305 Lecture 12.2 - Extending Modal Tableau

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Plan

- To introduce tableau for proving things in modal logics other than K.

Associated Reading

- Boxes and Diamonds, section 5.5-5.6.

K is special

The logic we have so far is very weak.

- It's the logic K, that puts no restrictions on the R-relation.
- Most applications of modal logic do have restrictions.
- How should we model them?

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- It's the logic K, that puts no restrictions on the R-relation.
- Most applications of modal logic do have restrictions.
- How should we model them?
- The answer is that we sort of model them one at a time.
- The rules are on page 80, and I'll end today with quickly mentioning a couple of them.

T - the logic of reflexive frames

Add two new rules.

1. If $\Box A$ is true at x, infer that A is true at x.
2. If $\Diamond A$ is false at x, infer that A is false at x.

That's it!

4 - the logic of transitive frames

Add two new rules

1. If □A is true at x, and x.y exists on the tree, add that □A is true at x.y. (You already should have added that A is true at x.y; that's the basic rule for □.
2. If ◇A is false at x, and x.y exists on the tree, add that ◇A is false at x.y. (You already should have added that A is false at x.y; that's the basic rule for □.)

$\Box A \to \Box \Box A$

This should fail in K - here's an open tableau for it.

1.	$\mathbb{F}\, 1, \Box A \to \Box \Box A$	Assumption
2.	$\mathbb{T}\, 1, \Box A$	$\to\mathbb{F}$, 1
3.	$\mathbb{F}\, 1, \Box \Box A$	$\to\mathbb{F}$, 1
4.	$\mathbb{F}\, 1.1, \Box A$	$\Box\mathbb{F}$, 3
5.	$\mathbb{T}\, 1.1, A$	$\Box\mathbb{T}$, 2
6.	$\mathbb{F}\, 1.1.1, A$	$\Box\mathbb{F}$, 4

$\Box A \rightarrow \Box \Box A$

But now let's apply the rules for 4 as well. After line 5 we need to make one new inference.

1.	$\mathbb{F}\, 1, \Box A \rightarrow \Box \Box A$	Assumption
2.	$\mathbb{T}\, 1, \Box A$	$\rightarrow\mathbb{F}$, 1
3.	$\mathbb{F}\, 1, \Box \Box A$	$\rightarrow\mathbb{F}$, 1
4.	$\mathbb{F}\, 1.1, \Box A$	$\Box\mathbb{F}$, 3
5.	$\mathbb{T}\, 1.1, A$	$\Box\mathbb{T}$, 2
6.	$\mathbb{T}\, 1.1, \Box A$	4$\Box$ 2
7.	$\mathbb{F}\, 1.1.1, A$	$\Box\mathbb{F}$, 4
8.	$\mathbb{T}\, 1.1.1, A$	4$\Box$ 6
	x	

For Next Time

We'll look more closely at the differences between different logics.