444 Lecture 5.6 - Best Responses

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Discuss how to think about best responses work once mixed strategies are on the table.

Bonanno, Section 6.4.

An example

	Left	Right
Up	3, 0	0, 0
Middle	2, 0	2, 0
Down	0, 0	3, 0

- Up is the best response to Left.
- Down is the best response to Right.
- Is Middle the best response to anything?

Best Responses

	Left	Right
Up	3, 0	0, 0
Middle	2, 0	2, 0
Down	0, 0	3, 0

Yes!

- Middle is the best response to the mixed strategy Left with probability 0.5, Right with probability 0.5.
- It gets 2, the other options have an expected return of 1.5.

	Left	Right
Up	3, 0	0, 0
Middle	2, 0	2, 0
Down	0, 0	3, 0

- Middle is the best thing to do if you know Column is going to flip a coin to decide what to do.

Varieties of Mixed Strategies

	Left	Right
Up	3, 0	0, 0
Middle	2, 0	2, 0
Down	0, 0	3, 0

- Middle is the best thing to do if you know Column is going to flip a coin to decide what to do.
- But it's also the best thing to do if you have no idea what Column is going to do, and the best you can do is say it's 50/50 what they are going to do.
- So it's actually pretty easy to think of situations where Middle is the smart play.

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- There is some probability distribution over the other player's strategies and ...
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- No strategy has a higher expected return than S given that probability distribution.
- Note that this allows for ties.
- Weakly dominated strategies can even be best responses in this sense.
- This definition also covers mixed strategies; they can also be best responses.

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A Surprising Theorem

- Say a strategy is **undominated** if no other strategy, pure or mixed, strongly dominates it.
- And it is a **best response** if it does as well as anything, given at least one probability distribution.
- Here's the surprising theorem:

  The strategies that are best responses are just the same strategies as those that are undominated.

Best Reponses

- This relates back to something I was saying in the last lecture.
- The strategies that are dominated by mixtures didn't seem to make sense – you could just play the mixtures.
- But here's another property that they have – they are never best responses.
- And if they are not best responses, no one can play them while maximising expected utility.
- Whatever probability you give to the other player's play, if you maximise expected utility you will play a best response.
- And you should maximise expected utility.

- I'll introduce a new solution concept, one that say playing a best response is not just necessary for rationality, it is also sufficient.