

Comments on Das

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COMPARISON OF EXPERIMENTS

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1. Summary

Bohnenblust, Shapley, and Sherman [2] have introduced a method of comparing two sampling procedures or experiments; essentially their concept is that one experiment α is more informative than a second experiment β , $\alpha \supset \beta$, if, for every possible risk function, any risk attainable with β is also attainable with α . If α is a sufficient statistic for a procedure equivalent to β , $\alpha \supset \beta$, it is shown that $\alpha \supset \beta$. In the case of dichotomies, the converse is proved. Whether \supset and \supset are equivalent in general is not known. Various properties of \supset and \supset are obtained, such as the following: if $\alpha \supset \beta$ and γ is independent of both, then the combination $(\alpha, \gamma) \supset (\beta, \gamma)$. An application to a problem in 2×2 tables is discussed.

2. Definitions

An *experiment* α is a set of N probability measures u_1, \dots, u_N on a Borel field \mathcal{B} of subsets of a space X . The N measures are considered as N possible distributions over X , and performing the experiment consists of observing a sample point $x \in X$. A *decision problem* is a pair (α, A) , where A is a bounded subset of N -space. The points $a \in A$ are considered as the possible actions open to the statistician; the loss from action $a = (a_1, \dots, a_N)$ is a_i if the actual distribution of x is u_i . A *decision procedure* f for (α, A) is a \mathcal{B} -measurable function from X into A , specifying the action a to be taken as a function of the sample point x obtained by the experiment. With every $f = [a_1(x), \dots, a_N(x)]$ is associated a loss vector

$$v(f) = \left(\int a_1(x) d u_1, \dots, \int a_N(x) d u_N \right);$$

the i -th component of $v(f)$ is the expected loss from f if x has distribution u_i . The range of $v(f)$ is a subset of N -space which we denote by $R_i(\alpha, A)$; the convex closure of $R_i(\alpha, A)$ will be denoted by $R(\alpha, A)$ and will be called the set of *attainable loss vectors* in (α, A) ; every vector in R is either attainable or approximable by a randomized mixture of $N + 1$ decision procedures.

THEOREM 1. $R(\alpha, A) = R(\alpha, A_1) = R_i(\alpha, A_1)$, where A_1 is the convex closure of A .

This theorem permits us to restrict attention to closed convex A , which we shall do in the following sections. The proof of the theorem will not be given here; it is straightforward except for the fact that $R(\alpha, A_1) = R_i(\alpha, A_1)$. This fact follows from the result that whenever A is closed, so is $R_i(\alpha, A)$, which has been proved elsewhere by the author [1].

David Blackwell, "Comparison of Experiments", Berkeley Symposium on Mathematical Statistics and Probability, 1951

Two Questions

1. Is the value of evidence thesis true?
2. Is Nilanjan's decision rule good?

Counterexamples to Value of Evidence

- When others know what kind of knowledge you have.

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- When knowledge comes to a group.

A Group Example

The group has to choose between these three options.

	p	$\sim p$
A	10	0
B	0	10
C	4	4

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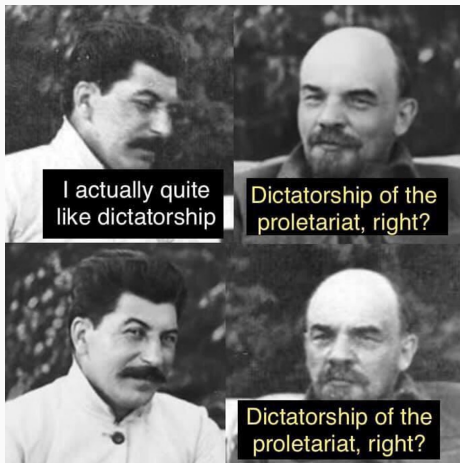
One Decision Rule

- We do whatever choice maximises the minimum expected utility across the group.
- We will do C whatever evidence comes in, but no one wants that either now or later.
- In fact everyone would pay 1 to avoid that outcome.

Other Decision Rules

- Let's try dictatorship.
- There might be an Arrowian argument that we'll be forced into dictatorship.
- Very hard to get a decision rule that is Paretian and Blackwellian other than dictatorship. (Does hard mean impossible? Good question - if someone wants to work this out/write this up, lmk.)

But It's a Good Dictatorship, Right



From qatsimai on the reddit forum historymemes

- One of the Pr on the credal committee gets to call the shots.
- This avoids incoherence, as long as the dictator is coherent.

Functionalist Worries

- What does it mean to say other Pr are even in the committee?

Am I Really on the Committee?



The dictator on the credal committee

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- But some inquiries run for decades.

Suggestion One: Politics are Everywhere

- The credal committee is a committee, and how it chooses is a political problem.
- Live with evidence being costly.
- We're used to that in committee choices already.

Suggestion Two: Irrationality is Everywhere

- Does each committee member regard the others as rational, assuming known conditionalisation?
- They regard the others as being procedurally rational, but perhaps not substantively rational.
- Each other is rational by the other's lights, but not by mine.
- So maybe I shouldn't be surprised that I want to keep information from them.

GREAT PAPER!

