444 Lecture 5.5 - Dominance by Mixture

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Plan

Discuss a new form of dominance reasoning - mixed strategy dominance.

Reading

Bonanno, Section 6.4.

Basic Example

	Left	Right
Up	3, 0	0, 0
Middle	1, 0	1, 0
Down	0, 0	3, 0

This is a bit boring for Column, but let's focus on Row for now.

Dominance Reasoning

	Left	Right
Up	3, 0	0, 0
Middle	1, 0	1, 0
Down	0, 0	3, 0

At first it looks like there are no dominated strategies.

- Up does worse than Middle and Down if Column plays Right, so it doesn't dominate anything.
- Middle does worse that Up if Column plays Left, and worse than Down if Column plays Right.
- Down does worse than both of them if Column plays Left.

Dominance Reasoning

	Left	Right
Up	3, 0	0, 0
Middle	1, 0	1, 0
Down	0, 0	3, 0

But compare these two strategies.

- Middle
- The mixed strategy of Up with probability 0.5, and Down with probability 0.5.

Dominance Reasoning

	Left	Right
Up	3, 0	0, 0
Middle	1, 0	1, 0
Down	0, 0	3, 0

- Middle gets an actual return of 1 whatever Column does.
- The mixed strategy gets an expected return of 1.5 whatever Column does.
- So it has a higher expected return given Left (1.5 > 1), and a higher expected return given Right (1.5 > 1).

Dominance Reasoning

	Left	Right
Up	3, 0	0, 0
Middle	1, 0	1, 0
Down	0, 0	3, 0

- If that happens, then we'll say that Middle is dominated by this mixture.
- When we're deleting dominated strategies, we should delete it too.

Nash and Dominance

- A strategy that is dominated by a mixture like this can never be part of a Nash equilibrium.
- After all, the player would be better off playing the mixture than playing it, so it fails the test that there is nothing better to do.
- So being able to find these dominating mixtures can be very helpful in working out what the Nash equilibrium is.

Rational Play and Dominance

- But even beyond that, it seems wrong to play strategies that are dominated in this way.
- If you're thinking about playing Middle (as Row), you increase your expected return by simply flipping a coin to choose between Left and Right.
- So that's what you should do.

For Next Time

We'll connect this expanded notion of dominance up to an expanded notion of best responses.