

Indecisive Decision Theory

Brian Weatherson

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1 Decisiveness

Say a decision theory is **decisive** iff for any decision problem, it says either:

1. There is a uniquely best choice, and rationality requires choosing it.; or
2. There is a non-singleton set of choices each of which is tied for being best, and each of which can be permissibly chosen.

A decision theory is **decisive over binary choices** iff it satisfies this condition for all decision problems where there are just two choices. Most decision theories in the literature are decisive, and of those that are not, most of them are at least decisive over binary choices. I'm going to argue that the correct decision theory, whatever it is, is indecisive. It is not, I'll argue, even decisive over binary choices.

My argument will focus on the following kind of problem. Player has a binary choice. There is a Demon who is very good, arbitrarily good, at predicting choices, and has already made a prediction about what Player will choose. Player's payoff is a function of what they choose, and what the Demon predicts. I'll use capital letters for Player's choices, and matching lower case letters for the Demon's predictions. Here is one very familiar example of the kind of case I have in mind.

Demonic Prisoners' Dilemma

	c	d
C	4	0
D	5	1

I've called this **Demonic Prisoners' Dilemma**. It's *Demonic* because there's a demon. And it's Prisoners' Dilemma because Player's payouts are taken from the payout table for Prisoners' Dilemma. I'll come back in the next section to the general strategy I'm using for

generating puzzles like this. If you're a philosopher reading this, you're probably used to thinking of Demonic Prisoners' Dilemma as Newcomb's Problem, but I want to use a productive naming convention, so I'm calling it Demonic Prisoners' Dilemma.

Here is another decision problem of the same broad kind.

Demonic Stag Hunt

	g	h
G	15	40
H	0	50

It's called Demonic Stag Hunt because it's got a Demon, and Player's payouts are like in a Stag Hunt.

The main decision problem I'm interested has the following structure: Player plays Demonic Prisoners' Dilemma, then Demonic Stag Hunt, and gets the sum of the payouts across the two games. More precisely, the following six things happen in this order.

1. Demon predicts what Player will do in Demonic Prisoners' Dilemma.
2. Player makes a choice in Demonic Prisoners' Dilemma.
3. Demon's prediction and Player's choices are revealed (to both Demon and Player), and Player receives their first payout.
4. Demon predicts what Player will do in Demonic Stag Hunt.
5. Player makes a choice in Demonic Stag Hunt.
6. Demon's prediction and Player's choices are revealed (to both Demon and Player), and Player receives their second payout.

I'm going to argue for the following claims.

1. If a decisive decision theory is correct, Player's choices at stages 2 and 5 should be exactly the choices they would make if they were playing these games as one-shot games, independent of any other interaction with the Demon.
2. Player's choice dispositions over the two games should be consistent with the choice they would make if they were choosing a **strategy** for playing the two games. I'm going to call this constraint Backwards Dynamic Consistency.
3. If 1 is true, then all existing decisive decision theories either violate Backwards Dynamic Consistency, or are independently objectionable.

So that's the plan, but before we start there are two pieces of important housekeeping.

First, the definition of decisiveness referred to options being tied. For the definition to be interesting, it can't just be that options are tied if each is rationally permissible. Then a decisive theory would just be one that either says one option is mandatory or many options are permissible. To solve this problem, I'll borrow a technique from Ruth Chang (2002). Some options are **tied** iff either is permissible, but this permissibility is sensitive to sweetening. That is, if options X and Y are tied, then for any positive ε , the agent prefers $X + \varepsilon$ to Y . If either choice is permissible even if X is 'sweetened,' i.e., replaced in the list of choices by $X + \varepsilon$, we'll say they aren't tied. My thesis then is that the correct decision theory says that sometimes there are multiple permissible options, and each of them would still be permissible if one of them was sweetened. Indeed, the argument will be that the very first decision problem I've stated is like that.

Second, there is an important term in the definition of decisiveness that I haven't clarified: **decision problem**. Informally, the argument assumes that in setting out Demonic Prisoners' Dilemma (aka Newcomb's Problem) and Demonic Stag Hunt I've described two decision problems. More formally, I'm assuming it suffices to specify a decision problem to describe the following four values.

- What choices the Player has;
- What possible states of the world there are (where it is understood that the choices of Player make no causal impact on which state is actual)¹;
- What the probability is of being in any state conditional on making each choice; and
- What return Player gets for each choice-state pair.

Most recent papers on decision theory do not precisely specify what they count as a decision problem, but they seem to implicitly share this assumption, since they will often describe a vignette that settles nothing beyond these four things as a decision problem. And that's what I did as well! And you should understand this as being part of the definition of decisiveness. And this implies that there are two ways to reject decisiveness.

First, say that these four conditions underspecify a real decision problem. In any real situation, decision theory has a decisive verdict, but it rests on information above and beyond the setting of these four values. Some versions of Causal Decision Theory go this route, though as we'll see not all do. So on this model, for any person playing Demonic Stag Hunt there is a decision theoretically correct thing for them to do (or the options are tied), but it's potentially a different thing for different people

Second, say that no matter how much one adds to the specification, there will be cases where

¹My personal preference is to understand states historically. For any proposition relevant to the decision, a state determines its truth value if it is about the past, or its chance at the start of deliberation if it is about the future. And then causal independence comes in from a separate presupposition that there is no backwards causation. But I definitely won't assume this picture of states here.

the correct decision theory is indecisive. This is the view I ultimately want to defend, and this paper is a part of the defence. But it's a proper part. In particular, it says nothing against the theories that say that once a problem like Demonic Stag Hunt is fully specified, there is a single correct option. That's an argument for another day. Today, we have enough to be getting on with.

2 Demonic Games

Here is a general recipe for generating a philosophically interesting decision problem. Take an interesting game, and replace one of the players with a demon. A demon, in the relevant context, can be defined in one of two more or less equivalent ways. One way is to say that the demon's payouts are 1 if in some sense they make the 'same' play as the other player, and 0 otherwise. Another is to say that each of their moves are predictions of what the other player will do, and conditional on any choice that player makes, the conditional probability of the demon making a correct prediction is arbitrarily high.

Let's illustrate this with a familiar game, Prisoners' Dilemma. I'm going to use slightly different payouts from those that Axelrod (1984) uses in his classic discussion, but not so different as to make the example unfamiliar. First, here is the game version.

Prisoners' Dilemma

	c	d
C	4, 4	0, 5
D	5, 0	1, 1

Now here's what happen if we demonize the payouts for column.

Prisoners' Dilemma with a Demon

	c	d
C	4, 1	0, 0
D	5, 0	1, 1

And here is what happens if we remove the demon altogether.

Demonic Prisoners' Dilemma

	c	d
C	4	0
D	5	1

And that's both Newcomb's Problem, and what I was calling Demonic Prisoners' Dilemma.

It isn't only Newcomb's Problem that can be generated in this way. Demonic Matching Pennies² is Death in Damascus (Gibbard and Harper 1978). Demonic Battle of the Sexes is Asymmetric Death in Damascus (Richter 1984). Demonic Chicken is Egan's Psychopath Button example (Egan 2007). A lot of the most notable examples in modern decision theory are Demonic versions of famous games.

When I'm introducing games in classes, I usually walk through five instances of two player, two options each, games. Four of them have already been mentioned in this section: Prisoners' Dilemma, Matching Pennies, Battle of the Sexes, and Chicken. But the fifth is probably the most interesting of the lot: Stag Hunt. Brian Skyrms has written extensively on why Stag Hunt is philosophically important (Skyrms 2001, 2004). I'll really just focus on one reason.

Here is an abstract form of a Stag Hunt game, where the options are G/g for Gather or H/h for Hunt. Actually, this is a table for a generic symmetric game; what makes it a Stag Hunt are the four constraints listed below.

Generic Stag Hunt

	g	h
G	x, x	y, z
H	z, y	w, w

- $x > z$
- $w > y$
- $w > x$
- $x + y > z + w$

²You can see examples of all these games, and all the game theoretic machinery I use throughout this paper, in any standard game theory textbook. My favorite such textbook is Bonanno (2018), which has the two advantages of being philosophically sophisticated and open access. I'm not going to include citations for every bit of textbook game theory I use; that seems about as appropriate as citing an undergrad logic textbook every time I use logic. But if you want more details on anything unfamiliar in this paper, that's where to look.

