HW 3: PMATH 945

Due Feb. 7 at 11:59 pm

(1) Consider the graded ring $R = \mathbb{Z}[x,y,z]/(zy^2 = x(x^2 - z^2))$.
   (a) Consider the set $V(pR) \subset \mathrm{Proj}(R)$ of homogeneous ideals containing pR for $p \in \mathbb{Z}$ prime. Describe the points of this set.
   (b) Consider the set $V(zR) \subset \mathrm{Proj}(R)$ of homogeneous ideals containing zR. Describe the points of this set.

(2) Fix a graded ring $S_\bullet$ and a homogeneous ideal I. Show that the following are equivalent:
   (a) $V(I) = \emptyset$.
   (b) If $\{f_i\}_{i \in I}$ are homogeneous generators of I, then $\bigcup_{i \in I} D^+(f_i) = \mathrm{Proj}(S)$.
   (c) $\sqrt{I} \supset S_+$.

(3) (a) Show that for a scheme X, $p \mapsto \overline{\{p\}}$ is a bijection between the points of the scheme and its irreducible closed subsets (the inverse of this map sends a closed subset to its generic point).
   (b) Show that a scheme X is integral if and only if it is reduced and irreducible.
   (c) Show that if X is integral and η is its generic point, then $\mathcal{O}_{X,\eta} = \mathrm{Frac}(R)$ for any affine open subset $\mathrm{Spec}(R) \subset X$.

(4) Let A be a Noetherian ring. Show that any projective A-scheme is Noetherian and of finite type over A.

Extra exercises (if the above wasn't enough): 4.3.F, 4.4.A, 4.5.F, 5.2.F, 5.3.E, 5.4.A, 5.4.I