

Symbolization

2026-01-12

Argument Forms

We're going to get a little clearer on the notions of 'form' and 'structure' from last time.

Associated Reading

forall x, chapter 4, "First Steps to Symbolization".

An argument is valid when

- The truth of the premises guarantees the truth of the conclusion.
- Equivalently, when it is impossible for the premises to be true and the conclusion not true.

An argument is formally valid if

- It is valid; and
- Any argument of the same form is valid.

In those cases, we infer that it is valid **because of** its form.

Three Big Questions About Form

1. Which words get treated as part of the form, and which get replaced by symbols when converting into formal notation?

Three Big Questions About Form

1. Which words get treated as part of the form, and which get replaced by symbols when converting into formal notation?
2. Which words have the same meaning and should be replaced by the same symbol, and which should be treated differently?

Three Big Questions About Form

1. Which words get treated as part of the form, and which get replaced by symbols when converting into formal notation?
2. Which words have the same meaning and should be replaced by the same symbol, and which should be treated differently?
3. How do we break the sentence into its constituent parts?

I'm going to have a fairly pluralist, pragmatist view on this question.

- This is not to say that pluralism or pragmatism is right - it's just that it's what I'll adopt. (It is to say that I think it's a reasonable approach.)
- For some purposes the 'formal words' are very few, like 'and', 'or', 'not' etc.
- For other purposes the set is more expansive.

- If you are studying the logic of causation, you might want to say that 'caused' is a formal word.
- And that would mean any two arguments that have this structure are of the same form.

A caused B.

B caused C.

\therefore A caused C.

- But for other purposes, ‘caused’ is just another binary relation between events.
- So those arguments will have the same form as these ones.

A happened near B.

B happened near C.

∴ A happened near C.

- What I mean by having a ‘pluralist, pragmatist’ view about form is that there is no objectively correct answer about whether these are the same form or not.
- For some purposes it will be helpful to treat them as having the same form.
- For other purposes it will not be.
- We’re going to start with a notion of form where lots and lots of things have the same form, so very few arguments are valid in virtue of form.

Three Big Questions About Form

1. Which words get treated as part of the form, and which get replaced by symbols when converting into formal notation?
2. Which words have the same meaning and should be replaced by the same symbol, and which should be treated differently?
3. How do we break the sentence into its constituent parts?

There are problems in both directions here.

- Sometimes we want to treat words with different spelling as representing the same meaning - e.g., 'kangaroo' and 'kangaroos'.
- And sometimes we want to treat words with the same spelling as expressing different meanings.

Some kinds of ambiguity are ‘resolved’ by treating the word as part of the formal system. Consider two possible interpretations of this sentence.

Nothing is better than world peace

“Nothing is better than world peace”

1. There is no thing such that it is better than world peace.

“Nothing is better than world peace”

1. There is no thing such that it is better than world peace.
2. It is better that there is nothing than that there is world peace.

- If you include words like ‘nothing’ as formal words, then the two readings of the sentence will have different formal representations.
- And the logic called ‘predicate logic’ does exactly that; it treats ‘nothing’ as one of the formal words.
- We’re not covering that in this course, though some of you have covered it elsewhere.
- Some more expansive accounts of what should be a formal word are moved by the thought that treating more words as formal removes some possible ambiguities.

Three Big Questions About Form

1. Which words get treated as part of the form, and which get replaced by symbols when converting into formal notation?
2. Which words have the same meaning and should be replaced by the same symbol, and which should be treated differently?
3. How do we break the sentence into its constituent parts?

1. Replacing words/sentences with letters.

Three Formal Tools

1. Replacing words/sentences with letters.
2. Using parentheses.

Three Formal Tools

1. Replacing words/sentences with letters.
2. Using parentheses.
3. Replacing 'formal words' with symbols.

Formal Tools

We're going to look at how to translate English sentences into a form where we can use the tools of propositional logic on them.

forall x, chapter 5, “Connectives”

The book goes into more detail than I'm going into here about the range of English sentences that can be translated (or at least symbolized) using the tools of propositional logic.

