COLLEGE

444 Lecture 23

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ASSIGNMENT

This week's assignment is about a signaling game.

- First Nature reveals a type (A or B);
- Then Proposer sends a signal (Left or Right);
- Then Responder, seeing the signal but not the state, chooses and action (Up or Down).

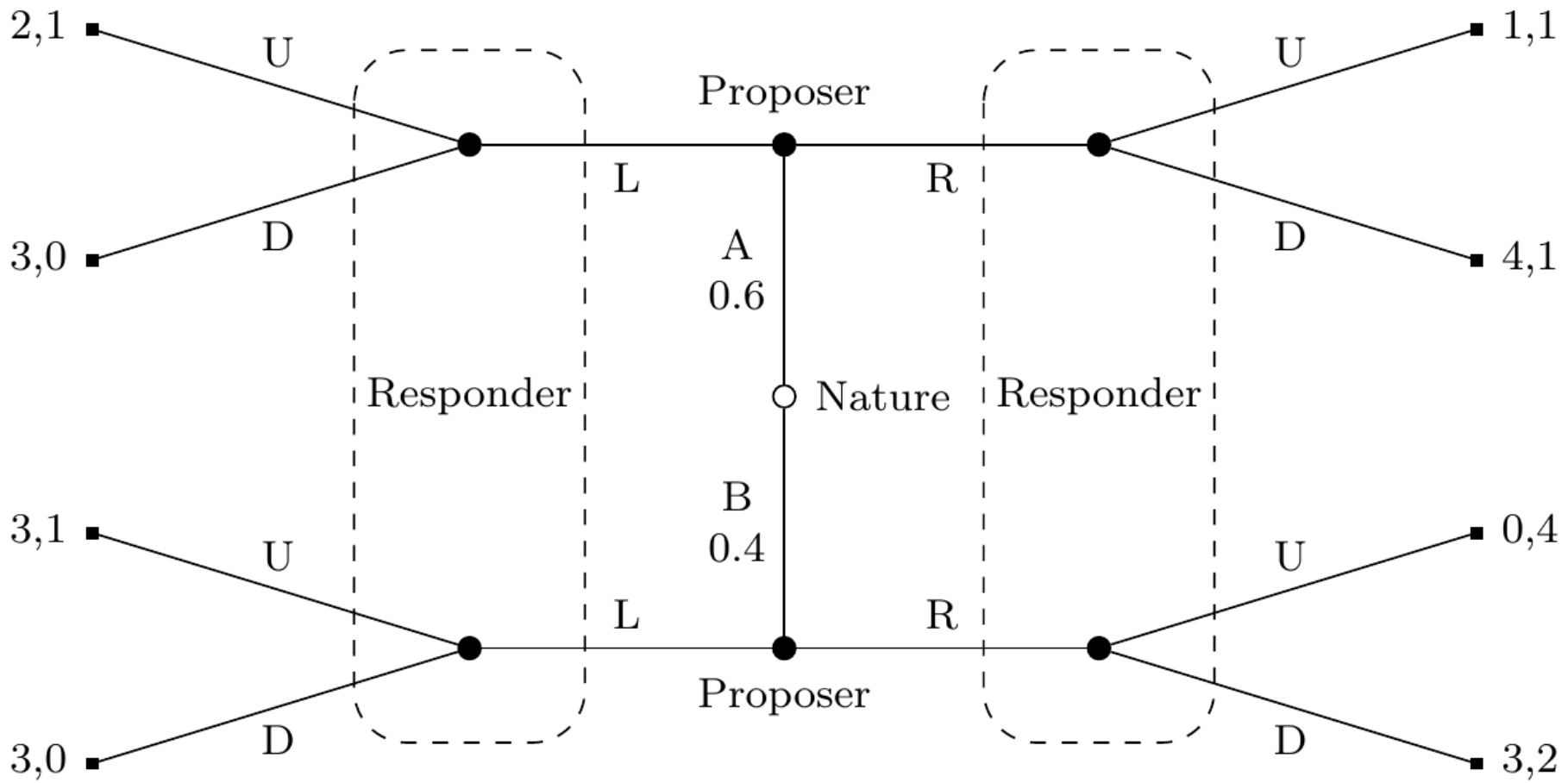

Figure 1: Tree for Weekly 5 practice version

Type	Proposer	Responder	Payouts
A	L	D	3, 0
A	L	U	2, 1
A	R	D	4, 1
A	R	U	1, 1
B	L	D	3, 0
B	L	U	3, 1
B	R	D	3, 2
B	R	U	0, 4

