ENAE441 Group Project

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1 Overview

In this project, the goal was to locate and estimate the orbit of two GPS satellites, GPS 10 and GPS 32, given the following variables:

- Pseudorange Data between the GNSS Satellite and three chosen Ground Stations.
- The Earth-Centered, Earth-Fixed Coordinates of the three chosen Ground Stations.
- A chosen date and time between January and October of 2019.

2 Work Done

2.1 Choosing the Ground Sites and the Date

2.1.1 Ground Sites

The following ground sites were chosen for this Project:

- GODE (Greenbelt, Maryland)
- NJTR (Trenton, New Jersy)
- LYCO (Middle of Pennsylvania)

September, 29, 2019 was chosen as the observation date for both GNSS satellite. We set t1 to 20:00 UTC Time for both satellites, as this was the time where the GNSS satellites were visible to all three of the ground stations. The end time, occurred twelve hours after t1. The date was chosen at the end of the allowable date range (January to October 2019). The idea behind choosing such a recent date was to improve the accuracy of our results by using the most recent data. The orbits of satellites change with time due to Perturbations and Drag. It is believed that the most recent data will give us the most recent orbital data which will be closest to the actual, current orbital elements of the GNSS satellite.

2.2 Trilateration

Trilateration was used by creating a function following the steps outlined in the book. There were issues with the first time of the first satellite, where it wouldn't converge. This is theorized to be due to the fact that the ground stations were too close together to provide far enough points to converge.

2.3 Position Vectors

The position vector was found by using a transformation matrix to change the coordinates from an ECEF frame to an IJK frame. This was done by taking the inverse of the Rz transformation matrix and then multiplying that to the ECEF vector. The resultant is the IJK vector. This is because the IJK frame and the ECEF frame share the same z axis so the transformation was simple.

2.4 Solving the Lambert Problem

2.5 Pseudoranges

3 Conclusion

3.1 Results

We were unable to get results due to the issue with trilateration of the first satellite time value.

3.2 Sources of Error

During the analysis, we had trouble getting the Trilateration Code for G32 to Converge at t1. The Trilateration code did converge for G32 at t2 and G10 at both t1 and t2. However, the position vectors we got out of the trilateration code were off from the position vectors by each ground site's sp3 files. We tested the trilateration code using an example from the textbook (Page 146) and Assignment 7 by inputting the given ranges and reference positions into the trilateration code and comparing the output of the code with the correct answers to the example problem and the homework. The output of the code matched the textbook and assignment answers, which led us to conclude that the code was written correctly, and is not the source of the error.

The error came from the chosen ground stations being too close to each other. When the ground stations were being selected, they were selected in a way where all three ground stations were as far away from each other as possible, but within 250 km of each other so that all three stations can receive the GNSS signal simultaneously. This Ground Station Selection method ended up becoming a source of error in our results. Trilateration works best when the reference locations are spread very far apart.

3.3 Limitations

One limitation of our analysis is that the trilateration code didn't include the transmitter clock error. This would have made the analysis much more accurate. GNSS trilateration is different from normal trilateration because GNSS trilateration includes the transmitter clock error. Including the transmitter clock error makes up for the time dilation GPS satellites go through in their orbits. (according to Special and General Relativity)

The analysis was also limited by the constrained range of dates given in the project. (January to October 2019) It was said before that the orbit of the GNSS satellite will change over time due to Drag and J2 Perturbations. Since the goal of the project is to measure the orbit of the GNSS satellite, it makes sense to get the most recent data from the ground stations, so that the results of the analysis (the estimated orbital elements of the chosen GNSS satellite) will be as up-to-date and accurate as possible.

If there is one thing about the project that can be changed to improve the accuracy of the results (and rid the analysis of this limitation), it would be to get rid of the constrained range of dates, so that students could pick a date from January 2019 to any day of 2019 where GNSS data is available. This will allow students to perform the analysis on more recent, up-to-date data and return more accurate results.