- We use letters, usually starting with p and then continuing through the alphabet, for 'atomic' sentences.
- forall x uses capital letters, others use lowercase. This is a matter of taste.

- “and” is represented by \wedge
- “or” is represented by \vee
- “not” is represented by \neg
- “if” is represented by \rightarrow
- “if and only if” is represented by \leftrightarrow

It will rain on Monday or it will rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

- $M \vee T$

Another Easy Example

It will rain on Monday and it will rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

- $M \wedge T$

Only Slightly Harder Example

If it rains on Monday, it will rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

- $M \rightarrow T$

And A Bit Harder Again

It will rain on Monday, but it won't rain on Tuesday

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday

Translation

- $M \wedge \neg T$

- As in math, these tell you what order to compute things in.
- They let you avoid the ambiguity in the following sentence.
It will rain on Monday and it will rain on Tuesday or it will rain on Wednesday.

Two Translations

Basic Sentences

- M = It will rain on Monday
- T = It will rain on Tuesday
- W = It will rain on Wednesday

Translations

1. $(M \wedge T) \vee W$
2. $M \wedge (T \vee W)$

What's the Difference?

1. $(M \wedge T) \vee W$
2. $M \wedge (T \vee W)$

What is a situation where one of these is true but not the other?

- If it only rains on Wednesday, then 1 is true but 2 is false.

What's the Difference?

1. $(M \wedge T) \vee W$
2. $M \wedge (T \vee W)$

What is a situation where one of these is true but not the other?

- If it only rains on Wednesday, then 1 is true but 2 is false.
- Are there any others?

Recursion

We're going to look at how and why we can iterate the translation procedures we've been investigating.

Associated Reading

forall x, chapter 6, "Sentences of TFL"

The language of propositional logic has some fairly simple composition rules.

- It says what the basic sentences are.
- It has some rules saying that if some things are sentences, so are some other things.

The effect is that there are an infinity of possible sentences.

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.
- Alex thinks that Kim thinks that Alex thinks that it will rain.

As speakers of a human language, you're used to this kind of recursion. All of these are sentences.

- It will rain.
- Alex thinks that it will rain.
- Kim thinks that Alex thinks that it will rain.
- Alex thinks that Kim thinks that Alex thinks that it will rain.
- And so on, to infinity, without adding any more words.

- If S is a sentence, and N is a name, then N *thinks that* S is a sentence.
- Note that the output of this rule can be the input to a new instance of it.

The letters P, Q, R , etc. are sentences.

If S and T are sentences, then so are:

1. $\neg S$
2. $S \vee T$
3. $S \wedge T$
4. $S \rightarrow T$
5. $S \leftrightarrow T$

So these are all sentences. (Note that I'm playing fast and loose with parentheses here.)

1. P
2. Q
3. $P \wedge Q$
4. $Q \rightarrow (P \wedge Q)$
5. $\neg P$
6. $Q \vee \neg P$
7. $(Q \rightarrow (P \wedge Q)) \leftrightarrow (Q \vee \neg P)$

The last one follows from the fact that 4 and 6 are sentences.

For any sentence you can make, there will be a 'last step' in the demonstration that it is a sentence.

- That last step will involve copying down 1 or 2 other sentences, and adding a connective.
- On the previous slide, you copy down 4 and 6, and put a \leftrightarrow between them.
- That connective you add is the **main connective** of the sentence.
- It covers all the material in the sentence.

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

- $P \wedge (Q \rightarrow R)$

The \wedge is the main connective because either side of it are

- P
- $(Q \rightarrow R)$

And they are both sentences.

A binary connective is the main connective if (and only if) either side of it are two complete sentences.