The first two constraints imply that $\langle G, g \rangle$ and $\langle H, h \rangle$ are both equilibria. This isn't like Prisoners' Dilemma, that only has one equilibrium. But it is like Prisoners' Dilemma in that there is a cooperative solution, in this case $\langle H, h \rangle$, but it isn't always easy to get to it. It isn't easy because there are at least two kinds of reasons to play G .

First, one might play G because one wants to minimise regret. Each play is a guess that the other player will do the same thing. If one plays G and guesses wrong, one loses $w - y$ compared to what one could have received. If one plays H and guesses wrong, one loses $x - z$. And the last constraint entails that $x - z > w - y$. So playing G minimises possible regret.

Second, one might want to maximise expected utility, given uncertainty about what the other player will do. Since one has no reason to think the other player will prefer g to h or vice versa - both are equilibria - maybe one should give each of them equal probability. And then it will turn out that G is the option with highest expected utility. Intuitively, H is a risky option and G is a safe option, and when in doubt, perhaps one should go for the safe option.

To be clear, I am not endorsing either of these arguments. I think both G and H are permissible moves in any version of Demonic Stag Hunt. Since all instances of Demonic Stag Hunt stay being instances of Demonic Stag Hunt under mild enough sweetening, just saying this about Demonic Stag Hunt is enough to make a theory indecisive. So Demonic Stag Hunt plays a few different roles in this story.

The arguments are distinct, even though in the two option game they are basically both consequences of $x + y > z + w$. In Stag Hunt type games with 3 or more options, the regret based approaches and the indifference over equilibria based approaches can lead to different choices. And there are ways of mixing and matching these two principles to get any number of other alternatives. In general, there are a lot of ways to motivate G , and these are not just conceptually distinct but lead to distinct recommendations in more complex games. But the arguments for choosing H all basically come down to *Better outcomes are better. Rocket emoji!*. So in this paper I'll spend a lot more time on decisive theories that recommend G than on decisive theories that recommend H . This isn't because these theories are necessarily better; just that they are more numerous.

3 Two Round Demonic Games

In a two round Demonic Game, Player plays one Demonic Game, the outcome of that game is revealed to both Player and the Demon, then Player plays another Demonic Game. Player's ultimate payout is the sum of their payouts in the two individual games.

In all Demonic Games, we assume that the Demon is very reliable at predicting Player's

choices, perhaps arbitrarily reliable. In a two round Demonic Game, we assume this, and we also assume that the Demon's errors, as few as they are, are randomly distributed. By that I mean that the Demon's probabilities of making a correct prediction in each of the two games are independent of each other. What could count as good evidence that the Demon's error probabilities are independent in this way is a tricky question, but presumably some evidence could do it. And we assume that Player has exactly that evidence.

This independence of the error probabilities has consequences for what recommendations decisive theories make in two round games. In general, one might not want to do the same thing in a two round game as one does in the two games taken individually. But for that to be the case, one of two things must happen.

One possibility is that there is a strategic benefit from some otherwise sub-optimal play in the first round. If a decision problem is defined the way we've defined it here, the only way that there could be such a strategic benefit is if the conditional probability of states (i.e., predictions) given choices could change depending on what one did in the first round. (The other three aspects of the second game clearly couldn't change, so if there is some change it must be here.) But the independence stipulation rules that out.

Another possibility is that something one learned from the first round could provide a reason to do something different in the second round. Call this an informational reason to change one's play in round two. But just like the strategic reason for changing one's play in round one, this could only happen if the conditional probability of states (i.e., predictions) given choices could change depending on what one did in the first round. And independence once again rules that out.

Note that this argument does not carry across to indecisive theories. One way to be indecisive is to think that there is some further information, behind the four things we've taken to describe a game here, that affect what choices one should make. And it is possible that there could be causal or evidential links between what one does in round one, and that 'further information' in round two. So given indecisiveness, it is possible there are strategic reasons for doing something in round 1 that one would not do in a one round game, or informational reasons for doing something in round 2 that one would not do in a one round game. And in fact I think that is the case - but it is something that only indecisive theories can accept.

We are going to be interested primarily in the following two round game: Demonic Prisoners' Dilemma plus Demonic Stag Hunt. And to make things concrete, I'll use the following payouts for the two games. First, Demonic Prisoners' Dilemma.

Demonic Prisoners' Dilemma

	c	d
C	4	0
D	5	1

Second, Demonic Stag Hunt.

Demonic Stag Hunt

	g	h
G	15	40
H	0	50

As you can see, I've made the stakes much higher in Demonic Stag Hunt than in Demonic Prisoners' Dilemma. Whether Player can cause the Demon to play *h* in round 2 is much more significant than whether they play *C* or *D* in round 1.

There are 16 possible outcomes to this game, depending on what Player and Demon do. They are given in this table.

Possible Outcomes of DPD+DSH

	cg	ch	dg	dh
CG	19	44	15	40
CH	4	54	0	50
DG	20	45	16	41
DH	5	55	1	51

But note that Player doesn't choose one of these four rows. In fact, it is wrong twice over to think that Player just makes one of these four choices. Player makes choices at two times, not at one. And if we want to model Player's choice at one time, we should model Player as choosing a strategy, not just a pair of moves.

4 Strategies

So what are strategies? In general, a strategy is a plan for playing a game that takes place over time. In the standard textbook treatment of strategies, they specify a choice for every node that might be reached in a game. And this turns out to require a lot of information, since nodes in games are individuated by their history.³ This means there are many more nodes than you might expect.

So think about a strategy for playing chess. There are two intuitive but mistaken ways to think about what a strategy, in this textbook sense, says a chess strategy does.

First, you might think that for any state of the board, it should specify a move. But in fact strategies must do more than that; they have to specify a move for each state of the board and each way of getting to that state. So a strategy should have separate entries for what to do after 1. e4, Nc6; 2. Nf3, e5 and what to do after 1. e4, e5; 2. Nf3, Nc6.

Second, you might think that a strategy for white specifies what to do from the initial position, then a strategy for how to respond to each move black might make, then a further strategy for how to respond to each subsequent move black might make, and so on until the game ends. And that kind of strategy would be unimaginably large, and a tiny fraction of what a real strategy does. A real strategy specifies what to do at every possible state of the game, including what to do in states that are excluded by earlier moves in this strategy. So if the strategy says to play e4 at move 1, it doesn't just say what to do at move 2 after ...e5, it says what to do at move 2 if the game starts 1. d4, d5.

In two stage Demonic Games, this means that a strategy has to specify five binary choices, so there are $2^5 = 32$ possible strategies. In the game we're primarily interested in, a strategy has to specify

1. What to do in Demonic Prisoners' Dilemma;
2. What to do in Demonic Stag Hunt if the first game ends $\langle C, c \rangle$.
3. What to do in Demonic Stag Hunt if the first game ends $\langle C, d \rangle$.
4. What to do in Demonic Stag Hunt if the first game ends $\langle D, c \rangle$.
5. What to do in Demonic Stag Hunt if the first game ends $\langle D, d \rangle$.

I'm generally going to write strategies as a string of five letters, which answer each of these five questions in order. So *CGHHG* is the strategy which plays *C* in the first game, then plays *G* in the second game if the Demon's prediction was correct in the first game, and *H* if it is was incorrect.

³To be more precise, a strategy specifies what to do at each information set, and information sets are individuated by the learning history of the player making the choice. The game we are playing is sort of a full information game, so this complication won't bother us.

Why do we want to specify strategies so finely? It turns out there are two reasons, one relatively obvious, one less obvious.

The obvious reason is that in round 2, Player has information about what the Demon has done in round 1, and we should at least in principle allow them to use it. Decisive theories say that there is nothing you can learn in round 1 that matters to round 2, but whether Decisive theories are correct is part of what's at issue here. So let's leave that as an open question.

The less obvious reason concerns off-path choices, i.e., what the strategy says to do if you didn't do the thing you planned to do at move 1. It turns out that different predictions the Demon makes about one's strategy can affect one's payout, even if those predictions only differ in off-path choices. So imagine one's strategy is *DG*GGG, i.e., Defect in Round 1, then Gather in Round 2 whatever happens in Round 1. And compare the payout one gets if the Demon predicts one is playing *CG*GGG, to if the Demon predicts one is playing *CG*GHG. In the first case, one gets 20 (5 in round 1, 15 in round 2). In the second case, the Demon predicts that one will play *H* in round 2, so trying to make a correct prediction, they will play *hh*. And now the payout will be 45.

