

444 Lecture 2.9 - Nash Equilibrium

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Plan

To introduce the most famous concept in game theory, Nash Equilibrium.

Reading

Bonanno, section 2.6.

John Nash



Figure 1: John Nash (via Hollywood)

- Nash Equilibrium is named after the American mathematician John Nash.
- Except I seem to have a picture of Russell Crowe here.

John Nash

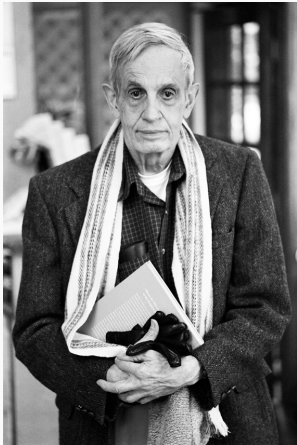


Figure 2: John Nash

- Nash Equilibrium is named after the American mathematician John Nash (1928-2015).
- It is the core concept of contemporary game theory.

Best Response

- We will build up to it in stages.
- The first important notion is that of a best response.
- Strategy S is a best response to strategies by the other players iff no other strategy can do better, given what the other players are doing.

Example

	Left	Center	Right
Up	4, 3	2, 0	0, 5
Middle	6, 2	0, 4	3, 1
Down	3, 0	2, 1	4, 2

- If Column plays Left, the best Row can do is play Middle.
- They get 6 that way, and 3 or 4 from other plays.
- So Middle is the best response to Left.

Example

	Left	Center	Right
Up	4, 3	2, 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2, 1	4, 2

- We will represent the fact that it is a best response by putting a box around the payout.
- There are all sorts of notations you'll see used for this; we'll just use a box.

Example

	Left	Center	Right
Up	4, 3	2, 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2, 1	4 , 2

- If Column plays Right, the best Row can do is play Down.
- So we'll put a Box around it as well.

Example

	Left	Center	Right
Up	4, 3	2 , 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2 , 1	4 , 2

- Now if Column plays Middle, Row has two options that are tied for best: Top and Bottom.
- They are both best responses.
- So we'll put boxes around each.

Example

	Left	Center	Right
Up	4, 3	2 , 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2 , 1	4 , 2

- I find it a little trickier to keep track of the best responses for Column, so I have to go a little slower.
- If Row plays Top, Column has a choice of 3 (if they play Left), 0 (if they play Middle), or 5 (if they play Right).
- 5 is best, so the best response is Right.

Example

	Left	Center	Right
Up	4, 3	2 , 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2 , 1	4 , 2

- If Row plays Middle, Column has a choice of 2 (if they play Left), 4 (if they play Middle), or 1 (if they play Right).
- 4 is best, so the best response is Middle.

Example

	Left	Center	Right
Up	4, 3	2 , 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2 , 1	4 , 2

- If Row plays Down, Column has a choice of 0 (if they play Left), 1 (if they play Middle), or 2 (if they play Right).
- 2 is best, so the best response is Right.
- We've now labelled all the (pure strategy) best responses.

Nash Equilibrium

- A strategy set for all the players is a Nash Equilibrium if each player is making a best response to what the others are doing.
- In these games, that means that both payoffs in the cell are boxed.

Nash Equilibrium

	Left	Center	Right
Up	4, 3	2 , 0	0, 5
Middle	6 , 2	0, 4	3, 1
Down	3, 0	2 , 1	4 , 2

- In this game, the unique Nash Equilibrium is Row plays Down, and Column plays Right.
- That's the only cell where both players are making a best response to the other players' strategy.

Nash Equilibrium

- The general idea is that some strategies form an equilibrium if no one could do better by unilaterally changing strategy.
- It's possible that players could do better if they both simultaneously changed - and we'll spend some time on cases where that happens.
- But everyone is doing as well as they can given what everyone else is doing.

For Next Time

- We'll think a bit about why this might be philosophically significant.