305 Lecture 5.3 - Rules for Not

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Plan

This lecture discusses the rules for negation.

Associated Reading

forall x, section 16.8.

A New Symbol

- $\bot$

Read this as 'contradiction', or 'the false'. It is a sentence that can't be true.

Contradiction and Negation

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- By proving some sentence and the negation of that very sentence.
- This is going to be our rule for proving things from a negation.

Neg-Elimination

m	$\neg\mathscr{A}$	
n	$\mathscr{A}$	
	$\bot$	$\neg\mathrm{E}\ m, n$

Neg-Elimination

From contradictory sentences, infer $\bot$.

How to Prove a Negation

Show that if the unnegated part were true, something absurd would follow.

Absurdity

- In engineering, a perpetual motion machine, or some other kind of free energy.

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- In engineering, a perpetual motion machine, or some other kind of free energy.
- In finance, a risk-free way to make a guaranteed profit.
- In logic, a sentence and its negation.

Neg-Introduction

$$
\begin{array}{r|l}
i & \begin{array}{|l} \mathscr{A} \\ \hline \end{array} \\
j & \begin{array}{|l} \bot \end{array} \\
 & \neg\mathscr{A} \qquad \neg\mathrm{I}\ i\text{–}j
\end{array}
$$

Neg-Introduction

- If A implies a contradiction, infer ¬A.

Indirect Proof

$$\begin{array}{r|l|ll} i & & \neg\mathscr{A} & \\ \hline j & & \bot & \\ & \mathscr{A} & & \text{IP } i\text{–}j \end{array}$$

Indirect Proof

- If ¬A implies a contradiction, infer A.

Explosion

$$
\begin{array}{r|ll}
m & \bot & \\
 & \mathscr{A} & \quad \mathrm{X}\ m
\end{array}
$$

Explosion

- A contradiction implies anything.
- Note that this rule is redundant; we can replicate it using Indirect Proof.
- I think they've added it because it is an interesting rule if you don't like Indirect Proof.

For Next Time

- That’s a lot of rules we’ve set out.
- We will start looking at how they work in practice.