Sta 141A Project: Auto-MPG Analysis

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Introduction

Fuel economy is defined as how much a car can travel per volume of fuel, which is usually measured as miles per gallon (mpg). Cars with low mpg have good fuel economy and vice versa. Logically, consumers prefer to own cars with good fuel economy, since they would spend less money on gas. In addition, cars with poor fuel economy consumes more gas, which, in turn, contributes to global warming; gas is ultimately a limited resource [1]. The effects of pollution produced from the consumption of gas can be mitigated by using cars with good fuel economy. The ability to predict a car's fuel economy based on a set of a given car's characteristics, or information on car models with good fuel economy, can allow individual to make a more informed decision when purchasing a car - a decision that can have a positive impact on both spending and global warming.

Dataset Description

The data set on fuel economy was obtained from kaggle: https://www.kaggle.com/uciml/autompg-dataset. Although it is data from the late 1900s, the features within the data set can provide us insight on what variables have impact on fuel economy.The dimensions of the data set are 398 by 9, meaning that we have 9 features within our data set. `Model year` is not useful for our purposes, so it will be dropped during the analysis. `Weight`, `acceleration`, `horsepower`, `displacement`, `cylinders` are all numerical features, while `origin` is categorical. `Weight`, `acceleration`, and `horsepower` are self-explanatory. `Cylinders` are indicative of the power of an engine, where more cylinders means more power but more consumption of gas. `Displacement` refers to how much volume of air and fuel moved through the cylinders of the engine.

Research Questions

- Is it appropriate to fit a linear model to this data? If so, what numerical variables have the most impact on mpg? For example, if increasing weight contributes the most to MPG, individuals should be wary about purchasing heavy cars, since it will lead to more consumption of fuel.
- Is it possible to build a predictive model with reasonable performance to predict a car's fuel economy? If it is possible, individuals will be able to make better car-purchasing decisions by inputting a car's features into the model and getting an estimated mpg.
- The origin column has Europe, Japan and USA encoded. Are cars from these regions similar, or are they completely different? Is there a region that tends to make cars with good fuel economy?
- What brand of cars are similar in terms of fuel economy and other features such as weight? Knowing this information can allow individuals to potentially buy cars with desired feature levels or even avoid buying cars with poor fuel economy.

These questions may be answered by using unsupervised and supervised learning methods.

