444 Lecture 3.2 - Backward Induction

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Plan

To introduce backward induction (or backwards induction - either will do).

Reading

Bonanno, section 3.2.

Class of Games We're Discussing

- Two-player

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- Two-player
- Turn-taking

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- Finite
- No hidden facts
- No randomness
- We'll start with zero-sum games, though drop this later.

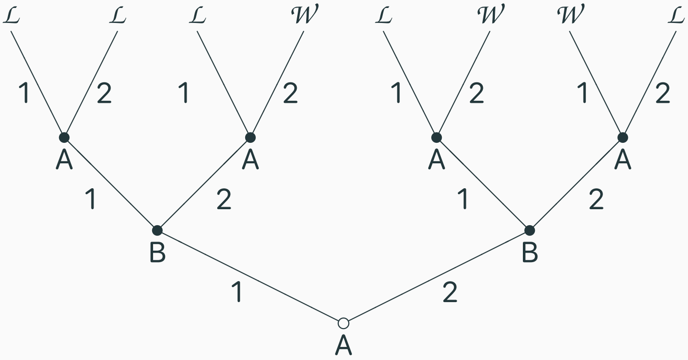
Five

- There are two players, who we'll call A and B.
- First A moves, then B, then finally A moves again.
- Each move involves announcing a number, 1 or 2.
- A wins if after the three moves, the numbers announced sum to 5.
- B wins otherwise.

Five

Question: How should you play this game?

Game Tree for Five



$\mathcal{W}$ means that A wins, and $\mathcal{L}$ means that B wins.

How to Solve These Games

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- Assume that they will make the choice that has higher value for them.
- Mark that choice, e.g., by doubling the line (as the textbook does) or bolding the line (as I'll do).
- If there are ties, mark both of the lines. (This gets more complicated once we leave zero-sum games.)

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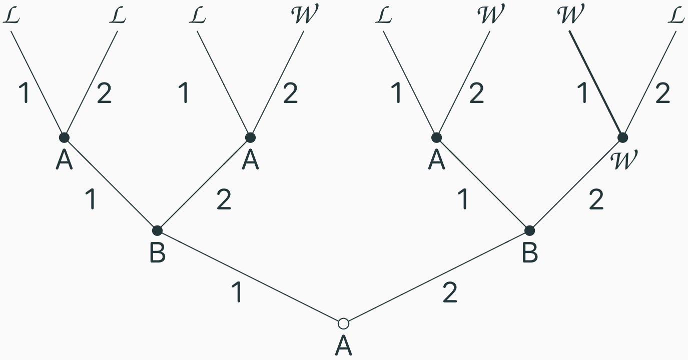
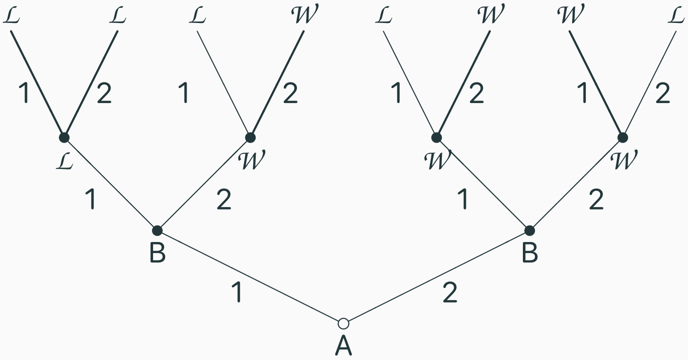
- So just the game assigns values to terminal nodes, we’ll now assign value to choice nodes.
- In **Five**, we’ll assign the value $\mathcal{W}$ to the top right node.

Five (after one step)



Five (after first level)



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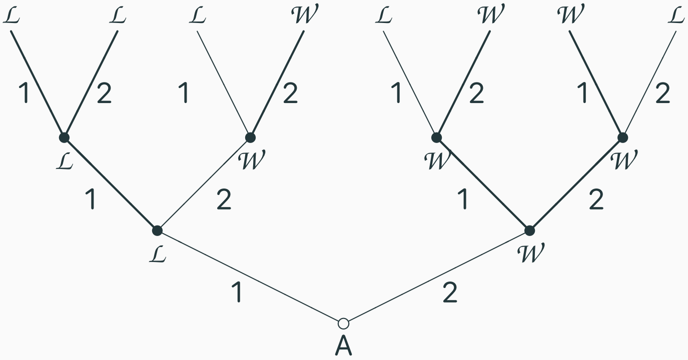
- We act as if B is choosing between terminal nodes.
- It is as if A doesn't have a choice - they will just make the choice that is best for them (i.e., worst for B).
- So B knows what the outcome of each choice will be.

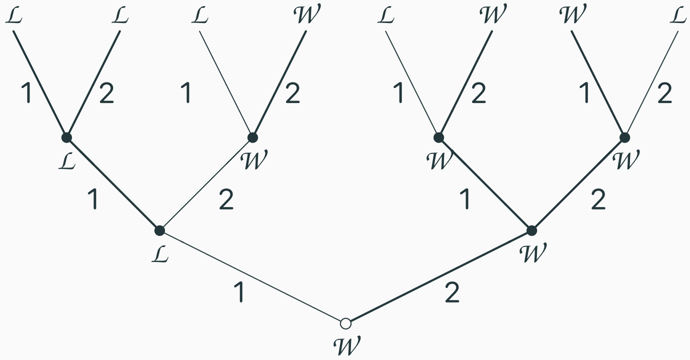
Five (After Two Rounds)



Five (After Two Rounds)

- So we act as if getting to the left hand node means B wins, and getting to the right hand node means A wins.
- And now we just have to make the choice for the initial node, using this fact.

Five (Full Graph)



Five - Full Analysis

- The equilibrium state of the game is that A wins.
- A plays 2 first.
- Then B can play anything they line.
- But whatever they do, A will win, by playing the opposite number.

Backwards Induction

- This process is called backwards induction.
- We start at the possible ends of the game.
- At each step, we assume that each player makes the best decision they can, on the assumption that later players will do the same thing.
- And eventually we can solve the game.