

444 Lecture 6.1 - Introducing Probability

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Week Plan

Pure revision/introduction of basics of probability

Totally Not Compulsory

- If you've taken any college class using probability before, then probably 100% of what I say this whole week will be pure revision.
- I bet many people in the class could teach all this stuff.
- That's fine - take a week off.
- But I really don't want anyone left behind before we dive into the more applied stuff.

Odds & Ends

Introducing Probability & Decision with a Visual Emphasis

Jonathan Weisberg

Preface

THIS textbook is for introductory philosophy courses on probability and inductive logic. It is based on a typical such course I teach at the University of Toronto, where we offer “Probability & Inductive Logic” in the second year, alongside the usual deductive logic intro.

The book assumes no deductive logic. The early chapters introduce the little that’s used. In fact almost no formal background is presumed, only very simple high school algebra.

Several well known predecessors inspired and shaped this book. Brian Skyrms’ *Choice & Chance* and Ian Hacking’s *An Introduction to Probability and Inductive Logic* were especially influential. Both texts are widely used with good reason—they are excellent. I’ve taught both myself many times, with great success. But this book blends my favourite aspects of each, organizing them in the sequence and style I prefer.

Figure: These slides are based off an open access textbook, Odds and Ends, available at <https://jonathanweisberg.org/vip/>

Plan

- This lecture introduces the basics of probability.

Associated Reading

Odds and Ends, Chapter 1

Basic Idea

- A probability function is a mapping from possibilities to numbers.
- The numbers must sum to one.
- Intuitively, the numbers measure how likely the possibilities are.

Sum to One

The math of probability functions is just the math of proportions. Ultimately, all we'll be doing is the same kind of math that you would do when thinking about things like

- What proportion of UM students are from North Carolina?
- What proportion of UM undergraduates are Tigers fans?

etc.

Three Big Questions

1. What to do with these numbers?

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2. Where these numbers come from?

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2. Where these numbers come from?
3. What do the numbers even mean?

A Simple Case

- Imagine that it is basketball season, and UM has planned to have both the women's and men's teams play on the same night.
- So at the end of the night there are four possible outcomes.

Made Up Probabilities

I'll stipulate that the probabilities of the four possible outcomes are given by this table.

	Men Win	Men Lose
Women Win	0.45	0.25
Women Lose	0.20	0.10

Another Representation

Here are the same numbers written a different way.

Women	Men	Probability
Win	Win	0.45
Win	Lose	0.25
Lose	Win	0.20
Lose	Lose	0.10

Say a possibility (for current purposes) is one of these maximally specific things that the probability is defined over.

- It is not really a complete possibility.
- It doesn't tell us the score, or the weather, or the results of the next election, or for that matter the results of the last election.
- But it tells us everything that's relevant to a particular inquiry.
- It is a lot like a line on a truth table.

Events

We will say an **event** is a proposition that can be defined using these possibilities. So here are some sample events.

- The women's team wins.
- The men's team wins.
- At least one Michigan team wins.
- The two teams have the same result.

Probability of Events

- An event is true at some possibilities, false at others.
- Each possibility gets a probability.
- The probability of an event is the sum of the probabilities of the possibilities where it is true.

Examples - Probability Women's Team Wins

Women	Men	Probability
Win	Win	0.45
Win	Lose	0.25
Lose	Win	0.20
Lose	Lose	0.10

- The women's team wins at lines 1 and 2.
- So its probability is
- $0.45 + 0.25 = 0.7$.

Examples - Probability Men's Team Wins

Women	Men	Probability
Win	Win	0.45
Win	Lose	0.25
Lose	Win	0.20
Lose	Lose	0.10

- The men's team wins at lines 1 and 3.
- So its probability is
- $0.45 + 0.20 = 0.65$.

Examples - At Least One Team Wins

Women	Men	Probability
Win	Win	0.45
Win	Lose	0.25
Lose	Win	0.20
Lose	Lose	0.10

- At least one team wins at lines 1, 2 and 3.
- So its probability is
- $0.45 + 0.25 + 0.20 = 0.90$.

Examples - Same Result in Each Game

Women	Men	Probability
Win	Win	0.45
Win	Lose	0.25
Lose	Win	0.20
Lose	Lose	0.10

- It is the same result in each game at lines 1 and 4.
- So its probability is
- $0.45 + 0.10 = 0.55$.

For Next Time

- We will look at some properties that all probability functions share.