A Code

A.1 Main Analysis

```
% Load data
  % clear all
  % clc
  % close all
  run set_data.m
  run set_parameters.m
6
  % Trilateration
  rho_t1_sat1 = Trilateration(rho_obsv_t1_sat1, ref_matrix, 1e-4, rho_njtr_t1_sat1)
  rho_t2_sat1 = Trilateration(rho_obsv_t2_sat1, ref_matrix, 1e-4, rho_njtr_t2_sat1)
10
  rho_t1_sat2 = Trilateration(rho_obsv_t1_sat2, ref_matrix, 1e-4, rho_njtr_t1_sat2)
  rho_t2_sat2 = Trilateration(rho_obsv_t2_sat2, ref_matrix, 1e-4, rho_njtr_t2_sat2)
12
13
  % Frame Conversion
14
  gst_t1 = GST(T1, t1);
  gst_t2_sat1 = GST(T2, t2_sat1);
16
  gst_t2_sat2 = GST(T2, t2_sat2);
```

```
ijk_t1_sat1 = eceftoijk(rho_njtr_t1_sat1, gst_t1)
19
   ijk_t2_sat1 = eceftoijk(rho_njtr_t2_sat1, gst_t1)
20
   ijk_t1_sat2 = eceftoijk(rho_njtr_t1_sat2, gst_t1)
21
   ijk_t2_sat2 = eceftoijk(rho_njtr_t2_sat2, gst_t1)
22
23
   % Lambert Solver
24
   dt1 = seconds(t_sat1(2) - t_sat1(1));
25
   dt2 = seconds(t_sat2(2) - t_sat2(1));
26
27
   [\,v01\,,\ v1\,]\ =\ Lambert\,(\,ijk\_t\,1\_s\,at\,1\ ,\ ijk\_t\,2\_s\,at\,1\ ,\ dt1\ ,\ 1\,,\ mu\_earth\ ,\ 1e-5)
28
   [v02, v2] = Lambert(ijk_t1_sat2, ijk_t2_sat2, dt2, 1, mu_earth, 1e-5)
29
   %% PRN
   mu = 3.986*10^5;
31
   tle1 = ['1 29486U 06042A]
                                  19272.00430744
                                                   -.00000001
                                                                 00000-0
                                                                           00000+0 0
                                                                                       9995 ; :
32
                                248.8813 0093935
                                                                         2.00560148 95342;;
            2 29486
                      54.9590
                                                      0.5082
                                                               31.6022
33
           [ '1 27663U 03005A
                                  19272.09385061
                                                    -.00000030
                                                                 00000-0
                                                                           00000+0 0
                                                                                        9998'
   tle2 =
34
            2 27663
                      56.2455
                                312.2850 \ 0106625
                                                     32.1585 \quad 330.1378
                                                                         2.00567379122108
35
   tle3 = ['1 \ 27663U \ 03005A]
                                  19272.09385061
                                                    -.00000030
                                                                 00000-0
                                                                           00000+0 0
                                                                                        9998 ';
36
            ^{\circ}2 27663
                       56.2455
                                312.2850 \ 0106625
                                                     32.1585 \quad 330.1378
                                                                         2.00567379122108'];
37
   tle4 = ['1 \ 24876U \ 97035A]
                                                                                        9997
                                  19272.18807153
                                                     .00000003
                                                                 00000-0
                                                                           00000+0 0
38
            2 24876
                      55.4568
                                194.4010 0036176
                                                     67.5612 \quad 292.8340
                                                                         2.00563944162551
39
   tle5 = ['1 38833U 12053A]
                                  19272.31629780
                                                    -.00000004
                                                                00000 - 0
                                                                           00000+0 0
40
            2 38833
                                                     33.5442 \quad 326.9658
                                                                         2.00559943 51133'];
                      53.7378
                                244.1349 0086285
41
           [ '1 25030U 97067A
                                  19272.33803917
                                                     .00000001
                                                                 00000-0
                                                                           00000+0 0
                                                                                        9994
42
                                                    231.6008 \ \ 293.5417
            2 25030
                      56.0213
                                256.2534 0190156
                                                                         2.00516989160455
43
   tle7 = ['1 \ 26690U \ 01004A]
                                  19272.35974556
                                                     .00000032
                                                                 00000 - 0
                                                                           00000+0 0
44
           26690
                                                   356.1546 \ 182.7140
                       53.2943
                                123.9467 \ 0001296
                                                                         1.91595261136520'];
            '1 39741U 14026A
                                  19272.45032143