If you think the Demon has probability 1 of making a correct prediction, and to make the math easier that is what I'm going to assume from now on, then it turns out that the strategies fall into 16 pairs. Within each pair, the two strategies have the same payouts no matter what the Demon does, and each other strategy has the same payout no matter which of the pair the Demon predicts one will play. So there isn't any practical difference between $Cx_1x_2x_3G$ and $Cx_1x_2x_3H$, or between $DGx_1x_2x_3$ and $DHx_1x_2x_3$, for any x_1, x_2, x_3 . For some purposes I'll treat these pairs as the 'same' strategy, but I'll make it clear when I'm doing this if anything at all turns on it.

To foreshadow a little bit, I'm going to be particularly interested in this strategy: *CH*GGG. This strategy Cooperates in round 1, then Hunts in round 2 if Demon correctly predicted Cooperation, and Gathers otherwise. This is a fairly interesting strategy for a few reasons. It has a very good return - 54 when the highest is 55. And even though *C* is strongly dominated by *D*, this strategy is not dominated by anything. Indeed, $\langle CHGGG, chggg \rangle$ is Nash equilibrium of the strategic form of the game. (And it's a subgame perfect equilibrium and a perfect Bayesian equilibrium, and satisfies any number of other solution concepts.) I'm not a Decisive decision theorist, so I don't want to say it's the one and only correct play in Demonic Prisoners' Dilemma followed by Demonic Stag Hunt. But I do think it's what I personally would play if forced to choose. And I think it's a rationally permissible choice. But no plausible decisive theory can find it as the correct choice.

Now I'm not going to argue from the intuitive plausibility of *CH*GGG to Indecisiveness. Instead, I'm going to use *CH*GGG, and occasionally *DG*GGH, to argue that a huge range of Deci-

sive theories are dynamically inconsistent. And they are dynamically inconsistent in a systematic enough way that we should be very sceptical that there is any dynamically consistent Decisive theory. But this style of argument raises an obvious question: what is dynamic consistency, and why should we care about it?

5 Dynamic Consistency

Think about the following two ways to get to the end of a two round Demonic Game. The natural way is that Player makes one choice, sees the outcome of that choice, then makes another choice. I'll say that when Player does this, they choose a **sequence** of moves. The less natural way is that Player is asked in advance what strategy they want to play, and then that strategy is carried out over the course of the game. I'll call this choosing a **strategy**. The two seem closely related, close enough that each of the following principles seem plausible.

Backwards Dynamic Consistency Any sequence that Player can rationally choose is part of some strategy that they can rationally choose.

Forwards Dynamic Consistency Any strategy that Player can rationally choose is such that choosing a sequence by doing what that strategy requires is rational.

The dynamic consistency argument I'm going to use in this paper relies on Backwards Dynamic Consistency. The argument is neutral on whether Forwards Dynamic Consistency is a true principle. Personally, I think it's false, and I'll say why in a bit. But the argument does not assume that it's false. (Actually, the argument would probably be strengthened if it were true – saying it is false is somewhat of a statement against interests here.)

I think Backwards Dynamic Consistency is more intuitive than any argument I could give for it, and certainly more intuitive than the intuitions about cases that usually ground arguments in decision theory. But there is one nice argument for Backwards Dynamic Consistency that has some persuasive force, and helps explain what it says, and even helps explain why it is a bit more plausible than Forwards Dynamic Consistency. So that argument is worth working through.

One way to get Player to choose a strategy is to show them all the possible strategies and ask them to pick one. But the forms you need to use for that kind of questionnaire grow really quickly. A better way is to ask Player several questions, the answers to which will collectively determine a strategy. So we could tell Player Demonic Prisoners' Dilemma plus Demonic Stag Hunt will be played tomorrow, but unfortunately they won't be able to make choices in real time. So they now have to make some choices that determine their strategy. In particular, they have to fill out this form.

Form for choosing a strategy in DPD+DSH

What will you do in		
Round 1	C <input type="checkbox"/>	D <input type="checkbox"/>
Round 2 if R1 ends Cc	G <input type="checkbox"/>	H <input type="checkbox"/>
Round 2 if R1 ends Cd	G <input type="checkbox"/>	H <input type="checkbox"/>
Round 2 if R1 ends Dc	G <input type="checkbox"/>	H <input type="checkbox"/>
Round 2 if R1 ends DD	G <input type="checkbox"/>	H <input type="checkbox"/>

Here's what it would take for Backwards Dynamic Consistency to fail. All the following would have to be true together.

1. Player rationally makes move M_1 in round 1.
2. Player rationally makes move M_2 in round 2 after seeing how round 1 ends.
3. There is no way to rationally fill in that form that includes saying M_1 to the first question, and M_2 to the conditional question that corresponds to how the game actually went in round 1.

But it is implausible this is true. In some good sense, the questions that we ask Player when we are finding what strategy they want just are the questions we ask them when the game is being played. To reject Backwards Dynamic Consistency is to say that there could be a situation S such that these two questions have different answers.

Question S1 What instruction would you like carried out if S happens?

Question S2 You're in S . What instruction would you like carried out?

But this way of putting it, that these two questions are basically the same question, makes it sound like both Forwards and Backwards Dynamic Consistency should be true. Now as I noted, I'm not assuming Forwards Dynamic Consistency is false. But I'm also not assuming that it is true. How is this viable given how similar the tasks of choosing a strategy and choosing a sequence look like?

Well, imagine that Player thinks that there is some situation that has probability 0 of arising. To be concrete, let's say it is that the first game ends Cd . And they also believe that were the first game to end Cd , then G would have higher expected utility than H . How should Player answer the third question, what to do if the first game ends Cd in the little survey above?

There are two arguments here that both seem plausible, but which point in opposite directions. The first says that since each answer to the survey is alike in expected utility, Player can choose whimsically. They are alike in expected utility because what one does at probability 0 events cannot make a difference to the expected utility of one's plan. The second

argument says that choosing G here could be better – if the first game does actually end Cd – and couldn’t be worse, so there is a weak dominance argument for choosing G that Player should follow. Personally, I think the expected utility consideration is stronger, but nothing in this paper turns on this particular question.

So there is a somewhat plausible argument that says there could be a situation where Question S1 is answered whimsically, although Question S2 has a uniquely correct answer. So maybe there is a permissible answer to Question S1 which is not a permissible answer to Question S2. But the theorist who denies Backwards Dynamic Consistency says that there is a permissible answer to Question S2 which is not even a permissible answer to Question S1. And there is no good reason to think that could be possible. So Backwards Dynamic Consistency is correct. Whatever sequence of moves you could rationally do, you could rationally give as part of your answers to the survey, and then rationally complete the rest of the survey.

But it turns out that a quite surprising variety of attempts to provide a decisive decision theory end up violating Backwards Dynamic Consistency. I’m going to go through a lot of examples in what follows. I’m not going to prove there is no decisive theory that is plausible and which doesn’t violate Backwards Dynamic Consistency. But I hope you’ll agree by the end that we have strong inductive evidence that no such theory exists.

At some level of generality, the argument I’m giving here is a variant on the argument against “Single Valued Solution Concepts” that David Pearce made in an old unpublished manuscript (Pearce 1983). The manuscript is cited in the paper where Pearce introduced the concept of a rationalizable strategy (Pearce 1984). The argument I’m giving grew out of an attempt to simply translate the argument of Pearce’s manuscript into the language of decision theory. And it is part of a larger project of applying the lesson’s of Pearce’s famous paper to decision theory. But that larger project will have to be carried out elsewhere; this paper is long enough as it is. Instead, let’s turn to seeing the conflict between various decision theories and Backwards Dynamic Consistency.

6 Evidential Decision Theory is Dynamically Inconsistent

As an illustration of these dynamic consistency principles, it’s worth walking through an argument that traditional Evidential Decision Theory (Evidential Decision Theory) violates Backwards Dynamic Consistency. This is not a new point. It is made in Gibbard and Harper (1978) using a variant of Newcomb’s Problem where the player has a chance to change their choice (for a small fee) after the demon’s predictions are announced. So I’m not offering a new argument against Evidential Decision Theory. Instead, I’m presenting this case because I find it helpful to see what the Dynamic Consistency Principles say by first starting

with a relatively simple case.

