Disjunctions of Universal Modals and Conditionals

Dean McHugh(✉)

Department of Philosophy and Institute for Logic, Language and Computation, University of Amsterdam, Amsterdam, The Netherlands
deanmchugh1@gmail.com

Abstract. This paper is concerned with disjunctions of universal modals, such as *You have to clean your room or you have to walk the dog*, and disjunctions of conditionals, such as *If Alice dances, Charlie will dance, or if Bob dances, Charlie will dance*. We aim to provide a uniform account of their surprising behaviour. Our proposal combines three independently attested components. Firstly, disjunction's dynamic effect, familiar from presupposition projection: when we evaluate a disjunction $A \vee B$, we typically interpret B assuming the negation of A, and optionally, also interpret A assuming the negation of B. Secondly, the fact that disjunctions often receive a conjunctive interpretation, familiar from free choice phenomena. Thirdly, that modal bases are restricted to local contexts, what Mandelkern has called 'bounded modality'.

1 Introduction

Remarkable things happen when disjunction joins universal modals and conditionals, as in (1) and (2).

(1) a. You must do clean your room or you must walk the dog.
  b. The keys must be in the drawer or John must have taken them.

(2) a. If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.
  b. If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

We study these two cases of disjunction in tandem since we believe that a uniform explanation underlies their surprising behaviour.

Disjunctions of universal modals such as *Must A or must B*, first discussed by Geurts [18], behave in a way unexpected from a simple application of classical logic: they can be asserted even when both disjuncts are false. You must clean your room or you must walk the dog, but it is not true that you must clean your room (you may walk the dog and leave your room as it is), and it is not true that you must walk the dog (you may clean your room and keep the dog home). The keys must be in the drawer or John must have taken them, but it is not true that they must be in the drawer (for John might have taken them), and it

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is not true that John must have taken them (for they might be in the drawer). Naturally, this violates classical logic. If both disjuncts are false, classical logic tells us that the disjunction as a whole must be false too.

Instead of the meaning we would expect from classical logic, *Must A or must B* somehow winds up meaning something akin to $Must(A \vee B)$. This is not how disjunctions of universal quantifiers usually behave.

(3) a. Everyone is in the kitchen or the garden.
b. $\not\equiv$ Everyone is in the kitchen or everyone is in the garden.

If some people are in the kitchen and the rest are in the garden, the first is true but the second false.[1] The problem becomes especially salient when we unpack the meaning of *must* according to a standard, Kratzerian semantics of modals.

(4) a. You must clean the kitchen or you must walk the dog.
b. $\not\equiv$ In every normatively best world you clean the kitchen, or in every normatively best world you walk the dog.

Our first goal is to account for this unexpected behaviour of disjunctions of universal modals.

Question 1. What are the truth-conditions of disjunctions of universal modals? In particular, why are they assertable even when both disjunctions are false?

Turning to disjunctions of conditionals, discussed by Woods [65], Geurts [18], and Khoo [30], we find an interesting contrast. When the antecedents are different—as in (2a), *If A, C or if B, C*—we readily perceive a conjunctive interpretation, with the sentence implying both disjuncts ($\rightsquigarrow$ denotes an intuitive felt inference, while remaining neutral on its precise status).

(2a) If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.
$\rightsquigarrow$ If Alice had come to the party, Charlie would have come.
$\rightsquigarrow$ If Bob had come, Charlie would have come.

However, when the antecedents are the same, as in (2b), the inference disappears.

[1] At least, the first *can* be true in this scenario; it also has a wide-scope reading on which it is equivalent to the second. This can be explained by the well-known scopal flexibility of disjunction, illustrated by (i) from Rooth and Partee [52].

(i) Mary is looking for a maid or a cook.

As Rooth and Partee observe, this has a reading suggested by the continuation "... but I don't know which", where the disjunction takes wide-scope, interpreted as "Mary is looking for a maid or Mary is looking for a cook".

Nonetheless, while (3a) has both narrow and wide scope readings, (3b) only has the wide scope reading.

(2b) If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.
$\not\rightsquigarrow$ If Alice had come to the party, Charlie would have come.
$\not\rightsquigarrow$ If Bob had come, Charlie would have come.

The two consequents are logically consistent: none of the information provided rules out that Charlie and Darius may both come to the party. Thus we cannot point to logical incompatibility as a reason why the inference disappears here.

Nor is the contrast explained by an exclusive reading of disjunction. One might propose that an exclusive reading blocks us from inferring in (2b) that both conditionals are true, since this is incompatible with the exclusive inference, and that since we cannot choose which conditional to not infer, by symmetry we infer neither. The exact same, however, may be said for (2a). This account does not rule out an exclusive reading of (2a), whereas we in fact infer that both conditionals are true. The possibility of an exclusive reading of disjunction does not account for the fact that (2a) is read conjunctively and (2b) disjunctively.

Nor can we trace any difference between the conditionals to the fact that one of them contains some redundancy, unnecessarily repeating material. Both (2a) and (2b) are on a par in terms of how much redundancy they contain—the meaning of each can equivalently be expressed with greater brevity, say, like so.

(5) a. If Alice or Bob had come to the party, Charlie would have come.
b. If Alice had come to the party, Charlie or Darius would have come.

Interestingly, the conjunctive interpretation can even arise when the antecedents are distinct. For instance, (6) has a conjunctive interpretation.

(6) If you had taken the morning train, you would have arrived before lunch. Or if you had taken the afternoon train, you would have arrived after lunch.

The contrast is robust across the various forms conditionals may take. It occurs regardless of the conditional's tense.[2] The contrast also appears with conditional readings of conjunctions.[3]

[2] For example, in each case below, as in (2), the (a)-sentences typically imply both conditionals, while the (b)-sentences do not.

(i) a. If Alice goes, Charlie goes, or if Bob goes, Charlie goes.
b. If Alice goes, Charlie goes, or if Alice goes, Darius goes.

(ii) a. If Alice goes, Charlie will go, or if Bob goes, Charlie will go.
b. If Alice goes, Charlie will go, or if Alice goes, Darius go will go.

(iii) a. If Alice went, Charlie would go, or if Bob went, Charlie would go.
b. If Alice went, Charlie would go, or if Alice went, Darius would go.

[3] For recent analyses of conditional conjunctions see von Fintel and Iatridou [17], Starr [59], Keshet and Medeiros [29], and Kaufmann and Whitman [28], among others.