- $P \wedge (Q \rightarrow R)$

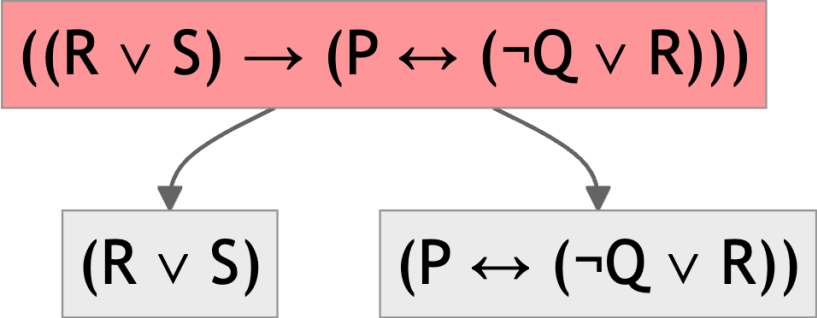
The \rightarrow is not the main connective because either side of it are

- $P \wedge (Q$

- $R)$

And they are not both sentences.

$$((R \vee S) \rightarrow (P \leftrightarrow (\neg Q \vee R)))$$

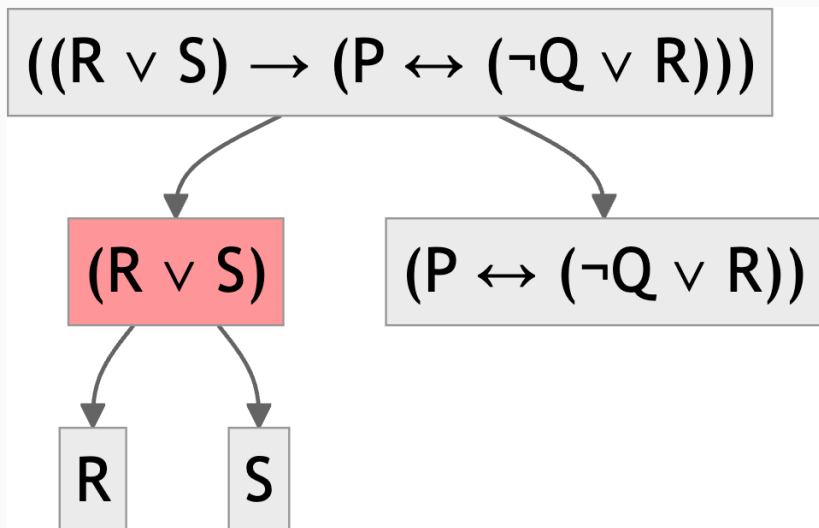
$$((R \vee S) \rightarrow (P \leftrightarrow (\neg Q \vee R)))$$


The diagram illustrates the decomposition of the logical formula $((R \vee S) \rightarrow (P \leftrightarrow (\neg Q \vee R)))$. Two curved arrows originate from the main formula box: one points to the antecedent $(R \vee S)$ and the other points to the consequent $(P \leftrightarrow (\neg Q \vee R))$.

