Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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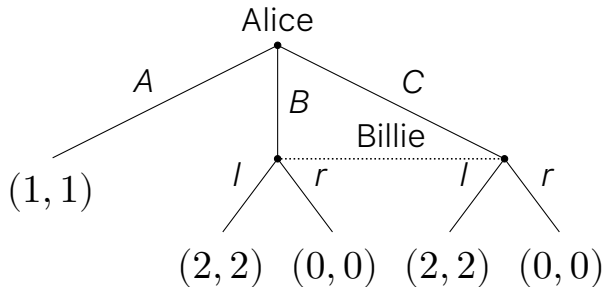# 444 Lecture 11

Games and Probability

Brian Weatherson

2/14/23

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Day Plan

Bayesian Equilibrium

Two Puzzles

Signaling Games

Bayesian Equilibrium
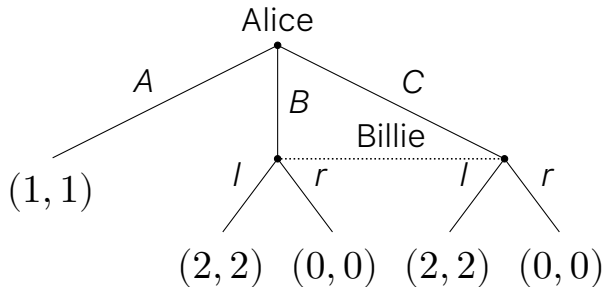○●○○○○○○○○

Two Puzzles
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Signaling Games
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What is Rational Here?

- Intuitively we should end up with one of the 2,2 outcomes.

What is Rational Here?

- Intuitively we should end up with one of the 2,2 outcomes.

- But how theoretically can we get that?

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Strategy Table

	l	r
A	1, 1	1, 1
B	2, 2	0, 0
C	2, 2	0, 0

Note that $\langle A, r \rangle$ is a Nash equilibrium.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Subgame Perfection
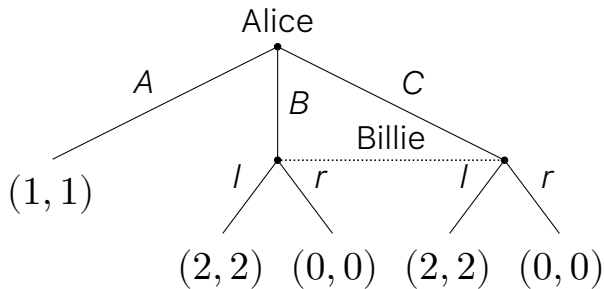
- There are no subgames (think about why) - so $\langle A, r \rangle$ is also subgame perfect.

Intuition

- It is absurd for Billie to play r if it gets that far.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Intuition

- It is absurd for Billie to play r if it gets that far.

- We need a theory that says this is absurd.

Intuition

- It is absurd for Billie to play r if it gets that far.

- We need a theory that says this is absurd.

- This is just the kind of thing subgame perfect equilibrium was introduced for, but it isn't working for technical reasons about the definition of subgames.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Intuition

- It is absurd for Billie to play r if it gets that far.

- We need a theory that says this is absurd.

- This is just the kind of thing subgame perfect equilibrium was introduced for, but it isn't working for technical reasons about the definition of subgames.

- I'm not sure if there is a completely standard solution here, but I wanted to set out an approach that's consistent with current philosophy.

Bayesian Equilibrium

- An equilibrium is a pair of behavioral dispositions.

Bayesian Equilibrium

- An equilibrium is a pair of behavioral dispositions.

- Each behavioral disposition gives a probability of each choice at each node the player may have to choose at.

Bayesian Equilibrium

- An equilibrium is a pair of behavioral dispositions.

- Each behavioral disposition gives a probability of each choice at each node the player may have to choose at.

- In equilibrium, each player has their own disposition from the pair, and believes (with certainty) that the other player has the other disposition from the pair.

