Four Problems in Decision Theory

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1 What is Decision Theory a Theory Of?

If you're reading a journal like this, you're probably familiar with seeing papers defending this or that decision theory. Familiar decision theories include:

- Causal Decision Theory (?; ?; ?; ?);
- Evidential Decision Theory (?);
- Benchmark theory (?);
- Risk-Weighted theory (?);
- Tournament Decision Theory (?); and
- Functional Decision Theory (?)

Other theories haven't had snappy 'isms' applied to them, such as the non-standard version of Causal Decision Theory that Dmitri Gallow (?) defends, or the pluralist decision theory that Jack Spencer (?) defends, or the broadly ratificationist theory that Melissa Fusco (?) defends

This paper isn't going to take sides between these nine or more theories.[1] Rather it is going to ask a prior pair of questions.

1. If these are the possible answers, what is the question? That is, what is the question to which decision theories are possible answers?
2. Why is that an interesting question? What do we gain by answering it?

On 1, I will argue that decision theories are answers to a question about what an ideal decider would do. The 'ideal' here is like the 'ideal' in a scientific idealisation, not the ideal in something like an ideal advisor moral theory. That is, the ideal decider is an idealisation

[1] The arguments here are intended to support a theory like Fusco's, but in a fairly roundabout way, but the connection between what I say here and Fusco's theory would take a paper as long as this one to set out.

in the sense of being simple, not in the sense of being perfect. The ideal decision maker is ideal in the same way that the point-masses in the ideal gas model are ideal; they are (relatively) simple to work with. The main opponent I have in mind is someone who says that in some sense decision theory tells us what decisions we should make.

On 2, I will argue that the point of asking this question is that these idealisations play important roles in explanatorily useful models of social interactions, such as the model of the used car market that George Akerlof (?) described. Here, the main opponent I have in mind is someone who says that decision theory is useful because it helps us make better decisions.

There is another pair of answers to this question which is interesting, but which I won't have a lot to say about here. David Lewis held that "central question of decision theory is: which choices are the ones that serve one's desires according to one's beliefs?" (?). That's not far from the view I have, though I'd say it's according to one's evidence. But I differ a bit more from Lewis as to the point of this activity. For him, a central role for decision theory is supplying a theory of constitutive rationality to an account of mental content (?). I think the resulting theory is too idealised to help there, and that's before we get to questions about whether we should accept the approach to mental content that requires constitutive rationality. That said, the view I'm defending is going to be in many ways like Lewis's: the big task of decision theory is describing an idealised system, not yet recommending it.

The nine theories I mentioned above disagree about a lot of things. In philosophy we typically spend our time looking at cases where theories agree. Not here! I will focus almost exclusively on two cases where those nine theories all say the same thing. I'll assume that whatever question they are asking, the correct answer to it in those two cases must agree with all nine theories. That will be enough to defend the view I want to defend, which is that a decision theory is correct iff is true in the right kind of idealisation.

2 Three Cases

2.1 Betting

Chooser has $110, and is in a sports betting shop. There is a basketball game about to start, between two teams they know to be equally matched. Chooser has three options: bet the $110 on Home, bet it on Away, keep money. If they bet and are right, they win $100 (plus get the money back they bet), if they are wrong, they lose the money. Given standard assumptions about how much Chooser likes money, all the decision theories I'm discussing say Chooser should not bet.

From this it follows that decision theory is not in the business of answering this question: *What action will produce the best outcome?*. We know, and so does Chooser, that the action that produces the best outcome is to bet on the winning team. Keeping their money in their pocket is the only action they know will be sub-optimal. And it's what decision theory says to do.

This is to say, decision theory is not axiology. It's not a theory of evaluating outcomes, and saying which is best. Axiology is a very important part of philosophy, but it's not what decision theorists are up to.

So far this will probably strike you, dear reader, as obvious. But there's another step, that I think will strike some people as nearly as obvious, that I'm at pains to resist. Some might say that decision theorists don't tell Chooser to bet on the winner because this is lousy advice. Chooser can't bet on the winner, at least not as such. That, I'll argue, would be a misstep. Decision theorists do not restrict themselves to answers that can be practically carried out.