Unsupervsed Learning Analysis

Hierarchical Clustering Analysis

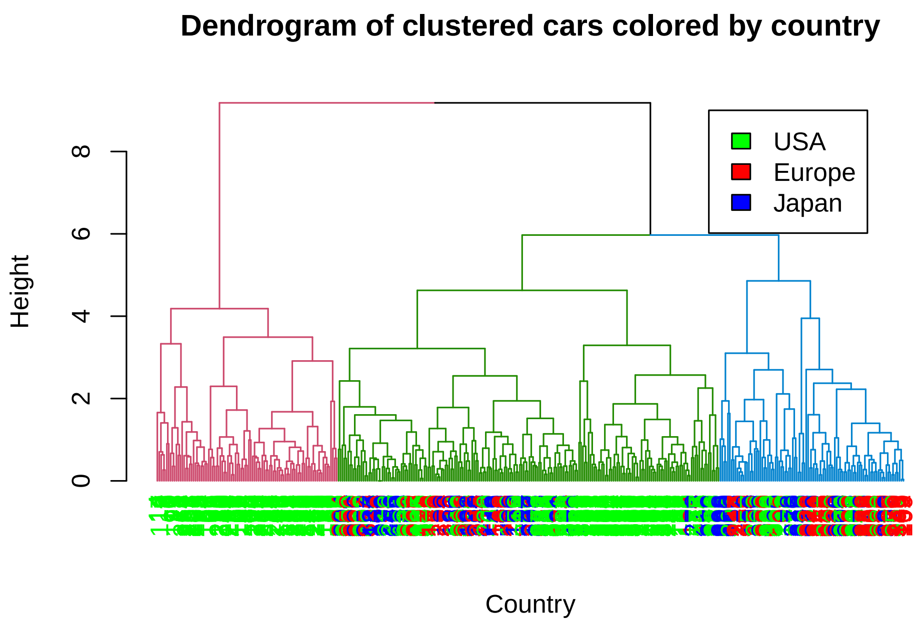


Figure 1: Hierarchical Clustering results

The numerical features were scaled to have standard deviation one and complete linkage was used to cluster the cars. From the dendrogram above, there seems to be three clusters. The cluster colored with red branches consists entirely of cars from the United States, while the other two clusters are mixed.

Table 1: Cluster Analysis

cluster	freq	mean.mpg	mean.displacement	mean.horsepower	mean.weight	mean.acceleration
1	95	14.46421	348.7895	162.4211	4150.474	12.58526
2	200	23.48550	165.0425	94.4700	2789.890	15.65550
3	97	32.16082	103.7732	68.3299	2215.876	18.20103

Table 2: Within Cluster Analysis

cluster	origin	freq	mean.mpg	mean.displacement	mean.horsepower	mean.weight	mean.acceleration
1	USA	95	14.46421	348.7895	162.42105	4150.474	12.58526
2	Europe	42	25.13095	111.3571	90.09524	2451.071	15.02619
2	Japan	37	26.23243	116.5676	95.45946	2453.919	14.73243
2	USA	121	22.07438	198.5000	95.68595	3010.231	16.15620
3	Europe	26	31.59615	106.8462	65.15385	2405.038	19.65000
3	Japan	42	34.16667	90.5000	66.07143	2016.238	17.44048
3	USA	29	29.76207	120.2414	74.44828	2335.414	18.00345

Principal Component analysis

```
##       Position    MPG Displacement Horsepower Weight Acceleration
## 1    Top Right Larger        Lower      Lower  Lower        Lower
```

```
## 2     Top Left  Lower       Larger     Larger Larger       Larger
## 3 Bottom Right Larger        Lower      Lower  Lower        Lower
## 4  Bottom Left  Lower       Larger     Larger Larger       Larger
```

