# 444 Lecture 11

## Games and Probability

Brian Weatherson

2/14/23

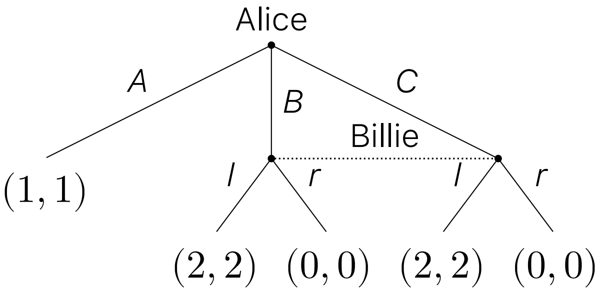
# Day Plan

Bayesian Equilibrium

Two Puzzles

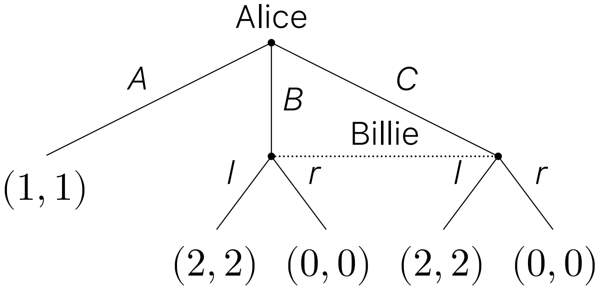
Signaling Games

# What is Rational Here?




- Intuitively we should end up with one of the 2,2 outcomes.

# What is Rational Here?



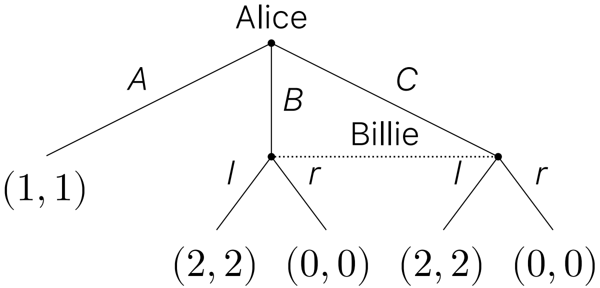

- Intuitively we should end up with one of the 2,2 outcomes.
- But how theoretically can we get that?

# Strategy Table

| | l | r |
|---|---|---|
| A | 1, 1 | 1, 1 |
| B | 2, 2 | 0, 0 |
| C | 2, 2 | 0, 0 |

Note that $\langle A, r \rangle$ is a Nash equilibrium.

# Subgame Perfection




- There are no subgames (think about why) - so $\langle A, r\rangle$ is also subgame perfect.

# Intuition

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- It is absurd for Billie to play r if it gets that far.
- We need a theory that says this is absurd.
- This is just the kind of thing subgame perfect equilibrium was introduced for, but it isn't working for technical reasons about the definition of subgames.
- I'm not sure if there is a completely standard solution here, but I wanted to set out an approach that's consistent with current philosophy.

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- Each behavioral disposition gives a probability of each choice at each node the player may have to choose at.
- In equilibrium, each player has their own disposition from the pair, and believes (with certainty) that the other player has the other disposition from the pair.

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- If there is a move made by Nature, each player has the correct probability for each of Nature's possible moves.
- At every stage, each Player maximises expected utility given their beliefs about the other.

# Re-formulation

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- Each player starts with a probability distribution over outcomes of the game.
- In equilibrium, these are the same, and the players have correct probabilities about the moves nature will make.

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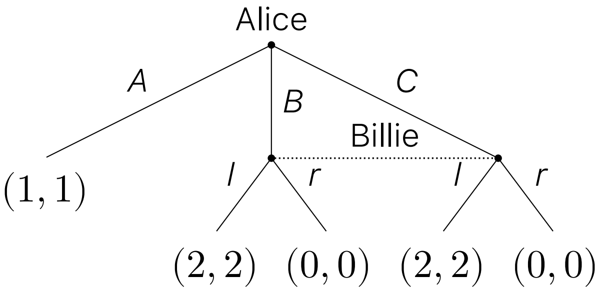
# Re-formulation

- When something happens (a move is revealed), the players update by conditionalisation if the event has positive probability.
- When something unexpected (probability zero) happens, the players just pick a new probability.
- In equilibrium, both players know how both players are disposed to react in each of these cases.
- Everyone is always maximising expected utility.




- At the B/C information set, Billie must have some probability between B and C.
- Whatever it is, l has higher expected utility that r.
- Alice knows this, so will choose B or C rather than A.