(7) a. Invite Alice and Charlie will go, or invite Bob and Charlie will go.
b. Invite Alice and Charlie will go, or invite Alice and Darius will go.

This shows that the preference for a conjunctive reading of *If A, C or if B, C* but a disjunctive reading of *If A, C or if A, D* is not specific to the conditional construction itself (say, the presence of *if*), but is rather a fact about conditional meaning broadly construed.

The conjunctive inference is cross-linguistically robust. Consider, for example, the following passage from the Book of Leviticus:

(8) And if a soul sin ... if he do not utter it, then he shall bear his iniquity.
Or if a soul touch any unclean thing ... he also shall be unclean, and guilty.
Or if he touch the uncleanness of man ... when he knoweth of it, then he shall be guilty. (Leviticus 5:1–3, King James Version, 1611).

This is most naturally read as a conjunction of conditionals. Cross-linguistically, a disjunction word links the clauses of Leviticus 5 in, for example, Mandarin Chinese *huò*, the original Hebrew *o*, Hungarian *vagy*, Icelandic *eða*, Māori *rānei*, Urdu *yâ*, Somali *ama*, Welsh *neu*, and Yoruba *tàbí*, suggesting that the conjunctive reading of disjunctions of conditionals is a cross-linguistically robust phenomenon.[4]

Our second goal is to capture this surprising behaviour of disjunctions of conditionals.

Question 2. What are the truth conditions of disjunctions of conditionals? In particular, why does *If A, C or if B, C* by default receive a conjunctive interpretation, while *If A, C or if A, D* does not?

There is an extensive literature on conditionals with disjunctive antecedents; in particular, on simplification of disjunctive antencedents—the inference from *if A or B*, *C* to *if A, C and if B, C*.[5] More recently, Khoo [31] and Klinedinst [32] consider the case of *if or if*-conditionals such as (9).[6]

[4] For sources see the Appendix.

[5] Among authors who argue for simplification's validity are Nute [47], Ellis, Jackson, and Pargetter [11], Warmbrōd [62], Fine [13], Starr [58], and Willer [64]. Among those who argue it is invalid are Nute [48], Bennett [3], van Rooij [51], Santorio [53], and Lassiter [38].

[6] Indeed, unlike with disjunctive antecedents, Starr [58] observes that *if or if*-conditionals *obligatorily* entail their simplifications, as shown by specificational disjunctive antecedent conditionals such as (i). (For discussion of specificational conditionals see Loewer [40], McKay and Inwagen [45], Nute [48], Bennett [3], and Klinedinst [34].)

(i) a. If John had taken the train or the metro, he would have taken the train.
b. #If John had taken the train or if he had taken the metro, he would have taken the train.

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(9) If you had taken the train or if you had taken the metro, you would have been on time.

a. $\rightsquigarrow$ If you had taken the train, you would have been on time.
b. $\rightsquigarrow$ If you had taken the metro, you would have been on time.

Apart from Woods [65], Geurts [18] and Khoo [31], there has, however, been little discussion of disjunctions of whole conditionals—our focus in this paper.

2 Motivation from Theories of Wide Scope Free Choice

The puzzling behaviour of disjunctions of universal modals becomes especially important in light of recent interest in wide scope free choice, the inference from $\Diamond A \vee \Diamond B$ to $\Diamond A \wedge \Diamond B$, from Zimmermann [66], and illustrated in (10).

(10) a. He might be in Brixton or he might be in Victoria.
(i) $\rightsquigarrow$ He might be in Brixton.
(ii) $\rightsquigarrow$ He might be in Victoria.
b. You may go to Brixton or you may go to Victoria.
(i) $\rightsquigarrow$ You may go to Brixton.
(ii) $\rightsquigarrow$ You may go to Victoria.

In contrast, $\Box A \vee \Box B$ does not imply $\Box A \wedge \Box B$.

(11) a. He must be in Brixton or he must be in Victoria.
(i) $\not\rightsquigarrow$ He must be in Brixton.
(ii) $\not\rightsquigarrow$ He must be in Victoria.
b. He must go to Brixton or he must go to Victoria.
(i) $\not\rightsquigarrow$ He must go to Brixton.
(ii) $\not\rightsquigarrow$ He must go to Victoria.

A number of theories today, such as Zimmermann [66], Geurts [18], and Aloni [1], aim to account for wide scope free choice, the inference from $\Diamond A \vee \Diamond B$ to $\Diamond A \wedge \Diamond B$. It is important to check, however, that these theories do not inadvertently predict the analogous inference from $\Box A \vee \Box B$ to $\Box A \wedge \Box B$.

For example, Zimmermann [66] and Geurts [18] derive wide scope free choice by proposing that disjunctions denote conjunctions of possibilities: $A \vee B$ means $\Diamond A \wedge \Diamond B$, and thus $\Box A \vee \Box B$ is equivalent to $\Diamond \Box A \wedge \Diamond \Box B$. By a further principle—Zimmerman's Authority Principle, stating that the agent is an authority on what is permitted—this is equivalent to $\Box A \wedge \Box B$. This prediction, Geurts [18, 388] notes, "is clearly wrong".

Aloni [1] derives wide scope free choice assuming a very different semantics of disjunction, but along the way uses a constraint similar to the Authority Principle, which she calls *Indisputability*: all worlds in the speaker's epistemic state agree on which worlds are (deontically/epistemically/...) possible. If the speaker's accessibility relation is indisputable, Aloni predicts that $\Box A \vee \Box B$ implies $\Box A \wedge \Box B$ (we provide a proof of this fact in the Appendix). Without further refinement, then, Aloni's account risks making the same "clearly wrong" prediction as Zimmermann and Geurts.

When we examine disjunctions of universal modals, we see that one may explicitly affirm authority/indisputability without *Must A or must B* meaning *Must A and must B*.

(12) I am your parent, so I make the rules. And I'm telling you that you have to do your homework or you have to walk the dog.
$\not\rightsquigarrow$ You have to do your homework and you have to walk the dog.

The epistemic case in (1b) is also problematic, since it is natural to assume that one is an authority on one's epistemic state.

(1b) The keys must be in the drawer or John must have taken them.
$\not\rightsquigarrow$ The keys must be in the drawer and John must have taken them.

Do accounts of wide scope free choice such as Geurts' and Aloni's really predict the unwelcome inference from *Must A or must B* to *Must A and must B*? Or is there more to the interpretation of disjunctions of universal modals than meets the eye? In what follows we propose that there is: additional factors influence their interpretation which renders (12) unproblematic for accounts that derive a conjunctive reading of disjunctions of universal modals.