                                                    -.00000030
                                                                 00000-0
                                                                           00000+0 0
   tle8 = [
                                                                                        9993;
46
            2 39741
                      55.9700
                                 67.1394 \ 0016102
                                                   289.8941
                                                               69.9389
                                                                         2.00564685 39320'];
47
   tle9 = ['1 \ 20959U \ 90103A]
                                                   +.00000035 +00000-0 +00000-0 0
                                  19272.48391087
48
            ^{\circ}2 20959 054.4426
                                136.9451 0017876 042.7358 317.5074 01.88156272209715;
49
   tle10 = [
              '1 32384U 07062A
                                   19272.48624791
                                                    -.00000087
                                                                  00000-0
                                                                            00000+0 0
                                                                                         9992
50
                                                                          2.00562300 86374'];
             ^{\circ}2 32384
                        56.4834
                                  11.0588 \ 0009646 \ 122.0956 \ 203.3162
51
   tle11 = ['1 32384U 07062A]
                                   19272.48624791
                                                     -.00000087
                                                                  00000 - 0
                                                                             00000+0 0
52
             2 \ 32384
                        56.4834
                                                    122.0956 \ 203.3162
                                                                          2.00562300 86374
                                  11.0588 0009646
53
   tle12 = |
                                                                            00000+0 0
             '1 28474U 04045A
                                   19272.50293218
                                                     -.00000038
                                                                  00000-0
                                                                                         9990
54
             2 28474
                                                                          2.00578637109238'];
                       54.7934
                                  63.5258 \ 0194068
                                                    261.9012 100.0304
55
   tle13 = ['1 \ 22231U \ 92079A]
                                   19272.53710043
                                                     +.00000004 +00000-0 +00000-0 0
56
             2 \ 22231 \ 055.1771
                                 198.1532 0032550
                                                     313.3900 \quad 046.3442 \quad 01.89107504015768
57
   tle14 = ['1 \ 40534U \ 15013A]
                                   19272.55094475
                                                     -.00000032
                                                                 00000-0
                                                                            00000+0 0
                                                                                         9991
58
             2 \ 40534
                        54.4286
                                 305.6404 0039273
                                                       3.5376 \ \ 356.5465
                                                                          2.00563196 33065']:
59
   tle15 = ['1 \ 40534U \ 15013A]
                                   19272.55094569
                                                     -.00000032 +00000-0 +00000-0 0
             2 40534 054.4286
                                 305.6405 \quad 0039258
                                                     003.5533 \ 356.5309 \ 02.00563180032636
61
   tle16 = ['1 35752U 09043A]
                                   19272.59462258
                                                      .00000032
                                                                  00000 - 0
                                                                            00000+0 0
                                                                                         9994 '
62
             2 35752
                        54.4690
                                 125.6613 \ 0057421
                                                      43.7292 \quad 316.6570
                                                                          2.00551811 74149'];
63
   tle17 = ['1 35752U 09043A]
                                   19272.59462258
                                                                  00000-0
                                                                            00000+0 0
                                                      .00000032
                                                                                         9994'
64
             ^{\circ}2 35752
                                                                          2.00551811 74149
                        54.4690
                                 125.6613 \quad 0057421
                                                      43.7292 316.6570
65
   tle18 = ['1 35752U 09043A]
                                   19272.59462258
                                                                  00000-0