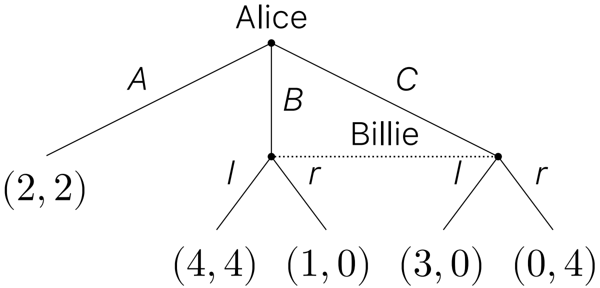
# Day Plan

Bayesian Equilibrium

Two Puzzles

Signaling Games

# First Puzzle




- I don't really know what to do here.

# Strategy Table

| | l | r |
|---|---|---|
| A | 2, 2 | 2, 2 |
| B | 4, 4 | 1, 0 |
| C | 3, 0 | 0, 4 |

Note that $\langle A, r \rangle$ is a Nash equilibrium, and B dominates C.

# Bayesian Equilibrium

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- Alice plays A.

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- So she will do what's best for her given this belief, i.e., play r.

# Bayesian Equilibrium

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- Alice plays A.
- Billie believes with probability 1 that Alice will play A.
- If Billie finds herself at the B/C set, she will believe that Alice played C.
- So she will do what's best for her given this belief, i.e., play r.
- Alice knows this.

# Bayesian Equilibrium

Alice plays A and gets 2, and she believes she would get 1 if she played B and 0 if she played C. So she's doing the thing that's best by her lights.

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- Billie is disposed to believe that if Alice does something weird (play B or C), she will do something really weird (play a dominated strategy).
- Is this an OK thing for Billie to believe?
- It seems a bit weird.

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- Maybe if Alice has not given Billie conclusive evidence that she will do something bizarre - i.e., play a dominated strategy - it's wrong to believe that she's done something bizarre.

# Wait a Minute!

- Maybe if Alice has not given Billie conclusive evidence that she will do something bizarre - i.e., play a dominated strategy - it's wrong to believe that she's done something bizarre.
- So if we get to B/C, Billie should believe we're at B, so should play I, and knowing that, Alice should play B.

# Which is right?

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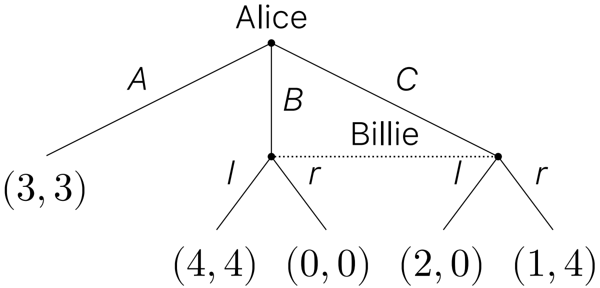
- I don’t really know.
- Here we’re already into contested territory.
- Let’s take one further step into contested territory.

# Second Puzzle




- I really don't know what to do here.

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- But maybe Alice should choose B.
- Note that A dominates C.
- So if Billie knows that Alice doesn't choose dominated options, and this survives learning that Alice has chosen B or C, then Billie will know that Alice has chosen B.
- And then Billie will choose l, so Alice should choose B.

# Two Big Questions

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1. Are there philosophical grounds to change our theory of equilibrium selection to make Alice choose B in both of these games?
2. If we want to do that, what's the best mathematical theory that generates the intended result?

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- We have to stop somewhere, and there are always more questions like this to ask and answer.
- Instead I'm going to a special class of games: signaling games.

# Day Plan

Bayesian Equilibrium

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Signaling Games

# Handout

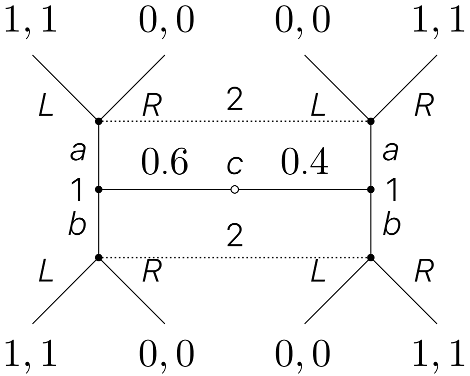
I used to do much more in this class on signaling games, but I dropped it because it's not relevant to the applications we're going to primarily work on in the second half.

- But there is a very long handout from the longer form of this unit on the website.

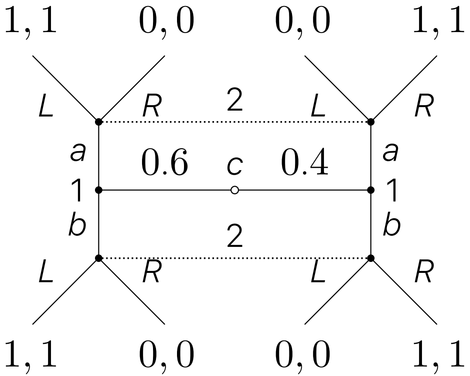
# Handout

I used to do much more in this class on signaling games, but I dropped it because it's not relevant to the applications we're going to primarily work on in the second half.