Bayesian Equilibrium

- If there is a move made by Nature, each player has the correct probability for each of Nature's possible moves.

Bayesian Equilibrium

- If there is a move made by Nature, each player has the correct probability for each of Nature's possible moves.

- At every stage, each Player maximises expected utility given their beliefs about the other.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Re-formulation

This is more or less equivalent.

- Each player starts with a probability distribution over outcomes of the game.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Re-formulation

This is more or less equivalent.

- Each player starts with a probability distribution over outcomes of the game.

- In equilibrium, these are the same, and the players have correct probabilities about the moves nature will make.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Re-formulation

- When something happens (a move is revealed), the players update by conditionalisation if the event has positive probability.

Re-formulation

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- When something unexpected (probability zero) happens, the players just pick a new probability.

Re-formulation

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- When something unexpected (probability zero) happens, the players just pick a new probability.

- In equilibrium, both players know how both players are disposed to react in each of these cases.

Re-formulation

- When something happens (a move is revealed), the players update by conditionalisation if the event has positive probability.

- When something unexpected (probability zero) happens, the players just pick a new probability.

- In equilibrium, both players know how both players are disposed to react in each of these cases.

- Everyone is always maximising expected utility.
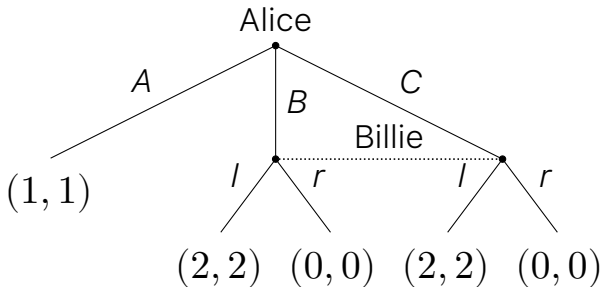
- At the B/C information set, Billie must have some probability between B and C.

- Whatever it is, l has higher expected utility that r.

- Alice knows this, so will choose B or C rather than A.

Day Plan

Bayesian Equilibrium

Two Puzzles

Signaling Games

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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First Puzzle
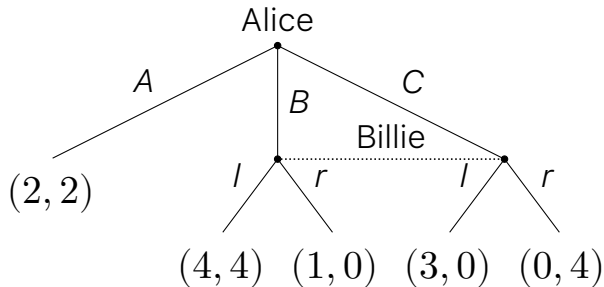
- I don't really know what to do here.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Strategy Table

	l	r
A	2, 2	2, 2
B	4, 4	1, 0
C	3, 0	0, 4

Note that $\langle A, r \rangle$ is a Nash equilibrium, and B dominates C.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Bayesian Equilibrium

The following is even a Bayesian Equilibrium

- Alice plays A.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Bayesian Equilibrium

The following is even a Bayesian Equilibrium

- Alice plays A.

- Billie believes with probability 1 that Alice will play A.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Bayesian Equilibrium

The following is even a Bayesian Equilibrium

- Alice plays A.

- Billie believes with probability 1 that Alice will play A.

- If Billie finds herself at the B/C set, she will believe that Alice played C.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Bayesian Equilibrium

The following is even a Bayesian Equilibrium

- Alice plays A.

- Billie believes with probability 1 that Alice will play A.

- If Billie finds herself at the B/C set, she will believe that Alice played C.

- So she will do what's best for her given this belief, i.e., play r.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Bayesian Equilibrium

The following is even a Bayesian Equilibrium

- Alice plays A.

- Billie believes with probability 1 that Alice will play A.

- If Billie finds herself at the B/C set, she will believe that Alice played C.

- So she will do what's best for her given this belief, i.e., play r.

