

Subgame Perfect Equilibrium

Brian Weatherson, February 8, 2018

Row and Column are going to play the following two-stage game. At stage 1, they will play this game, which has **games** as the ‘payouts’, i.e., as the results. So they could end up in one of the four games below, depending on what they choose.

	Left	Right
Up	Game 1	Game 2
Down	Game 3	Game 4

Depending on what happened in stage 1, they’ll play one of the following four games in stage 2.

Game 1

	Left	Right
Up	10, 0	0, 2
Down	0, 2	2, 0

Game 2

	Left	Right
Up	2, 0	0, 8
Down	0, 2	2, 0

Game 3

	Left	Right
Up	2, 0	0, 2
Down	0, 6	2, 0

Game 4

	Left	Right
Up	2, 0	0, 2
Down	0, 2	4, 0

Question

In the subgame perfect equilibrium, what is the probability that Row will receive the payoff 10?

Bonus Question

This is just if you are bored. An envelope has an amount of cash x in it. The amount x is drawn at random from a flat probability distribution over $[0, 100]$. Player A is shown the contents of the envelope; Player B is not. Both players know all these facts, and that for each player $U(x) = x$ - that is, that they have a non-declining marginal utility curve.

The envelope will be auctioned in a sealed bid auction. Each player will get to make one bid, in ignorance of the other player’s bid. The high bid will ‘win’; that is, they will pay the amount they bid, and win the auction. Neither player cares about winning as such; they just want money.

Find a Nash equilibrium for this game. That is, find a strategy for A and B such that neither can improve their expected outcome given the strategy the other player adopts.