                                                      .00000032
                                                                            00000+0 0
                                                                                         9994'
             ^{\circ}2 35752
                        54.4690
                                 125.6613 \quad 0057421
                                                      43.7292 \ 316.6570
                                                                          2.00551811 74149']:
67
   tle19 = ['1 35752U 09043A]
                                   19272.59462258
                                                      .00000032
                                                                  00000-0
                                                                             00000+0 0
                                                                                         9994;
68
             2 \ 35752
                        54.4690
                                 125.6613 \ 0057421
                                                      43.7292 \quad 316.6570
                                                                          2.00551811 74149
69
   tle20 = ['1 35752U 09043A]
                                   19272.59462258
                                                                  00000 - 0
                                                      .00000032
                                                                            00000+0 0
                                                                                         9994;
70
                                                                          2.00551811 74149'];
             ^{\circ}2 \quad 35752
                        54.4690
                                 125.6613 \ 0057421
                                                      43.7292 316.6570
71
   tle21 = ['1 \ 26605U \ 00071A]
                                   19272.69514253
72
                                                     +.00000007 +00000-0 +00000-0 0
                                                                                         9990;
             ^{\prime}2\ 26605\ 055.0312
                                 191.7874 0109855
                                                     249.7786 332.1308 02.00554402138357
73
   tle22 = ['1 \ 26605U \ 00071A]
                                   19272.69514253
                                                      .00000007
                                                                 00000-0 00000+0 0
```

```
2 26605
                                191.7874 0109855 249.7786 332.1308 2.00554402138346'];
                       55.0312
            [ '1 28474U 04045A
                                                   -.00000040 +00000-0 +00000-0 0 9993;
   tle23 =
                                  19272.75033867
76
             2 28474 054.7936
                                                   261.9054 278.6796 02.00578618109248'];
                                063.5158 0194091
77
   tle24 = ['1 \ 41019U \ 15062A]
                                  19272.76047173
                                                    .00000032
                                                               00000 - 0
                                                                          00000+0 0
78
            2 41019
                       55.2204
                                127.1099 0049663
                                                   205.3918 \ 154.4583
                                                                        2.00564610 28644'];
79
   tle 25 = ['1 \ 26407U \ 00040A]
                                  19272.76359852
                                                   -.00000037 +00000-0 +00000-0 0
                                                                                      9990;
80
             2 26407 056.1862
                               312.4812 0193416
                                                   276.6305 083.5227 02.00559992140769'];
81
   tle26 = ['1\ 38833U\ 12053A]
                                  19272.81488285
                                                   -.00000003 00000-0 00000+0 0
82
             '2 38833
                       53.7377
                                244.1143 \ 0086298
                                                    33.5504 \ \ 326.9594
                                                                        2.00559949 51143'];
   tle27 = ['1 38833U 12053A]
                                  19272.81488285
                                                   -.00000003 +00000-0 +00000-0 0
                                                                                      9995
84
             2 38833 053.7377
                                244.1143\ 0086298
                                                   033.5504 326.9594 02.00559949050366'];
85
   tle 28 = ['1 \ 28129U \ 03058A]
                                  19272.88447774
                                                    .00000024 \quad 00000-0
                                                                          00000+0 0
86
            28129
                                                                        2.00560713115612'];
                       53.1966
                                122.8369 \ 0071095
                                                   286.1149
                                                              73.1942
87
   tle29 = ['1 \ 40294U \ 14068A]
                                  19272.91473815
                                                    .00000031
                                                                00000 - 0
                                                                          00000+0 0