Player A has the following interaction with a demon. Tomorrow, A will be asked to choose the Left or Right box. The demon wants to predict A's choice. The demon will put \$1000 into whatever box they predict A will choose. And, A will be shown the contents of the boxes before they make their choice. This seems like an easy game – every decision theorist I know says that A should take the money. (And the demon will get their wish of correctly predicting A's choice.)

Player B has an interaction like Player A's, but with the following difference. The demon has the option of *passing*, and not predicting what choice B will make. The demon would prefer correct prediction to passing, and passing to incorrect prediction. If the demon passes, B will get \$2000, not just \$1000. But still, B will be shown how much is in each box. So almost all theorists will say that B should take the money, and since the demon knows this, the demon shouldn't pass. (I'll come back to theories that say B should pass up the money in the penultimate section of the paper.)

Player C is like Player B, except they will be at work all day tomorrow, and unable to make a choice. So they are asked to record in advance their strategy,. Now things get interesting. C has four options to choose between – since they have to announce left or right for each possible revelation of where the money is. And the demon has three choices, left, right or pass. So here's the game table, with Player C as Row and Demon as Column. (I'll write X-Y to mean for C to mean Do X if the money is in the left box, and Do Y if the money is in the right box. And I'll assume each \$1000 is worth 1 util to C.)

Threat Game			
	Left	Right	Pass
Left-Left	1, 1	0, 0	2, 0.5
Left-Right	1, 1	1, 1	2, 0.5
Right-Left	0, 0	0, 0	2, 0.5
Right-Right	0, 0	1, 1	2, 0.5

From what we've said about the demon, we can deduce the following conditional probabilities for predictions given strategies. (The second line is because the value for x is undetermined, but it's not going to matter given what comes next.)

Probabilities in Threat Game			
	Left	Right	Pass
Left-Left	1	0	0
Left-Right	x	$1 - x$	0
Right-Left	0	0	1
Right-Right	0	1	0

So the expected value of Right-Left is 2 and the expected value of the other three options is 1. So Evidential Decision Theory says that Player C should choose Right-Left.

And that's to say that the only rational strategy for Player B is Right-Left. But that's clearly not a rational sequence of moves, by the lights of Evidential Decision Theory, for Player B. The sequence that is rational at each time it is made is Left-Right.

So Evidential Decision Theory violates both Forwards and Backwards Dynamic Consistency. It violates Backwards Dynamic Consistency because it says the only sequence that is rational is Left-Right, but it does not say this is one of the rational strategies. And it violates Forwards Dynamic Consistency because it says that Right-Left is among the rational strategies, but it is not among the rational sequences.

And these violations seem like bad news for Evidential Decision Theory. If you ask the evidential decision theorist to compile a strategy by answering conditional questions, they will say they will turn the money down whatever the demon does. But if you ask them what to do once they see where the money is, they will take it. This looks like a bad combination of attitudes to have.

Now as I said at the start of this section, this point was made about Evidential Decision Theory over 40 years ago, and it clearly hasn't convinced everyone. So I don't expect this representation will change many more minds. But I hope it's a helpful guide to what violations of Backwards (and Forwards) Dynamic Consistency look like, and why they look, at least to many of us, to be problems.

7 Regret Based Strategies

In this section I'll argue that two recent attempts to land between Evidential and Causal Decision Theory are dynamically inconsistent. These are the theories developed by Ralph Wedgwood (2013) and Dmitri Gallow (2020). Both theories handle two option Demonic cases the same way, so let's start with what they have in common.

For any options X, Y , let $V_Y(X)$ be the (evidentially) expected value of X , on the assumption that the demon will act as if one has chosen Y . So $V_X(Y) - V_X(X)$ will be how much one subsequently regrets not choosing Y , when one actually chose X . In the two option case, but Wedgwood and Gallow say that one should choose X over Y so as to minimise expected regret. So one should choose X over Y if $V_Y(X) - V_Y(Y) > V_X(Y) - V_X(X)$, and be indifferent between the two options if they are equal. But the theories differ about what to do when there are more than two options.

Wedgwood says that one should evaluate each option X by comparing $V_X(X)$ to the set of values $\{V_X(Y) : Y \in O\}$, where O is the set of options. There are a few different comparisons one could make here – how does $V_X(X)$ compare to the maximum value of that set, or the minimum value, or the mean value, or the median value? It turns out it won't matter for our purposes what choice you make here. The general picture is that one wants to maximise $V_X(X) - f(\{V_X(Y) : Y \in O\})$, for one particular function f . Given a choice of f , Wedgwood calls $f(\{V_X(Y) : Y \in O\})$ the **benchmark** for X , and the aim is to maximise the surplus value of choosing X over its own benchmark. For simplicity, we'll say the benchmark is the mean value, but actually it doesn't matter which choice we make for what follows.

Given this setup, one should choose D in Demonic Prisoners' Dilemma, and G in Stag Hunt. But if you built the 32-by-32 strategy table for Demonic Prisoners' Dilemma plus Demonic Stag Hunt, the strategies with the highest surplus value over their own benchmark are $DGGGH$ and $DGGHH$. Just behind them are $CHGGG$ and $CHHGG$. And all the other strategies are, by the lights of Wedgwood's theories, much worse. So this theory violates Backwards Dynamic Consistency. If one is filling in the form, saying what one plans to do in various situations, one must say that one plans to play H in the situation one expects to find oneself in. But one must not play H when that situation arises.

Gallow's theory takes off from the two option case in a separate direction. Say that the measure $(V_X(X) - V_X(Y)) - (V_Y(Y) - V_Y(X))$ is a measure of the prima facie preferability of X over Y . As we saw above, this will be positive if X is strictly preferred to Y in the two option case; we won't be interested in the case where it is negative.

Now a simple theory would be to say that in general, whether X is preferred to Y is determined by whether this value is positive or negative. But as Gallow notes, this will lead to intransitivities. It is easy to come up with three option cases where, by this measure, X is preferred to Y , Y is preferred to Z , and Z is preferred to X . So Gallow offers a more sophisticated theory of all things considered preferability.

Here is Gallow's recipe for constructing a rank ordering of options. I'm going to simplify a bit and ignore how he handles ties, which require careful attention, but what I say about how to handle non-ties will be enough to give you the spirit of the theory. If we are trying

to construct a rank ordering of all options, we just need to construct a set of binary rankings $X > Y$ that satisfies transitivity. Gallow gives us a procedure for finding all those pairs. Start by taking all the pairs of options $\langle X, Y \rangle$ and rank them from highest to lowest by $(V_X(X) - V_X(Y)) - (V_Y(Y) - V_Y(X))$. Then work in steps down the list. At every step, find the pair $\langle X, Y \rangle$ with the highest such value you haven't processed. Add $X > Y$ to the master list of rankings. Then take the transitive closure of the list; if there are some options X_1, \dots, X_n such that the list contains $X_1 > X_2$ and ... and $X_{n-1} > X_n$, add $X_1 > X_n$ to the list. And if $X_n > X_1$ was on the list of prime facie preferences, delete it from that list. Now return to the (possibly) modified list of prima facie preferences, find the highest ranking preference that is neither added to the master list, nor deleted by this deletion procedure, add it to the master list, and continue. Eventually, one will have either $X > Y$ or $Y > X$ on the master list for all X, Y , and by construction the master list will be transitive.

I'm not going to work through here how this works for the 32-by-32 strategy game. But it turns out that the best option is *DGGGH*. And just behind is *CHGGG*. But when you apply the strategy to the two games individually, you get the clear verdicts that one should choose *D* in *Demonic Prisoners' Dilemma*, and *G* in *Stag Hunt*. So the theory is dynamically inconsistent in just the same way that Wedgwood's theory is. When filling in the form, one must say that one plans to play *H* in the situation one expects to find oneself in at the second game. But one must not play *H* when that situation arises.

8 Deliberational Strategies

So far I've argued, following Gibbard and Harper (1978), that evidential decision theory is dynamically inconsistent, and that two recent attempts to steer a middle path between evidential and causal decision theory are dynamically inconsistent. It's time to turn to causal decision theory.

