444 Lecture 3

Equilibrium

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Initial Idea

- If an option is strongly dominated, it shouldn't be chosen.
- In the simple case, if all options but one are strongly dominated, that one should be chosen.
- But we can say more than this.

Initial Idea

- If a strategy only makes sense if the other player plays a dominated strategy, then it doesn't make sense.
- Let's work through some examples to see how this works in practice.

Easy Example

	Left	Right
Up	4, 1	2, 2
Down	3, 3	1, 4

We can solve this using just domination.

Easy Example

	Left	Right
Up	4, 1	2, 2
Down	3, 3	1, 4

- Up dominates Down, so Row should play Up.
- Right dominates Left, so Column should play Right.
- So the solution is Up/Right.

Only Slightly Harder Example

	Left	Right
Up	4, 0	2, 1
Down	3, 1	1, 0

Now Column doesn't have a dominating option, but that doesn't stop us.

Only Slightly Harder Example

	Left	Right
Up	4, 0	2, 1
Down	3, 1	1, 0

- Up dominates Down, so Row should play Up.
- If Row is playing Up, Right is better than Left (1 beats 0).
- So since Row is playing Up, Column should play Right.
- So the solution (again) is Up/Right.

Iterated Dominance

	Left	Center	Right
Up	4, 2	3, 1	0, 0
Middle	3, 0	2, 2	1, 1
Down	2, 0	1, 0	0, 3

We can't immediately solve this with dominance, but we can in a few steps.

Iterated Dominance

	Left	Center	Right
Up	4, 2	3, 1	0, 0
Middle	3, 0	2, 2	1, 1
Down	2, 0	1, 0	0, 3

- Note first that Middle dominates Down.
- So Down should not be played.

Iterated Dominance

	Left	Center	Right
Up	4, 2	3, 1	0, 0
Middle	3, 0	2, 2	1, 1

- Here's what happens if we **delete** the dominated option Down.
- In fact, we might even act as if it is not there.

Iterated Dominance

	Left	Center	Right
Up	4, 2	3, 1	0, 0
Middle	3, 0	2, 2	1, 1

- Now Center dominates Right.
- It didn't a minute ago - Right is a better response to Down than Center is - but Down is deleted.
- So Right is out, and we'll delete it too.

Iterated Dominance

	Left	Center
Up	4, 2	3, 1
Middle	3, 0	2, 2

- In this game, Up dominates Middle.
- So Middle has to go.

Iterated Dominance

	Left	Center
Up	4, 2	3, 1

- And in this game, Left dominates Center.
- So the solution to the game is Up/Left.

General Strategy

- Start deleting dominated strategies.
- Then see if some strategies are dominated in the new version of the game.
- If you're lucky, the result will be that just one option for each player is left.
- If so, we'll call that the solution of the game.

Two Issues

- Order effects.
- Philosophical motivation.

Order Effects

- As Bonanno goes over, when deleting weakly dominated strategies, it matters what order you do the deletions in.
- Whether a strategy weakly dominates another at a point in the process might depend on how you got to that point.
- And the result is that different ways of applying the process lead to different 'solutions'.

Way Around This

- Bonanno says (as I think is standard) that you solve this by saying that at each stage, you delete every strategy that you possibly can.
- There is still an issue I think about why that deletion process is justified as opposed to some other.
- It does have the nice advantage of actually being a well defined process, so that's nice.

Philosophical Justification

- The bigger issue is that it is a little hard to say why we should care about the result of this procedure.
- Saying what's special about the result of this strategy is not completely obvious.
- Bonanno alludes to this - let's go over it in a bit more detail.

	Left	Center	Right
Up	1, 1	1, 1	0, 0
Middle	1, 1	0, 0	1, 0
Down	0, 0	0, 1	1, 1

- Middle weakly dominates Down, and Center weakly dominates Right.
- So let's delete them.

	Left	Center
Up	1, 1	1, 1
Middle	1, 1	0, 0

- Now Up weakly dominates Middle and Left weakly dominates Center.
- So the solution is Up/Left, right?
- Well, not so fast.