                                                                                      9990;
88
             2 \ 40294
                       55.2208
                                127.2854 \ 0023982
                                                    39.2856 \ \ 320.9842
                                                                        2.00560124 36006'];
89
                                                                          00000+0 0
   tle30 = ['1 \ 40294U \ 14068A]
                                  19272.91473815
                                                    .00000031
                                                                00000 - 0
90
                                                                       2.00560124 36006'];
             2 40294
                       55.2208
                                                    39.2856 \ \ 320.9842
                                127.2854 \ 0023982
91
   tle31 = ['1 29486U 06042A]
                                                   -.000000000 00000-0 00000+0 0 9996;
                                  19272.95201061
92
                                                     0.5417 355.8484 2.00560077 95359;;
                      54.9587
                                248.8433 0093968
             ^{\circ}2 29486
93
    [OE1] = TLE2OE(tle1, mu);
94
    OE2 = TLE2OE(tle2, mu);
95
         = TLE2OE(tle3, mu);
    OE31
96
    OE4] = TLE2OE(tle4, mu);
97
    OE5] = TLE2OE(tle5, mu);
98
         = TLE2OE(tle6, mu);
    OE6]
99
    OE7
         = TLE2OE(tle7, mu);
100
    OE8 = TLE2OE(tle8, mu);
101
    OE9] = TLE2OE(tle9, mu);
102
    OE10] = TLE2OE(tle10, mu);
103
    OE11]
          = TLE2OE(tle11, mu);
104
    OE12]
           = TLE2OE(tle12, mu);
105
          = TLE2OE(tle13, mu);
    OE13]
106
    OE14]
           = TLE2OE(tle14, mu);
107
           = TLE2OE(tle15, mu);
    OE15]
108
    OE16]
           = TLE2OE(tle16, mu);
109
    OE17]
           = TLE2OE(tle17, mu);
110
    OE18]
           = TLE2OE(tle18, mu);
111
    OE19]
           = TLE2OE(tle19, mu);
112
    OE20
          = TLE2OE(tle20, mu);
113
    OE21]
           = TLE2OE(tle21, mu);
114
    OE22
           = TLE2OE(tle22, mu);
115
    OE23
           = TLE2OE(tle23, mu);
116
    OE241
           = TLE2OE(tle24, mu);
117
           = TLE2OE(tle25, mu);
    OE25
118
    OE26]
           = TLE2OE(tle26, mu);
119
    OE27]
           = TLE2OE(tle27, mu);
120
    OE28]
           = TLE2OE(tle28, mu);
121
    OE29]
           = TLE2OE(tle29, mu);
122
    OE30]
          = TLE2OE(tle30, mu);
123
    [OE31] = TLE2OE(tle31, mu);
124
125
    states1 = OE2Cart (OE1, mu);
126
    states2]
              = OE2Cart (OE2, mu);
127
              = OE2Cart (OE3, mu);
    states3
128
              = OE2Cart (OE4, mu);
    states4
             = OE2Cart (OE5, mu):
    states 5
130
    [states6] = OE2Cart (OE6, mu);
```

```
= OE2Cart (OE7, mu);
    states7]
132
              = OE2Cart (OE8, mu);
    states8]
133
              = OE2Cart (OE9, mu);
    states91
134
    states10] = OE2Cart (OE10, mu);
135
    states11]
               = OE2Cart (OE11, mu);
136
               = OE2Cart (OE12, mu);
    states 12
137
    states13]
               = OE2Cart (OE13, mu);
138
               = OE2Cart (OE14, mu);
    states14]
139
    states15
               = OE2Cart
                           (OE15, mu);
140
    states16
               = OE2Cart
                           (OE16, mu);
141
    states17
               = OE2Cart (OE17, mu);
142
    states 18]
               = OE2Cart (OE18, mu);
143
               = OE2Cart
                           (OE19, mu);
    states 19]
144
    states 20
               = OE2Cart
                           (OE20, mu);
145
    states21
               = OE2Cart (OE21, mu);
146
    states 22
               = OE2Cart
                           (OE22, mu);
147
               = OE2Cart
                           (OE23, mu);
    states23]
148
               = OE2Cart
                           (OE24, mu);
    states24
149
               = OE2Cart (OE25, mu);
    states 25
150
    states26
               = OE2Cart (OE26, mu);
151
               = OE2Cart (OE27, mu);
    states27
152
    states 28
               = OE2Cart (OE28, mu);
153
    states 29]
               = OE2Cart (OE29, mu);
154
    states30]
               = OE2Cart (OE30, mu);
155
    states31
               = OE2Cart (OE31, mu);
156
   range = zeros(31, 1);
157
   range(1) = norm(states1);
158
   range(2) = norm(states2);
159
   range(3) = norm(states3);
160
   range(4) = norm(states4);
161
   range(5) = norm(states5);
162
   range(6) = norm(states6);
163
   range(7) = norm(states7);
164
   range(8) = norm(states8);
165
   range(9) = norm(states9);
166
   range(10) = norm(states10);
167
   range(11) = norm(states11);
168
   range(12) = norm(states 12);
169
   range(13) = norm(states13);
170
   range(14) = norm(states14);
171
   range(15) = norm(states15);
172
   range(16) = norm(states 16);
173
   range(17) = norm(states17);
174
   range(18) = norm(states18);
175
   range(19) = norm(states19);
176
   range(20) = norm(states 20);
177
   range(21) = norm(states21);
178
   range(22) = norm(states22);
179
   range(23) = norm(states23);
   range(24) = norm(states24);
181
   range(25) = norm(states25);
182
   range(26) = norm(states 26);
183
   range(27) = norm(states 27);
184
   range(28) = norm(states 28);
185
   range(29) = norm(states29);
186
   range(30) = norm(states 30);
187
   range(31) = norm(states31);
```

```
\begin{array}{lll} \operatorname{satg32} & \operatorname{satg32} = \left(24885.268756 + 25027.333239 + 24905.66238\right)/3 \\ \operatorname{satg10} & \operatorname{satg10} = \left(24741.064469 + 24965.346244 + 24859.33414\right)/3 \end{array}
```