At first glance, causal decision theories look like they can't pose a problem to my thesis that the correct theory is indecisive. That's because causal decision theories often take something beyond the acts, the states, the payouts, and the conditional probabilities of the states given the acts, to be crucial. This is even assuming that, as causal decision theorists insist, that the problem requires a specification that the states are causally independent of the acts. A lot of causal decision theories will insist that we also need the unconditional probabilities of the states in order to make a decision. And those theories will be, in my technical sense, indecisive. But some broadly causal theories do not require this. I'll look at two classes of such theories, one in this section and one in the next section.

Theories from the class I'm going to discuss in this section are not, to the best of my knowledge, actually defended anywhere in the literature. But the point of this paper is not to argue

that other philosophers are mistaken, but to argue that there is no plausible decisive decision theory. And to my mind, the biggest challenge to that conclusion comes from theories in this class. So I'm going to spend more time on them than on other theories. In this respect I'm following Gallow (2020), who also spends a fair bit of time arguing against this version of causal decision theory. (And indeed I'll appropriate one of Gallow's arguments in what follows.)

The theories I have in mind start with the important work of Brian Skyrms (1990) on the dynamics of deliberation. The resulting view is not going to be Skyrms's own view. Skyrms favors an indecisive view, and I'm not going to object to his positive view.⁴ But I will be interested in how a decisive theorist might appropriate Skyrms's machinery.

The part of that machinery we're going to focus on concerns updating one's credences during deliberation. There is a huge literature on updating credences during investigation. But that's not what we're interested in. Think about the process a good detective goes through. (A somewhat realistic one, not the idealizations of formal confirmation theory.) They investigate, and during the investigation gather a bunch of evidence. Then they reflect, and ask how the evidence fits together, and what conclusions it supports. Most work on credal updating has focussed on modeling the first step, the investigation. Skyrms is interested in modeling the second step, the deliberation. Now we're not doing detective stories here; we're trying to win money not solve a crime. But the same basic idea holds. Given what one knows about the case, one should reflect in a way that brings one's cognitive state into a kind of equilibrium.

Here is how Skyrms thinks we reach that equilibrium. I'll illustrate using Demonic Stag Hunt. (The one round game that is; I'll come back to the two round game very soon.) Imagine Player starts out thinking that it's 50/50 whether they'll Gather or Hunt. Since Player believes that Demon will do whatever they do, Player also thinks it's 50/50 whether Demon will play gather or hunt. Given that, the expected value to Player of Gather is 27.5, and the expected value of Hunt is 25, and Player's expected return is 26.25.

Now here's the crucial step. Since Gather has a return above expectations, and Hunt has a return below expectations, Player should adjust their credences about what they'll end up playing in the direction of the more successful strategy. They should "seek the good." There are a lot of ways to do this; here's one that Skyrms particularly likes.

For any strategy whose expected value is higher than Player's overall expected value, say that its **covetability** is the difference between those two expectations. For any other strategy, the covetability is 0. So in this situation, Gather has a covetability of 1.25, and Hunt

⁴It is an interesting question whether Skyrms's view is sufficiently indecisive, or whether it is too restrictive. That's a question for another, longer, paper.

has a covetability of 0. Now assume Player updates their credences using the following rule. In this rule, O is an arbitrary strategy, p_i is Player's probability at time t_i about what they'll play, c_i is the function from strategies to covetabilities at t_i , the sum in the denominator ranges over all strategies, and r is a measure of caution, that I'll say more about in a bit.

$$p_{i+1}(O) = \frac{rp_i(O) + c_i(O)}{r + \sum c(X)}$$

Let's set r to 2.5 to make the arithmetic easier. Then if p_0 of both Gather and Hunt was 0.5, we'll have $p_1(G) = \frac{2}{3}$, and $p_1(H) = \frac{1}{3}$. So at t_1 Player will have credence $\frac{2}{3}$ that demon will play gather. So the expected return of Gather will be $\frac{70}{3}$, the expected return of Hunt will be $\frac{50}{3}$, and the overall expected return will be $\frac{190}{9}$. So at t_2 the probability of Gather will rise a little more, the probability of Hunt will go down a little more, and this will continue until the Player is certain they will Gather.

This is what happens if Player is originally 50/50 about what they will do. But it's very dependent on that initial assumption. If Player had started off fairly confident that they would Hunt, and hence that demon would play hunt, then the resulting equilibrium would have been that they were certain they would Hunt. More precisely, that would happen if they started off with credence greater than 0.6 that they would Hunt. If they started off with credence exactly 0.6 that they would Hunt, they would be in an equilibrium already, and their credences would never change.

This is all well and good, but it doesn't feel like a decision theory yet. But there are a few simple ways to turn it into one. For example we could say (and Skyrms more or less does say) that any of these equilibrium states are rational, and so could ground rational choices. So in Demonic Stag Hunt, it's rational to Gather, rational to Hunt, and rational to play the mixed strategy Gather with probability 0.4, Hunt with probability 0.6. This last one is weird in a few ways; let's look at one of these.

Although there are three equilibria, they are not symmetric. They have different outcomes – one has an expectation of 50, one of 15, and the mixed strategy has an expectation of 40. We'll come back to theories that expectations to choose between the equilibria in the next section. For now, focus on what starting points get to those equilibria. Each equilibria has a 'basin of attraction': a set of initial probabilities that (given a choice of update function) lead to that equilibria. And these basins are of different sizes. Using a natural measure on the space of probability functions, the basin that leads to Gathering has measure 0.6, the basin that leads to Hunting has measure 0.4, and the basin that leads to the mixed strategy has measure 0. It's rather tempting to exclude equilibria whose basin has measure 0 from the set of permissible strategies. But ignore that for now, and focus on the pure strategy

equilibria. The basin that leads to Gather has two salient features that the basin that leads to Hunt lacks.

1. It is larger.
2. It includes the mid-point.

In two option choices these features usually go together, though not always, and they often come apart when there are more than two options. Either of these could be used to generate a decisive decision theory. That is, the following two Skyrms inspired theories are both broadly causal decision theories, and are both decisive in my sense.

1. In any decision problem, the rational choice is the equilibrium with the largest basin of attraction.
2. In any decision problem, the rational choice is the equilibrium whose basin of attraction includes the mid-point.

Like Gallow, I'm going to largely focus on the second of these, though I'll make some notes about the first from time to time. I'll call the second theory, the one I'm focussing on, the **midpoint theory**.

Let's bring this back to our two round game, Demonic Prisoners' Dilemma plus Demonic Stag Hunt. Midpoint theory says that one should Defect in Demonic Prisoners' Dilemma, and Gather in Demonic Stag Hunt. What does it say about the two stage game? Well, to answer that we need to precisify midpoint theory further. In particular, we need to answer two questions.

- What update function should we use?

It turns out this doesn't matter too much, as long as the function is sufficiently cautious. That is, the rule described above is fine as long as r is high enough. (A low value of r means that, at least for some games, one ends up oscillating between states rather than reaching equilibria; you need r low enough to make oscillations impossible.) The biggest question is this one.

- What strategies should we take seriously enough to give any positive probability to?

You might be interested in this question for boringly pragmatic reasons. The space of probability functions is an $n - 1$ dimension simplex, where n is the number of strategies taken seriously. So if there are 32 strategies, and you're trying to find the one with the largest basin of attraction, one needs to muck about with 31-dimensional geometry. And that is non-trivial.

But you might be interested in it for more philosophical reasons. If one can see that a strategy is sub-optimal, perhaps because it is dominated, or because it is not a possible equilib-

rium, it seems odd to even start the reflective process by having some probability one will end up there. So let's say that midpoint theory will involve starting with probability $\frac{1}{n}$ in each of the n sensible strategies. And the theory can be precisified in a few ways depending on what one counts as sensible.