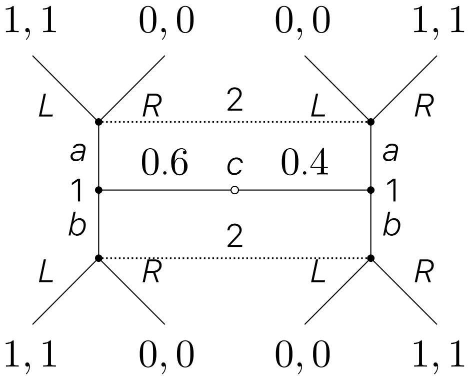
- But there is a very long handout from the longer form of this unit on the website.
- We really won't cover much of the stuff in it, but if you're interested in pointers to more material on this stuff, it could be interesting.



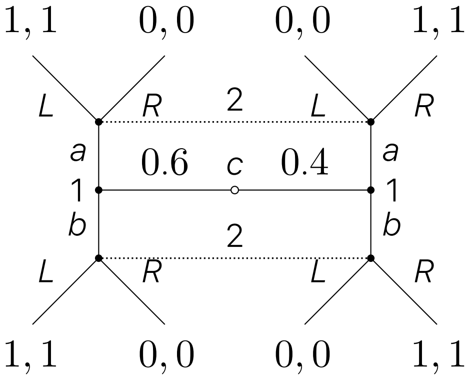

This is a basic signaling game.



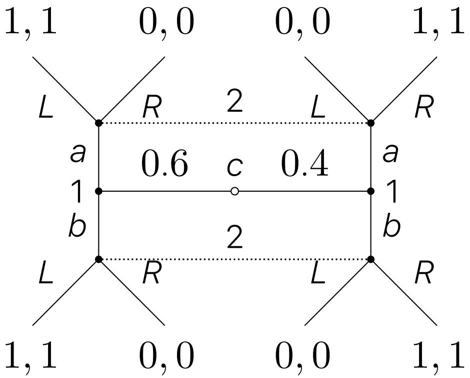

1. Nature moves left or right, and reveals this to Sender (1).



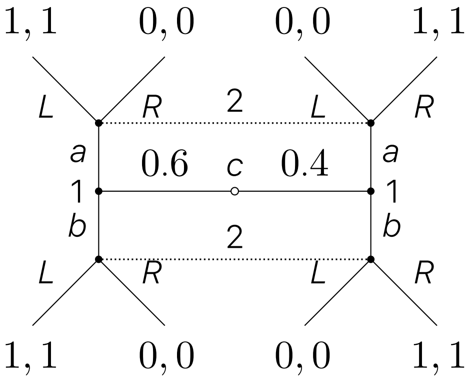

1. Nature moves left or right, and reveals this to Sender (1).
2. Sender moves up or down, and reveals this to Hearer (2).




1. Nature moves left or right, and reveals this to Sender (1).
2. Sender moves up or down, and reveals this to Hearer (2).
3. Hearer moves left or right.




- This is a purely cooperative version.




- This is a purely cooperative version.
- Each player gets 1 if Hearer makes same move as Nature, and 0 otherwise.

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3. Sender does the same thing no matter what. Hearer does Left no matter what.
4. Sender speaks randomly. Hearer does Left no matter what.

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- The third is a **pooling** equilibria. This is not great.
- The fourth is a **babbling** equilibria. This might be worse.

# Equilibria Types

When nature has only two possible states, there isn't much conceptual difference between pooling and babbling. But in the more general case, we want to distinguish between partially pooling equilibria and partially babbling equilibria.

# Conceptual Load

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- We're normally interested in games where each player is perfectly rational, and is basically a genius game player.
- But it's important for the purposes that signaling games are put that you can sustain a separating equilibrium when the players are really not very sophisticated.

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- Sender needs to be able to detect what Nature is doing, and differentially respond.
- Hearer needs to differentially respond to Sender's response – so they also need to be able to detect what Sender is doing, and use that detection to guide action.
- That's not nothing – two rocks couldn't do it – but it's not a lot.

# Tiny Players

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- Cells, for example, can detect some things and differentially respond.
- Certainly animals can do this.
- And probably a lot of plants can as well, at least for some possible signals.
- So you can use signaling games to explain all sorts of biological interactions, both between organisms and within them.

# For Next Time

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- But this isn't a (philosophy of) biology class, so I'll leave that to another day.
- Next time, we'll look at some important uses of signaling games in social sciences.