2.2 Salesman

We'll focus on a version of what Julia Robinson (?) called the travelling salesman problem. Given some points on a map, find the shortest path through them. We'll focus on the 257 cities shown on the map in Figure ??.

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Loading required package: tidyverse
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-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.1     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.0
v ggplot2   3.4.2     v tibble    3.2.1
v lubridate 1.9.2     v tidyr     1.3.0
v purrr     1.0.1
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
Loading required package: TSP

Loading required package: maps


Attaching package: 'maps'


The following object is masked from 'package:purrr':

    map


Loading required package: grid
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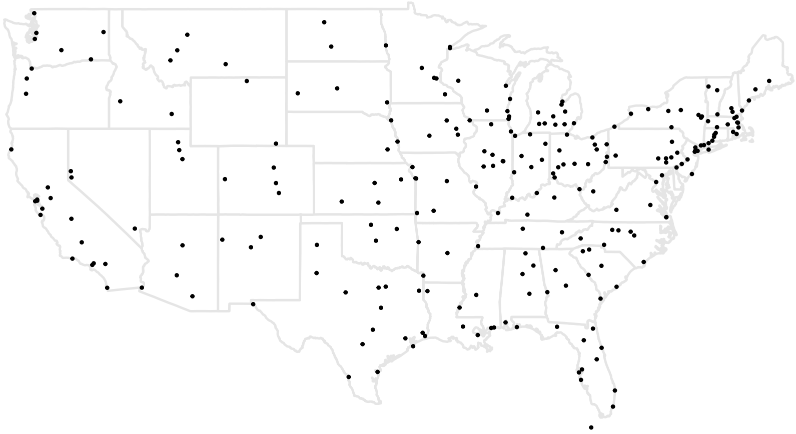

Figure 1: 257 American cites; we'll try to find the shortest path through them.

The task is to find the shortest path through those 257 cities.[2]

All nine of the decision theories I mentioned, and as far as I know every competitor to them in the philosophical literature, say the thing to do here is to draw whichever of the 256! possible paths is shortest. That is not particularly helpful advice. Unless you know a lot about problems like this, you can't draw the shortest path through the map. And least, you can't draw it as such. You can't draw it in the way that you can't enter the correct code on a locked phone (?).

One of the striking things about this puzzle is that it turns out there are some helpful things that can be said. One helpful bit of advice to someone trying to solve a problem like this is to use a Farthest Insertion Algorithm. Insertion algorithms say to start with a random city, then add cities to the path one at a time, at each time finding the point to insert the city into the existing path that adds the least distance. The Farthest Insertion Algorithm says that the city added at each stage is the one farthest from the existing path. Insertion algorithms in general produce pretty good paths in a very short amount of time - at least on normal computers. And the Farthest Insertion Algorithm is, most of the time, the best Insertion Algorithm to use. Figure ?? shows the result of one output of this algorithm.[3]

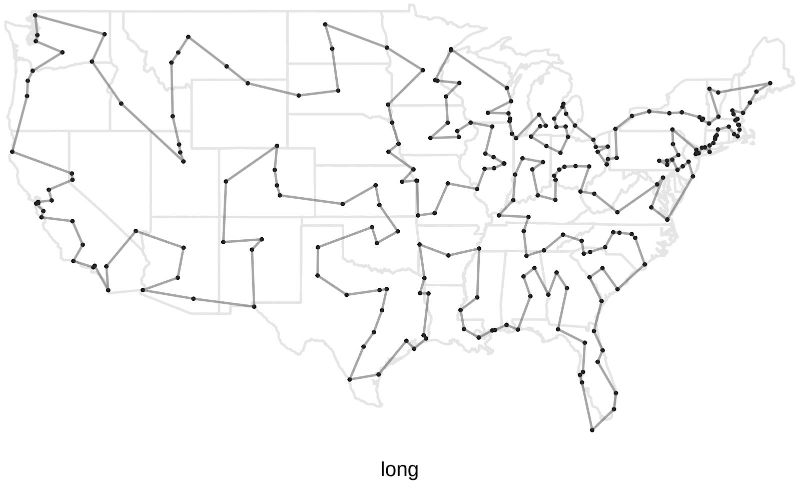

Figure 2: An output of the Farthest Insertion Algorithm, with a length of 21140 miles.

