444 Lecture 5.3 - Mixed Strategies in Equilibria

Brian Weatherson

Plan

Discuss the existence of mixed strategy equilibria.

Reading

Bonanno, section 6.2

Key Theorem

In any finite game in which all mixtures of strategies are available, there is at least one Nash equilibria.

Example

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Let's discuss this game for a bit. Does it have an equilibrium?

Example

	Left	Right
Up	$\boxed{2}, 0$	$0, \boxed{3}$
Down	$0, \boxed{1}$	$\boxed{1}, 0$

No Nash equilibrium in pure strategies.

A Strategy for Row

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Consider what happens if Row plays

- Up with probability 1/4;
- Down with probability 3/4.

Column's Expected Return

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Column's expected return from playing Left is

- 0 with probability 1/4 plus
- 1 with probability 3/4, i.e.,
- 3/4.

Column's Expected Return

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Column's expected return from playing Right is

- 3 with probability 1/4 plus
- 0 with probability 3/4, i.e.,
- 3/4.

Column's Expected Return

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

- So either way, Column's expected return from playing a pure strategy is $3/4$.
- And hence Column's expected return from playing any mixture of the two pure strategies is $3/4$.

A Strategy for Column

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Consider what happens if Column plays

- Left with probability 1/3;
- Right with probability 2/3.

Row's Expected Return

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Row's expected return from playing Up is

- 2 with probability 1/3 plus
- 0 with probability 2/3, i.e.,
- 2/3.

Column's Expected Return

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

Row's expected return from playing Down is

- 0 with probability 1/3 plus
- 1 with probability 2/3, i.e.,
- 2/3.

Column's Expected Return

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

- So either way, Row's expected return from playing a pure strategy is 2/3.
- And hence Row's expected return from playing any mixture of the two pure strategies is 2/3.

An Equilibria

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

- What happens if they both play the mixed strategies we've been discussing?

An Equilibria

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

- What happens if they both play the mixed strategies we've been discussing?
- It's an equilibria.

An Equilibria

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

- Whatever Column does, their expected return is 3/4.
- So they can't do better than play this mixed strategy.
- They can't do worse either, but the definition of equilibrium just requires that they can't do better.

An Equilibria

	Left	Right
Up	2, 0	0, 3
Down	0, 1	1, 0

- Whatever Row does, their expected return is 2/3.
- So they can't do better than play this mixed strategy.
- They can't do worse either, but the definition of equilibrium just requires that they can't do better.

Two General Points

1. There is always some equilibria like this (at least in finite games), even if it doesn’t look like it at first.

Two General Points

1. There is always some equilibria like this (at least in finite games), even if it doesn't look like it at first.
2. Typically, the way we find equilibria is making the other player indifferent between a bunch of options.

For Next Time

We will use that last point to work out how to compute the Nash equilibria for simple games.