Table 1: Payouts for Weekly 5 practice version

STRATEGIES

In this tree, Proposer has four possible strategies:

1. Left if A, Left if B (LL)
2. Left if A, Right if B (LR)
3. Right if A, Left if B (RL)
4. Right if A, Right if B (RR)

STRATEGIES

And Responder has four possible strategies

1. Up if Left, Up if Right (UU)
2. Up if Left, Down if Right (UD)
3. Down if Left, Up if Right (DU)
4. Down if Left, Down if Right (DD)

STRATEGY PAIRS

So the strategy table has four rows for Proposer, and four columns for Responder, and that means 16 cells.

If there was no randomness, then for any pair, you could tell what each player would get.

But there is randomness. So what do we do?

ANSWERS

I'll start with the answers, then work back to how to figure them out.

P1	DD	DU	UD	UU
LL	3, 0	3, 0	2.4, 1	2.4, 1
LR	3, 0.8	1.8, 1.6	2.4, 1.4	1.2, 2.2
RL	3.6, 0.6	1.8, 0.6	3.6, 1	1.8, 1
RR	3.6, 1.4	0.6, 2.2	3.6, 1.4	0.6, 2.2

Table 2: Expected values for Weekly 1 practice version

BOTTOM RIGHT

Let's start in the bottom right, the 0.6, 2.2.

The formula for P1's expected return is:

- Probability of A times P1's return if A happens (and they play these strategies); plus
- Probability of B times P1's return if B happens (and they play these strategies)

BOTTOM RIGHT

Let’s start in the bottom right, the 0.6, 2.2.

The formula for P2’s expected return is:

- Probability of A times P2’s return if A happens (and they play these strategies); plus
- Probability of B times P2’s return if B happens (and they play these strategies)

BOTTOM RIGHT

If A happens, and P1 plays RR, that means P1 chooses R (the first letter in P1's strategy).

If P1 chooses R, and P2 plays UU, that means P2 chooses U (the second letter in P1's strategy).

So the row of the table we're on is where A, then R, then U happen.

Type	Proposer	Responder	Payouts
A	L	D	3, 0
A	L	U	2, 1
A	R	D	4, 1
A	R	U	1, 1
B	L	D	3, 0
B	L	U	3, 1
B	R	D	3, 2
B	R	U	0, 4

And at that row P1 gets 1, and P2 gets 1.

BOTTOM RIGHT

If B happens, and P1 plays RR, that means P1 chooses R (the first letter in P1's strategy).

If P1 chooses R, and P2 plays UU, that means P2 chooses U (the second letter in P1's strategy).

So the row of the table we're on is where B, then R, then U happen.

Type	Proposer	Responder	Payouts
A	L	D	3, 0
A	L	U	2, 1
A	R	D	4, 1
A	R	U	1, 1
B	L	D	3, 0
B	L	U	3, 1
B	R	D	3, 2
B	R	U	0, 4

And at that row P1 gets 0, and P2 gets 4.

THE FORMULA

So P1's expected return is:

- 0.6 times 1, plus 0.4 times 0
- which is 0.6.

And P2's expected return is:

- 0.6 times 1 plus 0.4 times 4
- which is 2.2

RL/UD

Let’s do one more of these: the one where Player 1 plays RL and Player 2 plays UD.

The formulae are still the same, but we have to be a bit more careful about the applications.

RL/UD

As the table says, the results are 3.6 and 1.0. Let's work to get there.

The formula for P1's expected return is:

- Probability of A times P1's return if A happens (and they play these strategies); plus
- Probability of B times P1's return if B happens (and they play these strategies)

RL/UD

If A happens, and P1 plays RL, that means P1 chooses R (the first letter in P1's strategy).

If P1 chooses R, and P2 plays UD, that means P2 chooses D (the second letter in P1's strategy).

So the row of the table we're on is where A, then R, then D happen.