- Alice knows this.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Bayesian Equilibrium

Alice plays A and gets 2, and she believes she would get 1 if she played B and 0 if she played C. So she's doing the thing that's best by her lights.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Wait a Minute!

- Billie is disposed to believe that if Alice does something weird (play B or C), she will do something really weird (play a dominated strategy).

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Wait a Minute!

- Billie is disposed to believe that if Alice does something weird (play B or C), she will do something really weird (play a dominated strategy).

- Is this an OK thing for Billie to believe?

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Wait a Minute!

- Billie is disposed to believe that if Alice does something weird (play B or C), she will do something really weird (play a dominated strategy).

- Is this an OK thing for Billie to believe?

- It seems a bit weird.

Wait a Minute!

- Maybe if Alice has not given Billie conclusive evidence that she will do something bizarre – i.e., play a dominated strategy – it's wrong to believe that she's done something bizarre.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Wait a Minute!

- Maybe if Alice has not given Billie conclusive evidence that she will do something bizarre – i.e., play a dominated strategy – it's wrong to believe that she's done something bizarre.

- So if we get to B/C, Billie should believe we're at B, so should play l, and knowing that, Alice should play B.
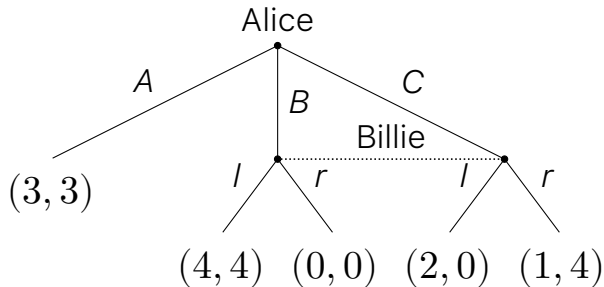
Which is right?

- I don't really know.

Which is right?

- I don't really know.

- Here we're already into contested territory.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Which is right?

- I don't really know.

- Here we're already into contested territory.

- Let's take one further step into contested territory.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Second Puzzle

- I really don't know what to do here.

Why is it Puzzling

- $\langle A, r \rangle$ is a Bayesian (and Nash and subgame perfect) equilibrium.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Why is it Puzzling

- $\langle A, r \rangle$ is a Bayesian (and Nash and subgame perfect) equilibrium.

- But maybe Alice should choose B.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Why is it Puzzling

- $\langle A, r \rangle$ is a Bayesian (and Nash and subgame perfect) equilibrium.

- But maybe Alice should choose B.

- Note that A dominates C.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Why is it Puzzling

- $\langle A, r \rangle$ is a Bayesian (and Nash and subgame perfect) equilibrium.

- But maybe Alice should choose B.

- Note that A dominates C.

- So if Billie knows that Alice doesn't choose dominated options, and this survives learning that Alice has chosen B or C, then Billie will know that Alice has chosen B.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Why is it Puzzling

- $\langle A, r \rangle$ is a Bayesian (and Nash and subgame perfect) equilibrium.

- But maybe Alice should choose B.

- Note that A dominates C.

- So if Billie knows that Alice doesn't choose dominated options, and this survives learning that Alice has chosen B or C, then Billie will know that Alice has chosen B.

- And then Billie will choose l, so Alice should choose B.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Two Big Questions

1. Are there philosophical grounds to change our theory of equilibrium selection to make Alice choose B in both of these games?

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Two Big Questions

1. Are there philosophical grounds to change our theory of equilibrium selection to make Alice choose B in both of these games?

2. If we want to do that, what's the best mathematical theory that generates the intended result?

Two Big Questions

- Both of these are hard questions, but not one's I'm going to address in this course.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Two Big Questions

- Both of these are hard questions, but not one's I'm going to address in this course.

- We have to stop somewhere, and there are always more questions like this to ask and answer.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Two Big Questions

- Both of these are hard questions, but not one's I'm going to address in this course.

- We have to stop somewhere, and there are always more questions like this to ask and answer.

