

# 444 Lecture 4.4 - Subgames

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# Plan

To describe the notion of a subgame.

# Reading

Bonanno, section 4.3.

## Definition

Roughly, a subgame is a part of the game that could be a complete game.

# First Constraint

A subgame has an initial node.

- All nodes in the subgame are downstream of that node.
- Remember that in general, there is only one way to get to a node.

# Histories

So don't think of nodes as like positions in a chess game.

- You can get to the same position multiple ways.
- Rather, think of them as like the history of the moves.

## Second Constraint

The subgame consists of all the nodes downstream of that initial node.

- If  $a$  is the initial node, and in the original game you can get from  $a$  to  $b$ , then  $b$  is in the subgame.

## Third Constraint

The subgame does not 'cut' any information sets.

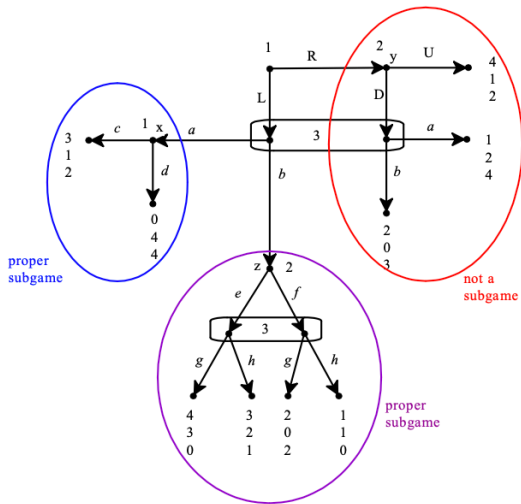
- If  $b$  is in the subgame, and  $c$  is in the information set that includes  $b$ , then  $c$  is in the subgame.



# Common Knowledge

Put another way,  $a$  has to be a point such that when you get to it, it is common knowledge you are there.

- It being in a singleton information set is necessary for that (if not, the player who has to play doesn't know you are there) but not sufficient.



A violation of third constraint

# Summing Up

- A subgame consists of all and only the points that are 'downstream' of some initial node.
- That initial node has to be such that if/when it is reached, it is common knowledge among the players that it is reached.

## For Next Time

- We will use this notion of a subgame to develop a new constraint on how to play games.