444 Lecture 5

Dyanmic Games

Brian Weatherson

1/19/23

Day Plan

Strategies in Dynamic Games

Incredible Threats

The Backward Induction Paradox

Zero-Sum Turn-Taking Games

A Strategy

- Imagine that you're coaching someone who is going to play a dynamic game.

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- And imagine that you're a really really controlling coach, and they have a good memory.

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A Strategy

- Imagine that you're coaching someone who is going to play a dynamic game.
- And imagine that you're a really really controlling coach, and they have a good memory.
- Then you could tell them what to do in every possible situation.
- That's a strategy.

Control Freaks

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- A strategy says what the player should do at every node in the tree.
- That includes nodes that are ruled out by their earlier play.
- So a strategy for chess might include the instructions “Open with e4, then if the first two moves are d4-d5, follow with c4.”

Why be a Control Freak?

- Strategies serve two roles.

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Why be a Control Freak?

- Strategies serve two roles.
- They get followed by one player.
- And they get reasoned about by the other players, in order to create their own strategies.
- And to play the latter role, sometimes you need these weird steps.

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- Sometimes in games we're interested in situations where there is a gap between giving an order and carrying it out.

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Why be a Control Freak?

- Sometimes in games we're interested in situations where there is a gap between giving an order and carrying it out.
- When opposing generals are watching a battle play out, they can't assume that their instructions will be carried out to the letter, or that what they see on the battlefield is the result of the other side carrying out instructions properly.
- In these cases, it is clear why a strategy should include "What to do if you haven't done what I said you should do so far."

Back to Nash

- Go back to the coaching metaphor.

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- Go back to the coaching metaphor.
- And imagine the other player in the dynamic game also has a coach.
- Then you and the other coach are playing a one-shot, simultaneous move game.
- Each of you have a lot of options - but you get one shot to choose one of them, and the other player makes their choice at the same time (more or less) as you.

Strategic (Normal) Form

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- For any two player game tree, we can write out all the strategies for each player.
- I mean, we can in principle.
- In chess there are probably more strategies than atoms in the universe, so in practice it would be hard, but in theory it can be done.

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- Once we have listed all the strategies for each player, we can form a table, and compute what would happen for each strategy pair.
- And then we can do all the fun stuff from last week, like eliminating dominated strategies, finding best responses and Nash equilibria etc.

The End of Time?

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- No, and next time we'll look at why.

The End of Time?

- So does that mean we don't need to think about dynamics at all?
- No, and next time we'll look at why.
- Some strategy pairs that make sense in a one-shot game don't seem to make sense in a dynamic game.

Day Plan

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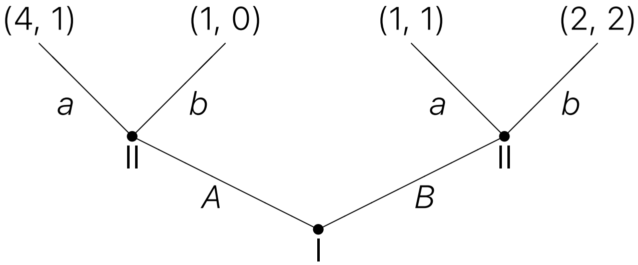
Threat Game



Strategies

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- A **strategy** for a game is a set of instructions for what to do at each node of a game.
- Even very small game trees there are a lot of possible strategies.
- If there are k possible nodes a player could have a choice at, and m possible moves at each of these nodes, then there are m^k possible strategies.
- Note that a strategy has to say what to do at nodes that are ruled out by your own prior moves.

Threat Game Strategies

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- For player I, there are just two strategies: *A* and *B*.
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- For player I, there are just two strategies: *A* and *B*.
- For player II, there are two nodes, and two possible choices at each node. So there are $2^2 = 4$ possible strategies.
- We'll write *xy* for the strategy of doing *x* in response to *A*, and *y* in response to *B*.
- And note I'm capitalising player I's moves, and using lower case for player II's moves, to make things clearer.

