444 Lecture 4.5 - Subgame Perfect Equilibrium

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Plan

To describe the notion of subgame perfect equilibrium.

Reading

Bonanno, section 4.4.

Definition

A set of strategies for each of the players is a subgame perfect equilibrium if and only if

- The set forms a Nash equilibrium.
- In every subgame, the set forms a Nash equilibrium.

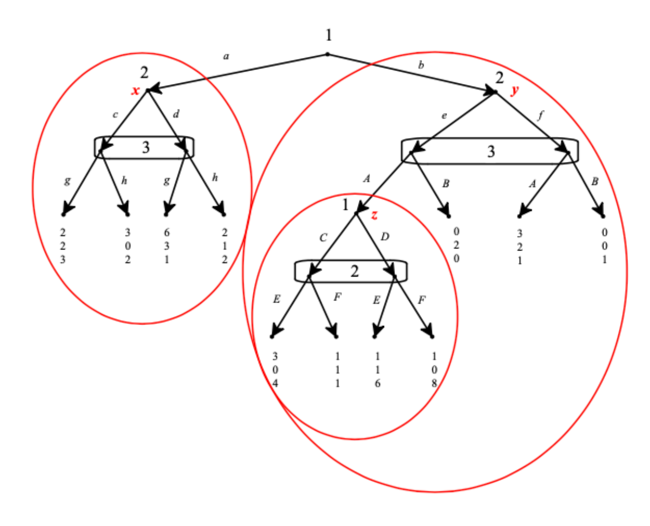
Subgame Perfect and Nash

The second clause is non-trivial.

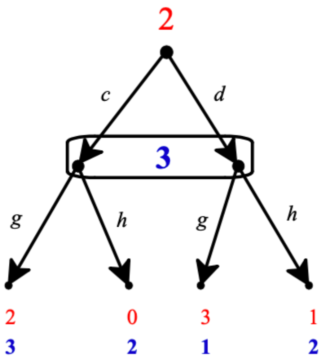
- It rules out players doing certain kinds of odd things at nodes that are not reached.
- At subgame perfect equilibrium, each player's strategies make sense given the other player's strategies, and they are disposed to keep making sense under different assumptions about what they might do.

Finding Subgame Perfect Equilibrium

- Find the minimal subgames.
- Act as if the initial node of that subgame is a terminal node, with its payouts being the equilibrium payouts of the subgame.
- If there are multiple equilibria, duplicate the tree enough times to cover each of them - you’ll have multiple subgame perfect equilibria.
- Repeat until you’ve covered the whole tree.