The path in Figure ?? is not bad, but with only a bit of extra computational work, one can do better. A fairly simple optimisation algorithm

[2] Include citation as to where I got the map, and the packages used in this, and to where I learned some of this stuff about TS problems.

[3] The algorithm is silent on which city you start with, and usually chooses this randomly.

takes a map as input, and then deletes pairs of edges at a time, and finds the shortest path of all possible paths with all but those two edges. The process continues until no improvements can be made by deleting two edges at a time, at which point you've found a somewhat resilient local minimum. Figure ?? is the output from applying this strategy to the path in Figure ??.

```
[1] FALSE
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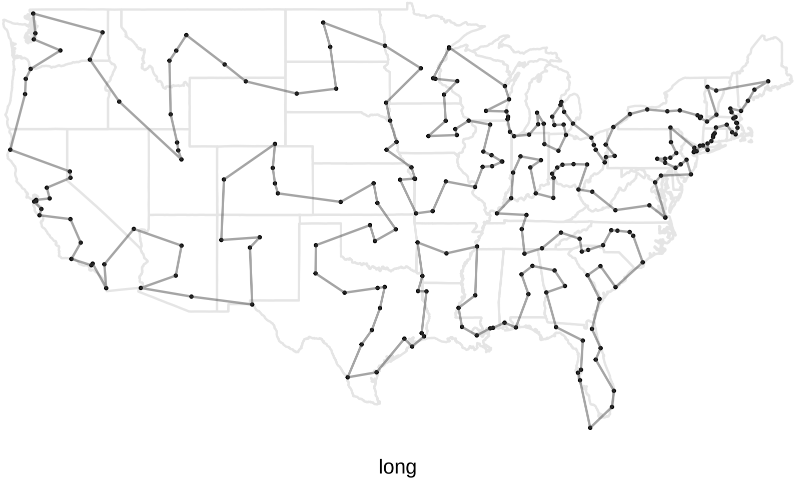

Figure 3: An output of the Farthest Insertion Algorithm, with a length of 21140 miles.

This optimisation tends to produce paths that look a lot like the original, but are somewhat shorter. For most practical purposes, the best advice you could give someone faced with a problem like this is to use a Farthest Insertion Algorithm, then optimise it in this way. Or, if they have a bit more time, they could do this a dozen or so times, and see if different starting cities led to slightly shorter paths.

While this is good advice, and indeed it's what most people should do, it's not typically what is optimal to do. For that reason, it's not what our nine decision theories would say to do. If one had unlimited and free computing power available, hacks like these would be pointless. One would simply look at all the possible paths, and see which was shortest. I do not have free, unlimited computing power, so I didn't do this. Using some black box algorithms I did not particularly understand, I was able to find a shorter path, however. It took some time, both of mine and my computer's, and for most purposes it would not have been worth the hassle of finding it. Still, just to show it exists, I've plotted it as Figure ??.

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[1] TRUE
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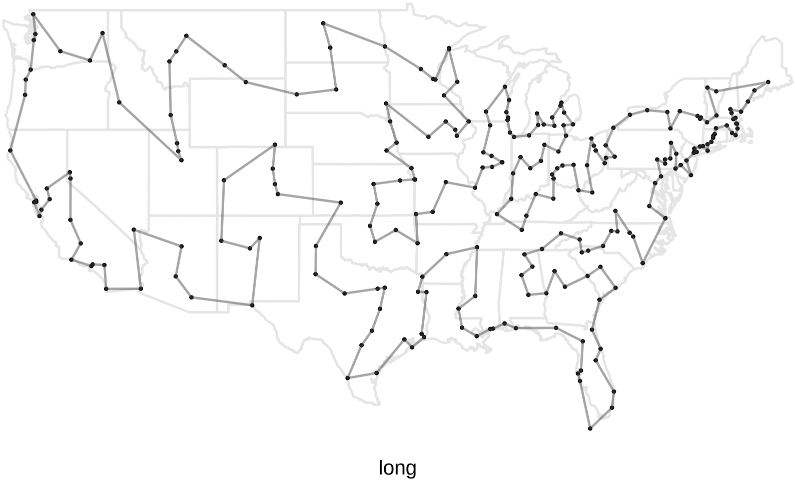

Figure 4: An output of the Farthest Insertion Algorithm, with a length of 21140 miles.

I'm not sure if Figure ?? is as short as possible, but I couldn't find a shorter one. Still, unless those 200 miles really make a difference, it wouldn't have been worth the trouble it took to find this map.

2.3 The Two Cases

Let's summarise these two cases in a table.

	Betting	Salesman
Best outcome	Bet on winner	Shortest path
Decision theory	Pass	Shortest path
Best advice	Pass	Learn algorithms

The first row says which action would produce the best outcome in the two cases. The third row says what advice one ought give someone who had to choose in the two cases. And the middle row says what all the decision theories say about the two cases. Notably, it agrees with neither the first nor third row. Decision theory is neither in the business of saying what will produce the best result, nor with giving the most useful advice. So what is it doing?

3 Decision Theory as Idealisation

Imagine a version of Chooser with, as Rousseau might have put it, their knowledge as it is, and their computational powers as they might be. That is, a version of Chooser who has unlimited, and free, computational powers, but no more knowledge of the world than the actually have - save what they learn by performing deductions from their existing knowledge.

