444 Lecture 4.2 - Structure of Information Sets

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Plan

To discuss some presuppositions of the theory of information sets.

Reading

This isn’t in the books; it’s not something game theorists discuss.

Three Features of Information Sets

1. Reflexive
2. Symmetric
3. Transitive

Reflexive

Each point is in its own information set.

- This seems fair enough; if you’re somewhere, then for all you know, you are there.

Symmetric

- If when you’re at a you might be at b, then
- When you’re at b you might be at a.
- In information set terms, if b is in a’s information set, then a is in b’s information set.

Symmetry

What this means is that what happens earlier in the game can’t affect a player’s powers of discrimination.

- This seems like an inappropriate assumption in, e.g., drinking games.
- At least in some games, one’s ability to discriminate between some options will be dependent on the path taken to get to those options.
- The standard treatment of partial information doesn’t allow us to represent this.

Transitive

- If when you’re at a you might be at b, and
- When you’re at b you might be at c, then
- When you’re at a you might be at c.
- In information set terms, if b is in a’s information set, and c is in b’s information set, then it must be that c is in a’s information set.

Transitivity

This rules out games where players can tell that they are in a certain ‘neighbourhood’. For example,

- Player 1 puts some jelly beans in a jar and gives it to Player 2.
- It matters to Player 2 how many jelly beans are in the jar, but she doesn’t have direct access to that.
- Still, she isn’t totally ignorant. She can see the jar and guess the number to the nearest, say 10.
- So if there are 160 jelly beans in the jar, she knows that there are between 150 and 170.
- And in fact, though Player 2 doesn’t know this, that’s true - there are 160 jelly beans.

Transitivity

- Let a be the node where there are 160 jelly beans in the jar.
- And b be the node where there are 150 jelly beans in the jar.
- And c be the node where there are 140 jelly beans in the jar.
- Player 2 knows she's not at c; her information set should exclude that.
- And her information set should include a - reflexivity guarantees that.
- Should it include b?

A Challenge

- On the one hand, it should - since from a all she knows is that there are between 150 and 170 jelly beans in the jar.
- On the other hand, it should not - since she can rule out c, and if b were actual, she could not rule out c.

What To Do

- In theory, there is an opening here for someone working out a theory of games with imperfect information that drops either the symmetry or transitivity assumption.
- In practice, no one has actually worked out that theory, and I'm not going to try teaching a non-existent theory.
- There is a little work on games involving "unawareness", which gets close to this, but it's way too novel a field to know where it will go.

For Next Time

- We will return to orthodoxy, and look at the notion of a strategy.