Type	Proposer	Responder	Payouts
A	L	D	3, 0
A	L	U	2, 1
A	R	D	4, 1
A	R	U	1, 1
B	L	D	3, 0
B	L	U	3, 1
B	R	D	3, 2
B	R	U	0, 4

And at that row P1 gets 4, and P2 gets 1.

RL/UD

If B happens, and P1 plays RL, that means P1 chooses L (the second letter in P1's strategy).

If P1 chooses L, and P2 plays UD, that means P2 chooses U (the first letter in P1's strategy).

So the row of the table we're on is where B, then L, then U happen.

Type	Proposer	Responder	Payouts
A	L	D	3, 0
A	L	U	2, 1
A	R	D	4, 1
A	R	U	1, 1
B	L	D	3, 0
B	L	U	3, 1
B	R	D	3, 2
B	R	U	0, 4

And at that row P1 gets 3, and P2 gets 1.

THE FORMULA

So P1's expected return is:

- 0.6 times 4, plus 0.4 times 3
- which is 3.6.

And P2's expected return is

- 0.6 times 1, plus 0.4 times 1
- which is 1

EQUILIBRIA

Now we have to find the equilibria for the game.

An equilibria is a pair of choices where neither party can do better by changing their view.

So the top right corner: LL, UU is an equilibrium.

EQUILIBRIA

If P1 plays LL, then P2 gets (in expectation)

- 0 if they play DD;
- 0 if they play DU;
- 1 if they play UD;
- 1 if they play UU.

They can't do better than playing UU. (Ties aren't a problem.)

EQUILIBRIA

If P2 plays UU, then P1 gets (in expectation)

- 2.4 if they play LL;
- 1.2 if they play LR;
- 1.8 if they play RL;
- 0.6 if they play RR.

They can’t do better than playing LL.

FINDING EQUILIBRIA

We don’t want to have to check all 16 cells this way. Fortunately, we can speed things up.

There are a few approaches - here’s one.

FINDING EQUILIBRIA

Go through each column, and find the highest score for P1.

That will be P1's **best response** to that strategy for P2.

Note that there may be more than one of these, if there are ties.

FINDING EQUILIBRIA

Then for each of the things you find, check whether P2 can do better by switching.

If so, it's not an equilibria.

If not, it is one.

And you'll find all the equilibria this way.

EQUILIBRIA

P1	DD	DU	UD	UU
LL	3, 0	3, 0	2.4, 1	2.4, 1
LR	3, 0.8	1.8, 1.6	2.4, 1.4	1.2, 2.2
RL	3.6, 0.6	1.8, 0.6	3.6, 1	1.8, 1
RR	3.6, 1.4	0.6, 2.2	3.6, 1.4	0.6, 2.2

Table 3: Expected values for Weekly 1 practice version

Applying this technique shows that LL,UU and RL,UD are the only equilibria.

POOLING/SEPARATING

An equilibrium is **pooling** if P1 does the same thing in each state. So P1 plays LL or RR.

An equilibrium is **separating** if P1 does different things when the states are different. So P2 plays LR or RL.

FURTHER QUESTIONS

If we were doing more game theory, there are more questions we could ask at this point.

Some equilibria that you can find this way are not intuitively sensible.

FURTHER QUESTIONS

Sometimes that's because they imply P2 will do things that no longer make sense once they see P1's move. The notion of **sequential equilibrium** was developed to rule out these.

For real life cases, there are times when it is sensible to bluff, but not sensible to stick with the bluff once it has been exposed. But the procedure we've outlined here might include equilibria where committing to the bluff *even when the cards are shown* might be an equilibrium.

FURTHER QUESTIONS

And into the really not on the exam stuff, there is the question of what to do with the equilibrium in the US-UK game where P1 believes that P2 believes that P1 will buy tea if and only if American.

It is somewhat less clear that this creats real life problems.

COLLEGE AND SIGNALS

SOME STYLISED FACTS

1. College is expensive, both in actual costs and opportunity costs.
2. College graduates get a lot more money over their lifetime than non-graduates.
3. Just getting into a fancy college does not trigger this higher lifetime earning.
4. Completing some but not all of a degree does not trigger this higher lifetime earning.
5. The increase earning to college graduates tends to get larger as people get older.