For any natural choice of update function, and of sensible strategies, the resulting theory is dynamically inconsistent. There are a lot of choices for each of these questions, and I'm not going to go through how they play out one by one. But there is a pattern. In the strategic form of the two round game, the resulting equilibrium is either *CHGGG* or *DGGGH*. That is, in the strategic form of the game, rationality according to the midpoint theory requires one to Hunt in round two (or at least to expect to Hunt with probability 1). But if one plays the game in real time according to the midpoint theory, one must Gather in round 2. This is dynamically inconsistent, and tells against midpoint theory.⁵

But there is a loophole here; there is one way to make the midpoint theory dynamically consistent. And it's a simple one. If we start by saying every theory is 'sensible,' that at t_0 each theory has probability $\frac{1}{32}$ in being played, the midpoint theory says that the rational choice in the strategic form of the game is *DGGGG*. (Assuming an appropriate choice of update function. But in fact given this starting point, lots of update functions will do.) That's dynamically consistent. And it's the first dynamically consistent decisive theory we've seen. So is this the theory a philosopher who wants a decisive theory should approve of?

There are three reasons for thinking this theory – midpoint theory that starts with every strategy having equal probability no matter how absurd the strategy is – can't be right. As you can possibly tell from the fact that I'm going to offer three reasons, I'm not sure any one of them on their own is decisive. And even after these reasons, I still think this theory is the best decisive theory on offer. But I think it's flawed for these three reasons.

The first reason is one that Gallow (2020) offers as a criticism. The theory handles clone options very badly. Imagine Player is a bit ambidextrous; they are right handed, but they can draw straight lines with their left hand. They can't, however, draw curves. And they have to write 'G' or 'H' to play (one round) Demonic Stag Hunt. So now they have three options in Demonic Stag Hunt.

- Write 'G' with their right hand.
- Write 'H' with their left hand.
- Write 'H' with their right hand.

Midpoint theory says that in this game, Player should play one of the latter two options.

⁵I would have liked to have a theory about just which combinations of update rule and sensible strategies led to *CHGGG* and which led to *DGGGH*, but I couldn't see any pattern to them. I suspect there is something useful to say here, but I don't know what it is.

Roughly, that's because they will start with a probability of $\frac{2}{3}$ that they will play 'H' one way or the other, and as long as they start with a probability greater than 0.6 that they'll play 'H,' that's what they should end up with. But it's absurd that 'cloning' one of the choices should change what they play.

The second reason relates to my term 'sensible.' Think about what the midpoint theory, with no restrictions on initial strategies, is saying. It says one should start with this ludicrous probability, where one thinks it is somewhat likely that one will play a strictly dominated strategy, then advance from there by steps that are each sensible movements from where one is. But there is nothing philosophically significant about the fact that a certain state is the endpoint of applying a rational process from an irrational starting point. It's like saying I should believe the sky is green because this follows by the rational rule of and-elimination from the starting point 'grass is blue and the sky is green.' So even if this theory is dynamically consistent, it is not philosophically coherent.

Now I should note at this point that Skyrms himself is not particularly sympathetic to this line of reasoning. He thinks it is fine (at least for some purposes) to include some absurdities in the initial state. After all, the point of the dynamic deliberative process is to weed out absurdities. And he does have a point; there is some amount of redundancy in the view that one should first delete the obviously bad strategies, then go through some process to delete the less obviously bad strategies. Why not just trust the process?

So let's turn to the third reason, which is that midpoint theory is also arguably dynamically inconsistent in a different case. I say 'arguably' because the case involves mixed strategies, and just how to understand dynamic consistency when mixed strategies are around is a tricky question.

The example involves playing the following two Demonic Anti-Coordination games in order, with the results of the first game being revealed to Player and demon before the second game is played.

First Anti-Coordination Game

	u	d
U	0	9
D	1	0

Second Anti-Coordination Game

	u	d
U	0	4
D	1	0

Read *U* as Up and *D* as Down. Each game on its own has a unique Skyrms equilibrium. We don't need to worry about update rules or basins or anything – the approach of seeking the good will end in the first game with having probability 0.9 of playing Up, and in the second game with having probability 0.8 of playing Up. So the strategic form of the game should have an equilibrium that consists of playing Up with probability 0.9 then, whatever happens in the first game, playing Up with probability 0.8 in the second game.

And that is one of the equilibria in the 32 strategy strategic form of the game. But it isn't the only equilibrium. There is another equilibrium that looks like this. (I'll use the same convention as above for describing strategies, so the 2nd through 5th letters describe what to do after the first game ends *Uu*, *Ud*, *Du* and *Dd* respectively.)

A mixed strategy for the two round anti-coordination game

Strategies	Probability
<i>UUUUU</i> , <i>UUUUD</i> , <i>UUUDU</i> , <i>UUUDD</i>	$\frac{51}{310}$
<i>UUDUU</i> , <i>UUDUD</i> , <i>UUDDU</i> , <i>UUDDD</i>	0
<i>UDUUU</i> , <i>UDUUD</i> , <i>UDDDU</i> , <i>UDUDD</i>	$\frac{51}{1240}$
<i>UDDUU</i> , <i>UDDUD</i> , <i>UDDDU</i> , <i>UDDDD</i>	0
<i>DUUUU</i> , <i>DUDUU</i> , <i>DDUUU</i> , <i>DDDUU</i>	$\frac{11}{310}$
<i>DUUUD</i> , <i>DUDUD</i> , <i>DDUUD</i> , <i>DDDUD</i>	0
<i>DUUDU</i> , <i>DUDDU</i> , <i>DDUDU</i> , <i>DDDDU</i>	$\frac{11}{1240}$
<i>DUUDD</i> , <i>DUDDD</i> , <i>DDUDD</i> , <i>DDDDD</i>	0

In words, here's what this strategy does.

- In round 1, play Up with probability $\frac{51}{62}$.
- If one gets a positive return in round 1, in round 2 play Up with probability 0.8.
- If one gets a zero return, in round 2 always play Up.

It isn't that hard to verify that this is an equilibrium. If one has this probability function over

the demon's choices, then each strategy with positive probability has an expected return of $\frac{809}{310}$, and the strategies with probability 0 have lower expected return. (Note this is a marginally higher expected return than playing the equilibrium mixed strategy each round, which returns 2.6, or $\frac{806}{310}$.)

What's surprising is that this equilibrium is the one that midpoint theory recommends in the strategic form of the game. Or, at least, if r is low enough for an equilibrium to be found, it recommends this equilibrium. (If r is too high, the process never reaches equilibrium, and instead oscillates between two somewhat strange strategies.) So midpoint theory says this is the one and only right strategy in the game.⁶

But this seems to make midpoint theory dynamically inconsistent. According to the strategic form of the game, midpoint theory says the one and only rational play in the first round is to play Up with probability $\frac{51}{62}$. But when the game is being played in real time, midpoint theory says the one and only rational play in the first round is to play Up with probability 0.9. In the strategic form of the game, midpoint theory says that it is never permissible to play Down in round 2 after getting a zero return in round 1. But it also says that in the real time version of the game, one should play Down in round 2 with probability 0.2, whatever happens in round 1. Now it isn't perfectly clear exactly how one should understand dynamic consistency when mixed strategies are being considered. But one way or the other, these results feel like they show midpoint theory is dynamically inconsistent.

So I don't think there is a way of converting the Skyrms dynamics to a plausible and consistent decisive theory by looking at basins of attractions. But obviously I haven't surveyed all of the ways that one could build a dynamic theory this way. Perhaps some future decisive theorist will find a way through the gaps in the argument above. And there are meant to be gaps. The argument here is essentially inductive - all these approaches fail, and that's probably because there isn't a successful approach to be found. But rather than try and shore up this inductive inference yet further, I'll turn to a different way that it has been suggested we use the Skyrms approach to develop a decisive theory - one that recommends Hunting in the one round Demonic Stag Hunt.

9 Choose the Best Equilibrium

So far I've spent most of the time considering solutions to Demonic Stag Hunt that recommend Gathering. It turns out we have to spend much less time on solutions that recommend Hunting. As I noted earlier, that's because the different approaches to getting to Hunting

⁶The basin of attraction for this equilibrium is very large, but I'm not quite sure how large. I think it has measure 1, but I haven't found a proof of this yet. If it does have measure 1, this has consequences for the view that says strategies with measure 0 are not rationally playable.

all have quite a bit in common.

One way to motivate Hunting starts with Jeffrey's version of a broadly evidential decision theory. As he noted in the second edition of *The Logic of Decision* (Jeffrey 1983), to handle real life Newcomb problems, it is natural to add side-constraints to evidential decision theory. Those constraints will rule out one-boxing, and the equivalent of one-boxing in various real life situations. Call such a view evidential decision theory plus side constraints. It won't matter for our purposes what the side constraints are, just that they don't rule out anything in Stag Hunt. So the view recommends Hunting.

