Seasonally Forced Epidemics and Bifurcation Diagram

K. Bodie Weedop

10/6/2019

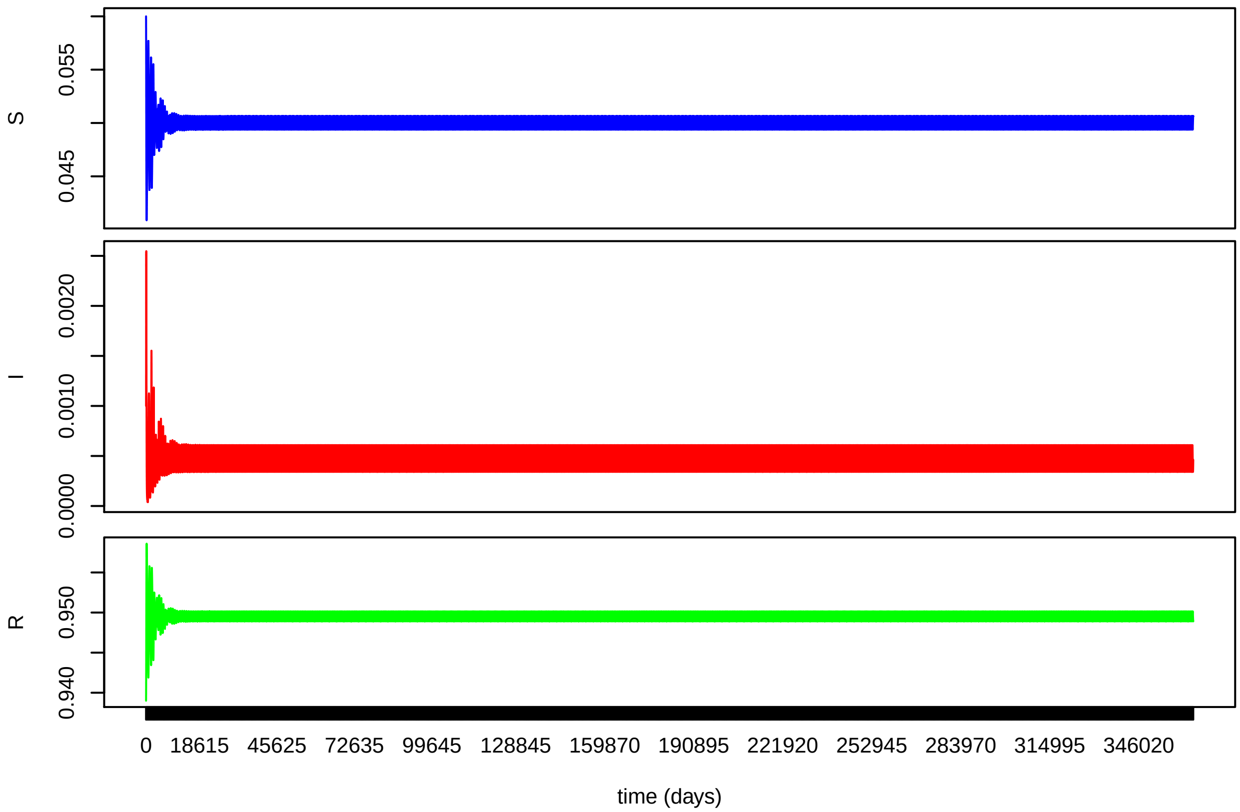
```r
# Function for diff. eqs.; Passed below to ode()
seasonal.sir <- function(t, vars, params) {
    # Getting initial values for variables
    s0 <- vars[1]
    i0 <- vars[2]
    r0 <- vars[3]

    # Getting value of beta(t) and other parameter values
    b_t <- betafx(beta0 = params[1], beta1 = params[2], t = t)
    gamma <- params[3]
    mu <- params[4]

    # Differential equations for model
    dS <- mu - b_t * s0 * i0 - mu * s0
    dI <- b_t * s0 * i0 - gamma * i0 - mu * i0
    dR <- gamma * i0 - mu * r0
    return(list(c(dS, dI, dR)))
}
```

```r
# Function for forcing function; Omega set so that oscillations are annual.
betafx <- function(beta0, beta1, t) {
    omega <- (2*pi)/365
    return(beta0 * (1 + beta1 * cos(omega * t)))
}
```