Decision theories describe what that version of Chooser would do in the problem that Chooser is facing. In the betting case, adding unlimited computing power doesn't tell you who is going to win the game. So that version of Chooser will still avoid betting. But in the Salesman case, adding unlimited computing power is enough to solve the problem. They don't even have to use any fancy techniques. To find the shortest path, all it takes is finding the length of each path, and sorting the results. The first requires nothing more that addition; at least if, as was the case here, we provided the computer with the distances between any pairs of cities as input. The second just requires being able to do a bubble sort, which is technically extremely simple. To be sure, doing all these additions, then doing a bubble sort on the results, will take longer than most human lives on the kinds of computers most people have available to them. But a version of Chooser with unlimited, free, computational power will do these computations no problem at all.

If we say that

3.1 Technical Detour

Most philosophical decision theory concerns decisions under uncertainty, not decisions like Salesman that are made under certainty.

- But the structure is still the same.

3.2 Technical Detour

They say that for each option, you should loop through the possible states of the world, in each case multiplying something (usually a probability) by something else (usually a utility), and then summing the results. Then you choose the maximum.

- That's exactly the same technical task as solving Salesman by brute force.[4]

3.3 Summary

Decision theory describes what a particular kind of idealised agent **will** do.

- I've bolded **will** because it's going to turn out that's the important modal to use here; as opposed to *should*.
- If there is any normativity here, it's in the **idealised** part of that sentence, not the modal.

4 Idealisations as Life Goals

4.1 A Modest Proposal

Decision theory is relevant to how we should act because:

1. It tells us that idealised people do use decision theory, and
2. We should try to be like idealised people.

C. We should try to use decision theory.

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[4] Actually one step harder because of the multiplication, but otherwise the same.

Figure 5: I think this stands for What Would Jeffrey Do?

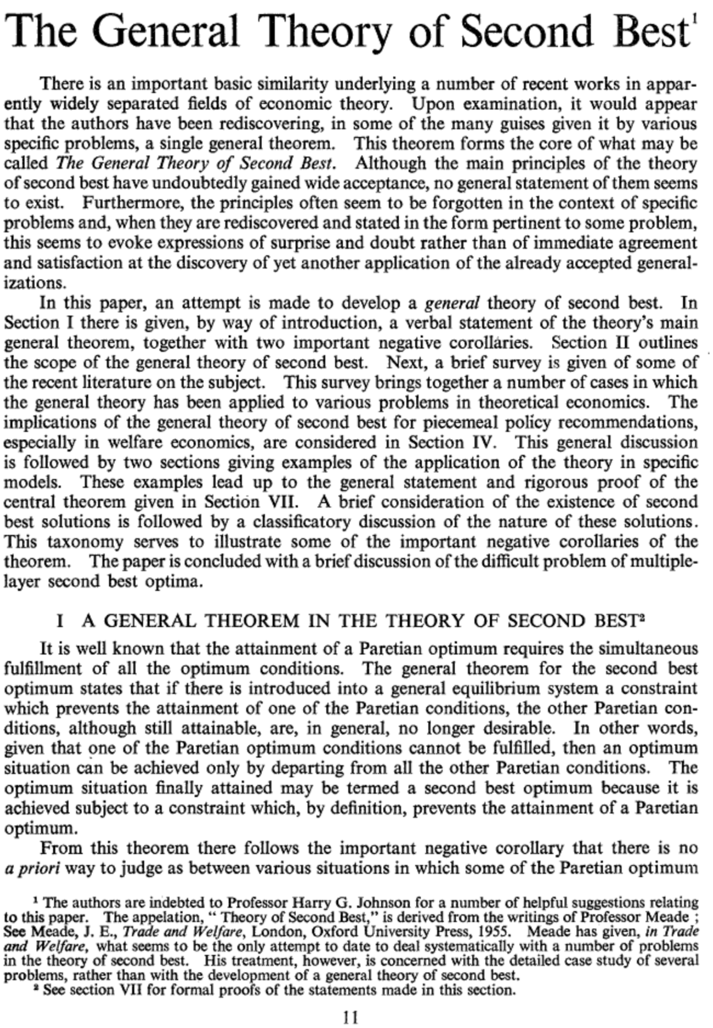
4.2 First Objection - Knowing the Inputs

To use decision theory as a guide to action, I need to know the utility of the possible states.

- Knowing ordering isn't enough, need cardinality of each utility.
- I can only ever tell that the utility of A is half way between that of B and C by thinking about whether A is better or worse to take than a 50/50 bet on B or C.
- I need to make decisions to get the inputs to decision theory.
- And I think this is the usual case.

4.3 Second Objection - The General Theory of the Second Best

In general, it's not true that one should try to approximate what the ideal is like.



The General Theory of Second Best[1]

