444 Lecture 6.4 - Conditional Probability

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Plan

- To introduce a key new concept: **conditional probability**.

Associated Reading

Odds and Ends, Chapter 6

What it is

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- We care about how likely it is given some other thing has happened or will happen.
- This might be because we want to plan.
- It might be because we want to compute overall probabilities.
- Or it might be because we've found something out, and want to know what it means for other likelihoods.

Prior Examples

We've already used some conditional probabilities.

- We already talked, for example, about the probability of the Fireflies winning conditional on them being in the final against the Bluebirds.

Inverting

But there are other questions we might want to ask as well.

- E.g., conditional on the Fireflies winning, how likely is it that they played the Huskies.

Intuitions

This isn't an easy question to answer intuitively.

- It is more likely that the Huskies will be actually in the final - because they are the better team.
- But it is more likely that the Fireflies will win against the Bluebirds - because they are weaker.
- It isn't always easy to intuitively balance these forces.

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- The left-hand side means "The probability of A given B."
- And the right-hand side says that this is equal to the probability of $A \wedge B$ divided by the probability of B.

Fireflies

$$\Pr(HR|FC) = \frac{\Pr(HR \wedge FC)}{\Pr(FC)}$$

That is, the probability that the Huskies are runners-up (HR) given that the Fireflies are champions (FC), is given by the formula on the right.

Fireflies

$$\Pr(HR|FC) = \frac{0.26}{0.26 + 0.116} = \frac{0.26}{0.376} \approx 0.691$$

So conditional on the Fireflies winning, it's just under 70% likely they beat the Huskies.

Updating

We are going to mostly assume that this philosophical claim is true.

$$Pr_B(A) = Pr(A|B)$$

The way to read that is saying that the unconditional probability of A after learning B equals the conditional probability of A given B. This claim - and it is a philosophical claim not a mathematical one - is a big part of why we care about conditional probability.

For Next Time

We're going to talk about when it is that two events are independent.