SOURCES

Lots of data: https://www.oecd-ilibrary.org/education/education-at-a-glance-2023_e13bef63-en

These facts: https://www.oecd-ilibrary.org/sites/9633d9f3-en/index.html?itemId=/content/component/9633d9f3-en#tablegrp-d1e7393-3a40fea345

Table A4.1. **Relative earnings of workers compared to those with upper secondary attainment, by educational attainment and age group (2021)**

Adults with income from employment (full-time full-year workers); upper secondary attainment for each age group = 100

	Below upper secondary			Post-secondary non-tertiary			Tertiary											
							Short-cycle tertiary			Bachelor's or equivalent			Master's, doctoral or equivalent			Total		
	25-34 year-olds	45-54 year-olds	25-64 year-olds	25-34 year-olds	45-54 year-olds	25-64 year-olds	25-34 year-olds	45-54 year-olds	25-64 year-olds	25-34 year-olds	45-54 year-olds	25-64 year-olds	25-34 year-olds	45-54 year-olds	25-64 year-olds	25-34 year-olds	45-54 year-olds	25-64 year-olds
OECD countries	(1)	(2)	(3)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
Australia	93	91	89	102	104	99	93	118	106	116	139	127	116	168	148	112	140	127
Austria	85	74	77	116	112	114	117	135	127	113	133	104	142	186	171	125	159	144
Belgium[1]	80	85	86	c	c	112	c	c	c	114	138	127	137	187	158	126	158	142
Canada[1]	87	73	79	129	110	113	109	113	112	141	145	141	153	166	161	133	135	134
Chile[1]	78	68	71	a	a	a	123	154	138	214	342	279	345	496	457	190	277	241
Colombia[1,2]	72	69	71	m	m	m	x(19)	x(20)	x(21)	x(19)	x(20)	x(21)	x(19)	x(20)	x(21)	194	275	237
Costa Rica	82	69	77	c	c	c	130	150	138	185	203	210	c	330	339	177	218	212
Czech Republic[1]	71	66	67	m	m	m	124	118	117	122	143	131	142	180	167	135	174	159
Denmark	92	90	90	c	119	124	104	116	110	110	116	113	129	161	144	117	132	124
Estonia	93	91	92	99	89	93	m	100	89	122	129	132	152	154	148	135	137	135
Finland[1]	100	99	100	113	114	116	c	116	122	112	130	122	138	172	163	122	143	139
France[1,2]	78	95	89	m	m	m	102	133	129	112	185	151	149	229	189	127	177	157
Germany	63	84	72	113	116	113	117	138	132	138	153	152	139	201	184	135	165	158
Greece[1]	80	76	81	100	106	102	c	167	162	113	133	132	186	169	170	123	139	138
Hungary	77	76	76	115	126	123	119	129	128	141	159	156	171	234	216	154	185	179
Iceland	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m
Ireland[1]	c	81	84	105	97	96	c	124	123	147	172	147	171	226	184	155	178	156
Israel[1]	83	70	75	a	a	a	115	120	117	162	173	157	187	202	206	158	171	164
Italy[1,2]	96	71	80	m	m	m	x(13)	x(14)	x(15)	121[d]	101[d]	102[d]	128	130	148	125	125	138
Japan	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m
Korea	91	84	82	a	a	a	107	121	111	117	153	135	151	193	177	115	150	132
Latvia	73[b]	110[b]	82[b]	59[b]	91[b]	82[b]	127[b]	166	157[b]	132[b]	167[b]	156[b]	202[b]	166[b]	170[b]	152[b]	166[b]	163[b]
Lithuania[1]	93	91	92	101	108	106	a	a	a	148	157	167	172	192	193	154	181	180
Luxembourg[3]	73	90	84	c	92	100	113	128	134	129	147	140	136	166	158	133	157	151
Mexico[1]	86	75	80	a	a	a	109	116	117	139	180	153	209	323	308	139	185	158
Netherlands	88	82	86	104	109	105	128	129	131	118	141	132	139	210	177	127	166	149
New Zealand	102	89	92	102	103	102	110	122	115	119	137	125	122	166	150	118	140	128
Norway	83	87	85	104	94	99	103	124	119	99	115	107	117	150	134	106	128	119
Poland[1]	89	85	86	97	104	101	m	m	m	129	155	140	140	178	162	136	174	157
Portugal	86	75	83	115	120	113	116	105	106	x(19)	x(20)	x(21)	x(19)	x(20)	x(21)	153	193	171
Slovak Republic	89	80	82	m	m	m	106	126	122	116	135	126	126	170	156	124	165	153
Slovenia	87	82	84	a	a	a	110	135	131	129	159	142	149	201	183	135	178	163
Spain[1]	87	78	80	c	c	110	116	118	112	155	143	146	165	196	185	148	161	155
Sweden	91	83	86	97	123	116	105	112	108	107	122	116	124	154	145	112	133	126
Switzerland[2]	84	76	80	m	m	m	x(13, 16)	x(14, 17)	x(15, 18)	125[d]	140[d]	130[d]	142[d]	183[d]	162[d]	132	161	145
Türkiye[3]	80	74	77	a	a	a	x(19)	x(20)	x(21)	x(19)	x(20)	x(21)	x(19)	x(20)	x(21)	136	188	153
United Kingdom	60	75	73	a	a	a	121	117	122	137	146	139	152	164	160	140	145	143
United States	100	75	78	m	m	m	107	105	110	162	169	169	200	217	221	162	172	173
OECD average	84	81	82	m	m	m	113	126	122	132	153	143	157	201	188	138	168	156