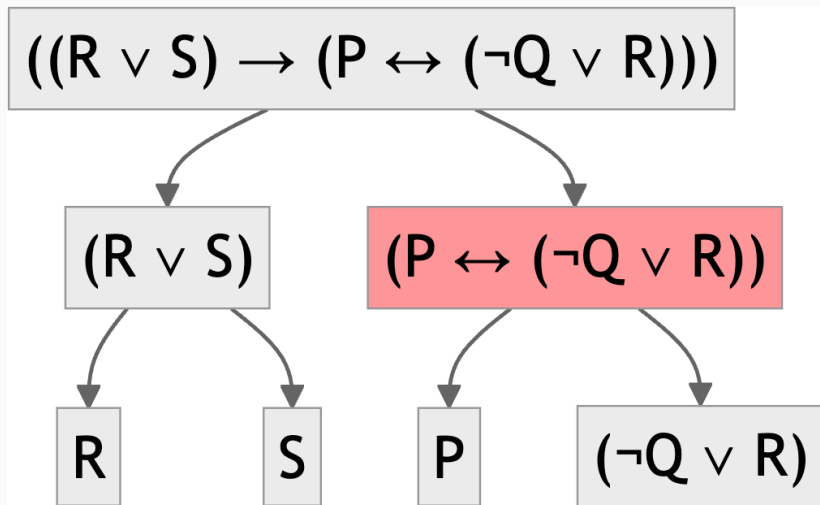
$$(R \vee S)$$

$$(P \leftrightarrow (\neg Q \vee R))$$

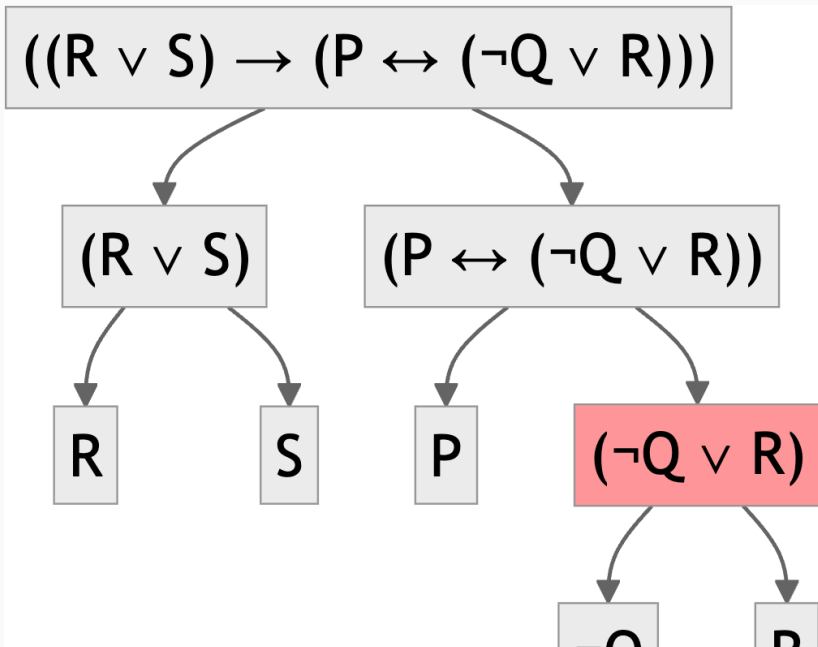
A Worked Example



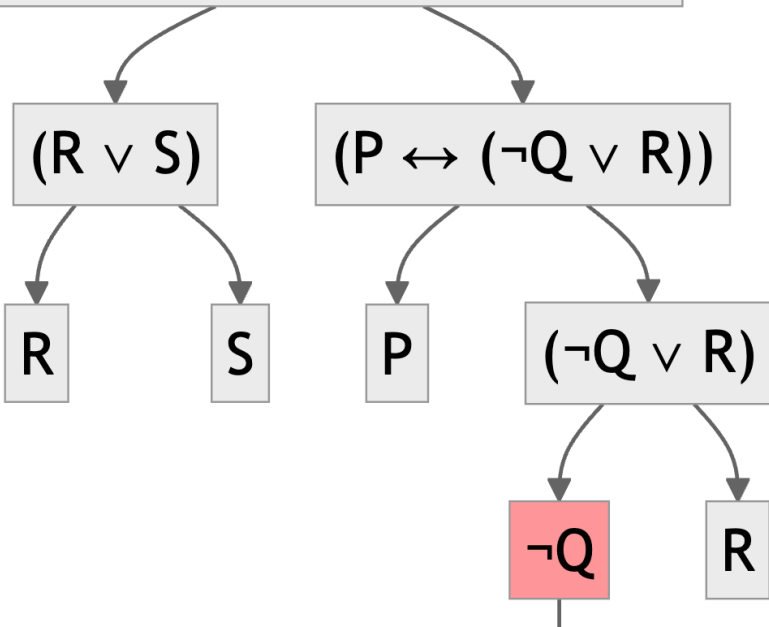
A Worked Example



A Worked Example



$((R \vee S) \rightarrow (P \leftrightarrow (\neg Q \vee R)))$



- We are going to start on how these connectives help us figure out the truth value of longer sentences.
- And we're going to use that to analyse arguments.