Or alternatively, one could start with the Skyrms approach to dynamic deliberation that was the focus of the last section. But instead of choosing between multiple equilibria by looking at properties of basins, as midpoint theory did, one could just choose the equilibria with the highest expected return. That's the broadly causal theory that Frank Arntzenius (2008) recommends. And it too recommends Hunting.

In a lot of cases, these two approaches will recommend very similar choices. That might be surprising if you think of one of them as broadly evidential and one as broadly causal. But a better way to think about them, I think, is to see them as reaching a natural compromise point between the two big approaches (evidential and causal) from different directions. And it's a good-making feature of a compromise that it can be reached by multiple paths in this way. Unfortunately, the resulting theory is dynamically inconsistent.

Go back to the two stage game, and consider the strategy *CHGGG*. If Player plays this strategy, and Demon predicts this, Player does very well. They get 54, which is almost the highest return in the game. And given Demon's prediction, they can't do better. Also, they can't do better in any circumstance where Demon predicts correctly. So this is an equilibrium, in Skyrms's sense, and ratifiable, in Jeffrey's sense. More generally, it satisfies any plausible side constraints one could add on to evidential decision theory. And it is the choice that evidential decision theory recommends. So it is the strategy any such theory should recommend. Strictly speaking, the theory says one should be indifferent between *CHGGG* and *CHGGH*; I won't fuss over this because what is going to matter is what one does at the very first step.

These decision theories recommend the strategy *CHGGG*. But they do not recommend choosing *C* in round 1 of a real life Demonic Prisoners' Dilemma plus Demonic Stag Hunt. They recommend choosing *D* in a stand alone version of Demonic Prisoners' Dilemma. Indeed, they are designed to give that recommendation. So they could only recommend playing *C* in round 1 of the two round game if there was a strategic benefit to playing *C*. But there is no strategic benefit. Playing *C* does not cause a change in any of the parameters that are relevant to the game in round 2. Indeed, it doesn't even provide evidence that changes one's

best estimates of those parameters. (Here it is crucial that we said the Demon's errors are probabilistically independent of each other.) So the theories say, inconsistently, to mark *C* when filling in the form, but to answer *D* when asked how one wants to play round 1. So they too are dynamically inconsistent.

It's possibly a failure of imagination on my part, but I really can't see any other motivation for Hunting than the fact that the Both Hunt equilibrium has a higher payout than the Both Gather. I'm not saying that's a bad motivation. If I personally was playing Demonic Stag Hunt, I'd probably Hunt, and for just this reason. What I am saying is that we can't, on pain of contradiction, say that it is a decisive reason to Hunt. The approach to decision theory that says it is a decisive reason offers inconsistent advice in the two stage game, and so is wrong. What we should say, and what indecisive theories do say, is that it's rationally permissible to Hunt for this reason, but it is also rationally permissible to Gather because Gathering reduces regret, or because Gathering has a larger Skyrmsian basin of attraction, or for any number of other reasons.

10 Existentialist Decision Theory

So far I've argued that decisive decision theories are dynamically incoherent. In this section I want to note one assumption that I've been making so far, and look at what happens if we drop that assumption. I call the assumption *existentialism*, but it takes a bit of explaining to see why that's a good name.

An existentialist decision theory says that each choice should be judged on its own, rather than as the contribution it makes to a strategy. Now a decision should be sensitive to the evidence; sometimes *X* is a better decision than *Y* because of what's happened earlier in the game. And a decision should be sensitive to long-run consequences; sometimes *X* is a better decision than *Y* because even though *Y* would have better short run consequences, *X* will promote valuable cooperation in the short run. But still, each decision should be made, and be judged, on its own, and not as part of a larger strategy.

The contrast to existentialism is what I'll call intellectualism. The intellectualist says that what makes a decision rational is that it is a manifestation of a rational long run strategy. The existentialism/intellectualism debate is orthogonal to the debate between evidential and causal decision theory, but it's very striking to see how it plays out if you assume evidential decision theory. The combination of intellectualism plus evidential decision theory says that in the cases that show evidential decision theory is dynamically inconsistent, one should take the choice that involves taking less money. And that's true even though at the time of the choice, one knows precisely how much one will get from each choice. That seems

bad, and I think it's a decisive reason to reject such theories.⁷

It's helpful to think of a strategy, a plan for what to do in all situations, as the essence of the chooser. And the sequence of choices the chooser makes in real time is their existence. So the intellectualist thinks that essence precedes existence; a choice is made good by its position in a grand strategy. And the existentialist denies this. They think that existence precedes essence, or at least that it does not proceed from essence.

The intellectualist position, whatever first order theory it is mixed with, immediately entails Backwards Dynamic Consistency and Forwards Dynamic Consistency. If what it is for a sequence of choices to be rational just is for it to be a manifestation of a rational strategy, then obviously there will be tight connections between rational sequences and rational strategies. But the reverse entailment is not true; the consistency norms do not entail intellectualism. One could, as I'll argue in a minute, hold on to the norms within a thoroughly existentialist framework.

At first glance, existentialism looks hostile to the very idea of inter-temporal norms on agency. And there is a super strong form of existentialism that denies all such norms. But there are three natural ways to moderate one's existentialism to allow for the possibility of such norms.

First, one could think that individual humans choose the units of time over which their choices will be assessed. Intellectualism is the view that the relevant unit is a life. The alternative to that need not be that each instant is to be assessed anew. So the existentialist can agree with Holton (2009) that it is good to make plans, that once a plan is made it is typically good to carry through with it, and even that if one chooses a plan, that plan should be assessed as a unit, instead of assessing the individual actions that make up a plan. Existentialism, in the sense I'm using (or co-opting) the expression, simply denies that planning in this sense is rationally mandatory, and in particular that there need to be life-plans.

Second, one could think that there are norms about the appropriate amount of fickleness in one's values and choices. It's consistent to say that each choice should be assessed on

⁷I think that intellectualism plus evidential decision theory is what Levinstein and Soares (2020) call 'functionalist decision theory.' I think, that is, that what they call choosing an algorithm is what I call choosing a strategy. But I'm not sure about this, since intellectualism plus evidential decision theory is a very strange theory. Note that it is strictly speaking indecisive in my sense. You can't tell what to do in such a theory given the setup of any decision problem. For any decision problem you like, including the one where the chooser simply has to choose more money rather than less, there is some possible pre-history of the problem where different choices will be rational. So if my interpretation is right, everything Levinstein and Soares say about what their theory recommends in one or another case is only correct given some substantive but unstated assumptions about the pre-history of the cases. That said, the broad approach they take does seem to be as anti-existentialist, in the ordinary sense of 'existentialist,' as it is possible to be. That is some evidence I guess in favor of this reading.

its own merits, but that a choice is bad in virtue of being excessively fickle. In that case one must say that it is the second choice of the fickle pair that's the bad one; that the chooser will later have different views is not a bad making feature of a particular choice. An advantage of this approach over intellectualism is that it allows that both stubbornness and fickleness are vices. The intellectualist thinks that the ideal agent has one grand plan and sticks to it through thick and thin. This doesn't sound great, and the existentialist is not committed to it.

Third, and most importantly, one could think that failures of dynamic consistency are not bad in themselves, but that they are evidence that one of the individual choices within them is bad. (I'm borrowing ideas from Christensen (1996) and Kolodny (2005) here.) If you know a person believes p and believes $\neg p$, you know they've made a mistake, even if you don't know when or where. And the mistake isn't (just) that they are incoherent; one of these two beliefs was ill-formed. This is the role that dynamic consistency norms play in the argument of this paper. We can see that a view is wrong by seeing that it is dynamically inconsistent. But the dynamic inconsistency is not constitutive of the wrongness; it is just how we see that the view is wrong.

That's the position I'm adopting here. The problem with the decisive views I've criticized is not that they are existentialist. Good theories are existentialist. It's that when you ask them the same question in two different guises, they give different answers. That's bad, and it's bad even if one thinks that good choosers do not need an essence, or a strategy, before they interact with the world.

