305 Lecture 3.5 - Introducing Truth Trees

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Plan

This lecture introduces a new way of testing for validity - truth trees.

Associated Reading

Boxes and Diamonds, section 2.1

What Tableaux Are

- A way for determining whether some combinations are logically possible.
- That can be used for determining whether some arguments are valid – if the truth of the premises and the falsity of the conclusion is not logically possible; then the argument is valid.

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- Each time one of those things implies that some other things must be the case, write those down too.
- For example, if you write down that $A \wedge B$ is true, also write down that A is true and that B is true.
- Each time there are multiple ways to make something you've written true, create multiple branches for those ways.
- For example, if you write down that $A \vee B$ is true, create a branch where A is true, and a branch where B is true.

Closing

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- The whole tableau is closed if every branch is closed.

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- If you are evaluating an argument, this means that the argument is valid.
- If you start the tableau by just saying that one sentence is false, the closure of the tableau means that that sentence is a logical truth.

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1. They don't grow exponentially when you increase the number of variables.
2. They can be generalised to things beyond propositional logic.

We are introducing them here because of point 2.

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Here is something the book doesn't make a big deal of, but is kind of important.

- A closed tableau can show that an argument is valid.
- An open **and completed** tableau can show that an argument is invalid.
- The trick here is that it's hard to tell when a tableau is completed in the relevant sense.
- This will be easier to illustrate in practice than in theory, so let's start building tableau up.

Signs

The system we are using is what is called a **signed tableau** system.

- That means that every line consists of two parts.
- The bigger, second, part is a sentence.
- The first part is a **sign**, which for now is a truth value.
- That is, it is either $\mathbb{T}$ or $\mathbb{F}$.

What Lines Mean

So each line either says that a particular sentence is true, or says that it is false.

- The book for some reason includes the word ‘might’ here.
- That’s misleading; what they should say is that each line says what is true given (a) the starting assumptions and (b) the assumptions we made for branching purposes.

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Both of these are somewhat idiosyncratic, though not abnormal. Unlike the truth tables, there just aren't well defined conventions for how to write these things out.

For Next Time

We will start looking at the rules that get used for building truth trees.