3 The Ingredients of Our Analysis

Our account has three, independently-motivated ingredients. Two are features of disjunction: its dynamic effect, and a conjunctive interpretation. The third relates the interpretation of modals to local contexts, what Mandelkern [42,43] calls *bounded modality*. We discuss each in turn.

3.1 The Dynamic Effect of Disjunction

The dynamic effect of disjunction is the fact that when we interpret a disjunction, we typically interpret the second disjunct assuming the negation of the first disjunct, and perhaps also symmetrically, interpret the first assuming the negation of the second. This is familiar from dynamic semantics (Heim [22], Veltman [61], Chierchia [9], and Beaver [2]), the wider literature on presupposition projection (Schlenker [54,55] and Chemla [8]), and what Klinedinst and Rothschild [33] call 'non-truth-tabular' disjunction.[7]

The primary evidence for disjunction's dynamic effect comes from presupposition projection. Karttunen [27] notes that both (13a) and its reordered variant (13b) do not presuppose that Jack has written letters.

[7] It is also similar to the contribution of *else*, though there are some differences between *or* and *or else* regarding which previous material can be negated, discussed by Webber et al. [63] and Meyer [46, 6–9].

(13) a. Either all of Jack's letters have been held up or he has not written any.
b. Either Jack has not written any letters or all of them have been held up.

The same point can be made with Evans' [12, 530] example in (14a), and Partee's bathroom sentences (appearing in Roberts [50]).[8]

(14) a. Either John does not own a donkey or he keeps it very quiet.
b. Either John keeps his donkey very quiet or he does not own a donkey.

(15) a. Either there's no bathroom in this house or it's in a funny place.
b. Either the bathroom is in a funny place or there's no bathroom here.

In these sentences, "all of Jack's letters have been held up" presupposes that there are such letters, "he keeps it very quiet" that John has a donkey, and "it's in a funny place" that the house has a bathroom. But the sentences as a whole do not: the presuppositions are filtered. A longstanding idea is that presuppositions must be satisfied in their local context (see Heim [23], Beaver [2], and Schlenker [55], among many others). Assuming that in a disjunction $A \vee B$, the local context of B entails $\neg A$, and the local context of A entails $\neg B$, the global context need not entail, respectively, that Jack has written letters, that John owns a donkey, and that the house has a bathroom. We therefore predict the correct presupposition filtering for these sentences.

A simple—but by no means the only—way to implement this idea is to introduce an additional parameter of interpretation representing the local context.[9]

$$[\![A \vee B]\!]^{w,c} = 1 \text{ iff } [\![A]\!]^{w,c} = 1 \quad \text{or } [\![B]\!]^{w,c\cap[\![\neg A]\!]^c} = 1 \qquad \text{ASYMMETRIC}$$

$$[\![A \vee B]\!]^{w,c} = 1 \text{ iff } [\![A]\!]^{w,c\cap[\![\neg B]\!]^c} = 1 \text{ or } [\![B]\!]^{w,c\cap[\![\neg A]\!]^c} = 1 \qquad \text{SYMMETRIC}$$

We will assume this additional parameter in what follows.

The pattern of presupposition projection remains when we add a universal modal to the second disjunct, as in (16), or both disjuncts, as in (17). We illustrate this here for Partee's bathroom sentences, though the same point applies to the other presupposition triggers considered above.

(16) a. There's no bathroom in this house or it must be in a funny place.
b. The bathroom is in a funny place or there must be no bathroom here.

(17) a. There must be no bathroom in this house or it must be in a funny place.

[8] Kalomoiros Schwarz [26] offer experimental evidence that presupposition filtering for disjunction is symmetric. For experimental evidence that presupposition filtering for conjunction is nonetheless asymmetric, see Mandelkern et al. [44].

[9] For any sentence A, $[\![A]\!]^c = \{w : [\![A]\!]^{w,c} = 1\}$ is the set of worlds where A is true.

b. The bathroom must be in a funny place or there must be no bathroom here.

In (16a), for example, we interpret "it must be in a funny place" assuming that the house has a bathroom. On the basis of these data, we propose that in $A \vee \Box B$ or $\Box A \vee \Box B$, the second disjunct may be interpreted assuming $\neg A$.[10]

Disjunctions of conditionals behave analogously. Given *If A, C, or if B, D*, we can interpret the second conditional assuming that A, the antecedent of the first, is false.

(18) a. If there's no bathroom in this house, I'll go home, or if it's in a funny place, I'll ask the host for directions.
b. If the bathroom is in a funny place, I'll ask the host for directions, or if there's no bathroom, I'll go home.

For example, in (18a) we interpret the second conditional assuming that the house has a bathroom.

Alternatively, given *If A, C, or if B, D*, we can interpret the second conditional assuming that C, the consequent of the first, is false.[11]

(19) a. If we hire architect A, our new office will have no bathroom, or if we hire architect B, it will be in a strange place.
b. If we hire architect B, our new office bathroom will be in a strange place, or if we hire architect A, our new office will have no bathroom.

In (19a), for example, we interpret the second conditional assuming that the new office has a bathroom. These provide evidence that when we interpret a disjunction of conditionals, the dynamic effect of disjunction can negate the antecedent or consequent of the first conditional.

Naturally, it remains possible for the disjunction's dynamic effect to negate the entire first conditional, as shown in (20) and (21).[12]

(20) The concrete stays intact if it is subjected to high pressure, or it is unsuitable for our purposes.
≡ The concrete stays intact if it is subjected to high pressure, or if it is not the case that it stays intact if it is subjected to high pressure, it is unsuitable for our purposes

(21) Ali will leave if Beth arrives, or they must have resolved their dispute.
a. ≡ Ali will leave if Beth arrives, or if it's not true that Ali will leave if Beth arrives, they must have resolved their dispute.

[10] It is an open and interesting question why it should be possible to negate the *prejacent* of a modal claim, rather than simply the whole modal claim. We discuss this in Sect. 6.

[11] I am grateful to Patrick Elliot, Alexandros Kalomoiros, Jacopo Romoli, and Yichi Zhang for helpful discussions of this example.

[12] (20) is modelled after the following example from Douven [10] of a left-nested conditional: "If this material becomes soft if it gets hot, it is not suited for our purposes.".

b. $\not\equiv$ Ali will leave if Beth arrives, or if Ali will not leave, they must have resolved their dispute.
c. $\not\equiv$ Ali will leave if Beth arrives, or if Beth will not arrive, they must have resolved their dispute.