A.2 Coordinate Transformation

```
function rijk = eceftoijk(r,gst)

This function takes in a column vector r containing the ECEF coordinates

Rz = \begin{bmatrix} \cos(gst) & \sin(gst) & 0; \\ -\sin(gst) & \cos(gst) & 0; \\ 0 & 0 & 1 \end{bmatrix};

rijk = inv(Rz)*r;
```

A.3 Lambert Solver

```
function [v0, v] = Lambert(r0, r, dt, direction, mu, tol_t)
  % This function is a lambert solver for elliptical solutions only
  % Inputs:
       % r0: initial position vector
4
       % r : final position vector
       % dt: time of flight
       \% direction: +1 for short way, -1 for long way about the orbit
       % mu: orbital parameter for whichever body is being orbited
       % tol_t: tolerance for which to break the loop
   % Outputs:
10
       % v0: Initial velocity vector
11
       % v : final velocity vector
12
   % Author: Blaire Weinberg
13
14
   cosdnu = dot(r0, r) / (norm(r0) * norm(r));
15
   A = \operatorname{direction} * \operatorname{sqrt}(\operatorname{norm}(r) * \operatorname{norm}(r0) * (1 + \operatorname{cosdnu}));
16
17
   if A == 0
18
        error ('Cannot Calculate Orbit, A = 0');
19
   end
20
21
   % Initial States
22
   psi_n = 0;
23
   c2 = 1/2;
24
   c3 = 1/6;
25
   psi_up = 4 * pi^2;
27
   psi_down = -4 * pi;
29
   dt_n = 0;
30
   loop = 0:
31
   loop_max = 100;
32
33
   while ((abs(dt - dt_n) > tol_t)&&(loop < loop_max))
34
       loop = loop + 1;
35
       yn = norm(r0) + norm(r) + (A * (psi_n * c3 - 1) / sqrt(c2));
36
       chi_n = sqrt(yn / c2);
37
       dt_n = (chi_n^3 * c3 + A * sqrt(yn)) / sqrt(mu);
38
39
        if dt_n \ll dt
40
            psi_down = psi_n;
41
        else
42
            psi_up = psi_n;
43
```

```
end
44
45
         psi_n = (psi_up + psi_down) / 2;
46
47
          if psi_n > 1e-6
48
           c2 = (1 - \cos(\operatorname{sqrt}(\operatorname{psi}_{-n}))) / \operatorname{psi}_{-n};
49
           c3 = (\operatorname{sqrt}(\operatorname{psi}_{-n}) - \sin(\operatorname{sqrt}(\operatorname{psi}_{-n}))) / \operatorname{sqrt}(\operatorname{psi}_{-n}^3);
50
          elseif psi_n < -1e-6
51
               c2 = (1 - \cosh(\operatorname{sqrt}(-\operatorname{psi}_{-n}))) / \operatorname{psi}_{-n};
52
               c3 = (-\operatorname{sqrt}(-\operatorname{psi}_{-n}) + \sinh(\operatorname{sqrt}(-\operatorname{psi}_{-n}))) / \operatorname{sqrt}((-\operatorname{psi}_{-n})^3);
53
          else
54
               c2 = 1/2;
55
               c3 = 1/6;
         end
57
    _{
m end}
58
59
    f = 1 - (yn / norm(r0));
60
   g = A * sqrt(yn / mu);
61
    gdot = 1 - (yn / norm(r));
63
   v0 = (r - f .* r0) ./ g;
   \mathbf{v} = (\mathbf{g} \mathbf{d} \mathbf{o} \mathbf{t} \cdot \mathbf{*} \mathbf{r} - \mathbf{r} \mathbf{0}) \cdot / \mathbf{g};
           Transformation Angle
   function [gst] = GST(T, t)
   % Determines the Greenwich Sidereal Time
   % Inputs:
3
         % T: Vector containing centuries since J2000 (UTC)
         % t: Vector containing seconds from midnight to the observation
5
   % Outputs:
         % gst: Greenwich Sidereal Time in degrees
   % Author: Blaire Weinberg
   w = 0.004178074; % deg/s
10
    gst0 = 100.4606184 + (36000.77004 \cdot T) + (0.000387933 \cdot T^2) - (2.583e - 8 \cdot T^3)
11
    gst = gst0 + (w .* t);
   A.5 General Orbital Parameters
   % Script to initialize global parameters/formatting for ENAE 404
   % Inludes things like fundamental constants, planetary radii, etc.
   % Author: Blaire Weinberg
   % Formatting
   format long g
    clc
    clear all
   % Fundamental Constants
10
    c = 3e5; % km/s
11
12
   % Gravitational Parameters
   mu_earth = 3.986e5; % km<sup>3</sup> / s<sup>2</sup>
14
   mu_mars = 4.305 e4; % km^3/s^2
   mu_saturn = 3.7931187e7; % km<sup>3</sup>/s<sup>2</sup>
```

```
mu_moon = 0.00490e6; % km^3/s^2 (nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.
      html)
   mu_jupiter = 126.687e6; % km<sup>2</sup>/s<sup>2</sup> (nssdc.gsfc.nasa.gov/planetary/factsheet/
18
      jupiterfact.html)
  mu_mercury = 0.022032e6; % km<sup>2</sup>/s<sup>2</sup> (https://nssdc.gsfc.nasa.gov/planetary/factsheet