	Left	Center	Right
Up	1, 1	1, 1	0, 0
Middle	1, 1	0, 0	1, 0
Down	0, 0	0, 1	1, 1

- Think about it from Row's perspective.
- We have an argument that Column will play Left.
- If that argument works, Row shouldn't prefer Up - they should be indifferent between Up and Middle.

	Left	Center	Right
Up	1, 1	1, 1	0, 0
Middle	1, 1	0, 0	1, 0
Down	0, 0	0, 1	1, 1

- Why does the argument say to play Up then?
- The answer is that Middle is risky. In the game after deletion, Middle has a risk of getting 0, but Up is sure to get 1.

	Left	Center	Right
Up	1, 1	1, 1	0, 0
Middle	1, 1	0, 0	1, 0
Down	0, 0	0, 1	1, 1

- But look at the original game - Up is risky too!
- I think this makes it hard to philosophically defend IDWDS

Example

Two trucks have to choose where they will sell ice-cream on a particular beach. There are 7 locations to choose from, which we'll number 0, 1, ..., 5, 6. Spot 0 is at the left end of the beach, Spot 6 is at the right end of the beach, and the other spots are equally spaced in between. There are 10 people at each location. Each of them will buy ice-cream. If one truck is closer, they will buy ice-cream from that truck. If two trucks are equally close, then 5 of them will buy ice-cream from one truck, and 5 from the other. Each truck aims to maximise the amount of ice-cream it sells. Where should the trucks end up?

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

Think about why each of these payoffs is correct.

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

The highlighted values show that 1 strongly dominates 0.

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

But 2 doesn't dominate 1, because 1 is a better response to 0.

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

The game is symmetric around 3, so 5 also dominates 6 (and is not dominated by 4).

	0	1	2	3	4	5	6
0	35, 35	10, 60	15, 55	20, 50	25, 45	30, 40	35, 35
1	60, 10	35, 35	20, 50	25, 45	30, 40	35, 35	40, 30
2	55, 15	50, 20	35, 35	30, 40	35, 35	40, 30	45, 25
3	50, 20	45, 25	40, 30	35, 35	40, 30	45, 25	50, 20
4	45, 25	40, 30	35, 35	30, 40	35, 35	50, 20	55, 15
5	40, 30	35, 35	30, 40	25, 55	20, 50	35, 35	60, 10
6	35, 35	30, 40	25, 45	20, 50	15, 55	10, 60	35, 35

- And the game is symmetric for Row/Column, so 0 and 6 are dominated for Column too.

	1	2	3	4	5
1	35, 35	20, 50	25, 45	30, 40	35, 35
2	50, 20	35, 35	30, 40	35, 35	40, 30
3	45, 25	40, 30	35, 35	40, 30	45, 25
4	40, 30	35, 35	30, 40	35, 35	50, 20
5	35, 35	30, 40	25, 55	20, 50	35, 35

Here's what it looks like after those dominated strategies are removed.

	1	2	3	4	5
1	35, 35	20, 50	25, 45	30, 40	35, 35
2	50, 20	35, 35	30, 40	35, 35	40, 30
3	45, 25	40, 30	35, 35	40, 30	45, 25
4	40, 30	35, 35	30, 40	35, 35	50, 20
5	35, 35	30, 40	25, 55	20, 50	35, 35

Note now that 2 dominates 1 - since 0 is removed, and 4 dominates 5. And this holds for both Row and Column.

	2	3	4
2	35, 35	30, 40	35, 35
3	40, 30	35, 35	40, 30
4	35, 35	30, 40	35, 35

And in this game, 3 is the strictly dominant option for each player.

	1	2	3	4	5
1	35, 35	20, 50	25, 45	30, 40	35, 35
2	50, 20	35, 35	30, 40	35, 35	40, 30
3	45, 25	40, 30	35, 35	40, 30	45, 25
4	40, 30	35, 35	30, 40	35, 35	50, 20
5	35, 35	30, 40	25, 55	20, 50	35, 35

- I started with 7 because that's literally what would fit on the screen, but the same form of reasoning would work for any (odd) number of slots on the beach, as long as the people are evenly distributed.

Hotelling

Figure: Harold Hotelling

The game I've described here is a version of a model originally described by Harold Hotelling (1895-1973)

Feature Space

- Hotelling was less interested in physical location than location in feature space.
- He wanted an explanation of why the products of competing firms tended to be like one another.