11 Conclusions and Further Research

I've argued that a whole bunch of decisive theories fail, in systematic ways, to be dynamically consistent. I haven't offered anything like a proof that there could not be a plausible theory that is both decisive and dynamically consistent. Since some intellectualist theories are decisive and dynamically consistent, maybe there is a way to make one of them plausible. But by now the prospects should look grim.

So if decisive theories are bad, what should a good indecisive theory look like? We've already seen one plausible indecisive theory: the theory that says all Skyrms equilibria can be rationally chosen. But is it the only plausible indecisive theory? Is it ultimately plausible at all?

I think the way to finding a plausible indecisive theory goes via answering the following five questions.

First, does decision theory start with what the chooser believes, or with what they should

believe? If Player is certain that the red box has more money, but they have conclusive evidence that the blue box has more money, which box does decision theory say that they should choose? David Lewis (2020a) says that the answer is ‘the red box’; it is just the theory about how to turn beliefs and desires into action. I’m more sympathetic to the arguments that Nomy Arpaly (2003) makes that the theory of rational choice should not pay any special attention to the chooser’s beliefs. What’s rational to choose in a situation is a function of what’s rational to believe in that situation, not what one actually believes.

Second, in a given situation, how many different beliefs are rational? The Uniqueness thesis says the answer is one. Permissivism says that Uniqueness is false, and for some propositions in some situations, there are multiple rational attitudes to have. See Kopec and Titelbaum (2016) for a good survey of the issues, Schultheis (2018) for a recent argument for Uniqueness, and Callahan (2021) for a recent argument for Permissivism.⁸ I’m on the Permissivist side of this debate.

Now if you think decision theory should be sensitive to rational beliefs rather than actual beliefs, and you think Permissivism is true, you’re committed to indecisiveness. You won’t even need demons. After all, any situation where any credence in p between x and y is permissible will mean there are multiple bets at distinct odds on p that rationality neither requires taking nor requires passing. I think this is a perfectly sound argument for indecisiveness, but I didn’t lean on it here because the premises are considerably less secure than Backwards Dynamic Consistency.

But there is a third question that needs answering before we can offer a plausible indecisive theory: what is a mixed strategy? Relatedly, what role do mixed strategies have in the correct decision theory? This is a rather vexed question, and an important one. Almost all recent arguments against causal decision theory seem, to my eyes at least, to turn on attributing a bad theory of mixed strategies to the causal decision theorist. But that’s a topic for another paper.⁹

Note one thing I haven’t said so far, and won’t say in what follows. I don’t say that the way to find the correct indecisive theory is to come up with a bunch of cases, consult our intuitions about them, and then see which theory can match at least 80% of those intuitions. (Or whatever percentage we are working with this week.) That is a dubious approach in general, but around here it is close to incoherent.

⁸Interestingly, Callahan connects Permissivism to existentialism. I suspect there are deep and unexplored connections between the issues raised in the previous section and the Uniqueness/Permissivism debate.

⁹I haven’t given a positive theory here because it’s a big question, but I think the story must include the following two factors. First, playing a mixed strategy is just what Lewis (2020b) calls using a tie-breaking procedure. Second, the output of such a tie-breaking procedure is in principle unpredictable by anything that doesn’t time travel.

Most contemporary work in decision theory starts with the assumption that when there are no demons around (or anything else vaguely demonic), expected utility maximisation is the correct decision theory. And then theorists will start rolling out fantastic cases involving demons or predictors or lesions or genes or twins or triplets or whatever is in fashion. And they will ask what extension of expected utility theory best tracks intuitions about these cases. But this seems like a very dubious strategy, since intuitions about cases will not lead one to expected utility theory in the first place. Trying to match intuitions about cases like the Allais or Ellsberg paradoxes will lead one to prefer some non-standard theory like the one developed by John Quiggin (1982) or Lara Buchak (2013). It seems very unlikely that the best way to extend a counterintuitive theory like expected utility maximisation is by consulting intuitions about puzzle cases. It is much better to ask what principles we want our theory to endorse, and work towards a theory that satisfies those principles. And that is the methodology I have adopted here.

I've relied heavily in this paper on one such principle: Backwards Dynamic Consistency. I'll end by describing one more principle, and noting two questions it raises. The principle is that a decider should be a probabilist, and that they should maximise expected utility. More precisely, it says that if the states are $\{S_1, \dots, S_m\}$, and the choices are $\{O_1, \dots, O_n\}$, then O_i is a permissible choice just in case there is some probability function Pr such that

$$\sum_{k=1}^m V(S_k \wedge O_i) Pr(S_k) \geq \sum_{k=1}^m V(S_k \wedge O_j) Pr(S_k)$$

for all $j \in \{1, \dots, n\}$. Even if the subjective probability of the state is affected by the choice one makes, there should be some probability function that the chooser ends up with, and their choice should make sense by the lights of that probability function. Note that if we assume that the chooser can select any mixed strategy from among their choices, there is guaranteed to be at least one strategy that satisfies this requirement, even if one thinks the states are choices of a demon who can predict one's strategy.¹⁰

So that seems to me like a minimal constraint on choices. As Pearce (1984) shows, it is equivalent to the requirement that one not make a choice that is strictly dominated by some other choice, or by some mixture of other choices. (This result is hardly obvious, but it turns out to be a reasonably straightforward consequence of the existence of Nash equilibria for all finite zero-sum games.) That's hardly an uncontroversial principle, but it is also one I'm happy to adopt. If you're still on board, there are two more questions that we need to answer before we finish our decision theory.

¹⁰If the demon can predict what one will do on a given occasion while playing a mixed strategy, this guarantee may fail. But assuming what I said in the last footnote about mixed strategies, that would mean we're in the realm of backwards causation, and the states are not causally independent of the actions.

Are all further constraints on rational decisions representable as constraints on the *Pr* in this principle? There surely are some further constraints on rational decisions. If you're offered a bet at even money on whether I will become Canadian President next week, the only rational thing to do is to decline it. And that's true even though there is a *Pr* such that taking the bet maximises expected utility. But that *Pr* is completely irrational given your evidence. So *Do something that maximises expected utility given some probability* is too liberal a rule; we need to say something about the *Pr*. Do we need to say more than that? My answer is no, though I'm not even going to start defending that here.¹¹

The Canadian Presidency examples suggests that there are constraints on *Pr* that are external to decision theory. You shouldn't take that bet because you shouldn't have probability above 0.5 that I'll become Canadian President next week. The order of explanation runs from the (ir)rationality of the credal state to the (ir)rationality of the decision. Our fifth and final question is, are there any cases where the order of explanation goes the other way? Arntzenius (2008) argued that one should have credences such that the highest value equilibrium was also the choice that maximised expected utility. That's an example of a constraint on *Pr* where the order of explanation runs from decisions to beliefs. I argued against that principle, but not because of a systematic reason to think that the order of explanation can't run that way. Instead I argued that this particular principle was dynamically incoherent. That leaves open the general question of whether any such principles, where constraints on decisions explain constraints on belief, are right.

The long term goal of the project behind this paper is to argue that there are no such principles. The only constraints on rational decision are that one should maximise expected utility given some *Pr*, and this *Pr* should satisfy independently motivated epistemic requirements. Now I haven't come close to arguing for that here, and it's a very strong claim. Given everything else I've said, it basically amounts to the claim that the theory of equilibrium selection has no role to play in normative decision theory. It may have a central role to play in descriptive decision theory, in explaining why people end up at a certain equilibrium. But it can't justify that equilibrium, since any equilibrium could be rationally justified.

But all of this is for future work. The aim of this paper has been to open up the possibility of an indecisive, i.e., permissive, decision theory. Decisive decision theories have to take a stand on Demonic Stag Hunt, but it seems surprisingly hard to find a plausible way to take a stand on it in a dynamically coherent way. All the decisive theories I've considered across a wide range of traditions, have ended up saying that there is a clearly right thing to do if *p* is correct, but it's not the thing they will say if you tell them *p* is true and ask them what to do. That's incoherent, so those theories are incorrect. The best explanation of this range

¹¹Note that if you say no to this question, and you think that probabilities have to be real-valued, then you're committed to weak dominance not having a role to play in decision theory. So this is a non-trivial question.

of indecisive theories being incorrect, I suggest, is that no decisive theory is correct. The correct decision theory is indecisive.

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