Negating the entire first disjunct is not very interesting for present purposes since it has the same truth conditions as when the dynamic effect does not apply. One way to see this is to note that $A \vee B$, $A \vee (\neg A \rightarrow B)$, and $(\neg B \rightarrow A) \vee (\neg A \rightarrow B)$ are all classically equivalent, where $\rightarrow$ denotes the material conditional (that is, $A \rightarrow B$ is equivalent to $\neg A \vee B$).[13]

3.2 Conjunctive Interpretations of Disjunction

The second ingredient of our proposal is that disjunction may receive a conjunctive interpretation, familiar from the literature on wide scope free choice (Zimmermann [66], Geurts [18], and Aloni [1]), already encountered in Sect. 2, and Klinedinst and Rothschild's [33] non-truth-tabular disjunction. In what follows we remain neutral on what exact mechanism is responsible for wide scope free choice; for example, Aloni's [1] state-based account or Goldstein's [20] homogeneous dynamic semantics. Provided we have some mechanism to strengthen disjunctions into conjunctions, one may add it as a module to our account.

3.3 Bounded Modality

For our account to succeed, we need to relate the interpretation of modal statements to local contexts. This is the third and final ingredient of our proposal: the interpretation of modal statements is restricted to local contexts. Mandelkern [42,43] independently argues for this constraint, calling it *bounded modality*.

On a simple semantics of modals via accessibility relations, we can implement the constraint by requiring that modals are evaluated at the accessible worlds compatible with the local context, so that $\Box A$ is true just in case A is true at every accessible world compatible with the local context. Formally, let W to be the set of possible worlds, R the relevant accessibility relation for modals, and $R[w] = \{w' \in W : wRw'\}$ the set of accessible worlds at w.[14] *Must* A is true at

[13] Granted, the material conditional is likely not the most natural restriction device by which to represent the effect of local contexts. Nonetheless, however one represents the restriction provided by local contexts, it is plausible that disjunctions will have the same truth conditions regardless of whether or not the local context for the second disjunct entails the negation of the first, and whether or not the local context for the first entails the negation of the second. Indeed, precisely this assumption—that the addition of local contexts preserves truth conditions—is a cornerstone of the most comprehensive theory of local contexts currently available: Schlenker's [55] algorithm.

[14] We assume that for each modal there is an associated accessibility relation; to distinguish them one may choose to add indices to modals and accessibility relations, *à la* von Fintel [14].

a world w just in case A is true at every world compatible with the local context that is accessible from w.[15]

(22) **Bounded Kripke.**
$[\![\Box A]\!]^{c,w}$ is true just in case $R[w] \cap c \subseteq [\![A]\!]^c$.

On a more sophisticated story, such as Kratzer's [36] doubly-relative account, we implement boundedness by restricting the modal base to the local context. Kratzer [35] proposes that *Must* A holds just in case A is true at every world in the modal base ranked highest according to the ordering source: $[\![\Box A]\!]^{w,f,g,c}$ is true just in case $\max_{g(w)} \bigcap f(w) \subseteq [\![A]\!]^{f,g,c}$.[16] On the bounded version, *Must* A is true at a world w just in case A is true at every world in the modal base at w *compatible with the local context* that is ranked highest according to the ordering source at w.

(23) **Bounded Kratzer.**
$[\![\Box A]\!]^{f,g,c,w}$ is true just in case $\max_{g(w)}((\bigcap f(w)) \cap c) \subseteq [\![A]\!]^{f,g,c}$.

We can derive bounded modality from independent observations. For one may think of the local context as itself a kind of presupposition. At every stage of interpretation we presuppose that the actual world is compatible with the local context. The local context is, if you like, the arch-presupposition: the presupposition that entails all others at each stage of interpretation.

Heim [24] notes that, when we interpret a modal statement, we assume that its presuppositions hold throughout the modal base.

(24) a. Patrick wants to sell his cello.
b. If John had attended the party too, ...

As Heim observes, (24a) presupposes that in all of the worlds compatible with Partick's beliefs, he owns a cello. It is assertable, say, even if Patrick is only under the misconception that he owns a cello. Similarly, when it is already in the

[15] There are two noteworthy differences between our entry in (22) and the bounded theory of Mandelkern [42,43]. For Mandelkern, modal statements *presuppose* that the modal domain is included in the local context, whereas we implement this restriction via the *truth conditions* of modal statements. Our simplification is purely for reasons of simplicity: when showing results for our system, we can assume a uniform modal base, without needing to compare various modal bases under different restrictions. One may straightforwardly implement our account on a more nuanced story, such as Mandelkern's, which distinguishes between truth and definedness conditions. The second difference is that we allow the local context to restrict the prejacent of a modal, while Mandelkern does not consider this possibility.

[16] We let $[\![A]\!]^{f,g,c} = \{w' : [\![A]\!]^{w',f,g,c} = 1\}$ denote the set of worlds where A is true, and $[\![\neg A]\!]^{f,g,c} = W \setminus \{w' : [\![A]\!]^{w',f,g,c} = 1\}$ the set of worlds where A is false, with respect to f, g, and c. For every set of worlds $X \subseteq W$, we let $\max_P(X) = \{w \in X : w' <_P w \text{ for no } w' \in X\}$, where for any set of propositions P, $w <_P w'$ holds just in case for all $p \in P$ such that p is true at w', p is true at w (for discussion see von Fintel and Heim [16]).

common ground that Mary attended, when we interpret (24b) we only consider cases where Mary still attended. We could not assert, for example, *If John had attended the party too, Mary would not have attended*, when Mary is among the salient individuals picked out by *too*.

It is typically assumed that bouletic predicates such as *want* have a doxastic modal base: they are restricted to worlds compatible with the attitude holder's beliefs (Heim [24] and von Fintel [15]). Then (24a) shows that the presuppositions of a modal statement must be satisfied in the modal base. (24b) makes the same point with a counterfactual.

So we have two observations. The local context is itself a presupposition, and modal statements are evaluated at modal bases restricted to worlds where the statement's presuppositions hold. With these we derive bounded modality.

4 Disjunctions of Universal Modals

4.1 Putting the Ingredients Together

Putting the above ingredients together, on our proposal the meaning of disjunctions of universal modals *Must A or must B* as in (1) can be paraphrased as follows, with an asymmetric and symmetric dynamic effect, respectively.

