

Chapter 12 Objective Probabilities

12.1 Credences and Norms

We ended last time with looking at the idea that the probabilities in expected utility calculations should be subjective. As it is sometimes put, they should be degrees of belief. Or, as it is also sometimes put, they should be credences. We noted that under this interpretation, the maximise expected utility rule doesn't put any constraints on certain simple decisions. That's because we use the rule to calculate what credences are, and then use the very same credences to say what the rule requires. But the rule isn't useless. It puts constraints, often sharp constraints, on sets of decisions. In this respect it is more like the rule *Have Consistent Beliefs* than like the rule *Believe What's True*, or *Believe What Your Evidence Supports*. And we compared it to *procedural*, as opposed to *substantive* norms.

What's left from all that are two large questions.

- Do we get the *right* procedural/consistency constraints from the expected utility rule? In particular (a) should credences be probabilities, and (b) should we make complex decisions by the expected utility rule? We'll look a bit in what follows at each of these questions.
- Is a purely procedural constraint all we're looking for in a decision theory?

And intuitively the answer to the second question is **No**. Let's consider a particular case. Alex is very confident that the Kansas City Royals will win baseball's World Series next year. In fact, Alex's credence in this is 0.9, very close to 1. Unfortunately, there is little reason for this confidence. Kansas City has been one of the worst teams in baseball for many years, the players they have next year will be largely the same as the players they had when doing poorly this year, and many other teams have players who have performed much much better. Even if Kansas City were a good team, there are 30 teams in baseball, and relatively random events play a big role in baseball, making it unwise to be too confident that any one team will win.

Now, Alex is offered a bet that leads to a \$1 win if Kansas City win the World Series, and a \$1 loss if they do not. The expected return of that bet, given Alex's credences, is +80c. So should Alex make the bet?

Intuitively, Alex should not. It's true that given Alex's credences, the bet is a good one. But it's also true that Alex has crazy credences. Given more sensible credences, the bet has a negative expected return. So Alex should not make the bet.

It's worth stepping away from probabilities, expected values and the like to think about this in a simpler context. Imagine a person has some crazy beliefs about what is an effective way to get some good end. And assume they, quite properly, want that good end. In fact, however, acting on their crazy beliefs will be counterproductive; it will just make things worse for everyone. And their evidence supports this. Should they act on their beliefs? Intuitively not. To be sure, if they didn't act on their beliefs, there would be some inconsistency between their beliefs and their actions. But inconsistency isn't the worst thing in the world. They should, instead, have different beliefs.

Similarly Alex should have different credences in the case in question. The question, what should Alex do given these credences, seems less interesting than the question, what should Alex do? And that's what we'll look at.

12.2 Evidential Probability

We get a better sense of what an agent should do if we look not to what credences they have, but to what credences they *should* have. Let's try to formalise this as the credences they would have if they were perfectly rational.

Remember credences are still being measured by betting behaviour, but now it is betting behaviour under the assumption of perfect rationality. So the probability of p is the highest price the agent would pay for a bet that pays \$1 if p , if they were perfectly rational. The thing that should be done then is the thing that has the highest expected utility, relative to this probability function. In the simple case where the choice is between taking and declining a bet, this becomes a relatively boring theory - you should take the bet if you would take the bet if you were perfectly rational. In the case of more complicated decisions, it becomes a much more substantive theory. (We'll see examples of this in later weeks.)

But actually we've said enough to give us two philosophical puzzles.

The first concerns whether there determinately is a thing that you would do if you were perfectly rational. Consider a case where you have quite a bit of evidence for and against p . Different rational people will evaluate the evidence in different ways. Some people will evaluate p as being more likely than not, and so take a bet at 50/50 odds on p . Others will consider the evidence against p to be stronger, and hence decline a bet at 50/50 odds. It seems possible that both sides in such a dispute could be perfectly rational.

The danger here is that if we define rational credences as the credences a perfectly rational person would have, we might not have a precise definition. There may be many different credences that a perfectly rational person would have. That's bad news for a purported definition of rational credence.

The other concerns cases where p is about your own rationality. Let's say p is the proposition that you are perfectly rational. Then if you were perfectly rational, your credence in this would probably be quite high. But that's not the rational credence for you to have right now in p . You should be highly confident that you, like every other human being on the planet, are susceptible to all kinds of failures of rationality. So it seems like a mistake in general to set your credences to what they would be were you perfectly rational.

What seems better in general is to proportion your credences to the evidence. The rational credences are the ones that best reflect the evidence you have in favour of various propositions. The idea here is to generate what's usually called an **evidential probability**. The probability of each proposition is a measure of how strongly it is supported by the evidence.

That's different from what a rational person would believe in two respects. For one thing, there is a fact about how strongly the evidence supports p , even if different people might disagree about just how strongly that is. For another thing, it isn't true that the evidence supports that you are perfectly rational, even though you would believe that if you were perfectly rational. So the two objections we just mentioned are not an issue here.

From now on then, when we talk about probability in the context of expected utility, we'll talk about evidential probabilities. There's an issue, one we'll return to later, about whether we can numerically measure strengths of evidence. That is, there's an issue about whether strengths of evidence are the right kind of thing to be put on a numerical scale. Even if they are, there's a tricky issue about how we can even guess what they are. I'm going to cheat a little here. Despite the arguments above that evidential probabilities can't be *identified* with betting odds of

perfectly rational agents, I'm going to assume that, unless we have reason to the contrary, those betting odds will be our first approximation. So when we have to guess what the evidential probability of p is, we'll start with what odds a perfectly rational agent (with your evidence) would look for before betting on p .

