444 Lecture 2.8 - Ice Cream

Brian Weatherson

Plan

To look at a famous example of iterated deletion.

This isn't in the book.

Example

Two trucks have to choose where they will sell ice-cream on a particular beach. There are 7 locations to choose from, which we'll number 0, 1, ..., 5, 6. Spot 0 is at the left end of the beach, Spot 6 is at the right end of the beach, and the other spots are equally spaced in between. There are 10 people at each location. Each of them will buy ice-cream. If one truck is closer, they will buy ice-cream from that truck. If two trucks are equally close, then 5 of them will buy ice-cream from one truck, and 5 from the other. Each truck aims to maximise the amount of ice-cream it sells. Where should the trucks end up?

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

- Think about why each of these payoffs is correct.

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

- I’ve highlighted Row’s payoffs for 0 and 1 to make it clear that 1 strongly dominates 0.

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

- But 2 doesn't dominate 1, because 1 is a better response to 0 than 2 is.

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

- The game is symmetric around 3, so 5 also dominates 6 (and is not dominated by 4).

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

- And the game is symmetric for Row/Column, so 0 and 6 are dominated for Column too.

	1	2	3	4	5
1	35, 35	20, 50	25, 45	30, 40	35, 35
2	50, 20	35, 35	30, 40	35, 35	40, 30
3	45, 25	40, 30	35, 35	40, 30	45, 25
4	40, 30	35, 35	30, 40	35, 35	50, 20
5	35, 35	30, 40	25, 55	20, 50	35, 35

- Here’s what it looks like after those dominated strategies are removed.

	1	2	3	4	5
1	35, 35	20, 50	25, 45	30, 40	35, 35
2	50, 20	35, 35	30, 40	35, 35	40, 30
3	45, 25	40, 30	35, 35	40, 30	45, 25
4	40, 30	35, 35	30, 40	35, 35	50, 20
5	35, 35	30, 40	25, 55	20, 50	35, 35

- Note now that 2 dominates 1 - since 0 is removed, and 4 dominates 5.
- And this holds for both Row and Column.

	2	3	4
2	35, 35	30, 40	35, 35
3	40, 30	35, 35	40, 30
4	35, 35	30, 40	35, 35

- And in this game, 3 is the strictly dominant option for each player.

	1	2	3	4	5
1	35, 35	20, 50	25, 45	30, 40	35, 35
2	50, 20	35, 35	30, 40	35, 35	40, 30
3	45, 25	40, 30	35, 35	40, 30	45, 25
4	40, 30	35, 35	30, 40	35, 35	50, 20
5	35, 35	30, 40	25, 55	20, 50	35, 35

- I started with 7 because that's literally what would fit on the screen, but the same form of reasoning would work for any (odd) number of slots on the beach, as long as the people are evenly distributed.

Hotelling

Harold Hotelling

The game I've described here is a version of a model originally described by Harold Hotelling (1895-1973)

Feature Space

- Hotelling was less interested in physical location than location in feature space.
- He wanted an explanation of why the products of competing firms tended to be like one another.

Political Versions

- Games like this have become favorite tools of political scientists, arguing why political parties tended (at least in the 20th century!) to move towards the median.
- You have to be careful about the payoffs here; political parties don't want to maximise votes, they want to maximise win probability and policy outcomes.
- It turns out under a lot of assumptions you still get something like Hotelling's result, though it is sensitive to a lot of factors.

Rationality Assumptions

- Finally, I want to briefly flag the rationality assumptions this argument needs.
- As long as the players are rational, they won't play 0/6.
- As long as they know the other player is rational, they won't play 1/5.
- But to rule out 2/4, we need something stronger. We need that they each know that the other knows that each is rational.
- For longer beaches, we need even stronger assumptions. And those assumptions may be implausible.

For Next Time

- We will look at the most famous concept in game theory: Nash Equilibrium.