444 Lecture 5.4 - Finding Mixed Strategy Equilibria

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Plan

Discuss how we can find mixed strategy equilibria.

Reading

Bonanno, Section 6.3.

Basic Idea

- In equilibria, the other player is willing to play a mixed strategy.
- That requires that they be indifferent between other strategies.
- So we find the equilibria by finding the mixture that makes them indifferent.

Example

	Left	Right
Up	5, 2	1, 3
Down	2, 2	3, 0

- You can see fairly quicklythat there is no pure strategy equilibria.
- So the equilibria must be a mixed strategy equilibria.

Theory

What does it take for Column to play a mixed strategy in equilibria?

- Assume that Left has a higher expected return than Right.
- The expected return of a mixed strategy is a weighted average of the expected returns of Left and Right.
- If Left has a higher expected return than Right, that weighted average will be strictly between the expected returns of Left and Right.
- And that means it can't be an equilibrium, since in equilibrium there is no alternative with a higher expected return.

Theory

What does it take for Column to play a mixed strategy in equilibria?

- Assume that Left has a higher expected return than Right.
- The expected return of a mixed strategy is a weighted average of the expected returns of Left and Right.
- If Left has a higher expected return than Right, that weighted average will be strictly between the expected returns of Left and Right.
- And that means it can't be an equilibrium, since in equilibrium there is no alternative with a higher expected return.
- And the same reasoning shows Right can't have a higher expected return than Left.

Theory

So we are trying to find the mixture such that Column is indifferent between Left and Right.

- The other crucial thing to remember is that probabilities add to 1.
- So when working out Row's strategy, there is only one variable.
- Once we set the probability of Row playing Up to x, that sets all the probabilities, because the probablity of playing Down is 1 – x.

A Note

I'm only going to go over cases where the mixed strategy equilibrium involves a mixture of two pure strategies.

- There are cases where the mixed strategy equilibrium involves mixtures of 3 or more pure strategies.
- Rock, Paper, Scissors is the simplest such example.
- But in general the math of calculating these is considerably fancier than what we'll be doing, and I'll stick to cases where the mixed strategy equilibrium only involves 2 pure strategies.

Back to the Example

	Left	Right
Up	5, 2	1, 3
Down	2, 2	3, 0

- Assume Row plays Up with probability x, and Down with probability 1 – x.
- Our job is to find an x such that the expected return of Left and Right is the same.

Left and Right

	Left	Right
Up	5, 2	1, 3
Down	2, 2	3, 0

- The expected return of Left is $2x + 2(1 - x)$, i.e., 2.
- The expected return of Right is $3x + 0(1 - x)$, i.e., $3x$.

Row's Strategy.

	Left	Right
Up	5, 2	1, 3
Down	2, 2	3, 0

- So $2 = 3x$, so $x = 2/3$.
- So Row's strategy is to play Up with probability $2/3$, and hence Down with probability $1/3$.

- The expected return of Right is $3x + 0(1 - x)$, i.e., $3x$.

Onto Column

	Left	Right
Up	5, 2	1, 3
Down	2, 2	3, 0

- Assume Column plays Left with probability x, and Right with probability 1 – x.
- Our job is to find an x such that the expected return of Up and Down is the same.

Left and Right

	Left	Right
Up	5, 2	1, 3
Down	2, 2	3, 0

- The expected return of Up is $5x + 1(1 - x)$, i.e., $4x + 1$.
- The expected return of Down is $2x + 3(1 - x)$, i.e., $3 - x$.

Column's Strategy.

$$4x + 1 = 3 - x$$

$$5x + 1 = 3$$

$$5x = 2$$

$$x = 2/5$$

So Column's strategy is to play Left with probability $2/5$, and hence Right with probability $3/5$.

Takeaways

- To find a player's move probabilities in equilibria, look to the other player's payouts.
- Try to make the other player indifferent between their choices.

Extra Steps

- I'm not going to go over more complicated examples on the slides, but there is an extra step you can do (and which we can discuss in class if you're interested).
- Sometimes you can find the mixed strategy equilibria of a game with more than 2 moves by first deleting **strongly** dominated strategies.
- Bonanno works through an example like this.
- I'm going to come back to it later, but for now I'll just stick to this example.

For Next Time

I'm going to start on an idea I want to work through very slowly, and spend a bit of time on - the idea that a mixture of strategies can dominate another strategy.