Relative wage by educational attainment

US SPECIFIC

For most of these facts we can get very resilient data cross-nationally.

But there's one group, which will be important in what follows, for which we really have to focus on US data.

That's the 'some college' group.

SOME COLLEGE

Educational attainment in the United States (2018)[5]

Education	Age 25 and over	Age 25-30
High school diploma or GED	89.80%	92.95%
Some college	61.28%	66.34%
Associate degree	45.16%	46.72%
Bachelor's degree	34.98%	36.98%
Master's degree	13.04%	9.01%
Professional degree	3.47%	2.02%
Doctorate	2.03%	1.12%

Educational Attainment in the US (wiki)

Note the gap between ‘some college’ and ‘associate degree’ (which means at least associate).

To the best of my knowledge, this gap is many times higher in the US than in any other rich country.

GENDER

I'm not going to go into the gender splits in these numbers, but I think the rough picture is this.

The college premium is particularly high for men.

Not breaking things down by gender tends to *under-state* the premium because more women go to college and women earn less.

QUESTIONS

1. Why do people go to college?
2. Why do employers pay so much extra for college graduates?

TWO COMPLICATIONS

We should include as contrasts things that are really far from reality - part of the job is to explain why those things are far from reality.

Ideally, we'd like explanations where people are basically rational.

Maybe we'll have to give up on the latter, but let's try.

SIMPLE ANSWER

People go to college because it means they get more money than not going, and people like money.

Two problems:

1. Still leaves the question of why they get paid this money.
2. Leaves the question of why they don’t do something else, e.g., turn up to job interviews with admission letters and nothing else.

DISCUSSION

What do you think?

Why do people go to college, and why do employers pay so much for college grads?

THREE MODELS

1. Selection
2. Human Capital
3. Signaling

SELECTION

- Colleges only admit smart people.
- Employers want to hire smart people, but are kind of useless at telling who is smart, and find it easier to defer this decision to university admissions committees.
- So the college wage premium is explained by intelligence (or something like it) being the common cause of being admitted to college, and being paid the big bucks.

HUMAN CAPITAL

- Going to college means you acquire skills that are valued by employers.
- It's worth going to college because the payoff in higher salary is more than the cost of acquiring those skills.
- Employers pay more for college grads because they value those skills.

SIGNALING

- Going to college is costly and unpleasant, and no one really wants to do it, and no one gets value from it.
- But it's more costly and unpleasant for less intelligent people.
- Going to college sends a signal to employers that you're one of the people who can cope with college.
- Employers like this because they want to hire intelligent people.

THREE MODELS

All of these models explain the existence of a college wage premium.

They aren’t completely incompatible - the truth may contain some measure of each.

We’ll go over next time much more detail about which of them explain the various facts.

PUZZLES

Just getting into college isn't valuable:

- This is tricky for the selection effect to explain.

Doing 2-3 years of college isn't valuable:

- This is tricky for the human capital theory to explain.

The college wage premium rises over time:

- This is tricky for the signaling theory to explain.

Much more to follow!