There is an important basic similarity underlying a number of recent works in apparently widely separated fields of economic theory. Upon examination, it would appear that the authors have been rediscovering, in some of the many guises given it by various specific problems, a single general theorem. This theorem forms the core of what may be called *The General Theory of Second Best*. Although the main principles of the theory of second best have undoubtedly gained wide acceptance, no general statement of them seems to exist. Furthermore, the principles often seem to be forgotten in the context of specific problems and, when they are rediscovered and stated in the form pertinent to some problem, this seems to evoke expressions of surprise and doubt rather than of immediate agreement and satisfaction at the discovery of yet another application of the already accepted generalizations.

In this paper, an attempt is made to develop a *general* theory of second best. In Section I there is given, by way of introduction, a verbal statement of the theory's main general theorem, together with two important negative corollaries. Section II outlines the scope of the general theory of second best. Next, a brief survey is given of some of the recent literature on the subject. This survey brings together a number of cases in which the general theory has been applied to various problems in theoretical economics. The implications of the general theory of second best for piecemeal policy recommendations, especially in welfare economics, are considered in Section IV. This general discussion is followed by two sections giving examples of the application of the theory in specific models. These examples lead up to the general statement and rigorous proof of the central theorem given in Section VII. A brief consideration of the existence of second best solutions is followed by a classificatory discussion of the nature of these solutions. This taxonomy serves to illustrate some of the important negative corollaries of the theorem. The paper is concluded with a brief discussion of the difficult problem of multiple-layer second best optima.

I A GENERAL THEOREM IN THE THEORY OF SECOND BEST[2]

It is well known that the attainment of a Paretian optimum requires the simultaneous fulfillment of all the optimum conditions. The general theorem for the second best optimum states that if there is introduced into a general equilibrium system a constraint which prevents the attainment of one of the Paretian conditions, the other Paretian conditions, although still attainable, are, in general, no longer desirable. In other words, given that one of the Paretian optimum conditions cannot be fulfilled, then an optimum situation can be achieved only by departing from all the other Paretian conditions. The optimum situation finally attained may be termed a second best optimum because it is achieved subject to a constraint which, by definition, prevents the attainment of a Paretian optimum.

From this theorem there follows the important negative corollary that there is no *a priori* way to judge as between various situations in which some of the Paretian optimum

[1] The authors are indebted to Professor Harry G. Johnson for a number of helpful suggestions relating to this paper. The appelation, " Theory of Second Best," is derived from the writings of Professor Meade ; See Meade, J. E., *Trade and Welfare*, London, Oxford University Press, 1955. Meade has given, *in Trade and Welfare*, what seems to be the only attempt to date to deal systematically with a number of problems in the theory of second best. His treatment, however, is concerned with the detailed case study of several problems, rather than with the development of a general theory of second best.

[2] See section VII for formal proofs of the statements made in this section.

Figure 6: *The General Theory of the Second Best*, by R. G. Lipsey and Kelvin Lancaster, The Review of Economic Studies, 1956