Political Versions

- Games like this have become favorite tools of political scientists, arguing why political parties tended (at least in the 20th century!) to move towards the median.
- You have to be careful about the payoffs here; political parties don't want to maximise votes, they want to maximise win probability and policy outcomes.
- It turns out under a lot of assumptions you still get something like Hotelling's result, though it is sensitive to a lot of factors.

Rationality Assumptions

- Finally, I want to briefly flag the rationality assumptions this argument needs.
- As long as the players are rational, they won't play 0/6.
- As long as they know the other player is rational, they won't play 1/5.

Rationality Assumptions

- But to rule out 2/4, we need something stronger. We need that they each know that the other knows that each is rational.
- For longer beaches, we need even stronger assumptions. And those assumptions may be implausible.

John Nash

Figure: John Nash (via Hollywood)

- Nash Equilibrium is named after the American mathematician John Nash.
- Except I seem to have a picture of Russell Crowe here.

John Nash

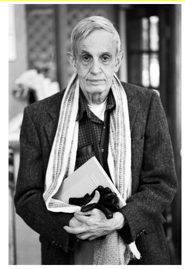

Figure: John Nash

- Nash Equilibrium is named after the American mathematician John Nash (1928-2015).
- It is the core concept of contemporary game theory.

Best Response

- We will build up to it in stages.
- The first important notion is that of a best response.
- Strategy *S* is a best response to strategies by the other players iff no other strategy can do better, given what the other players are doing.

	Left	Center	Right
Up	4, 3	2, 0	0, 5
Middle	6, 2	0, 4	3, 1
Down	3, 0	2, 1	4, 2

- If Column plays Left, the best Row can do is play Middle.
- They get 6 that way, and 3 or 4 from other plays.
- So Middle is the best response to Left.

	Left	Center	Right
Up	4, 3	2, 0	0, 5
Middle	$\boxed{6}$,2	0, 4	3, 1
Down	3, 0	2, 1	4, 2

- We will represent the fact that it is a best response by putting a box around the payout.
- There are all sorts of notations you'll see used for this; we'll just use a box.

	Left	Center	Right
Up	4, 3	2, 0	0, 5
Middle	$\boxed{6}$,2	0, 4	3, 1
Down	3, 0	2, 1	$\boxed{4}$,2

- If Column plays Right, the best Row can do is play Down.
- So we'll put a Box around it as well.

	Left	Center	Right
Up	4, 3	$\boxed{2}$,0	0, 5
Middle	$\boxed{6}$,2	0, 4	3, 1
Down	3, 0	$\boxed{2}$,1	$\boxed{4}$,2

- Now if Column plays Middle, Row has two options that are tied for best: Top and Bottom.
- They are both best responses.
- So we'll put boxes around each.

	Left	Center	Right
Up	4, 3	$\boxed{2}$,0	0,$\boxed{5}$
Middle	$\boxed{6}$,2	0, 4	3, 1
Down	3, 0	$\boxed{2}$,1	$\boxed{4}$,2

- I find it a little trickier to keep track of the best responses for Column, so I have to go a little slower.
- If Row plays Top, Column has a choice of 3 (if they play Left), 0 (if they play Middle), or 5 (if they play Right).
- 5 is best, so the best response is Right.

	Left	Center	Right
Up	4, 3	$\boxed{2}$,0	0,$\boxed{5}$
Middle	$\boxed{6}$,2	0,$\boxed{4}$	3, 1
Down	3, 0	$\boxed{2}$,1	$\boxed{4}$,2

- If Row plays Middle, Column has a choice of 2 (if they play Left), 4 (if they play Middle), or 1 (if they play Right).
- 4 is best, so the best response is Middle.

	Left	Center	Right
Up	4, 3	$\boxed{2}$,0	0,$\boxed{5}$
Middle	$\boxed{6}$,2	0,$\boxed{4}$	3, 1
Down	3, 0	$\boxed{2}$,1	$\boxed{4}$, $\boxed{2}$

- If Row plays Down, Column has a choice of 0 (if they play Left), 1 (if they play Middle), or 2 (if they play Right).
- 2 is best, so the best response is Right.
- We've now labelled all the (pure strategy) best responses.

