

444 Lecture 2.3 - Ordinal and Cardinal Utility

Brian Weatherson

Plan

- Explain the difference between ordinal and cardinal utility.

Associated Reading

Bonanno, section 2.1.

A utility function (for a particular agent) is a mapping U from situations to numbers satisfying this constraint.

- $U(S_1) > U(S_2)$ iff the agent is better off in S_1 than in S_2 .

This isn't part of the formal theory, but we usually implicitly assume (at least in our narratives), the following principle.

The agent is better off in S_1 than in S_2 iff, given a choice and assuming they are fully informed, they prefer being in S_1 to S_2 .

That is, we'll usually speak as if a radically subjectivist view of welfare is correct. I've been doing this already, and I'm going to keep doing it.

Ordinal Utility

- When we say that we're working with **ordinal** utility functions, really the only principle that applies is the one from two slides back.
- Higher utilities are better, i.e., are preferred.
- The term **ordinal** should make you think of 'orders'; all an ordinal utility function does is provide a rank **ordering** of the outcomes.

Two Functions

So if we're working in ordinal utility, these two functions describe the same underlying reality.

	U_1	U_2
O_1	1	1
O_2	2	10
O_3	3	500
O_4	4	7329

Cardinal Utility

- In cardinal utility theory, the differences between the numbers matter.
- The numbers now express quantities, and the two functions from the previous slide do not represent the same underlying reality.

Cardinal Utility (Detail)

- There is a fussy point here that's worth going over.
- Even cardinal utility functions don't come with a scale.
- So two functions with different numbers in them can still express the same underlying reality.

Cardinal Utility (Detail)

The standard way to put this is that (cardinal) utility is defined only up to a **positive, affine transformation**. That means that if U_1 and U_2 are related by the following formula, then they represent the same state of affairs.

$$U_2(o) = aU_1(o) + b \text{ where } a > 0$$

Celsius and Fahrenheit

- The main real world cases of scales that are related in this way are temperature scales.
- To convert between Celsius and Fahrenheit you use the formula $F = 1.8C + 32$.
- But the scales are just two ways of representing the same physical reality.

Cardinal Utility (Detail)

- So there is no such thing as one outcome being twice as good as another.
- But we can say a lot of things about differences.

Cardinal Utility (Detail)

- If the difference between O_1 and O_2 is the same as the difference between O_2 and O_3 , that will stay the same under any positive affine transformation.

Cardinal Utility (Detail)

- If the difference between O_1 and O_2 is the same as the difference between O_2 and O_3 , that will stay the same under any positive affine transformation.
- Indeed, for any k , if the difference between O_1 and O_2 is k times the difference between O_2 and O_3 , that will stay the same under any positive affine transformation.

For Next Time

- We will start on section 2.2.