      /mercuryfact.html)
  sunfact.html)
21
  % Radii
22
   r_earth = 6378; \% km
23
  r_moon = 1738.1; % km (nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html)
   r_jupiter = 71492; % km (nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html)
   r_mercury = 2439.7; % km (https://nssdc.gsfc.nasa.gov/planetary/factsheet/
      mercuryfact.html)
  r_mars = 3396.2; % km (https://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html
   r_{-geo} = 42164; \% \text{ km}
28
  % Semi-major axes
30
   a_mercury = 57.91e6; % km (https://nssdc.gsfc.nasa.gov/planetary/factsheet/
31
      mercuryfact.html)
   a_earth = 149.6e6; % km (https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.
32
   a_jupiter = 778.57e6; % km (nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html
33
34
  % Mass
35
  m_mercury = 0.33011e24; % kg (https://nssdc.gsfc.nasa.gov/planetary/factsheet/
36
      mercuryfact.html)
   m_jupiter = 1898.19e24; % kg (nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.
37
      html)
  m_sun = 1988500e24; % kg (nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html)
38
  % Gravitational Parameters
   g_earth = 9.798e-3; % km/s (https://nssdc.gsfc.nasa.gov/planetary/factsheet/
      earthfact.html)
        Project-Specific Data
  A.6
  % This script contains all of the data pulled from online for the project
  % Author: Blaire Weinberg
3
  % G32
   t_{sat1} = [datetime(2019, 09, 29, 20, 0, 0);
             datetime (2019, 09, 30, 3, 12, 30); % YY-MM-DD hh:mm: ss
6
  % [GODE; LYCO; NJTR]
   rho_obsv_t1_sat1 = [25051697.711; 25250311.013; 25300673.18]; \% m
   rho_obsv_t2_sat1 = [24885268.756; 25027333.239; 24905662.38]; \% m
10
11
   rho\_gode\_t1\_sat1 = [-16495.751222; -20282.441992; -4990.466293]; \% m
12
   rho_gode_t2_sat1 = [20213.786583; -14550.961176; -8980.879524]; \% m
13
14
   rho_1 y co_1 t 1_s at 1 = [-16495.751222; -20282.441992; -4990.466293]; \% m
15
   rho_1yco_1t2_sat1 = [20213.786583; -14550.961176;
                                                      -8980.879524]; % m
16
17
   rho_njtr_t1_sat1 = [-16495.751222; -20282.441992; -4990.466293]; % m
```

```
rho_njtr_t2_sat1 = [20213.786583; -14550.961176; -8980.879524]; \% m
19
     % G10
21
      t_{sat2} = [datetime(2019, 09, 29, 20, 0, 0);
                           datetime (2019, 09, 30, 1, 22, 30); % YY-MM-DD hh:mm: ss
23
24
     % [GODE; LYCO; NJTR]
25
      rho_obsv_t1_sat2 = [23211963.276; 23183895.657; 23332124.46]; \% m
26
      rho_{-}obsv_{-}t2_{-}sat2 = [24741064.469; 24965346.244; 24859334.14]; \% m
28
      rho\_gode\_t1\_sat2 = [-16660.548228; -11669.484643; 17253.892228]; \% m
29
      rho\_gode\_t2\_sat2 = [11443.232098; -19758.768196; -13321.664005]; \% m
30
      rho_1 y co_1 t_1 sat_2 = [-16660.548228; -11669.484643; 17253.892228]; \% m
32
      rho_1 y co_1 t 2_s at 2 = [11443.232098; -19758.768196; -13321.664005]; \% m
33
34
      rho_njtr_t1_sat2 = [-16660.548228; -11669.484643; 17253.892228]; \% m
35
      rho_njtr_t2_sat2 = [11443.232098; -19758.768196; -13321.664005]; \% m
36
     % Ground Stations
38
     % ECEF XYZ
     GODE = [1130774.439; -4831255.071; 3994200.558]; \% m
     LYCO = [1080274.057; -4680045.229; 4182682.6660]; \% m
     NJTR = [1278261.278; -4703708.0040; 4099884.5550]; \% m
43
      ref_matrix = [GODE'; LYCO'; NJTR'];
44
45
     % Times
46
      t1 = 20 * 3600; \% s
47
      t2\_sat1 = (3 * 3600) + (12 * 60) + 30; \% s
      t2\_sat2 = 3600 + (22 * 60) + 30; \% s
49
     T1 = (7181 + 29 + 0.5) / 32525;
51
     T2 = (7181 + 30 + 0.5) / 32525;
      A.7
                  Trilateration
     function [r] = Trilateration (rho_obsv, ref, tolerance, r_guess)
     % Determines the location of a satellite given reference information
     % Inputs:
 3
             % rho_obsv: vector containing observed ranges
             % ref: matrix containing all of the [X Y Z] information for the
             \% reference locations for the observations
             % r_guess: guess [X Y Z] vector for where the satellite should be located
     % Outputs:
 9