12.3 Objective Chances

There is another kind of probability that theorists are often interested in, one that plays a particularly important role in modern physics. Classical physics was, or at least was thought to be, deterministic. Once the setup of the universe at a time t was set, the laws of nature determined what would happen after t . Modern physics is not deterministic. The laws don't determine, say, how long it will take for an unstable particle to decay. Rather, all the laws say is that the particle has such-and-such a chance of decaying in a certain time period. You might have heard references to the half-life of different radioactive particles; this is the time in which the particle has a $\frac{1}{2}$ probability of decaying.

What are these probabilities that the scientists are talking about? Let's call them 'chances' to give them a name. So the question is, what is the status of chances. We know chances aren't evidential probabilities. We know this for three reasons.

One is that it is a tricky empirical question whether any event has any chance other than 0 or 1. It is now something of a scientific consensus that some events are indeed chancy. But this relies on some careful scientific investigation. It isn't something we can tell from our armchairs. But we can tell from just thinking about decisions under uncertainty that the evidential probability of some outcomes is between 0 and 1.

Another is that, as chances are often conceived, events taking place in the past do not, right now, have chances other than 0 or 1. There might have been, at a point in the past, some intermediate chance of a particle decaying. But if we're now asking about whether a particle did decay or not in the last hour, then either it did decay, and its chance is 0, or it did not decay, and its chance is 1. (I should note that not everyone thinks about chances in quite this way, but it is a common way to think about them.) There are many events that took place in the past, however, whose evidential probability is between 0 and 1. For instance, if we're trying to meet up a friend, and hence trying to figure out where the friend might have gone to, we'll think about, and assign evidential probabilities to, various paths the friend might have taken in the past. These thoughts won't be thoughts about chances in the physicists' sense; they'll be about evidential probabilities.

Finally, chances are objective. The evidential probability that p is true might be different for me than for you. For instance, the evidence she has might make it quite likely for the juror that the suspect is guilty, even if he is not. But the evidence the suspect has makes it extremely likely that he is innocent. Evidential probabilities differ between different people. Chances do not. Someone might not know what the chance of a particular outcome is, but what they are ignorant of is a matter of objective fact.

The upshot seems to be that chances are quite different things from evidential probabilities, and the best thing to do is simply to take them to be distinct basic concepts.

12.4 The Principal Principle and Direct Inference

Although chances and evidential probabilities are distinct, it seems they stand in some close relation. If a trustworthy physicist tells you that a particle has an 0.8 chance of decaying in the next hour, then it seems your credences should be brought into line with what the physicists say. This idea has been dubbed the Principal Principle, because it is the

main principle linking chances and credences. If we use Pr for evidential probabilities, and Ch for objective chances in the physicists' sense, then the idea behind the principle is this.

Principal Principle $Pr(p|Ch(p) = x) = x$

That is, the probability of p , conditional on the chance of p being x , is x .

The Principal Principle may need to be qualified. If your evidence also includes that p , then even if the chance of p is 0.8, perhaps your credence in p should be 1. After all, p is literally evident to you. But perhaps it is impossible for p to be part of your evidence while its chance is less than 1. The examples given in the literature of how this could come about are literally spectacular. Perhaps God tells you that p is true. Or perhaps a fortune teller with a crystal ball sees that it is true. Or something equally bizarre happens. Any suggested exceptions to the principle have been really outlandish. So whether the principle is true for all possible people in all possible worlds, it seems to hold for us around here.

Chances, as the physicists think of them, are not frequencies. It might be possible to compute the theoretical chance of a rare kind of particle not decaying over the course of an hour, even though the particle is so rare, and so unstable, that no such particle has ever survived an hour. In that case the frequency of survival (i.e. the proportion of all such particles that do actually survive an hour) is 0, but physical theory might tell us that the chance is greater than 0. Nevertheless chances are like frequencies in some respects.

One such respect is that chances are objective. Just as the chance of a particle decay is an objective fact, one that we might or might not be aware of, the frequency of particle decay is also an objective fact that we might or might not be aware of. Neither of these facts are in any way relative to the evidence of a particular agent, the way that evidential probabilities are.

And just like chances, frequencies might seem to put a constraint on credences. Consider a case where the only thing you know about a is that it is G . And you know that the frequency of F -hood among G s is x . For instance, let a be a person you've never met, G be the property of being a 74 year old male smoker, and F the property of surviving 10 more years. Then you might imagine knowing the survival statistics, but knowing nothing else about the person. In that case, it's very tempting to think the probability that a is F is x . In our example, we'd be identifying the probability of this person surviving with the frequency of survival among people of the same type.

This inference from frequencies to probabilities is sometimes called "Direct Inference". It is, at least on the surface, a lot like the Principal Principle. But it is a fair bit more contentious. We'll say a bit more about this once we've looked about probabilities of events with infinite possibility spaces. But for now just note that it is really rather rare that all we know about an individual can be summed up in one statistic like this. Even if the direct inference can be philosophically justified (and I'm a little unsure that it can be) it will rarely be applicable. So it is less important than the Principal Principle.

We'll often invoke the Principal Principle tacitly in setting up problems. That is, when I want to set up a problem where the probabilities of the various outcomes are given, I'll often use objective chances to fix the probabilities of various states. We'll use the direct inference more sparingly, because it isn't as clearly useful.