$\forall x \exists y\, Syx \vdash \forall x \exists y\, Sxy$

1	$\forall x \exists y\, Syx$		
2	$\neg \forall x \exists y\, Sxy$	✓	
3	$\exists x \neg \exists y\, Sxy$	✓	2, $\neg\forall$
4	$\neg \exists y\, S1y$	✓	3, $\exists$
5	$\forall y \neg S1y$		4, $\neg\exists$
6	$\exists y\, Sy1$	✓	1, $\forall$
7	$\neg S11$		5, $\forall$
8	$S21$		6, $\exists$
9	$\exists y\, Sy2$	✓	1, $\forall$
10	$\neg S12$		5, $\forall$
11	$S32$		9, $\exists$
12	$\exists y\, Sy3$	✓	1, $\forall$
13	$\neg S13$		5, $\forall$
14	$S43$		12, $\exists$

Reflexive $\begin{tikzcd} \cdot \arrow[loop above] \end{tikzcd}$

Serial — No dead ends

Symmetric $\cdot \longleftrightarrow \cdot$

Transitive $\begin{tikzcd} \cdot \arrow[r] \arrow[rr, bend right] & \cdot \arrow[r] & \cdot \end{tikzcd}$

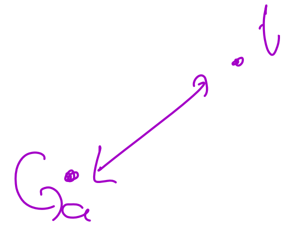
1	$\forall x\, Sxx$
2	$\neg \forall x \exists y\, Sxy$
3	$\exists x \neg \exists y\, Sxy$
4	$\neg \exists y\, Say$
5	$\forall y\, \neg Say$
6	$\neg Saa$
7	Saa
	$\times$

To prove: Saa

Since S is serial, there is a b such that Sab

Since S is symmetric, and Sab, it follows that Sba

Since S is transitive and Sab and Sba, Saa



1 $\forall x \exists y \, Sxy$

2 $\forall x \forall y \, (Sxy \supset Syx)$

3 $\forall x \forall y \forall z \, ((Sxy \wedge Syz) \supset Sxz)$

4 $\neg \forall x \, Sxx$ ✓

5 $\exists x \, \neg Sxx$ — 4, $\neg\forall$

6 $\neg Saa$ — 5, $\exists$

7 $\exists y \, Say$ — 1, $\forall$

8 Sab — 7, $\exists$

9 $Sab \supset Sba$ — 2, $\forall$

10 $\neg Sab$ | Sba — 9, $\supset$

× 8, 10 (left branch)

11 $(Sab \wedge Sba) \supset Saa$ — 3, $\forall$

12 $\neg (Sab \wedge Sba)$ | Saa — 11, $\supset$

× 6, 12 (right branch, under Saa)

13 $\neg Sab$ | $\neg Sba$

× 8, 13 | × 10, 13

$\forall x \exists y \, Sxy, \ \forall x \forall y \, (Sxy \supset Syx), \ \forall x \forall y \forall z \, ((Sxy \wedge Syz) \supset Sxz) \vdash \forall x \, Sxx$

There are at least 2 cats

$$\exists x \exists y (Cx \wedge Cy \wedge x \neq y)$$

$$\exists x \exists y (Cx \wedge Cy)$$
$$\exists y (Cf \wedge Cy)$$
$$Cf \wedge Cf$$

1. $a = b$
2. $\ldots a \ldots$

C. $\ldots b \ldots$

1. $a = b$
2. Dam

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3. Dbm