This is one of the most philosophically important economics papers ever published.

4.4 Second Best

Often times, the right thing to do is something whose value consists in mitigating the costs of our other flaws.

- We should, especially in high stakes settings, stop and have a little think before acting.
- The "ideal agent" of decision theory never stops to have a think.
- Stopping is costly, and **they** don't gain anything from it.

4.5 Second Best

- The ideal agent does lots of things we don't do.
- They always take reasonable hedges against costly possibilities, and they never stop to have a think.
- Knowing that the ideal agent is F doesn't tell us whether we should try to be F unless we also know that F is more like the first of these than the second.
- And decision theory, in **anything like its current form**, is not particularly helpful on this score.

4.6 Third Objection - The Yoda Objection

Decision theory doesn't say what one should try or not try, it says what one should do.

- So it's weird to infer something about trying from a theory about doing.

4.7 Yoda

I think there's something importantly right about this - decision theory gives criteria of correctness not methods of deliberation - but that in turn shows us why it might be useful.

5 Idealisations as Models

5.1 Two Notions of Idealisation

In philosophy we use the word ‘idealisation’ for two rather different kinds of thing.

1. Perfect
2. Simple

The point particles in ideal gas theory are not perfect - having volume is not an imperfection.

Nor are they things to aim for - high school chemistry does not imply a rule: **Smaller the better**.

But they are simple.

5.2 Idealisations in Decision Theory

Decision theory provides idealisations in the second sense - they are **simplifications**.

- Just like the point masses we use in the ideal gas law, they say not what should happen, but what would happen in the absence of certain complications.

5.3 Idealisations in Decision Theory

Why do I say this idealisation is a simplification not a perfection?

1. Allocating zero seconds to hard math problems is not a perfection.
2. The idealised self isn’t absolutely perfect - they have very restricted information.

5.4 Idealisations in Decision Theory

The idealised self that gets used is god-like in one respect - computational ability - but human-like in another - informational awareness.

- That's a common feature of idealised models.
- You abstract away from one feature, but not others.

5.5 Why Care?

That's what we do, but why do we do it?

- Because sometimes these models are enlightening.
- Sometimes, the fact that we have computational limitations is not relevant to predicting/explaining/understanding what we will do.

5.6 Really, Why Care?

It's tempting to identify these with high stakes situations, since those are ones where we'll throw enough computational resources at the problem that we have god-like powers.

- But that isn't quite right.
- In some high stakes cases, we also throw enough investigative resources at the problem that holding actual knowledge fixed is a bad modelling assumption.

5.7 Informational Limitations

What we need are cases where there are principled limitations to our informational capacities, such as,

1. Cases where the information concerns the future; or
2. Cases where someone has (or may have) just as strong an incentive to hide information from us.

I'll end with a discussion of an important instance of the second.[5]

[5] Photo of George Akerlof on next slide by Yan Chi Vinci Chow.

5.8 Akerlof on Lemons