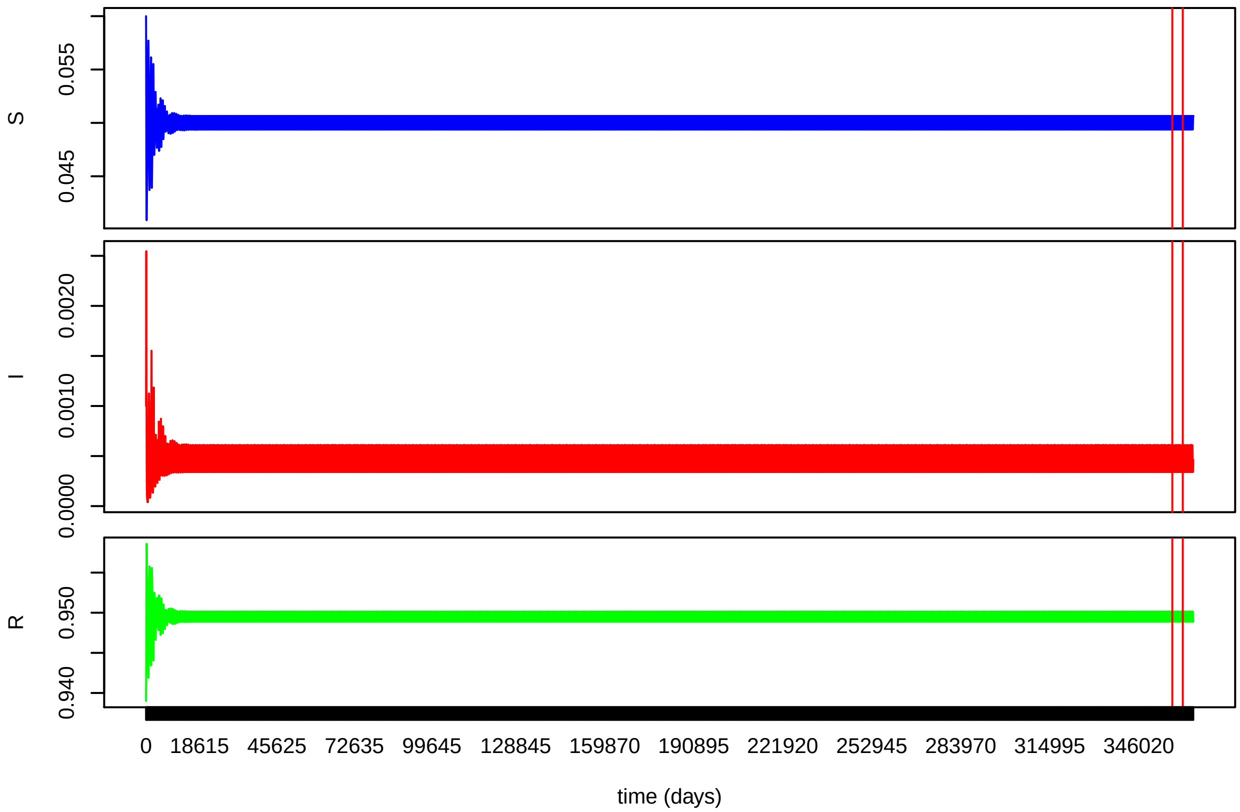
```r
# Initial values for variables
s0 <- 0.06
i0 <- 0.001
r0 <- 1-s0-i0
init.vars <- c(S = s0, I = i0, R = r0)

# Time to run model simulation; Allowed to run for 100 years.
years <- 1000
t_range <- seq(0, years*365, 1)

# Parameter values
b0 <- 1.6
b1 <- 0.05
g <- 0.08
m <- 1/(70*365)
params <- c(beta0 = b0, beta1 = b1, gamma = g, mu = m)
```