Threat Game Strategies

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1. *aa* - Do *a* no matter what.
2. *ab* - Do whatever player I does.
3. *ba* - Do the opposite of what player I does.
4. *bb* - Do *b* no matter what.

The strategies for the players determine the outcome. Here is the table for the game, given the strategies.

	aa	ab	ba	bb
A	4, 1	4, 1	1, 0	1, 0
B	1, 1	2, 2	1, 1	2, 2

	aa	ab	ba	bb
A	$\boxed{4}$, $\boxed{1}$	$\boxed{4}$, $\boxed{1}$	$\boxed{1}$, 0	1, 0
B	1, 1	$\boxed{2}$, $\boxed{2}$	$\boxed{1}$, 1	$\boxed{2}$, $\boxed{2}$

I've put boxes around the best responses, so you can see there are three Nash equilibria.

1. A, aa - with result 4, 1

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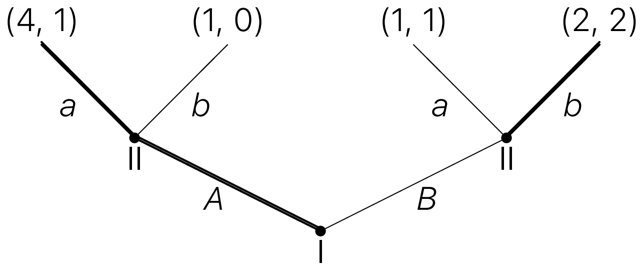
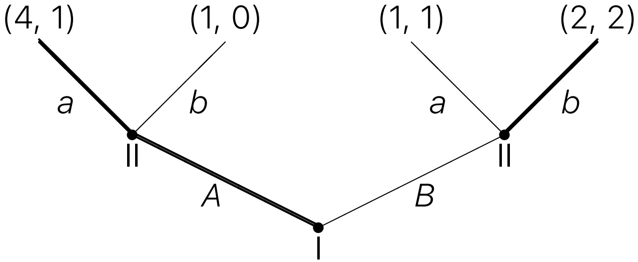
2. A, ab - with result 4, 1
3. B, bb - with result 2, 2



- I've bolded the best moves at each node, assuming backward induction.



- I’ve bolded the best moves at each node, assuming backward induction.
- The path of best moves is the (in this case unique) backward induction solution.

Threat Game

- There are three Nash equilibria of the game: strategy pairs that no one can improve on by unilaterally changing strategy.

Threat Game

- There are three Nash equilibria of the game: strategy pairs that no one can improve on by unilaterally changing strategy.
- There is just one backward induction solution of the game: a strategy pair where everyone does the best they can **at every node** assuming others play rationally at every node.

Incredible Threats

What makes $\langle B, bb \rangle$ a Nash equilibrium is that Player II can make the following speech.

> "I'm going to play b whatever you do. I want that 2 payout, and I'm going to get it. And since I'm going to play b whatever you do, you're better off playing B. That way you'll get 2, when you'd only get 1 if you played A. And you can tell I'm not bluffing because this strategy makes sense for me. Since you'll play B, since I'm committed to always playing b, it's in my best interests to stick to this strategy."

Incredible Threats

What makes $\langle B, bb \rangle$ not subgame perfect, what makes it an incredible threat, is that A can make the following reply.

"That's an interesting plan. And if it was just a strategic game, I might even believe it. But the problem for you is that you have to stick to that bluff once you know that it's been called. To commit to always playing b means playing b even when you know I've played A. And I don't reckon you'll do it – it's worse for me (which doesn't matter), and it's worse for you (which does). If we were just choosing strategies, I might just about believe that you would adopt a disposition that's bad in some circumstances in the hope that by adopting it, you'll guarantee that those circumstances don't arise. But when you have to play in real time, I don't think you can do it."

Incredible Threats

So I plays A, and they end up at the 4,1 outcome.

Day Plan

Strategies in Dynamic Games

Incredible Threats

The Backward Induction Paradox

Zero-Sum Turn-Taking Games

Reading

- No required reading, but if you want to see more, read "The Backward Induction Paradox" by Philip Pettit and Robert Sugden, *Journal of Philosophy* 1989.