(25) ASYMMETRIC DYNAMIC EFFECT + CONJUNCTIVE INTERPRETATION
  a. You must do clean your room and if you do not clean your room, you must walk the dog.
  b. The keys must be in the drawer and if they are not in the drawer, John must have taken them.

(26) SYMMETRIC DYNAMIC EFFECT + CONJUNCTIVE INTERPRETATION
  a. If you do not walk the dog, you must do clean your room, and if you do not clean your room, you must walk the dog.
  b. If John has not taken the keys, they must be in the drawer, and if they are not in the drawer, John must have taken them.

We propose that both the asymmetric and symmetric readings are available, with the asymmetric reading giving priority to the first disjunct and the second true at more remote possibilities—what Schwager [56] calls the *or else* effect. For example, the asymmetric reading allows us to predict the following contrast.

(27) a. You have to pay the bill, or you have to go to jail.
  b. ??You have to go to jail, or you have to pay the bill.

Given an asymmetric dynamic interpretation and conjunctive reading, these wind up with a meaning we may paraphrase as follows, which pattern the same as (27) in their acceptability.

(28) a. You have to pay the bill. If you do not pay the bill, you have to go to jail.

b. ?? You have to go to jail. If you do not go to jail, you have to pay the bill.

Following Meyer [46], we adopt Kratzer's [36] analysis of modality, whereby worlds are ordered along a dimension according to their flavour (such as epistemic, or in this case, deontic). $\Box A$ is true at a world w just in case A is true at all worlds in the modal base that come closest to the ideal according to w. In addition, the contribution of *if*-clauses is to restrict the modal base (Kratzer [37]). Thus (28a) says that at all of the normatively best worlds, you pay the bill, and at all of the normatively best worlds where you don't, you go to jail.

In this way sentences like (28a) are of a piece with well-known counterexamples to antecedent strengthening for conditionals, such as Goodman's [21] classic example: we can accept *If I struck this match, it would light* while denying *If I struck this match and it were wet, it would light*. (28a) is true while (28b) is false since all else being equal, worlds where you pay the bill are better—closer to our deontic ideals—than worlds where you go to jail.

In contrast, on the symmetric reading both options are viewed on a par, with neither having priority over the other. This is shown in examples like (29), where order seems not to matter.

(29) *For Ali to win the game, the die must land on multiple of three.*

a. Ali has to roll a three or he has to roll a six.
b. Ali has to roll a six or he has to roll a three.

Given a symmetric dynamic interpretation and conjunctive reading of conjunction, these can be paraphrased as follows.

(30) a. If Ali doesn't roll a six, he has to roll a three, and if he doesn't roll a three, he has to roll a six.
b. If Ali doesn't roll a three, he has to roll a six, and if he doesn't roll a six, he has to roll a three.

This proposal solves the problem on behalf of Zimmermann's, Geurts' and Aloni's theories of wide scope free choice. Instead of trying to block the unwanted conjunctive reading of $\Box A \vee \Box B$ as $\Box A \wedge \Box B$, we embrace it. While the conjunctive reading initially appeared too strong, disjunction's dynamic effect weakens it, resulting in plausible predictions for what disjunctions of universal modals intuitively mean.

4.2 The Equivalence of *Must A or Must B* and *Must*(*A or B*)

Given a symmetric dynamic effect we can derive the equivalence of $\Box A \vee \Box B$ and $\Box(A \vee B)$. Generally speaking, the key observation is that $d \cap \overline{a} \subseteq b$ is equivalent to $d \subseteq a \cup b$ for any sets a, b and d (where $\overline{a}$ denotes a's complement). Taking d to be the modal domain, a the proposition expressed by A and b that of B, we derive the equivalence of $\Box A \vee \Box B$ and $\Box(A \vee B)$.

Let us first show this on the bounded Kripkean semantics. As a convenient shorthand, let us write $\Box_A B$ to denote that $\Box B$ is evaluated at the global context restricted by A.

$$[\![\Box_A B]\!]^{w,f,g,c} \quad = \quad [\![\Box B]\!]^{w,f,g,c\cap[\![A]\!]^{f,g,c}}$$

It follows that $\Box_{\neg A} B$ is equivalent to $\Box(\neg A \rightarrow B)$, where $\rightarrow$ is the material conditional, which is equivalent to $\Box(A \vee B)$. Hence $\Box_{\neg B} A \wedge \Box_{\neg A} B$ is equivalent to $\Box(A \vee B)$.

$$\begin{array}{ll} & \Box_{\neg B} A \wedge \Box_{\neg A} B \\ \equiv & \Box(\neg B \rightarrow A) \wedge \Box(\neg A \rightarrow B) \\ \equiv & \Box(A \vee B) \wedge \Box(A \vee B) \\ \equiv & \Box(A \vee B) \end{array}$$

To show this equivalence on the bounded Kratzerian semantics, we need two further assumptions. The first is that $\neg A$ and $\neg B$ are each possible: $\neg\Box A$ and $\neg\Box B$. The second is a principle we will call *Strengthening with a Possibility*, named after a corresponding rule in the logic of conditionals: given $\neg(\textit{if } A, \neg B)$, *if* A, C implies *if* A *and* B, C.

Strengthening with a Possibility (SP).
If $(\max_P X) \cap Y$ is nonempty, then $\max_P(X \cap Y) = (\max_P X) \cap Y$.

Strengthening with a Possibility says that if we find a world where Y is true among the closest X-worlds, then the closest X-and-Y-worlds are just the closest X-worlds where Y is true. This does not hold in general on ordering approaches to modality. It corresponds to *almost connectedness*: for any worlds w, w', w'', if $w \leq_P w''$ then either $w \leq_P w'$ or $w' \leq_P w''$ (Veltman [60, 103]). It is automatically satisfied by the sphere models of Lewis [39], though there are—to my mind convincing—counterexamples to it (Ginsberg [19, 50], Boylan and Schultheis [4,5]). Nonetheless, in the simple kinds of modal statements we have considered, we can reasonably expect it to hold.

