$\neg \Box (A \land \neg \Box A)$ **(in KT)**

Build a Tableau

To Check Whether it is Valid

Hypothesis

$\neg \Box (A \wedge \neg \Box A)$ is a theorem of KT.

- So we can use all the rules, plus the special rules for T.

$\neg \Box (A \land \neg \Box A)$

1.	$1, \mathbb{F} \quad \neg \Box (A \land \neg \Box A)$	Assumption

Start with it being false at 1.

$\neg \Box (A \wedge \neg \Box A)$

1.	$1, \mathbb{F}$	$\neg \Box (A \wedge \neg \Box A)$ ✓	Assumption
2.	$1, \mathbb{T}$	$\Box (A \wedge \neg \Box A)$	$\neg\mathbb{F}$, 1

Turn false $\neg$ false into true unnegated sentence.

$\neg\Box(A \wedge \neg\Box A)$

1.	$1, \mathbb{F}\quad \neg\Box(A \wedge \neg\Box A)$ ✓	Assumption
2.	$1, \mathbb{T}\quad \Box(A \wedge \neg\Box A)$	$\neg\mathbb{T}$, 1
3.	$1, \mathbb{T}\quad A \wedge \neg\Box A$	$T\Box$ 2

It's KT, so whatever is necessary is true.

$\neg\Box(A \wedge \neg\Box A)$

1.	$1, \mathbb{F} \quad \neg\Box(A \wedge \neg\Box A)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box(A \wedge \neg\Box A)$	$\neg\mathbb{T}, 1$
3.	$1, \mathbb{T} \quad A \wedge \neg\Box A$ ✓	$T\Box 2$
4.	$1, \mathbb{T} \quad A$	$\wedge\mathbb{T}, 3$
5.	$1, \mathbb{T} \quad \neg\Box A$	$\wedge\mathbb{T}, 3$

Both sides of a conjunction have to be true.

$\neg \Box (A \wedge \neg \Box A)$

1.	$1, \mathbb{F} \quad \neg \Box (A \wedge \neg \Box A)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box (A \wedge \neg \Box A)$	$\neg\mathbb{T}, 1$
3.	$1, \mathbb{T} \quad A \wedge \neg \Box A$ ✓	$\mathrm{T}\Box\, 2$
4.	$1, \mathbb{T} \quad A$	$\wedge\mathbb{T}, 3$
5.	$1, \mathbb{T} \quad \neg \Box A$ ✓	$\wedge\mathbb{T}, 3$
6.	$1, \mathbb{F} \quad \Box A$	$\neg\mathbb{F}, 5$

Whatever isn't true is false.

$\neg\Box(A \land \neg\Box A)$

1.	1, $\mathbb{F}$	$\neg\Box(A \land \neg\Box A)$ ✓	Assumption
2.	1, $\mathbb{T}$	$\Box(A \land \neg\Box A)$	$\neg\mathbb{T}$, 1
3.	1, $\mathbb{T}$	$A \land \neg\Box A$ ✓	$\mathrm{T}\Box$ 2
4.	1, $\mathbb{T}$	A	$\land\mathbb{T}$, 3
5.	1, $\mathbb{T}$	$\neg\Box A$ ✓	$\land\mathbb{T}$, 3
6.	1, $\mathbb{F}$	$\Box A$ ✓	$\neg\mathbb{F}$, 5
7.	1.1, $\mathbb{F}$	A	$\Box\mathbb{F}$, 6

A false Box sentence requires a world that makes it false. What can we do now?

$\neg\Box(A \wedge \neg\Box A)$

1.	$1, \mathbb{F} \quad \neg\Box(A \wedge \neg\Box A)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box(A \wedge \neg\Box A)$	$\neg\mathbb{T}$, 1
3.	$1, \mathbb{T} \quad A \wedge \neg\Box A$ ✓	T $\Box$ 2
4.	$1, \mathbb{T} \quad A$	$\wedge\mathbb{T}$, 3
5.	$1, \mathbb{T} \quad \neg\Box A$ ✓	$\wedge\mathbb{T}$, 3
6.	$1, \mathbb{F} \quad \Box A$ ✓	$\neg\mathbb{F}$, 5
7.	$1.1, \mathbb{F} \quad A$	$\Box\mathbb{F}$, 6
8.	$1.1, \mathbb{T} \quad A \wedge \neg\Box A$	$\Box\mathbb{T}$, 2

Line 3 is a $\Box$ sentence, so the unboxed part applies everywhere accessible.

$\neg \Box (A \wedge \neg \Box A)$

1.	$1, \mathbb{F} \quad \neg \Box (A \wedge \neg \Box A)$ ✓	Assumption
2.	$1, \mathbb{T} \quad \Box (A \wedge \neg \Box A)$	$\neg\mathbb{T}, 1$
3.	$1, \mathbb{T} \quad A \wedge \neg \Box A$ ✓	$T \Box 2$
4.	$1, \mathbb{T} \quad A$	$\wedge\mathbb{T}, 3$
5.	$1, \mathbb{T} \quad \neg \Box A$ ✓	$\wedge\mathbb{T}, 3$
6.	$1, \mathbb{F} \quad \Box A$ ✓	$\neg\mathbb{F}, 5$
7.	$1.1, \mathbb{F} \quad A$	$\Box\mathbb{F}, 6$
8.	$1.1, \mathbb{T} \quad A \wedge \neg \Box A$ ✓	$\Box\mathbb{T}, 2$
9.	$1.1, \mathbb{T} \quad A$	$\wedge\mathbb{T}, 8$
	x	

And both parts of an and sentence have to be true - though the first is enough to close this tree.

Philosophical Consequences

- This is a philosophically interesting result.
- If $\Box$ means "Is known by X", then it says that X can't know of a particular proposition that it is true but they don't know it.
- Assuming that X is not omniscient, so there are some things they don't know, this means there are limits to X's knowledge.

Idealism

- Let B = “Brian doesn’t know that B is true”. (And assume, for now, that this kind of self-reference makes sense.
- That’s surely true. My knowing it implies a contradiction in the way we’ve just shown.
- So we all know it, because we can all follow the proof.

Idealism

- Let B = “Brian doesn’t know that B is true”. (And assume, for now, that this kind of self-reference makes sense.
- That’s surely true. My knowing it implies a contradiction in the way we’ve just shown.
- So we all know it, because we can all follow the proof.
- Except wait - it’s true, and it says I don’t know it, so I must not know it.
- So there’s something you all know that I don’t know.
- Though just why I don’t know it is something of a mystery, since I do have a well-supported (and correct) belief that it is true.