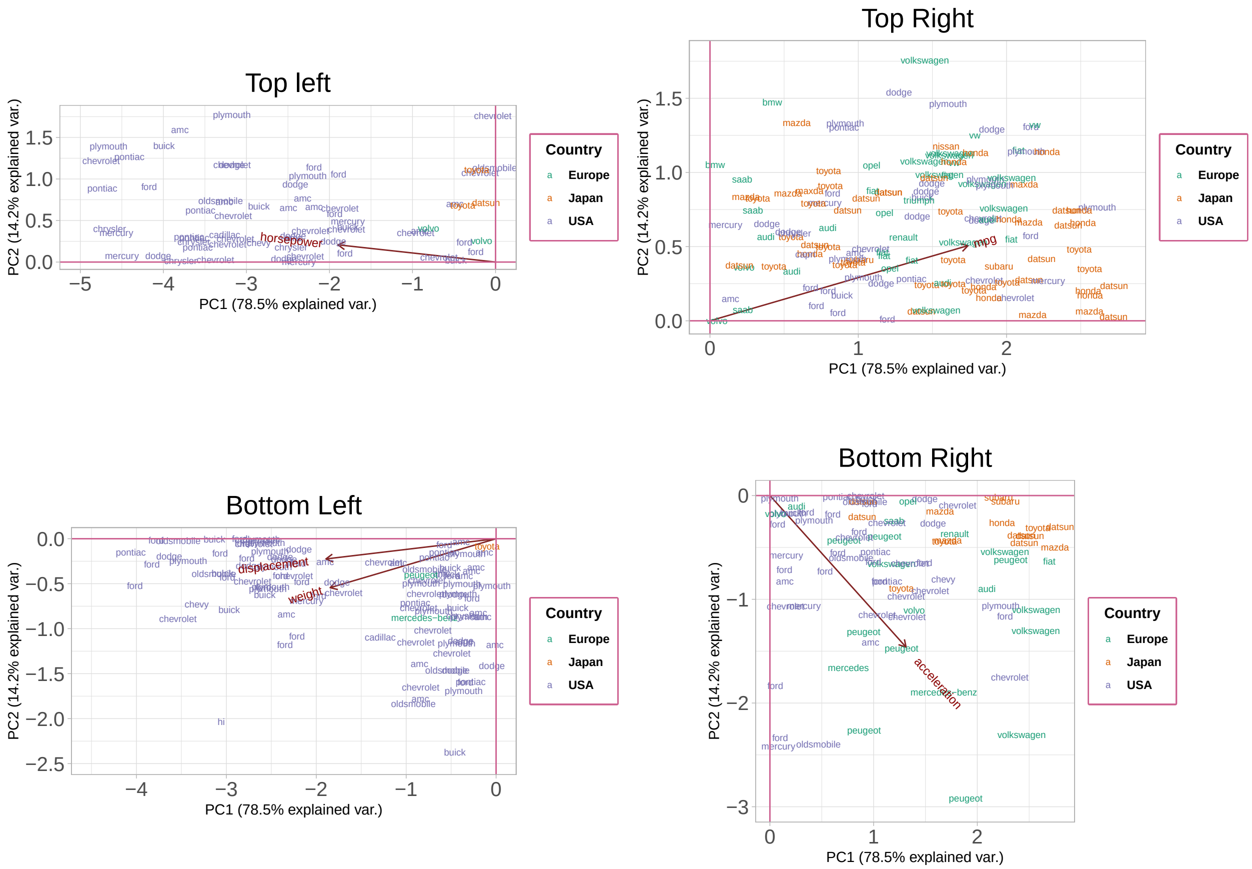
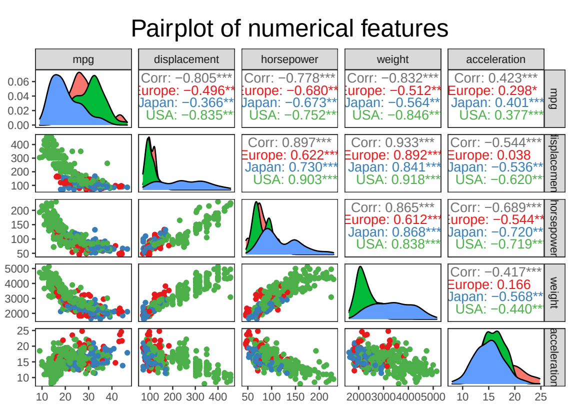
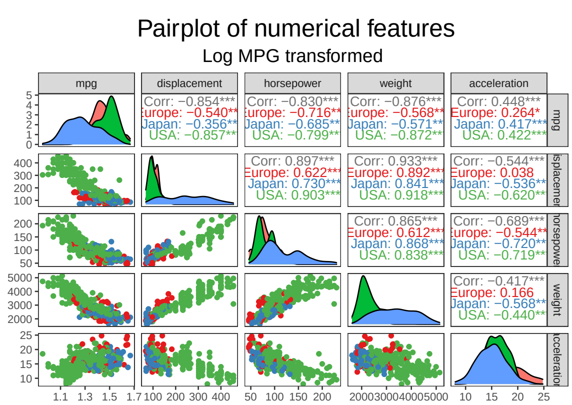


