444 Lecture 3.1 - Trees

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Plan

To introduce games that take place over time.

Bonanno, section 3.1.

- The tables we discussed last week represent games where each player moves once, and those moves are simultaneous.
- But few games are like that.
- We need a way to represent games that take time.

Trees

- We do that with trees.
- A tree represents all the ways that a game that takes place over time could go.

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- Some nodes are **terminal nodes**; they represent that the game has ended.
- Each terminal node has a payout for each of the players.
- At any other node, either a player moves, or Nature 'moves'.
- One of the non-terminal nodes is special: it is the node where the game starts.

Branches

- Each non-terminal node has branches, leading to other nodes.
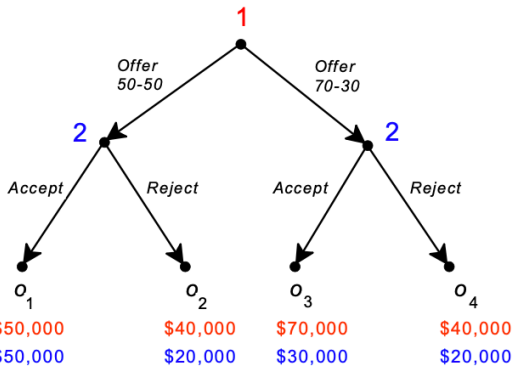- A move at a node is always a choice of branches.

Figure 1: Example from Bonanno

- There are two players, 1 and 2.
- Each player moves once.
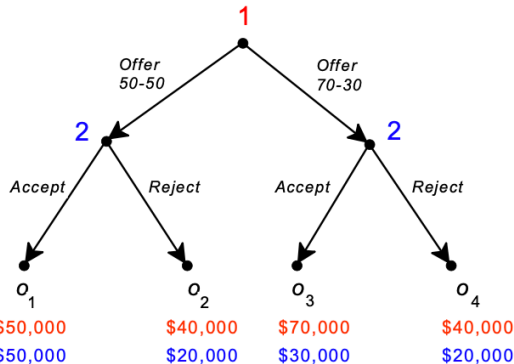- First 1 moves, then 2 moves, then the game ends.

Figure 2: Example from Bonanno

- Some books use a special notation for the initial node, such as having an open circle rather than a closed circle.
- Bonanno doesn't, which is a bit odd.
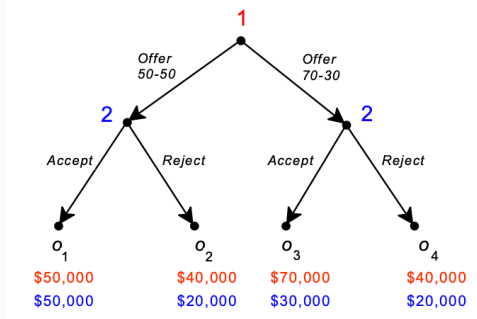- But it's clear in context what the initial node is.

Figure 3: Example from Bonanno

- As he goes on to note, this isn't really a tree yet.
- It describes the physical outcomes of the game at each terminal node, but not the **payoffs**.
- There is a natural function from outcomes to payoffs – more money equals more utility – but it is not a compulsory interpretation.

Future Additions

- Moves by Nature

- Moves by Nature
- Moves under uncertainty

We will think about how players should play these games.