

The main topic for today is identity.

Identity is a two place predicate, but it has some special features.

To reflect that, we write it using 'infix' notation.

So we write " $a = b$ ", rather than " $=ab$ ".

And we write " $a \neq b$ " as shorthand for " $\neg a = b$ ".

Identity is part of logic.

This means that it has rules in tableau, and constraints on models.

The constraints on models are simple.

" $a = b$ " is true in a model iff $v(a)$ and $v(b)$ are the same thing.

So if you are trying to build a model where " $a=b$ " is true, make them the same.

There are two rules in tableau, both of them distinctive.

At any time, for any name 'a' in the tableau, you can write " $a=a$ ".

And if the branch includes " $a=b$ " or " $b=a$ ", and an **atomic** sentence $A(a)$, you can write $A(b)$.

This is the first rule that has two different lines as input.

Note that from " Saa " and " $a=b$ ", the rule lets you infer three things:

- Sab
- Sba
- Sbb

$\text{Pa} \mid - \exists x(x = a \wedge Px)$

1. Pa

2. $\neg \exists x(x = a \wedge Px)$

$\exists x(Px \wedge \forall y(Py \supset x = y)), \exists x(Px \wedge Qx) \mid - \forall y(Py \supset Qy)$

1. $\exists x(Px \wedge \forall y(Py \supset x = y))$

2. $\exists x(Px \wedge Qx)$

3. $\neg \forall y(Py \supset Qy)$

Here is an invalid one to show how to build models

$$\exists x P_x, a=b \mid - \exists x Q_x$$

1. $\exists x P_x$
2. $a=b$
3. $\neg \exists x Q_x$