Referee report on
"Epistemic Permissiveness and Symmetric Games"
Prepared for *Principia*

Summary: This paper provides a philosophical argument against the Uniqueness thesis, and in favour of the Permissiveness thesis, in games. The Uniqueness thesis states that there is at most one rational attitude that players in a game can take, whereas Permissiveness can be viewed as the negation of this thesis. If the game at hand is symmetric, the Uniqueness thesis implies that all players should engage in the same, symmetric Nash equilibrium. The paper points out that this logical consequence is problematic, by investigating a number of "problematic" symmetric games. In the Chicken Game, for instance, the only symmetric Nash equilibrium is one in mixed strategies, in which both players are indifferent between their two choices. However, the symmetric Nash equilibrium imposes that both players assign probabilities 0.99 and 0.01 to the opponent's strategies, thereby clearly viewing one strategy as more plausible than the other, despite the fact that both strategies yield the same expected utility. The paper shows that the Uniqueness thesis requires both players to assign precisely these probabilities to the opponent's strategies, which seems rather implausible. The paper also dicusses two infinite symmetric games, explored in Fey (2012) and Xefteris (2015), which only contain asymmetric Nash equilibria (even in mixed strategies). The paper uses these two games as an additional argument against the Uniqueness thesis, as in those games the Uniqueness thesis, in combination with rationality, cannot be maintained. Indeed, the Uniqueness thesis together with rationality would lead the players to a symmetric Nash equilibrium, which is not present in these two games.

Evaluation: I have read this paper with great interest. As a researcher working on the foundations of game theory, I find the issue of Uniqueness versus Permissiveness in games very intriguing. The argument adopted in this paper – to criticize Uniqueness on the basis of symmetric games – seems very well justified. But as a game theorist, I sometimes had problems following some arguments in the philosophical debate. I will comment more on this below. And since I am not a philosopher, I find it difficult to judge the quality and content of the philosphical discussions in the paper. Therefore, most of my comments below will be on the game-theoretic content. As you will see, I think the paper should definitely refer to Nash's (1951) original result on the existence of symmetric equilibria in finite symmetric games. Also, the paper could benefit from discussing the relation between

the Uniqueness thesis on the one hand, and the concept of simple belief hierarchies (Perea (2012)) on the other hand. But overall, I am quite enthusiastic about the paper, and believe that an appropriately revised version could be published in *Principia.*

Main comments:

1. *Nash's (1951) result on finite symmetric games*

Nash (1951) proves, in Theorem 2, that every finite symmetric game (that is, containing finitely many choices for the players) contains at least one symmetric Nash equilibrium in mixed strategies. I believe the paper should definitely cite this paper, and take this existence result as a benchmark for the discussion in the paper. Nash's result implies that the non-existence of symmetric equilibria in symmetric games, as is the case in the examples from Fey (2012) and Xefteris (2015), can only happen if the game at hand contains infinitely many choices for at least one of the players. This fact may open the door to a potentially interesting philosophical discussion that could be added to the paper. Apparently, every symmetric game with finitely many choices allows the Uniqueness thesis to be applied, at least to some extent, since a symmetric Nash equilibrium in mixed strategies is guaranteed to exist, and this equilibrium may be viewed as the consequence of the Uniqueness thesis in combination with rationality. On the other hand, the move to infinitely many choices may cause the Uniqueness thesis to fail, as symmetric equilibria may no longer exist. But then, philosophically, what is the crucial difference between finite and infinite symmetric games, that causes the Uniqueness thesis to fail in the second class, while still allowing it in the first class? Perhaps the paper could expand on this issue, which could perhaps lead to an interesting philosophical discussion.

2. *Relation with simple belief hierarchies*

The Uniqueness thesis states that there is at most one rational attitude that players in a game can take. An idea in game theory that comes close to this is the concept of a *simple belief hierarchy,* as discussed in Perea (2012, Chapter 4). A player with a simple belief hierarchy believes that each of his opponents is correct about the actual beliefs he holds, and in the case of three players or more believes that every opponent holds the same belief as he has about a third player. In a sense, a player with a simple belief hierarchy views his own belief hierarchy as the unique focal point that all players in the game agree on. This, at least intuitively, is rather similar to the Uniqueness thesis. Moreover, it is shown in Perea (2012, Theorem 4.2.2) that common belief in rationality in combination with a simple belief hierarchy

epistemically characterizes the concept of Nash equilibrium. It would be interesting if the paper could elaborate on the similarity and differences between the Uniqueness thesis on the one hand, and the concept of a simple belief hierarchy on the other hand. The relation with Nash equilibrium could play an important role in this comparison.

Smaller comments:

3. *Unclear sentences*

As mentioned above, I had difficulties following some of the sentences, and philosophical arguments, in the paper. Here are some examples:

(a) Page 1, line 4: "... if permissivism is false." What does it mean? Please explain.
(b) Page 1, lines 7 and 8: "... if permissivism is false there is no rational way to play the game, although intuitively the game could be rationally played." I don't understand what this means. Please elaborate.
(c) Page 1, line 5 of second paragraph: "... there are two possible rational attitutes." Why two? Please explain.
(d) Page 8, lines 4 and 5: "If the players ... Uniqueness." Why is this? Please clarify.

4. Page 3, last line: "a symmetric equilibria" should be "asymmetric equilibria in mixed strategies".

5. Page 7, Footnote, last line: I think that "... is a Nash equilibrium" should be "... is a symmetric Nash equilibrium". Is this right?

References

[1] Nash, J.F. (1951), Non-cooperative games, *Annals of Mathematics* **54**, 286–295.

[2] Perea, A. (2012), *Epistemic Game Theory: Reasoning and Choice,* Cambridge University Press.