Nash Equilibrium

- A strategy set for all the players is a Nash Equilibrium if each player is making a best response to what the others are doing.
- In these games, that means that both payoffs in the cell are boxed.

Nash Equilibrium

	Left	Center	Right
Up	4, 3	$\boxed{2}$,0	0,$\boxed{5}$
Middle	$\boxed{6}$,2	0,$\boxed{4}$	3, 1
Down	3, 0	$\boxed{2}$,1	$\boxed{4}$, $\boxed{2}$

- In this game, the unique Nash Equilibrium is Row plays Down, and Column plays Right.
- That's the only cell where both players are making a best response to the other players' strategy.

Nash Equilibrium

- The general idea is that some strategies form an equilibrium if no one could do better by unilaterally changing strategy.
- It's possible that players could do better if they both simultaneously changed - and we'll spend some time on cases where that happens.
- But everyone is doing as well as they can given what everyone else is doing.

A Philosophical Claim

In any game where it is common knowledge that all the players are rational, every player will play a strategy that forms part of a Nash Equilibrium.

Status of Nash

- I think most economists and political scientists accept something like this.
- But I think philosophers who work on game theory more often do not accept it.

Arguments for Nash

- Oddly, it's hard to find canonical arguments for the importance of Nash.
- It's so deeply embedded in game theory that it doesn't get discussed in research articles, more in textbooks.
- Bonanno has a passage on page 40 that you could (perhaps uncharitably) count as a contribution to that genre.

Transparency of Reason Interpretation

If players are all "equally rational" and Player 2 reaches the conclusion that she should play y, then Player 1 must be able to duplicate Player 2's reasoning process and come to the same conclusion; it follows that Player 1's choice of strategy is not rational unless it is a strategy x that is optimal against y. A similar argument applies to Player 2's choice of strategy (y must be optimal against x) and thus (x,y) is a Nash equilibrium.

Transparency of Reason Interpretation

- This doesn't look like a good argument for the Philosophical Claim.
- All it shows is the weaker claim that if there is a uniquely rational play for each player, those plays will form a Nash Equilibrium.

Viable recommendation interpretation

Imagine that a third party makes a public recommendation to each player on what strategy to play; then no player will have an incentive to deviate from the recommendation (if she believes that the other players will follow the recommendation) if and only if the recommended strategy profile is a Nash equilibrium.

Viable recommendation interpretation

- Again, this argument only works if the third party makes a unique recommendation.
- If the third party says "Do one of these three things", there is no argument that all three have to be Nash.

Self-enforcing agreement interpretation

Imagine that the players are able to communicate before playing the game and reach a non-binding agreement expressed as a strategy profile s; then no player will have an incentive to deviate from the agreement (if she believes that the other player will follow the agreement) if and only if s is a Nash equilibrium.

Self-enforcing agreement interpretation

- This is right as far as it goes, but doesn't help defend the philosophical claim in cases where no communication is possible.
- And here it is particularly notable that Bonanno's purposes are not quite the same as mine.

No regret interpretation

s is a Nash equilibrium if there is no player who, after observing the opponent's choice, regrets his own choice (in the sense that he could have done better with a different strategy of his, given the observed strategy of the opponent).

- This is a very good clear definition of what Nash is, but hard to see how it's a an argument for the importance of Nash.

Other Arguments

- You might be being spied on.
- The other player might be a mind-reader.
- You might be playing repeatedly, and so your strategy will be (more or less) revealed.

Repeated Games

- The last one is, I think, the main reason in practice people care about Nash.
- But it turns out for one important game, the Prisoners Dilemma, it is arguable that in the repeated game you should not play the Nash equilibrium.

Prisoners' Dilemma

	Coop	Defect
Coop	3, 3	0, $\boxed{5}$
Defect	$\boxed{5}$, 0	$\boxed{1}$, $\boxed{1}$

- The only Nash equilibrium is both players defect.
- And personally, I think in the one-shot game they should both defect.
- But it is not at all obvious they should defect in the repeated game.
- We will return to this point a lot in a few weeks.

For Next Week

- We will start looking at chapter 3, on games that have sequential moves.