444 Lecture 6.2 - About Probability Functions

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Plan

- This lecture looks at some general features of probability functions, and looks at some ways to think probabilistically about real world events.

Associated Reading

Odds and Ends, Chapter 5

Scale

$$0 \leq \Pr(A) \leq 1$$

Negation

$$\Pr(\neg A) = 1 - \Pr(A)$$

Excluded Middle

$$\Pr(A) + \Pr(\neg A) = 1$$

Partition

Some events $A_1, \ldots A_n$ form a partition if, necessarily, exactly one of them is true.

- So they are **exclusive** - you can't have any two of them both be true.
- And they are **exhaustive** - you have to have at least one true.

Partition

If $A_1, \ldots A_n$ form a partition then

$$\Pr(A_1) + \cdots + \Pr(A_n) = 1$$

Exclusive

If A, B are exclusive

$$\Pr(A \vee B) = \Pr(A) + \Pr(B)$$

General Principle

$$\Pr(A) + \Pr(B) = \Pr(A \vee B) + \Pr(A \wedge B)$$

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It's worth thinking through why this is true in terms of possibilities.

Trees

- Often, we can't just write down numbers for the possibilities.
- But we can write down, or at least make reasonable guesses about, numbers for certain events.
- And we can think about how things are likely to go given those events happen.
- This is the tree structure that is used in Odds and Ends.

Academic Forecasting

- So let's say you're interested in the probability that we have in person classes all the way through Fall semester.
- The vaccine rollout is going much better than I ever expected.
- But we saw how bad things got last Thanksgiving and Christmas, and you can see why that might make you worried.
- But it's hard to put into numbers how it affects things.

One Method

- Divide up the state space.
- Work out the probability of being in one or other part of the space.
- Work out the probability of classes running all semester in each part of the space.
- Add things up.

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- You could divide up the space by how many cases there are in Michigan.
- Or you could divide up the space by how many other universities shut down.
- Or, and let's work with this one, you could divide it up by how many people are vaccinated by the the start of term.

Three States

1. Only high risk staff and students are vaccinated.
2. More than just the high risk folks are vaccinated, but there are still shortages.
3. Vaccine for all.

Two Step Process

1. Work out (or at least estimate) probability of each state.
2. Work out probability of the university staying open within each of those states.

This will involve a lot of guesswork - do not make investment decisions on the basis of the numbers I'm about to use because they are really pulled out of the air - but it's a very helpful heuristic to at least approximate the reality.

Probabilities of States

This could be embarrassingly out of date by the time we even do the classes, but let's say the probabilities look like this.

- High risk only - 20%, or 0.2.
- Limited quantities of vaccine - 50%, or 0.5.
- Unlimited quantities of vaccine - 30%, or 0.3.

Class Probabilities

Then we want to work out how likely it is that we get a full semester of classes in given each of these three possibilities.

High Risk Only

Given how badly things went last Fall, I'm not sure what we'd do in this situation. Maybe we'd muddle through and it would work. But I suspect that it's most likely we'd be planning to shut down at Thanksgiving, so not strictly a full semester.

- Let's call it a 30% chance of staying open all semester in that situation.

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- I really, and I can't stress this enough, want to emphasise that I'm making these numbers up for the purposes of an in class example.

Limited Vaccine

This starts to feel more like something where we can get through the term. For one thing, we might just tought it out even if there is a bit of an outbreak. For another, I said limited in September; maybe by October/November it would be unlimited. So it feels pretty likely.

- Let's say there is an 80% chance we make it all the way through semester in that situation, and only 20% that we don't.

Unlimited vaccine

Now it's hard to see why the university would shut down. Rather than fuss about the details, let's just simplify and say that in this (very unlikely) scenario, it's 100% likely that we'll get all classes in.

The Giant Table

	Full Semester	Some shutdown
No vaccine	$0.2 \times 0.3 = 0.06$	$0.2 \times 0.7 = 0.14$
Limited vaccine	$0.5 \times 0.8 = 0.40$	$0.5 \times 0.2 = 0.10$
Unlimited vaccine	$0.3 \times 1.0 = 0.30$	$0.3 \times 0.0 = 0.00$

So the probability that we get a full semester in (given these literally made up assumptions) is $0.06 + 0.40 + 0.30 = 0.76$, or 76%.

Probabilities and Errors

- The error bars on that calculation are massive.
- But it's a kind of sanity check on how you think things are going.
- Especially in situations where only a handful of paths lead to a salient result (like in playoffs in sports, or when thinking about the likelihood of a particular challenger winning), doing the tree even with favorable numbers can give you a conservative estimate of some probability.

Three Cases Where This is More Precise

1. Probabilities are stipulated
2. Probabilities are due to well understood chance processes (like gambling devices)
3. Probabilities are derived from very large data sets

For Next Time

- We will look at an example where the probabilities are indeed stipulated.