305 Lecture 10.6 - Convergence Theorems

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Plan

- We’re going to talk about why different priors might not matter - because they usually converge to the same thing.

Associated Reading

This isn’t in the book; we’ll return to its narrative next time.

Big Picture

- Maybe there is no one true prior.
- But not anything goes.
- And the ones that are ok are all such that they will converge to the truth given enough evidence.

Convergence

- I am really not going to go over the details of this.
- But it turns out there are a large class of functions with the following feature.
- According to any function in the class, the probability that evidence will come in that makes every function in the class get arbitrarily close is very high.

Intuitive Case

Imagine that I know a coin is biased in 1 of 2 ways.

1. Each flip has probability 0.8 of landing heads.
2. Each flip has probability 0.2 of landing heads.

Then I get to flip the coin 100 times. What will happen?

Convergence

- On scenario 1, the probability that I'll get at least 60 heads is greater than 0.99999.
- But on scenaio 2, the probability of that is less than 10^{-10}.
- So if I start out 50/50 between the options, and get more than 60 heads, I'll end up massively leaning towards scenario 1.

Convergence

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- But on scenaio 2, the probability of that is less than 10^{-10}.
- So if I start out 50/50 between the options, and get more than 60 heads, I'll end up massively leaning towards scenario 1.
- But imagine someone else starts out thinking that option 2 is really likely - 0.99 likely and option 1 only 0.01.
- They will also get to the right view after 100 trials - even 60 heads (which is really low on scenario 1) would be enough to change the probabilities.

Extreme Example

- What if we started with a really extreme view, that the probability of option 1 is 10^{-30}?

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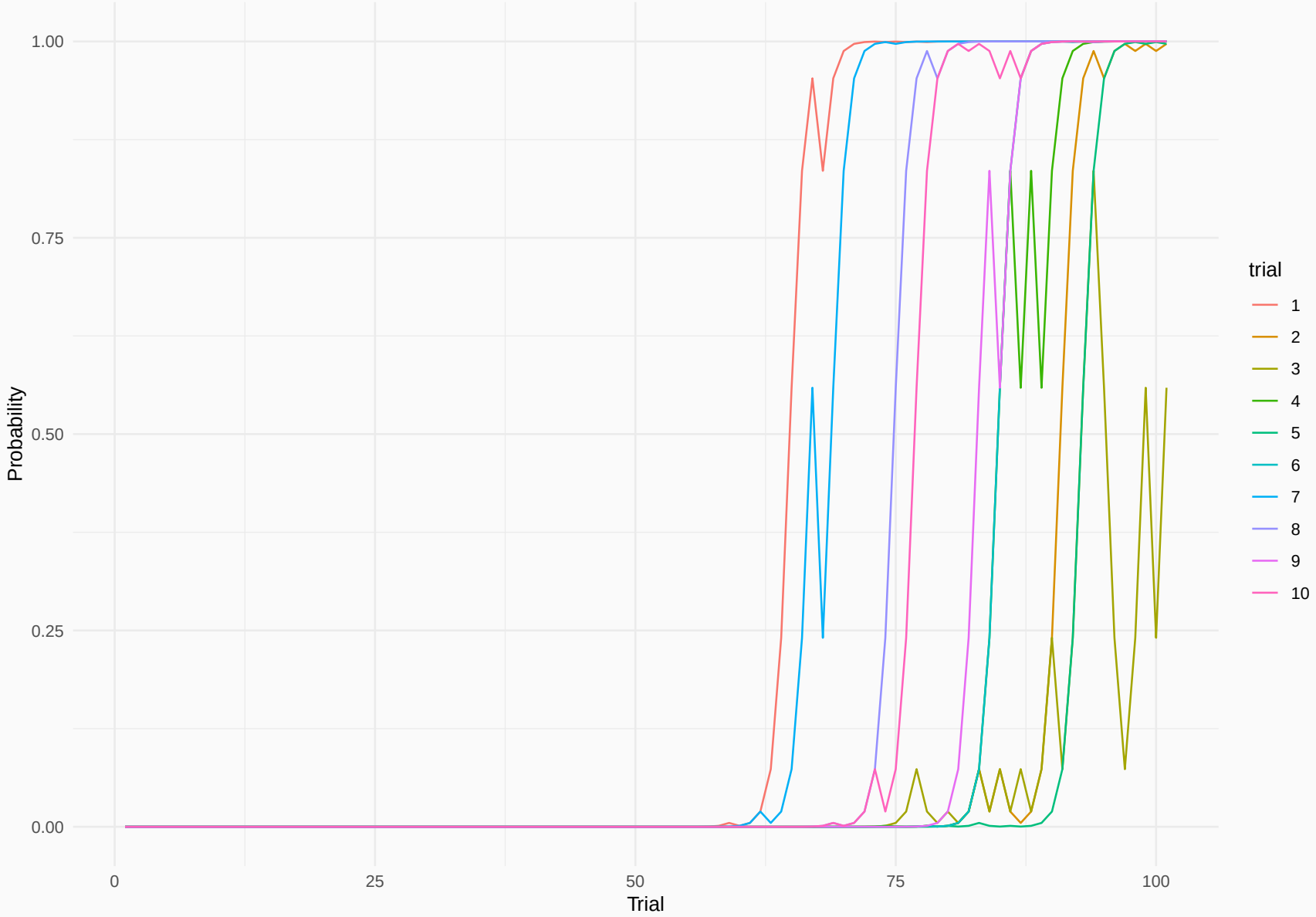
- Well 100 coin flips would probably still be enough.
- The next slide shows what happens to the probability of option 1 for 10 experimenters who start out with that low probability, and what their probability for option 1 is after each coin flip.

Example


1.00
0.75
0.50
0.25
0.00
Probability
0
25
50
75
100
Trial
trial
1
2
3
4
5
6
7
8
9
10

General Principle

As long as we don’t start with probability 0 for one or other scenario, get enough evidence and we’ll converge to the correct scenario.

Two Problem Cases

1. There isn't enough evidence around. This is a big problem in thinking about history, and also about social sciences.
2. People do start with probability 0 for various scenarios.

Optimistic Take

- These two problems won't arise very often.
- So updating by conditionalisation will lead us to converge.
- That's the sense in which we get objectivity; subjective priors that are sufficiently responsive to the evidence end up being objective enough.

For Next Time

- We will end this unit by looking at a common scientific practice – significance testing.