444 Lecture 5.2 - Mixed Strategies

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Plan

Introduce the idea of mixed strategies

Reading

Bonanno, section 6.2

Core Idea

Flip a coin!

Motivation



Rock! Paper!! Scissors!!!

Payout

If Row plays 60% Up and 40% Down, and Column plays Left, then Row's expected payout will be

- 0.6 times the payout for Up/Left
- 0.4 times the payout for Down/Left

Payout

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- 0.6 times the payout for Up/Left
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When we evaluate a mixed strategy, we evaluate it by its expected payout, not its actual payout.

Payout

If Row plays 60% Up and 40% Down, and Column plays Left, then Column's payout will be

- 0.6 times the payout for Left/Up
- 0.4 times the payout for Left/Down

Payout

If Row plays 60% Up and 40% Down, and Column plays Left, then Column's payout will be

- 0.6 times the payout for Left/Up
- 0.4 times the payout for Left/Down

We have to use expected values for evaluating even **pure** strategies like Left, once one player plays mixtures.

Both Mixing

What if Column also mixes, playing 70% Left and 30% Right.

- Then Row's expected payout is 0.7 times the **expected** payout of their mixed strategy when Column plays Left, plus 0.3 times the expected payout of their mixed strategy when Column plays right.
- We have to use expected values twice over to get the philosophically important value of this strategy.

Independence

- What we're assuming here is that the probabilities of each player are **independent**.
- If they roll 20-sided dice or whatever, they roll different ones.
- There is an interesting part of game theory where we drop that assumption, but I'm not going to cover it.
- This is the part that deals with so-called **correlated equilibrium**.
- Bonanno doesn't cover this, and I won't either.

What is a Mixed Strategy

At least three interpretations.

1. Random device
2. Frequencies
3. Epistemic

Random Device

- I mean, you literally roll a die, flip a coin, use something on your phone or something.
- This is the traditional interpretation, but it’s become less popular.

Frequencies

- In the long-run, your frequency of each play tracks the probabilities in the strategy.
- And more specifically, your frequency of each play in each identifiable situation tracks the probabilities.
- Alternating rock-then-paper-then-scissors isn't a way to play the mixed strategy 1/3 for each move.
- Relatedly, you will lose.
- On the frequency interpretation of mixed strategies, playing that mixed strategy means that after rock, you'll play 1/3 rock, 1/3 paper, 1/3 scissors, and after paper-then-scissors, you'll play ..., and so on.

Epistemic

- A rational onlooker will not be able to figure out what you will do/are doing, beyond saying that you're playing each strategy with the specified probability.
- It doesn't matter how you get there, as long as that's all that an onlooker (or co-player) can figure out.

Mixtures Always Available

- Game theorists standardly assume that if some moves are available, so is any mixed strategy built out of them.
- Philosophers do not assume this, and often think it's the least plausible thing about game theory.
- Is this assume plausible?

What's a Mixed Theory

- On the random device interpretation, it's very implausible.
- Not all game players have arbitrary random devices available.
- But on the other interpretations, it is more plausible.
- A good player should be able to hide their strategy, even in repeated plays.

For Next Time

We will look at a striking consequence of the existence of mixed strategies - every game has a Nash equilibrium.