Assuming the bounded Kratzerian semantics in (23), Strengthening with a Possibility guarantees that if there is a world where A is true among the closest worlds, then $\Box_A B$ is equivalent to $\Box(A \rightarrow B)$.[17]

$$SP \wedge \Diamond A \quad \rightarrow \quad (\Box_A B \leftrightarrow \Box(A \rightarrow B))$$

[17] Let us prove this here. Pick any parameters w, f, g, c. To simplify notation, for any sentence A let $|A| = [\![A]\!]^{f,g,c}$ be the set of worlds where A is true, $D = \max_{g(w)}((\bigcap f(w)) \cap c)$, the modal domain, that is, the closest worlds in the modal base restricted to the global context, and let $D_A = \max_{g(w)}((\bigcap f(w)) \cap c \cap |A|)$ be the modal domain where the local context is restricted by A. Suppose $\Diamond A$ is true at w. Then $D \cap |A|$ is nonempty, so by Strengthening with a Possibility, $D_A = D \cap |A|$. Hence $\Box_A B$ is true at w iff $D_A \subseteq |B|$ iff $D \cap |A| \subseteq |B|$ iff $D \subseteq |\neg A| \cup |B|$ iff $\Box(A \rightarrow B)$ is true at w.

Hence if $\neg A$ is possible, $\Box_{\neg A}B$ is equivalent to $\Box(\neg A \rightarrow B)$, which is equivalent to $\Box(A \vee B)$, and if $\neg B$ is possible, $\Box_{\neg B}A$ is also equivalent to $\Box(A \vee B)$. Hence at any world where $\neg A$ and $\neg B$ are possible, $\Box_{\neg B}A \wedge \Box_{\neg A}B$ is equivalent to $\Box(A \vee B)$.

$$SP \wedge \Diamond\neg A \wedge \Diamond\neg B \quad \rightarrow \quad ((\Box_{\neg B}A \wedge \Box_{\neg A}B) \leftrightarrow \Box(A \vee B))$$

Returning to our original examples in (1), this accounts for the following equivalences.

(31) a. You must do clean your room or you must walk the dog.
 b. You must do clean your room or walk the dog.

(32) a. The keys must be in the drawer or John must have taken them.
 b. It must be that the keys are in the drawer or John took them.

Happily, our account does not inadvertently predict this equivalence for universal quantifiers in general, such as in (3), repeated below.

(33) a. Everyone is in the kitchen or the garden.
 b. Everyone is in the kitchen or everyone is in the garden.

A local context is a piece of information—in possible worlds semantics, a set of worlds—rather than a set of individuals. A set of individuals (say, the set of salient people outside the kitchen) is simply not the kind of thing that can serve as a local context.

5 Disjunctions of Conditionals

Turning to conditionals, the very same combination of disjunction's dynamic effect and conjunctive interpretation accounts for the contrast in (2), repeated below, where the first appears to be interpreted conjunctively and the second not.

(2) a. If Alice had come to the party, Charlie would have come. Or if Bob had come, Charlie would have come.
 b. If Alice had come to the party, Charlie would have come. Or if Alice had come, Darius would have come.

In principle there are a number of options for which clause is negated by the dynamic effect of disjunction: the whole conditional, the antecedent or the consequent. As discussed, negating the whole conditional is possible, but not very interesting for our purposes since it is classically equivalent to an interpretation without any dynamic effect.

Putting aside the option of negating the whole conditional, we find that in (2a), negating the previous consequent is not an option since it would violate a general ban on triviality; the second conditional would assert that if Bob but not Charlie had come, Charlie would have come. If there are relevant possibilities where Bob but not Charlie comes, this is trivially false. If there are not, the conditional either suffers from presupposition failure or is vacuously true, depending on one's view. Either way, the conditional is uninformative, violating principles of conversation. However, negating the previous antecedent is perfectly possible, stating that if Alice but not Bob had come to the party, Charlie would have.

(34) If A, C or if B, C

a. If A, C and if B and $\neg C$, C NEGATING THE CONSEQUENT: ✗
b. If A, C and if B and $\neg A$, C NEGATING THE ANTECEDENT: ✓

In (2b) the situation is reversed. Negating the antecedent is not an option since it would also violate a general ban on triviality; the second conditional would assert, vacuously, that if Alice but not Alice had come, Darius would have come. However, negating the previous consequent is an option, stating that if Alice but not Charlie had come, Darius would have.

(35) If A, C or if A, D

a. If A, C and if A and $\neg C$, D NEGATING THE CONSEQUENT: ✓
b. If A, C and if A and $\neg A$, D NEGATING THE ANTECEDENT: ✗

Given an asymmetric dynamic effect, then, the sentences in (2) have interpretations which we may paraphrase as follows.

(36) ASYMMETRIC DYNAMIC EFFECT + CONJUNCTIVE INTERPRETATION

a. If Alice had come to the party, Charlie would have come. And if Bob but not Alice had come, Charlie would have come.
b. If Alice had come to the party, Charlie would have come. And if Alice but not Charlie had come, Darius would have come.

And given a symmetric dynamic effect, they have interpretations which we may paraphrase as:

(37) SYMMETRIC DYNAMIC EFFECT + CONJUNCTIVE INTERPRETATION

a. If Alice but not Bob had come to the party, Charlie would have come. And if Bob but not Alice had come, Charlie would have come.
b. If Alice but not Darius had come to the party, Charlie would have come. And if Alice but not Charlie had come, Darius would have come.

These are plausible predictions for what (2) mean.

This also correctly predicts the interpretation of (6), where the antecedent and consequents are distinct. For example, on a symmetric dynamic effect on the antecedent, the meaning of (6) can be paraphrased as in (38).

(38) If you had taken the morning train (and not taken the evening train), you would have arrived before lunch. And if you had taken the evening

train (and not taken the morning train), you would have arrived after lunch.

This solves our puzzle regarding why (2a) appeared to be interpreted conjunctively but (2b) disjunctively. In fact both are read conjunctively. The restriction provided by local contexts results in the apparent divergence in readings. This has the welcome consquence of dissolving the tricky question of explaining why the conjunctive inference applies when the antecedents are the same but disappears when the consequents are the same. In fact the inference applies across the board.

5.1 The Equivalence of *if A, C or if A, D* and *if A, C or D*

Following Kratzer [37], we assume the conditionals feature a modal (overt or covert), and that conditional antecedents restrict the modal base.

$$[\![\textit{if } A, C]\!]^{w,f,g,c} \quad = \quad [\![C]\!]^{w,f+A,g,c}$$

where $f + A$ is given by $f + A(w) = f(w) \cup \{[\![A]\!]^{f,g,c}\}$ for every world w.

