

$\Diamond A \rightarrow \Box \Diamond A$ (in S5)

Build a Tableau

To Check Whether it is Valid

Hypothesis

$\Diamond A \rightarrow \Box \Diamond A$ is a theorem of S5.

- We are going to use simplified S5 because normal S5 is really painful for cases like this one.
- Note that if we can prove this, we'll have shown in effect that in S5, all true modal claims are necessarily true. That's the distinctive feature of S5; modality itself is never contingent.

$$\Diamond A \rightarrow \Box \Diamond A$$

1. 1, \mathbb{F} $\Diamond A \rightarrow \Box \Diamond A$ Assumption

Start with it being false at 1.

$$\Diamond A \rightarrow \Box \Diamond A$$

- | | | |
|----|---|-----------------------------|
| 1. | $1, \mathbb{F} \quad \Diamond A \rightarrow \Box \Diamond A \checkmark$ | Assumption |
| 2. | $1, \mathbb{T} \quad \Diamond A$ | $\rightarrow \mathbb{F}, 1$ |
| 3. | $1, \mathbb{F} \quad \Box \Diamond A$ | $\rightarrow \mathbb{F}, 1$ |

This feels familiar by now.

$$\Diamond A \rightarrow \Box \Diamond A$$

1.	1, \mathbb{F} $\Diamond A \rightarrow \Box \Diamond A$ ✓	Assumption
2.	1, \mathbb{T} $\Diamond A$ ✓	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Box \Diamond A$ ✓	$\rightarrow \mathbb{F}, 1$
4.	2, \mathbb{T} A	$\Diamond \mathbb{T}, 2$
5.	3, \mathbb{F} $\Diamond A$	$\Box \mathbb{F}, 3$

- True \Diamond sentences and false \Box sentences have to be made true somewhere.
- Not necessarily the same 'somewhere'.

$$\Diamond A \rightarrow \Box \Diamond A$$

1.	1, \mathbb{F} $\Diamond A \rightarrow \Box \Diamond A$ ✓	Assumption
2.	1, \mathbb{T} $\Diamond A$ ✓	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Box \Diamond A$ ✓	$\rightarrow \mathbb{F}, 1$
4.	2, \mathbb{T} A	$\Diamond \mathbb{T}, 2$
5.	3, \mathbb{F} $\Diamond A$	$\Box \mathbb{F}, 3$
6.	2, \mathbb{F} A	$\Diamond \mathbb{F}, 5$

- False \Diamond sentences are false everywhere.
- That is, the part inside the \Diamond is false everywhere.

$$\Diamond A \rightarrow \Box \Diamond A$$

1.	1, \mathbb{F} $\Diamond A \rightarrow \Box \Diamond A$ ✓	Assumption
2.	1, \mathbb{T} $\Diamond A$ ✓	$\rightarrow \mathbb{F}, 1$
3.	1, \mathbb{F} $\Box \Diamond A$ ✓	$\rightarrow \mathbb{F}, 1$
4.	2, \mathbb{T} A	$\Diamond \mathbb{T}, 2$
5.	3, \mathbb{F} $\Diamond A$	$\Box \mathbb{F}, 3$
6.	2, \mathbb{F} A	$\Diamond \mathbb{F}, 5$
	x	

And so it closes, so this is a theorem.