Choice and Sets of Utility Functions

We have a set of basic options. An option is some member of that set, or a lottery with known probabilities whose prizes are the basic options.

We also have a set L of utility functions. Each utility function assigns a value to each basic option, and then values each lottery as the expected value of the basic options it contains.

There is a choice function C that takes options as input and returns a subset of those options. The choice function is related to L in the following way. For any set S of options C(S) is the set of options in S that are chosen by at least one utility function in L. In other words, C(S) is the set of options that are preferred to at least one other option in S by at least one utility function in L.

Here are four properties that a choice function might satisfy:

α: $(x \in C(S) \wedge x \in T \wedge T \subseteq S) \rightarrow x \in C(T)$

- If x is chosen from S and T is a subset of S that contains x, then x is also chosen from T.

β: $(x \in C(T) \wedge y \in C(T) \wedge T \subseteq S) \rightarrow (x \in C(S) \leftrightarrow y \in C(S))$

- If x and y are both chosen from T, and T is a subset of S, then x is chosen from S if and only if y is chosen from S.

γ: $(x \in C(S) \wedge x \in C(T)) \rightarrow (x \in C(S \cup T))$

- If x is chosen from both S and T, then x is chosen from the union of S and T.

δ: $(x \in C(T) \wedge y \in C(T) \wedge T \subseteq S) \rightarrow (\{y\} \neq C(S))$

- If x and y are both chosen from T, and T is a subset of S, then y is not the only option chosen from S.

Question: Which of these properties are satisfied by the choice function C?

Further question: What properties are satisfied by the choice function C if we assume that L is a set of utility functions that are all strictly increasing? What properties guarantee that there will be a set L of strictly increasing utility functions that satisfies the properties of C?