             % r: [X Y Z] vector containing the iterated solution for where the
10
             % satellite is
11
12
     % Author: Blaire Weinberg
13
14
     r = r_g uess;
15
16
      rho\_pred = sqrt((ref(:, 1)-r(1,1)).^2+(ref(:, 2)-r(2,1)).^2+(ref(:, 3)-r(3,1)).^2);
17
      e = (rho\_obsv-rho\_pred);
      while (norm(e)>tolerance)
19
              A = [(r(1,1)-ref(1, 1))/rho\_pred(1), (r(2,1)-ref(1, 2))/rho\_pred(1), (r(3,1)-ref(1, 2))/rho\_
20
                     (1, 3)/rho_pred(1);
```

```
(r(1,1)-ref(2, 1))/rho\_pred(2), (r(2,1)-ref(2, 2))/rho\_pred(2), (r(3,1)-ref(2, 2))/rho\_pred(
21
                                                       (2, 3)/rho_pred(2);
                                           (r(1,1)-ref(3, 1))/rho\_pred(3), (r(2,1)-ref(3, 2))/rho\_pred(3), (r(3,1)-ref(3, 2))/rho\_pred(
22
                                                       (3, 3)/rho_pred(3);
                        rho_{-}pred = sqrt((ref(:, 1)-r(1,1)).^2+(ref(:, 2)-r(2,1)).^2+(ref(:, 3)-r(3,1))
23
                        e = (rho_obsv-rho_pred);
24
                        r = r + (A \setminus e);
25
                        disp(e);
26
         end
27
                              OE to Cartesian Transformation
          A.8
         function [states] = OE2Cart (oe, mu)
         % Converts orbital elements to cartesian coordinates
         % Inputs:
                       % oe: array of orbital elements
  4
                                      %[a, e, i, OMEGA, omega, nu]
  5
                       % mu: constant set in 'set_parameters'
         % Outputs:
                       \% states: current state
                                     % [x y z xdot ydot zdot]
10
         % Determine position vector
11
          p = oe(1) * (1 - oe(2)^2);
         r = p / (1 + (oe(2) * cosd(oe(6))));
13
          r = [r * cosd(oe(6)); r * sind(oe(6)); 0];
15
         % Determine velocity vector
         v = sqrt(mu / p) .* [-sind(oe(6)); oe(2) + cosd(oe(6)); 0];
17
 18
         % Create rotation matrix
19
          Rotate = [cosd(oe(5)) sind(oe(5)) 0; -sind(oe(5)) cosd(oe(5)) 0; 0 0 1] \dots
20
                                       *[1 \ 0 \ 0; \ 0 \ \cos d(oe(3)) \ \sin d(oe(3)); \ 0 \ -\sin d(oe(3)) \ \cos d(oe(3))] \ \dots
21
                                       *[\cos d(oe(4)) \sin d(oe(4)) 0; -\sin d(oe(4)) \cos d(oe(4)) 0; 0 0 1];
22
          Rotate = Rotate;
23
24
         % Rotate r, v vectors
         r = Rotate * r;
26
         v = Rotate * v;
27
28
         % Set output to a standard state space vector
         states = [r];
                              TLE to OE Transformation
         A.9
         function [OE] = TLE2OE(TLE, mu)
         i = str2double(TLE(2, 9:16));
        OMEGA = str2double(TLE(2,18:25));
         e = str2double(strcat('0.', TLE(2,27:33)));
         omega = str2double(TLE(2, 35:42));
         M = str2double(TLE(2, 44:51));
         n = str2double(TLE(2, 53:63)) * 2 * pi / (3600 * 24);
         a = nthroot(mu / n^2, 3);
         TA = rad2deg(M+(2*e-.25*e^3)*sin(M)+1.2*e^2*sin(2*M)+(13/12)*e^3*sin(3*M));
```

OE = [a; e; i; OMEGA; omega; TA];

B Group Work Distribution

B.1 Tyler Chotoo

- List of the five digit SDC numbers for all the NAVSTAR satellites.
- Found the satellites corresponding to our chosen PRN.
- Did part five of the analysis.
- Contributed to writing the report in the work done section.
- Attempted to find results

B.2 Abubakr Hamid

•

B.3 Richard Quarles

- Ground Site Determination (using the CORS Map)
- Data Collection from the CORS Website
- Pseudo-range and Ground Site ECEF Determination (using the Observation files)
- Spreadsheet
- Wrote the Overview and 2,1.1, created the Results Tables, and contributed to the Sources of Error subjection of the report, as well as the Limitations subjection.

B.4 Blaire Weinberg

- Lambert Solver Code §A.3
- Transformation Angle Code §A.4
- General Orbit Parameters Code §A.5
- Project Specific Data Code §A.6
- Trilateration Code §A.7
- Transformation Code §A.9 §A.8
- Getting Actual Positions ??
- Trilateration Analysis 2.2