

## Comments on Das

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## COMPARISON OF EXPERIMENTS

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### 1. Summary

Bohnenblust, Shapley, and Sherman [2] have introduced a method of comparing two sampling procedures or experiments; essentially their concept is that one experiment  $\alpha$  is more informative than a second experiment  $\beta$ ,  $\alpha \supset \beta$ , if, for every possible risk function, any risk attainable with  $\beta$  is also attainable with  $\alpha$ . If  $\alpha$  is a sufficient statistic for a procedure equivalent to  $\beta$ ,  $\alpha \supset \beta$ , it is shown that  $\alpha \supset \beta$ . In the case of dichotomies, the converse is proved. Whether  $\supset$  and  $\supset$  are equivalent in general is not known. Various properties of  $\supset$  and  $\supset$  are obtained, such as the following: if  $\alpha \supset \beta$  and  $\gamma$  is independent of both, then the combination  $(\alpha, \gamma) \supset (\beta, \gamma)$ . An application to a problem in  $2 \times 2$  tables is discussed.

### 2. Definitions

An *experiment*  $\alpha$  is a set of  $N$  probability measures  $u_1, \dots, u_N$  on a Borel field  $B$  of subsets of a space  $X$ . The  $N$  measures are considered as  $N$  possible distributions over  $X$ , and performing the experiment consists of observing a sample point  $x \in X$ . A *decision problem* is a pair  $(\alpha, A)$ , where  $A$  is a bounded subset of  $N$ -space. The points  $a \in A$  are considered as the possible actions open to the statistician; the loss from action  $a = (a_1, \dots, a_N)$  is  $a_i$  if the actual distribution of  $x$  is  $u_i$ . A *decision procedure*  $f$  for  $(\alpha, A)$  is a  $B$ -measurable function from  $X$  into  $A$ , specifying the action  $a$  to be taken as a function of the sample point  $x$  obtained by the experiment. With every  $f = [a_1(x), \dots, a_N(x)]$  is associated a loss vector

$$v(f) = \left( \int a_1(x) du_1, \dots, \int a_N(x) du_N \right);$$

the  $i$ -th component of  $v(f)$  is the expected loss from  $f$  if  $x$  has distribution  $u_i$ . The range of  $v(f)$  is a subset of  $N$ -space which we denote by  $R_i(\alpha, A)$ ; the convex closure of  $R_i(\alpha, A)$  will be denoted by  $R(\alpha, A)$  and will be called the set of *attainable loss vectors* in  $(\alpha, A)$ ; every vector in  $R$  is either attainable or approximable by a randomized mixture of  $N + 1$  decision procedures.

**THEOREM 1.**  $R(\alpha, A) = R(\alpha, A_1) = R(\alpha, A_1)$ , where  $A_1$  is the convex closure of  $A$ .

This theorem permits us to restrict attention to closed convex  $A$ , which we shall do in the following sections. The proof of the theorem will not be given here; it is straightforward except for the fact that  $R(\alpha, A_1) = R(\alpha, A_1)$ . This fact follows from the result that whenever  $A$  is closed, so is  $R(\alpha, A)$ , which has been proved elsewhere by the author [1].

David Blackwell, "Comparison of Experiments", Berkeley Symposium on Mathematical Statistics and Probability, 1951

## Two Questions

1. Is the value of evidence thesis true?
2. Is Nilanjan's decision rule good?

## Counterexamples to Value of Evidence

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- When knowledge affects value, e.g., spoilers.
- When knowledge comes to a group.

## A Group Example

The group has to choose between these three options.

	p	$\sim p$
A	10	0
B	0	10
C	4	4

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- We do whatever choice maximises the minimum expected utility across the group.



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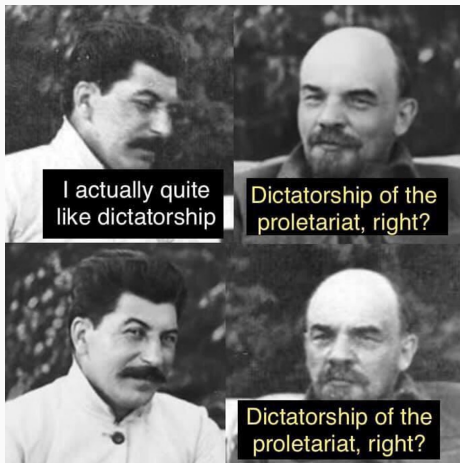
# One Decision Rule

- We do whatever choice maximises the minimum expected utility across the group.
- We will do C whatever evidence comes in, but no one wants that either now or later.
- In fact everyone would pay 1 to avoid that outcome.

## Other Decision Rules

- Let's try dictatorship.
- There might be an Arrowian argument that we'll be forced into dictatorship.
- Very hard to get a decision rule that is Paretian and Bayesian other than dictatorship. (Does hard mean impossible? Good question - if someone wants to work this out/write this up, lmk.)

## But It's a Good Dictatorship, Right



From qatsimai on the reddit forum historymemes

- One of the Pr on the credal committee gets to call the shots.
- This avoids incoherence, as long as the dictator is coherent.

## Functionalist Worries

- What does it mean to say other Pr are even in the committee?

# Am I Really on the Committee?



The dictator on the credal committee

- The solution is to say that the dictator only stays in the job for the length of an inquiry, then there is a new lottery.



- The solution is to say that the dictator only stays in the job for the length of an inquiry, then there is a new lottery.
- But some inquiries run for decades.

## Suggestion One: Politics are Everywhere

- The credal committee is a committee, and how it chooses is a political problem.
- Live with evidence being costly.
- We're used to that in committee choices already.

## Suggestion Two: Irrationality is Everywhere

- Does each committee member regard the others as rational, assuming known conditionalisation?
- They regard the others as being procedurally rational, but perhaps not substantively rational.
- Each other is rational by the other's lights, but not by mine.
- So maybe I shouldn't be surprised that I want to keep information from them.

**GREAT PAPER!**