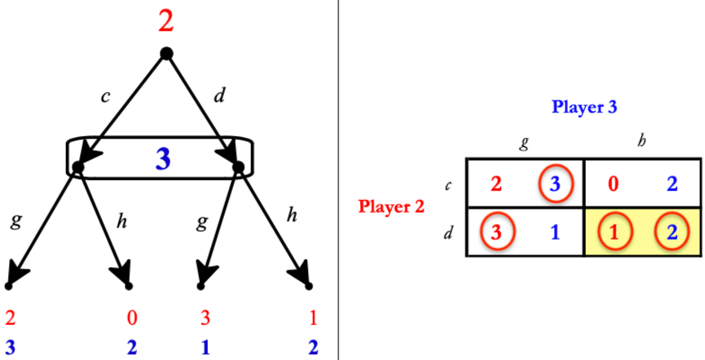


The large game

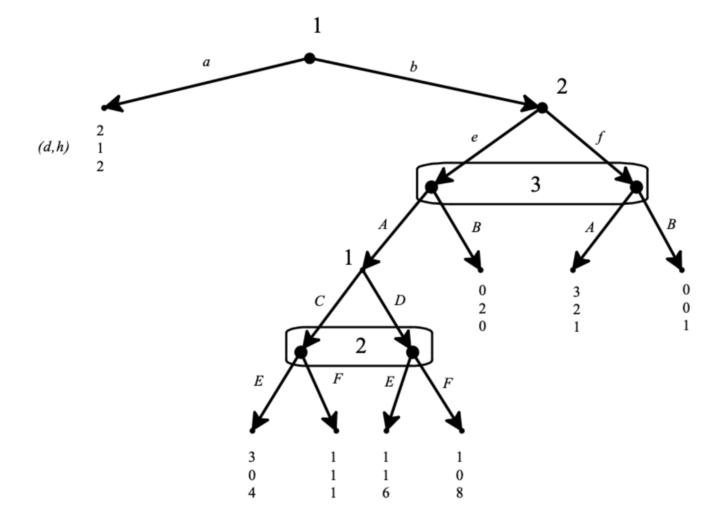


		Player 3	
		g	*h*
Player 2	*c*	2, 3	0, 2
	d	3, 1	1, 2

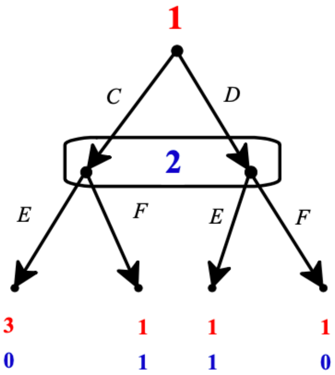
The left subgame



The left subgame with labeled best responses

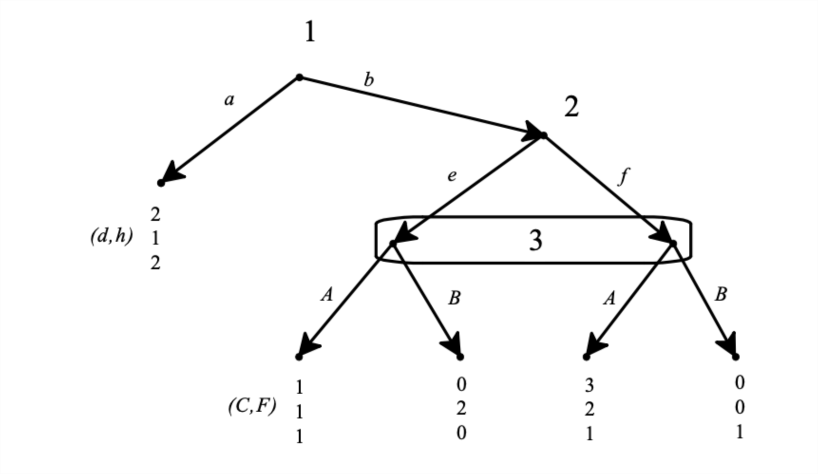


The big game with reduced left subgame



		Player 2	
		E	F
Player 1	C	3 0	1 1
	D	1 1	1 0

The middle subgame with labeled best responses



The big game with reduced middle subgame

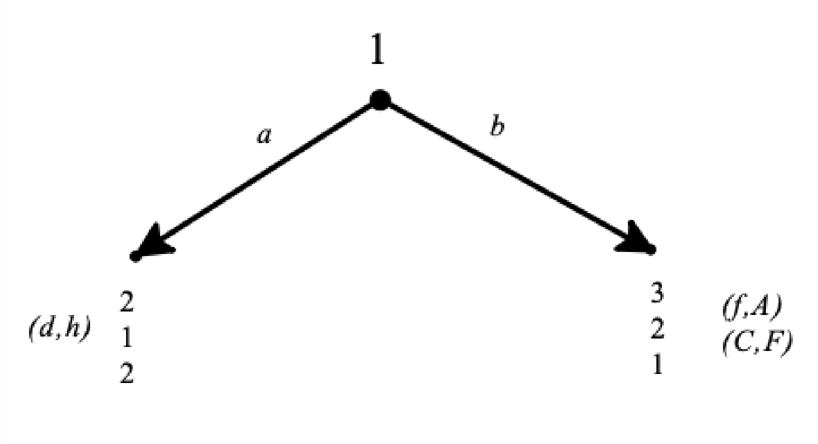
The right subgame

	A	B
e	1, 1	2, 0
f	2, 1	0, 1

- Player 2 is row
- Player 3 is column
- Player 1 is ignored, because they have no moves

The right subgame with labeled best responses

	A	B
e	1, $\boxed{1}$	$\boxed{2}$, 0
f	$\boxed{2}$, $\boxed{1}$	0, $\boxed{1}$



The big game with reduced right subgame

- Only Nash equilibrium is Player 1 plays b.

Summary

So the subgame perfect equilibrium is:

- Player 1 plays b, C.
- Player 2 plays d, f, F.
- Player 3 plays h, A.

And the payouts are reverse order of their names: 3, 2, 1.

For Next Time

We will start looking at games with cardinal utility.