quad \Diamond B$		$\wedge\mathbb{T}, 2$
6.	$1.1, \mathbb{T} \quad A$		$\Diamond\mathbb{T}, 4$
7.	$1.1, \mathbb{F} \quad A \wedge B$ ✓		$\Diamond\mathbb{F}, 3$
8.	$1.1, \mathbb{F} \quad A$	$1.1, \mathbb{F} \quad B$	$\wedge\mathbb{F}, 7$
	x		

Two ways for $A \wedge B$ to fail; only one is possible.

$(\Diamond A \land \Diamond B) \to \Diamond(A \land B)$

1.	$1, \mathbb{F}\quad (\Diamond A \land \Diamond B) \to \Diamond(A \land B)$ ✓		Assumption
2.	$1, \mathbb{T}\quad \Diamond A \land \Diamond B$ ✓		$\to\mathbb{F}, 1$
3.	$1, \mathbb{F}\quad \Diamond(A \land B)$		$\to\mathbb{F}, 1$
4.	$1, \mathbb{T}\quad \Diamond A$ ✓		$\land\mathbb{T}, 2$
5.	$1, \mathbb{T}\quad \Diamond B$ ✓		$\land\mathbb{T}, 2$
6.	$1.1, \mathbb{T}\quad A$		$\Diamond\mathbb{T}, 4$
7.	$1.1, \mathbb{F}\quad A \land B$ ✓		$\Diamond\mathbb{F}, 3$
8.	$1.1, \mathbb{F}\quad A$	$1.1, \mathbb{F}\quad B$	$\land\mathbb{F}, 7$
9.	x	$1.2, \mathbb{T}\quad B$	$\Diamond\mathbb{T}, 5$

B has to be possible, so we'll say it is possible at 1.2 - using a name we haven't already used.

$(\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$

1.	$1, \mathbb{F}\quad (\Diamond A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B)$ ✓		Assumption
2.	$1, \mathbb{T}\quad \Diamond A \wedge \Diamond B$ ✓		$\rightarrow\mathbb{F}, 1$
3.	$1, \mathbb{F}\quad \Diamond (A \wedge B)$		$\rightarrow\mathbb{F}, 1$
4.	$1, \mathbb{T}\quad \Diamond A$ ✓		$\wedge\mathbb{T}, 2$
5.	$1, \mathbb{T}\quad \Diamond B$ ✓		$\wedge\mathbb{T}, 2$
6.	$1.1, \mathbb{T}\quad A$		$\Diamond\mathbb{T}, 4$
7.	$1.1, \mathbb{F}\quad A \wedge B$ ✓		$\Diamond\mathbb{F}, 3$
8.	$1.1, \mathbb{F}\quad A$	$1.1, \mathbb{F}\quad B$	$\wedge\mathbb{F}, 7$
9.	x	$1.2, \mathbb{T}\quad B$	$\Diamond\mathbb{T}, 5$
10.		$1.2, \mathbb{F}\quad A \wedge B$	$\Diamond\mathbb{F}, 3$

$A \wedge B$ is still impossible.

$(\Diamond A \land \Diamond B) \rightarrow \Diamond(A \land B)$

1.	$1, \mathbb{F} \quad (\Diamond A \land \Diamond B) \rightarrow \Diamond(A \land B)$ ✓			Assumption
2.	$1, \mathbb{T} \quad \Diamond A \land \Diamond B$ ✓			$\rightarrow\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond (A \land B)$			$\rightarrow\mathbb{F}, 1$
4.	$1, \mathbb{T} \quad \Diamond A$ ✓			$\land\mathbb{T}, 2$
5.	$1, \mathbb{T} \quad \Diamond B$ ✓			$\land\mathbb{T}, 2$
6.	$1.1, \mathbb{T} \quad A$			$\Diamond\mathbb{T}, 4$
7.	$1.1, \mathbb{F} \quad A \land B$ ✓			$\Diamond\mathbb{F}, 3$
8.	$1.1, \mathbb{F} \quad A$	$1.1, \mathbb{F} \quad B$		$\land\mathbb{F}, 7$
9.	x	$1.2, \mathbb{T} \quad B$		$\Diamond\mathbb{T}, 5$
10.		$1.2, \mathbb{F} \quad A \land B$ ✓		$\Diamond\mathbb{F}, 3$
11.		$1.2, \mathbb{F} \quad A$	$1.2, \mathbb{F} \quad B$	$\land\mathbb{F}, 7$
			x	

And we're done. All the rules have been applied - though you really need to check this - and the middle branch does not close.

A Model

When the tableau doesn't close, you should be able to build a model where the sentence is false at w_1.

In fact, you should be able to read it off the tree.

A Model

- Three worlds, $w_1, w_{1.1}, w_{1.2}$.
- The only accessibility relations are $w_1 R w_{1.1}$ and $w_1 R w_{1.2}$.
- In general, the accessibility relations are the ones required by the restrictions (but there are none here), plus the ones required by the numbering, i.e., $w_x R w_{x.y}$.
- A is true at $w_{1.1}$ and false at $w_{1.2}$.
- B is true at $w_{1.2}$ and false at $w_{1.1}$.
- It doesn't matter what atomics are true at w_1.
- But both $\Diamond A$ and $\Diamond B$ will be true at w_1, while $\Diamond(A \wedge B)$ is false there.