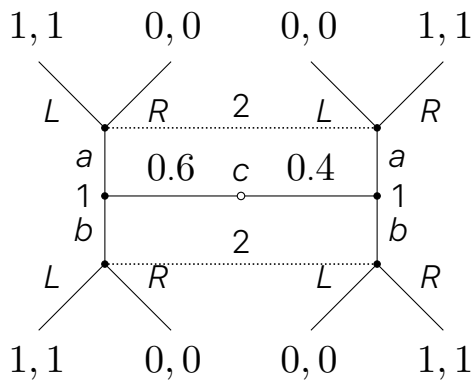- Instead I'm going to a special class of games: signaling games.

Bayesian Equilibrium
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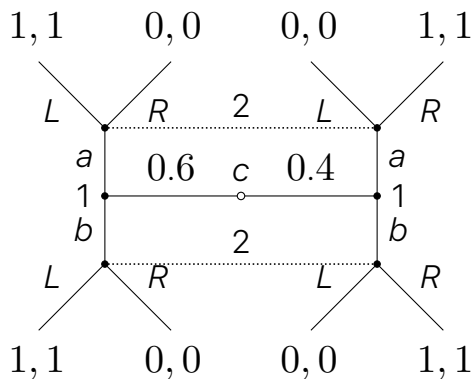
Two Puzzles
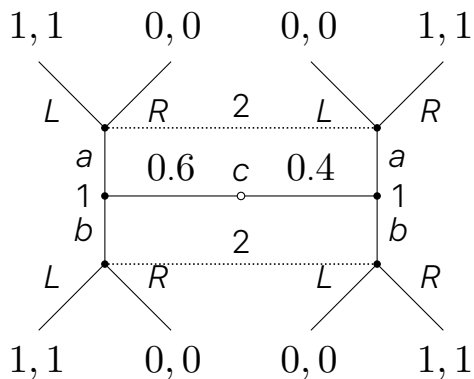○○○○○○○○○○○○○

Signaling Games
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Day Plan
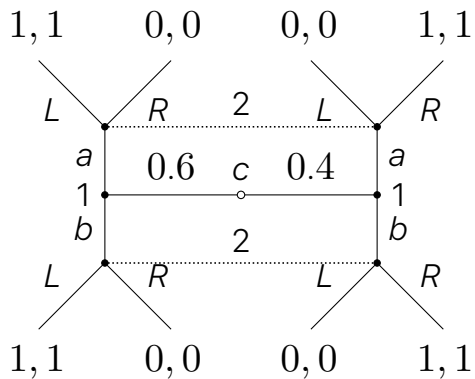
This is a basic signaling game.
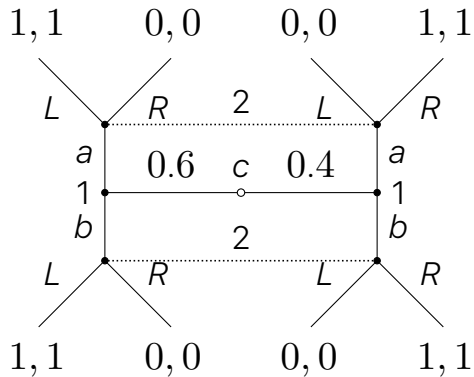
1. Nature moves left or right, and reveals this to Sender (1).

1. Nature moves left or right, and reveals this to Sender (1).

2. Sender moves up or down, and reveals this to Hearer (2).

1. Nature moves left or right, and reveals this to Sender (1).

2. Sender moves up or down, and reveals this to Hearer (2).

3. Hearer moves left or right.

- This is a purely cooperative version.
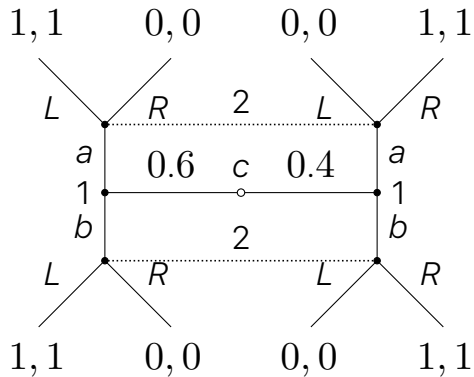
- This is a purely cooperative version.
- Each player gets 1 if Hearer makes same move as Nature, and 0 otherwise.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Four Equilibria