Backward Induction in Economics

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Backward Induction in Economics

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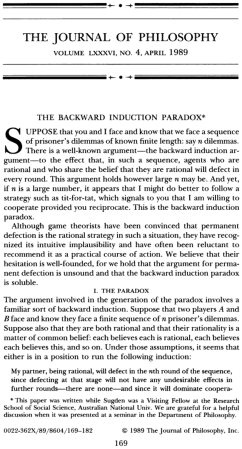
- What he meant, and what’s true, is that among mainstream economists, this is more controversial than whether the reasoning behind it is sound.
- In philosophy there is somewhat more controversy.

The Backward Induction Paradox



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THE BACKWARD INDUCTION PARADOX*

SUPPOSE that you and I face and know that we face a sequence of prisoner's dilemmas of known finite length: say *n* dilemmas. There is a well-known argument—the backward induction argument—to the effect that, in such a sequence, agents who are rational and who share the belief that they are rational will defect in every round. This argument holds however large *n* may be. And yet, if *n* is a large number, it appears that I might do better to follow a strategy such as tit-for-tat, which signals to you that I am willing to cooperate provided you reciprocate. This is the backward induction paradox.

Although game theorists have been convinced that permanent defection is the rational strategy in such a situation, they have recognized its intuitive implausibility and have often been reluctant to recommend it as a practical course of action. We believe that their hesitation is well-founded, for we hold that the argument for permanent defection is unsound and that the backward induction paradox is soluble.

I. THE PARADOX

The argument involved in the generation of the paradox involves a familiar sort of backward induction. Suppose that two players *A* and *B* face and know they face a finite sequence of *n* prisoner's dilemmas. Suppose also that they are both rational and that their rationality is a matter of common belief: each believes each is rational, each believes each believes this, and so on. Under those assumptions, it seems that either is in a position to run the following induction:

> My partner, being rational, will defect in the *n*th round of the sequence, since defecting at that stage will not have any undesirable effects in further rounds—there are none—and since it will dominate coopera-

* This paper was written while Sugden was a Visiting Fellow at the Research School of Social Science, Australian National Univ. We are grateful for a helpful discussion when it was presented at a seminar in the Department of Philosophy.

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Pettit and Sugden's Paper

That's in part due to this paper.

Iterated Prisoners Dilemma

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Iterated Prisoners Dilemma

- It's time to get on the table a game we'll be spending some time on: Iterated Prisoners Dilemma.
- It turns out the central event in the history of the study of this game happened at the University of Michigan, but that's a story for another day.
- A and B will play 100 rounds of the game on the next slide

Axelrod's Version of Prisoners Dilemma

	Coop	Defect
Coop	3, 3	0, 5
Defect	5, 0	1, 1

Scoring

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- This is still a non-competitive game: they are trying to maximise points, not maximise lead over the other.
- But the points add up over all the rounds. (And they don't decay or melt.)
- So each party wants to maximise their sum score over 100 plays of the game.

Strategy

- At each play, each party knows what the other did on all the previous rounds.

Strategy

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- The strategic form of this is impossibly big; even the two round game has 32 strategies per player, so 1024 cells.

One Shot Reasoning

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3. So either way, I'm better off Defecting.
4. So, I'm better off Defecting.

Repeated Play

But in round one of a repeated game, the following reasoning also looks sound.

1. The best outcome in the long run is if we both Cooperate as much as possible.

Repeated Play

But in round one of a repeated game, the following reasoning also looks sound.

1. The best outcome in the long run is if we both Cooperate as much as possible.
2. A plausible way to get that would be to signal that I will Cooperate if, but only if, the other player does.

Repeated Play

3. A natural way to implement that is to start Cooperating, then Defect when the other player does (this strategy has become known as Tit-for-Tat).

Repeated Play

3. A natural way to implement that is to start Cooperating, then Defect when the other player does (this strategy has become known as Tit-for-Tat).
4. So at round 1 I'll cooperate - if the other player is thinking the same way as me, we'll both make a lot of utility, and relative to how much there is to gain, it's only a small loss if I'm wrong.