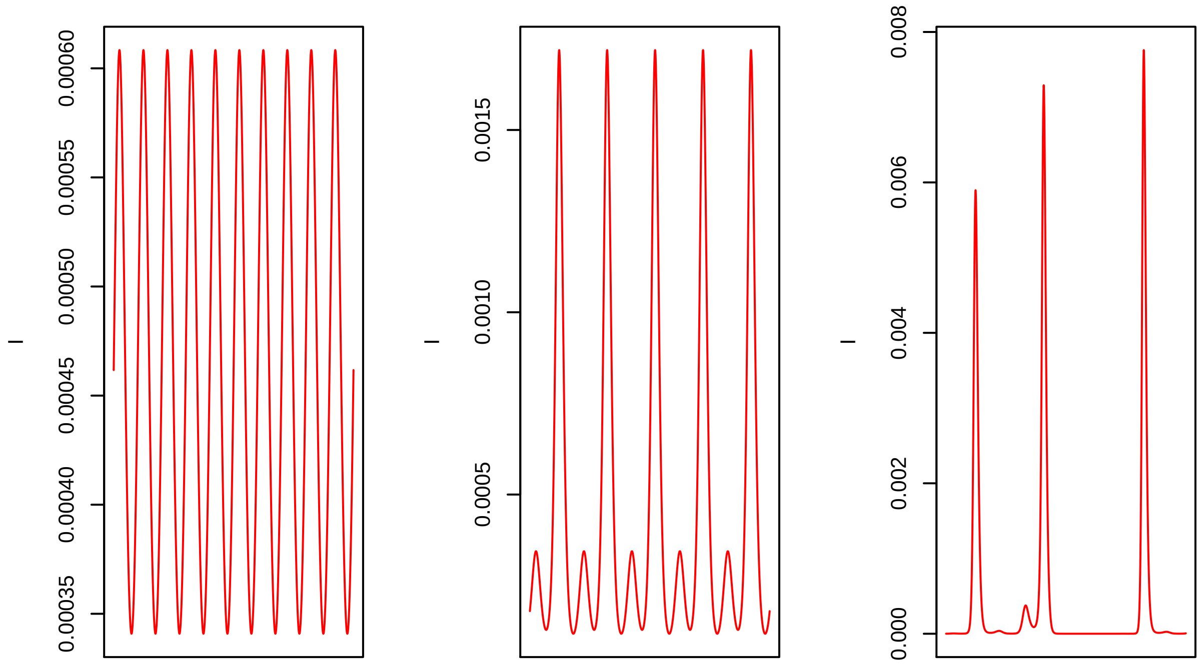
```r
# Run simulation
out <- ode(y = init.vars,
           times = t_range,
           func = seasonal.sir,
           parms = params)
```

```r
# Plotting
# Each variable (S, I, R) will be plotted individually over total time
#    simulation allowed to run.
par(mfrow = c(3,1),
    pin = c(10, 1.5),
    mai = c(0, 0.6, 0, 0.1),
    mar = c(0.5, 4, 2, 0.5),
    oma = c(0.5, 0, 0, 0.1))
plot(out[, 2] ~ out[, 1],
     type = "l",
     col = "blue",
     xaxt = "n",
     xlab = "",
     ylab = "S")
par(mar = c(0.5, 4, 0, 0.5))
plot(out[, 3] ~ out[, 1],
     type = "l",
     col = "red",
     xaxt = "n",
     xlab = "",
     ylab = "I")
par(mar = c(4, 4, 0.5, 0.5))
plot(out[, 4] ~ out[, 1],
     type = "l",
     col = "green",
     xlab = "time (days)",
     xaxt = "n",
     ylab = "R")
xticks <- seq(0, years*365, 365)
axis(1, at = xticks)
```



I selected a ten year window (years 980 through 990) of the simulation where the oscillations had stabilized. The window selected can be seen indicated with the red lines on the same plot below.






```
bif.points <- function(amp) {
    # Parameter values
    b0 <- 1.6
    b1 <- amp
    g <- 0.08
    m <- 1/(70*365)
    params <- c(beta0 = b0, beta1 = b1, gamma = g, mu = m)

    # Run simulation
    out <- ode(y = init.vars,
               times = t_range,
               func = seasonal.sir,
               parms = params)

    # Ten year window
    data.window <- as.data.frame(out[(window[1]*365+1):(window[2]*365+1),])
    # Sampling regular intervals of the ten year window
    samples <- rep(NA, 10)
    for (i in 1:(length(samples))) {
        samples[i] <- round(data.window$I[365*i], 9)
    }
    return(unique(samples))
}
```

```
# Different values for amplitude of seasonality
b1_vals <- seq(0.01, 0.30, 0.001)

dat <- data.frame()
for (i in b1_vals) {
    temp <- bif.points(i)
    for(j in temp) {
        dat <- rbind(dat, c(i, j))
```

```
    }
}

colnames(dat) <- c("beta1", "I")

plot(log10(dat$I) ~ dat$beta1,
     xlab = expression(paste("Amplitude of Seasonality (", beta[1], ")")),
     ylab = "Log10 Proportion Infected (I)",
     pch = "'")
```