1. Sender does Up if Left, Down if Right. Hearer does what Nature does.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Four Equilibria

1. Sender does Up if Left, Down if Right. Hearer does what Nature does.

2. Sender does Up if Right, Down if Left. Hearer does what Nature does.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Four Equilibria

1. Sender does Up if Left, Down if Right. Hearer does what Nature does.

2. Sender does Up if Right, Down if Left. Hearer does what Nature does.

3. Sender does the same thing no matter what. Hearer does Left no matter what.

Four Equilibria

1. Sender does Up if Left, Down if Right. Hearer does what Nature does.

2. Sender does Up if Right, Down if Left. Hearer does what Nature does.

3. Sender does the same thing no matter what. Hearer does Left no matter what.

4. Sender speaks randomly. Hearer does Left no matter what.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Equilibria Types

- The first two are **separating** equilibria. This is what we hope happens!

Equilibria Types

- The first two are **separating** equilibria. This is what we hope happens!

- The third is a **pooling** equilibria. This is not great.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Equilibria Types

- The first two are **separating** equilibria. This is what we hope happens!

- The third is a **pooling** equilibria. This is not great.

- The fourth is a **babbling** equilibria. This might be worse.

Bayesian Equilibrium
oooooooooooo

Two Puzzles
ooooooooooooo

Signaling Games
ooooooo●ooooo

Equilibria Types

When nature has only two possible states, there isn't much conceptual difference between pooling and babbling. But in the more general case, we want to distinguish between partially pooling equilibria and partially babbling equilibria.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Conceptual Load

- We're normally interested in games where each player is perfectly rational, and is basically a genius game player.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Conceptual Load

- We're normally interested in games where each player is perfectly rational, and is basically a genius game player.

- But it's important for the purposes that signaling games are put that you can sustain a separating equilibrium when the players are really not very sophisticated.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Conceptual Load

- Sender needs to be able to detect what Nature is doing, and differentially respond.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Conceptual Load

- Sender needs to be able to detect what Nature is doing, and differentially respond.

- Hearer needs to differentially respond to Sender's response – so they also need to be able to detect what Sender is doing, and use that detection to guide action.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Conceptual Load

- Sender needs to be able to detect what Nature is doing, and differentially respond.

- Hearer needs to differentially respond to Sender's response – so they also need to be able to detect what Sender is doing, and use that detection to guide action.

- That's not nothing – two rocks couldn't do it – but it's not a lot.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
○○○○○○○○○○●○

Tiny Players

- Cells, for example, can detect some things and differentially respond.

Bayesian Equilibrium
OOOOOOOOOO

Two Puzzles
OOOOOOOOOOOO

Signaling Games
OOOOOOOOOO●O

Tiny Players

- Cells, for example, can detect some things and differentially respond.

- Certainly animals can do this.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Tiny Players

- Cells, for example, can detect some things and differentially respond.

- Certainly animals can do this.

- And probably a lot of plants can as well, at least for some possible signals.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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Tiny Players

- Cells, for example, can detect some things and differentially respond.

- Certainly animals can do this.

- And probably a lot of plants can as well, at least for some possible signals.

- So you can use signaling games to explain all sorts of biological interactions, both between organisms and within them.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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For Next Time

- But this isn't a (philosophy of) biology class, so I'll leave that to another day.

Bayesian Equilibrium
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Two Puzzles
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Signaling Games
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For Next Time

- But this isn't a (philosophy of) biology class, so I'll leave that to another day.

- Next time, we'll look at some important uses of signaling games in social sciences.