Backward Induction

But there is a counter argument.

1. At round 100, there is no signalling value of Cooperating; I just get more from Defecting.

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2. Everyone knows this is true.
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4. Everyone knows this is true.

Backward Induction

But there is a counter argument.

1. At round 100, there is no signalling value of Cooperating; I just get more from Defecting.
2. Everyone knows this is true.
3. So at round 99, there is no signalling value of Cooperating; the other player will Defect at round 100 whatever I do at 99.
4. Everyone knows this is true.
5. So at round 98, there is no signalling value of Cooperating;...

Temporary Conclusion

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- Backward induction suggests that we should defect every round.
- Eventually there will be no signalling benefit to cooperation, and backward induction pushes the moment where that happens back to the start of the game.

Pettit and Sugden

This reasoning is self-defeating.

- Imagine I'm thinking about cooperating for signalling purposes at round one.

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Pettit and Sugden

This reasoning is self-defeating.

- Imagine I'm thinking about cooperating for signalling purposes at round one.
- I might worry that the other player will defect come what may at round 2 because of the backward induction argument.
- But the premises of the backward induction argument imply that I'll defect at round 1.

Pettit and Sugden

- And at round 2, the other player will know that I did not actually defect at round 1.

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- So I should only worry if I think the other player will use an argument whose premises they know to be false.

Pettit and Sugden

- And at round 2, the other player will know that I did not actually defect at round 1.
- So I should only worry if I think the other player will use an argument whose premises they know to be false.
- And that's not something to worry about.

Short Version

To give up on cooperation requires believing that the other player will think as follows.

- Game theoretic rationality requires defection at every round, so that's what the other player will do from round 3 onwards, so I may as well defect.

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Short Version

To give up on cooperation requires believing that the other player will think as follows.

- Game theoretic rationality requires defection at every round, so that's what the other player will do from round 3 onwards, so I may as well defect.
- And I know that the other player will do what's game theoretically rational even though they totally did not do that the very last time I interacted with them.
- That's absurd.

Game Theorists Respond

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- You should always think the other player is rational.
- If you observe a departure from rationality, you should assume it is a performance error, not a competence error (to use Chomsky's terminology).
- Or, to use the terminology of game theorists, you should assume it was a "trembling hand" error.

A Further Puzzle

- The argument for defecting at round 100 is unaffected by Pettit and Sugden's argument, you should totally defect then.
- And I'm not sure that the argument for defecting at round 99 is affected either.
- Is round 98 different?
- If you are convinced by their argment that the backward induction argument fails in general, when does it start failing?

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Zero-Sum Turn-Taking Games

A Famous Theorem

Every turn taking zero sum game has a **value**.

The Class of Games Under Discussion

- Two-player
- Turn-taking
- Finite
- No hidden facts
- No randomness
- Zero-sum

That is a lot of restrictions, but it includes classic games like chess, checkers, go, othello and more.

Theorem

Every one of these games has a **value**.

- The value of the game is an outcome that each player can guarantee that they get at least that good a result.

Theorem

Every one of these games has a **value**.

- The value of the game is an outcome that each player can guarantee that they get at least that good a result.
- Since the game is zero-sum, it follows that no player can guarantee that they do better.

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- The value of that node is the best value of the subsequent nodes, by the lights of the player who has to play.
- So if player 1 has to play, and one nodes leads to a draw, and the other to Player 2 winning, the value of that node is draw.

Proof Schema

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- Since the game is finite, a finite number of steps of this process will assign a value to every node in the tree.
- So eventually the initial node will get a value.
- And that's the value of the game.

Proof Schema

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Proof Schema

- The way we've constructed that value shows that each player always has a path to ensure they never do worse.
- If it is their move, there will be a move that preserves value.
- And if it is the other players' move, the value is by definition the most harm they can do with that move.

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Consequences

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- Whatever the tree is, we know the table will have one of the following two features.

1. There is a row where every result is that Player 1 wins; or
2. There is a column where every result is that Player 2 wins.

