305 Lecture 12.1 - Modal Tableau

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Plan

- To introduce tableau for proving things in modal logic.

Associated Reading

- Boxes and Diamonds, section 5.1

Modal Tableau

One big difference:

- On each line, we include reference to a world.
- The line says that a particular proposition is true or false at a world.
- The tableau only close if the tableau says the same proposition is both true and false at the same world.

Referring to Worlds

We refer to a world with a string of numbers, such as 1.2.1.3.

- The string tells you something (but not everything) about R relations.
- World x can always access world x.y.
- So there is an R-relation from 1.2.1 to 1.2.1.3, and indeed to 1.2.1.x for any x.
- These don't exhaust the R-relations; perhaps there is also an R-relation from 1.2.1.3 back to 1.2.1.
- But the relation from x to x.y is guaranteed.
- Note that worlds can be picked out by multiple strings - we do not assume that 1.1 and 1.2 are different, though we don't assume they are the same either.

Rules

The rules for the old connectives stay as they are.

- The only difference is that you have to note which world you are in.
- So if you have that $A \wedge B$ is true in 1.4.7, then you have to write down that A is true in 1.4.7, and that B is true in 1.4.7.
- And if $A \vee B$ is true in 1.6.8 you have to have two branches, one where A is true in 1.6.8, and the other where B is true in 1.6.8.
- But otherwise things are as they were before.

Rules for □

If □A is true at x, then for any x.y that is already on the tree, we can infer that A is true at x.y.

- Note: If there is no x.y on the tree, we can't assume that there is one.
- □A can be true at a world because that world can't access any other world.

Rules for □ (cont)

If □A is false at x, then we have to add a **new** x.y, and make A false at x.y.

- It is very important that x.y be new.
- We know that A is false at some accessible world, but we don't know which one.
- For any given world, A might be true there, as long as it is false somewhere.
- That's why we use a new number.
- Remember that it might be that multiple strings refer to the same world.

Rules for $\Diamond$

If $\Diamond A$ is true at x, then we have to add a **new** x.y, and make A true at x.y.

- It is very important that x.y be new.
- We know that A is true at some accessible world, but we don't know which one.
- For any given world, A might be false there, as long as it is false somewhere.
- That's why we use a new number.
- Remember that it might be that multiple strings refer to the same world.

Rules for $\Diamond$ (cont)

If $\Diamond A$ is false at x, then for any x.y that is already on the tree, we can infer that A is false at x.y.

- Note: If there is no x.y on the tree, we can't assume that there is one.
- $\Diamond A$ can be false at a world because that world can't access any other world.

$(\Box A \wedge \Box B) \to \Box(A \wedge B)$

1.	$1, \mathbb{F}\ \ (\Box A \wedge \Box B) \to \Box(A \wedge B)$		Assumption
2.	$1, \mathbb{T}\ \ \Box A \wedge \Box B$		$\to\mathbb{F}, 1$
3.	$1, \mathbb{F}\ \ \Box(A \wedge B)$		$\to\mathbb{F}, 1$
4.	$1, \mathbb{T}\ \ \Box A$		$\wedge\mathbb{T}, 2$
5.	$1, \mathbb{T}\ \ \Box B$		$\wedge\mathbb{T}, 2$
6.	$1.1, \mathbb{F}\ \ A \wedge B$		$\Box\mathbb{F}, 3$
7.	$1.1, \mathbb{F}\ \ A$	$1.1, \mathbb{F}\ \ B$	$\wedge\mathbb{F}, 6$
8.	$1.1, \mathbb{T}\ \ A$	$1.1, \mathbb{T}\ \ B$	$\Box\mathbb{T}, 4; \Box\mathbb{T}, 5$
	x	x	

$\Diamond(A \vee B) \to (\Diamond A \vee \Diamond B)$

1.	$1, \mathbb{F} \quad \Diamond(A \vee B) \to (\Diamond A \vee \Diamond B)$		Assumption
2.	$1, \mathbb{T} \quad \Diamond(A \vee B)$		$\to\mathbb{F}, 1$
3.	$1, \mathbb{F} \quad \Diamond A \vee \Diamond B$		$\to\mathbb{F}, 1$
4.	$1, \mathbb{F} \quad \Diamond A$		$\vee\mathbb{F}, 3$
5.	$1, \mathbb{F} \quad \Diamond B$		$\vee\mathbb{F}, 3$
6.	$1.1, \mathbb{T} \quad A \vee B$		$\Diamond\mathbb{T}, 2$
7.	$1.1, \mathbb{T} \quad A$	$1.1, \mathbb{T} \quad B$	$\vee\mathbb{T}, 6$
8.	$1.1, \mathbb{F} \quad A$	$1.1, \mathbb{F} \quad B$	$\Diamond\mathbb{F}, 4; \Diamond\mathbb{F}, 5$
	x	x	

For Next Time

We'll look at how to extend this approach to other logics.