Note that on our bounded semantics of modals in (23), we could equivalently write that conditional antecedents restrict the local context: *if* $A, \Box C$ is equivalent to $\Box_A C$. Then assuming a symmetric dynamic effect of disjunction and a conjunctive interpretation, *if A, C or if A, D* expresses $\Box_{A\wedge\neg D} C \wedge \Box_{A\wedge\neg C} D$. Under the same assumptions as before—Strengthening with a Possibility and that $\Diamond_A \neg C$ and $\Diamond_A \neg D$ are both true—we predict $\Box_{A\wedge\neg D} C$ to be equivalent to $\Box_A(\neg D \rightarrow C)$, and hence to $\Box_A(C \vee D)$; similarly, we predict $\Box_{A\wedge\neg C} D$ to be equivalent to $\Box_A(C \vee D)$. Then their conjunction is also equivalent to $\Box_A(C \vee D)$.

$$SP \wedge \Diamond_A \neg C \wedge \Diamond_A \neg D \quad \rightarrow \quad ((\Box_{A\wedge\neg D} C \wedge \Box_{A\wedge\neg C} D) \leftrightarrow \Box_A(C \vee D))$$

Thus assuming Strengthening with a Possibility, $\Diamond_A \neg C$ and $\Diamond_A \neg D$, we predict *if A, C or if A, D* to be equivalent to *if A, C or D*. This prediction is borne out.

(39) a. If Alice had come to the party, Charlie would have come, or if Alice had come to the party, Darius would have come.
b. If Alice had come to the party, Charlie or Darius would have come.

Importantly, we derive this equivalence assuming that disjunctions of conditionals receive a conjunctive interpretation across the board. This resolves the puzzle of explaining why *If A, C or if B, C* seems to receive a conjunctive interpretation while *If A, C or if A, D* seems not to. In fact both are read conjunctively, but the symmetric dynamic effect of disjunction weakens them, rendering *If A, C or if A, D* equivalent to *If A, C or D*.

A further test of this prediction comes from (40), modelled after Quine's [49] Bizet–Verdi example.

(40) *Coin A landed heads, coin B landed tails.*

a. If both coins had landed on the same side, they both would have landed heads, or if they had landed on the same side, they both would have landed tails.
b. If both coins had landed on the same side, they would have landed both heads or both tails.

These are naturally felt to be equivalent, as predicted on this account.[18]

6 Negating the Entire Disjunct or a Part Thereof

We have proposed that the prejacent of a modal statement is available for restriction by local contexts: when we interpret $\Box A \vee \Box B$, we may evaluate each disjunct assuming that the other disjunct is false, or assuming that *prejacent* of the other disjunct (that is, A and B) is false. Furthermore, we proposed that in a conditional, the antecedent and consequent are both available for restriction. This is not predicted by standard approaches to local contexts, such as Schlenker's [55] algorithm, but there is independent evidence that these restrictions are available. In Sect. 3.1 we have seen independent evidence for this assumption from presupposition projection. Furthermore, Meyer [46] observes the same behaviour for *or else* sentences with modals, offering examples such as (41), found in Heim and Kratzer [25].

(41) Pronouns must be generated with an index or else they will be uninterpretatable.

As Meyer points out, the second disjunct is interpreted assuming that pronouns are not generated with an index, rather than that they merely need not be generated with an index.

The same observation applies to plain disjunctions without *else*, as in (42a), which receives the same interpretation as (41).

(42) a. Pronouns must be generated with an index or they will be uninterpretatable.

[18] As with all cases of wide-scope free choice, the first also has an ignorance reading—brought out by the continuation *... but I don't know which*—on which they are not equivalent. We can express this reading on the present proposal as $\Box A \vee \Box B$, without the addition of local contexts, or with local contexts where the local context is the negation of the entire other disjunct: asymmetrically as $\Box A \vee \Box_{\neg\Box A} B$, and symmetrically as $\Box_{\neg\Box B} A \vee \Box_{\neg\Box A} B$.

Note further that the equivalence of *If A, C or if A, D* and *If A, C or D* is also predicted by selectional approaches to conditionals, which evaluate the consequent at a unique selected world where the antecedent holds (see, for example, Stalnaker [57], Cariani and Santorio [7], Cariani [6], and Mandelkern [41]). We therefore cannot take the intuitive equivalence of (40a) and (40b) as a decisive point in favour of our proposal, though it is nonetheless a welcome result that the correct prediction also follows on our proposal.

Example (43), from the film *The Terminator*, illustrates the point with a naturally-occurring example. John comes back from the future to tell Sarah:

(43) You must survive or I will never exist.

This says that if Sarah does not survive, John will never exist, rather than that if Sarah is not required to survive, John will never exist. This suggests that both the modal statement as a whole and its prejacent are in general accessible for restriction by local contexts.

Plain modal statements appear to be special in this respect. Compare:

(44) a. You must clean your room or you must walk the dog.
b. You are required to clean your room or you are required to walk the dog.
c. According to the rules, you must clean your room, or according to the rules, you must walk the dog.

We typically read (44a) as equivalent to *You must clean your room or walk the dog*. In contrast, (44b) and (44c) seem to be more naturally read as saying that one of the disjuncts is required, but the speaker is unsure which.[19]

We see the same with epistemic modals.

(45) a. The keys must be in the drawer or John must have taken them.
b. I'm certain that the keys are in the drawer or I'm certain that John has taken them.
c. According to my information, the keys must be in the drawer, or according to my information, John must have taken them.

(45a) has an easily accessible reading on which it is equivalent to $\Box(A \vee B)$, but this reading appears harder to access in (45b) and (45c).

One possible, simple explanation for these differences comes from pragmatic pressure to avoid redundancy. By adding more material, such that *according to the rules* or *it is required that*, it becomes more costly to assert $\Box A \vee \Box B$ rather than $\Box(A \vee B)$. With greater cost, we are more likely to assume that there must be a good reason for this additional cost, and so assume that the speaker does not intend to communicate the simpler $\Box(A \vee B)$. As we have seen, $\Box_{\neg B} A \vee \Box_{\neg A} B$ is equivalent to $\Box(A \vee B)$. However, the alternative available parse, $\Box_{\neg \Box B} A \vee \Box_{\neg \Box A} B$ is not equivalent to $\Box(A \vee B)$, with greater cost we would therefore be more likely to opt for the second reading. I will leave this as a speculative idea for now.

Zooming out, there is also evidence that given a disjunction $A \vee B$, the local context for B cannot be restricted to just any part of A whatsoever. Meyer

[19] The empirical picture here is subtle. I do not rule out that (44b) and (44c) may have a reading equivalent to $\Box(A \vee B)$, further empirical testing would prove beneficial.