Consequences

- In chess, at least one player has a strategy that guarantees not losing.
- For some chess like games this is not a surprise – if player 2 had a winning strategy, player 1 could as it were execute it first. This isn't quite right for chess, but no one really thinks that player 2 has a winning strategy in chess.
- Probably the value of chess is a draw, but this isn't known yet.

Solved Games

Solved games [edit]

Awari (a game of the Mancala family)

The variant of Oware allowing game ending "grand slams" was strongly solved by Henri Bal and John Romein at the Vrije Universiteit in Amsterdam, Netherlands (2002). Either player can force the game into a draw.

Chopsticks

The second player can always force a win.[citation needed]

Connect Four

Solved first by James D. Allen on October 1, 1988, and independently by Victor Allis on October 16, 1988.[6] The first player can force a win. Strongly solved by John Tromp's 8-ply database[7] (Feb 4, 1995). Weakly solved for all boardsizes where width+height is at most 15 (as well as 8×8 in late 2015)[6] (Feb 18, 2006).

English draughts (checkers)

This 8×8 variant of draughts was **weakly solved** on April 29, 2007, by the team of Jonathan Schaeffer. From the standard starting position, both players can guarantee a draw with perfect play.[8] Checkers is the largest game that has been solved to date, with a search space of 5×10^{20}.[9] The number of calculations involved was 10^{14}, which were done over a period of 18 years. The process involved from 200 desktop computers at its peak down to around 50.[10]

Fanorona

Weakly solved by Maarten Schadd. The game is a draw.[citation needed]

Free gomoku

Solved by Victor Allis (1993). The first player can force a win without opening rules.

Ghost

Solved by Alan Frank using the *Official Scrabble Players Dictionary* in 1987.[citation needed]

Guess Who?

Strongly solved by Mihai Nica in 2016.[11] The first player has a 63% chance of winning under optimal play by both sides.

Hex

- A strategy-stealing argument (as used by John Nash) shows that all square board sizes cannot be lost by the first player. Combined with a proof of the impossibility of a draw this shows that the game is ultra-weak solved as a first player win.
- Strongly solved by several computers for board sizes up to 6×6.
- Jing Yang has demonstrated a winning strategy (weak solution) for board sizes 7×7, 8×8 and 9×9.
- A winning strategy for Hex with swapping is known for the 7×7 board.
- Strongly solving Hex on an $N \times N$ board is unlikely as the problem has been shown to be PSPACE-complete.
- If Hex is played on an $N \times (N+1)$ board then the player who has the shorter distance to connect can always win by a simple pairing strategy, even with the disadvantage of playing second.
- A weak solution is known for all opening moves on the 8×8 board.[12]

Hexapawn

3×3 variant solved as a win for black, several other larger variants also solved.[13]

Kalah

Most variants solved by Geoffrey Irving, Jeroen Donkers and Jos Uiterwijk (2000) except Kalah (6/6). The (6/6) variant was solved by Anders Carstensen (2011). Strong first-player advantage was proven in most cases.[14][15] Mark Rawlings, of Gaithersburg, MD, has quantified the magnitude of the first player win in the (6/6) variant (2015). After creation of 39 GB of endgame databases, searches totaling 106 days of CPU time and over 55 trillion nodes, it was proven that, with perfect play, the first player wins by 2. Note that all these results refer to the Empty-pit Capture variant and therefore are of very limited interest for the standard game. Analysis of the standard rule game has now been posted for Kalah(6,4), which is a win by 8 for the first player, and Kalah(6,5), which is a win by 10 for the first player. Analysis of Kalah(6,6) with the standard rules is on-going, however, it has been proven that it is a win by at least 4 for the first player.

L game

Easily solvable. Either player can force the game into a draw.

Losing chess

Weakly solved as a win for white beginning with 1. e3.[16]

Maharajah and the Sepoys

This asymmetrical game is a win for the sepoys player with correct play.

Nim

Strongly solved.

Wikipedia page on solved games

For Next Week

We will start chapter 4, on games involving hidden information.