[46] presents the following argument against this idea, based on presupposition projection.[20]

(46) Either they didn't remind John to bring his ID or Bill regretted that he didn't (bring it).
Presupposes: *John didn't bring his ID* (Meyer [46], example 22)

If we assume that *John brought his ID* is a subpart of *They didn't remind John to bring his ID*, then we might expect that the second disjunction could be interpreted assuming that he didn't bring his ID, in which case the presupposition that John didn't bring his ID would incorrectly predicted to be filtered.

Thankfully, the present proposal does not require that any part of a disjunct is available for restriction, but merely the more limited claim that in a plain modal statement, *Must A or must B*, the prejacent of each modal is available for restriction, and that in a conditional, the antecedent and consequent are both available for restriction. At present we lack a general theory of what parts of a statement are accessible for restriction by local contexts, though the data in (44) and (45) help point us toward an answer.

7 Conclusion

Disjunctions of universal modals and disjunctions of conditionals give rise to surprising effects, ones unexpected from the perspective of classical logic. We often read *Must A or must B* as equivalent to *Must*(*A or B*). And while *if A, C or if B, C* intuitively implies each conditional, *if A, C or if A, D* does not.

We have proposed a uniform account of these data. Our account combined three independently motivated ingredients: the dynamic effect of disjunction (familiar from presupposition projection), a conjunctive interpretation (familiar from wide scope free choice), and bounded modality (familiar from Mandelkern's [42] account of *might*). As we have seen, these three ingredients together with some mild auxiliary assumptions, we derive the equivalence of *Must A or must*

[20] An anonymous reviewer helpfully points out that the same point applies to conjunction:

(i) Mary believes that Bill will come, and Sue knows that Bill will come.

It is standardly assumed that the local context for B in A *and* B entails A. If it were possible for local context to entail any subpart of A whatsoever, then the local context for the second conjunct could entail that Bill will come (rather than the full conjunct, Mary believes that Bill will come). This would incorrectly predict presupposition that Bill will come to be filtered.

It appears that an analogous point can be made for disjunction.

(ii) Mary believes that Bill will come, or Sue knows that he won't.

This seems to presuppose that Bill will not come, though the judgement is not perfectly robust and would benefit from further experimental investigation.

B and *Must*(A *or* B), and explain why *if A, C or if B, C* is read conjunctively, while *if A, C or if A, D* appears to be read disjunctively. In fact, both are read conjunctively, but the dynamic effect of disjunction weakens the latter, rendering it equivalent to *if A, C or D*.

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Appendix

The Inference from $\Box A \vee \Box B$ to $\Box A \wedge \Box B$ in Aloni's (2023) System

Let $M = (W, R, V)$ be a Kripke model and $s \subseteq W$ a set of worlds. For any world $w \in W$ let $R[w] = \{w' : wRw'\}$ be the set of worlds accessible from w. Aloni introduces a constraint (called 'neglect-zero') requiring each state to be nonempty, represented by a constant NE. She proposes that sentences are interpreted by a process of pragmatic enrichment that avoids empty interpretations; for example, a disjunction $A \vee B$ is interpreted as $(A \wedge \text{NE}) \vee (B \wedge \text{NE})$.

$$\begin{array}{rll} M, s \models A \vee B & \text{iff} & \text{there are states } t \text{ and } t' \text{ such that } s = t \cup t', \\ & & M, t \models A \text{ and } M, t' \models B \\ M, s \models A \wedge B & \text{iff} & M, s \models A \text{ and } M, s \models B \\ M, s \models \Box A & \text{iff} & \text{for all } w \in s, M, R[w] \models A \\ M, s \models \text{NE} & \text{iff} & s \neq \emptyset \end{array}$$

We call R *indisputable* in (M, s) just in case for all $w, v \in s$, $R[w] = R[v]$.

Given these semantic clauses and the assumption that R is indisputable in (M, s), it follows that disjunctions of universal modals imply their conjunctions: $(\Box A \wedge \text{NE}) \vee (\Box B \wedge \text{NE})$ implies $(\Box A \wedge \text{NE}) \wedge (\Box B \wedge \text{NE})$. For $M, s \models (\Box A \wedge \text{NE}) \vee (\Box B \wedge \text{NE})$ holds just in case there are non-empty states t and t' such that $s = t \cup t'$, (i) for all $w \in t$, $M, R[w] \models A$ and (ii) for all $w' \in t'$, $M, R[w'] \models B$. Since t and t' are nonempty, there are $w \in t$ and $w' \in t'$, and by indisputability, $R[w] = R[w']$. To show that $M, s \models \Box A \wedge \Box B$, pick any $v \in s$. By indisputability, $R[v] = R[w] = R[w']$. Then $M, R[v] \models A$ and $M, R[v] \models B$, so $M, s \models \Box A \wedge \Box B$, and since s is nonempty, also $M, s \models (\Box A \wedge \text{NE}) \wedge (\Box B \wedge \text{NE})$.

Links to Online Materials

- Mandarin Chinese *huò* https://web.archive.org/web/20220425194146/https://www.biblegateway.com/passage/?search=Leviticus+5&version=CNVT

– English *or* http://web.archive.org/web/20220408125727/https://www.biblegateway.com/passage/?search=Leviticus+5&version=KJV
– Hebrew *o* https://web.archive.org/web/20220425194625/https://www.biblegateway.com/passage/?search=Leviticus+5&version=WLC
– Hungarian *vagy* https://web.archive.org/web/20220425194655/https://www.biblegateway.com/passage/?search=Leviticus+5&version=KAR
– Icelandic *eða* https://web.archive.org/web/20220425194740/https://www.biblegateway.com/passage/?search=Leviticus+5&version=ICELAND
– Māori *rānei* http://web.archive.org/web/20201127071407/https://www.biblegateway.com/passage/?search=Leviticus+5&version=MAORI
– Urdu *yâ* https://web.archive.org/web/20220425195430/https://www.biblegateway.com/passage/?search=Leviticus+5&version=ERV-UR
– Somali *ama* https://web.archive.org/web/20220425195432/https://www.biblegateway.com/passage/?search=Leviticus+5&version=SOM
– Welsh *neu* http://web.archive.org/web/20220425195453/https://www.biblegateway.com/passage/?search=Leviticus+5&version=BWM
– Yoruba *tàbí* https://web.archive.org/web/20220425195612/https://www.biblegateway.com/passage/?search=Leviticus+5&version=BYO
– Example (43) https://web.archive.org/web/20241222204426/https://getyarn.io/yarn-clip/dfd80466-cd89-421b-b3df-9201282cd90f

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