Figure 1

Supervised Learning Analysis

Appropriateness of a linear model





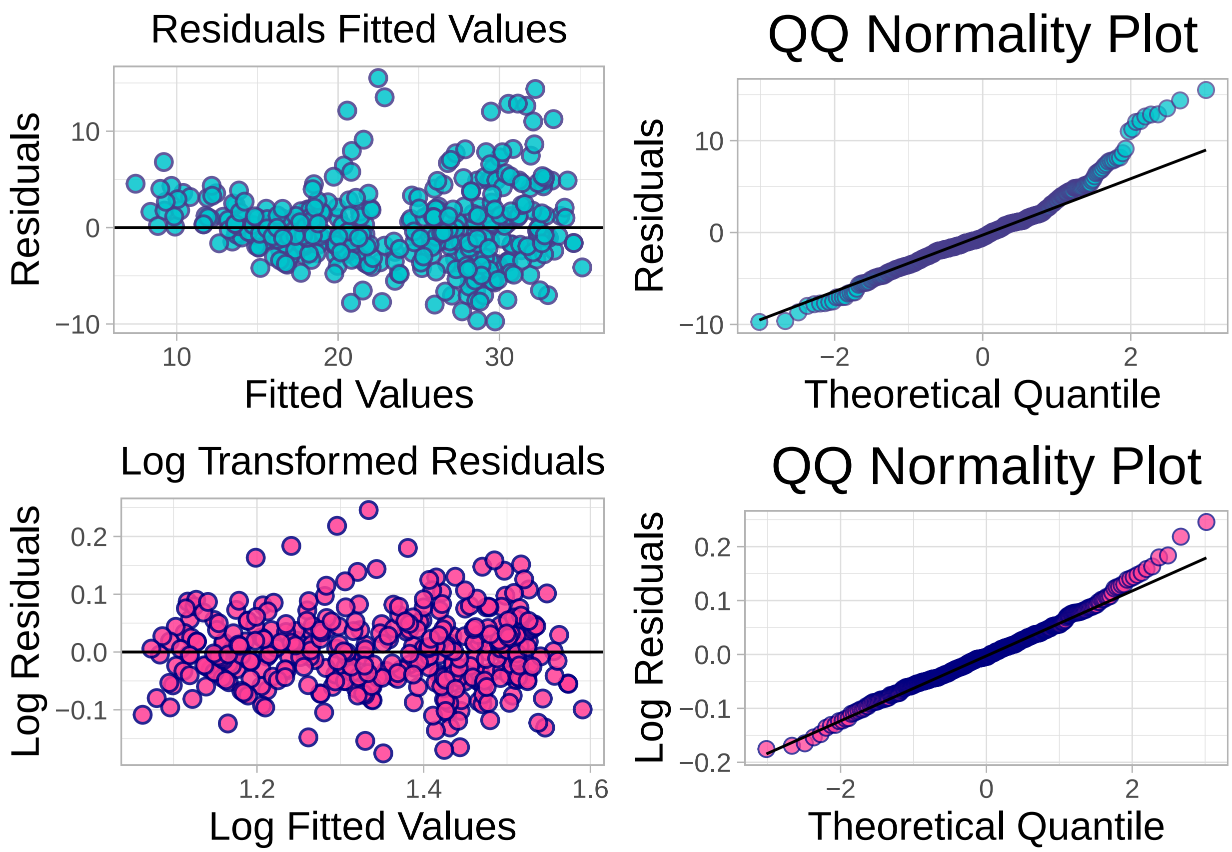

```
##
## Call:
## lm(formula = mpg ~ ., data = data[-c(8)])
##
## Residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -9.7287 -2.3413 -0.5307  1.7955 15.5121
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  43.9236266  2.4378221  18.018  < 2e-16 ***
## cylinders6   -4.0784024  0.8358754  -4.879 1.57e-06 ***
## cylinders8   -2.1875310  1.5136786  -1.445  0.14923
## displacement  0.0125671  0.0087680   1.433  0.15259
## horsepower   -0.0822381  0.0165086  -4.982 9.57e-07 ***
## weight       -0.0041588  0.0007832  -5.310 1.86e-07 ***
## acceleration -0.0353203  0.1183637  -0.298  0.76556
## originJapan   1.9720779  0.6722152   2.934  0.00355 **
## originUSA    -0.5203577  0.6810074  -0.764  0.44528
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.987 on 383 degrees of freedom
## Multiple R-squared:  0.7444, Adjusted R-squared:  0.739
## F-statistic: 139.4 on 8 and 383 DF,  p-value: < 2.2e-16
```

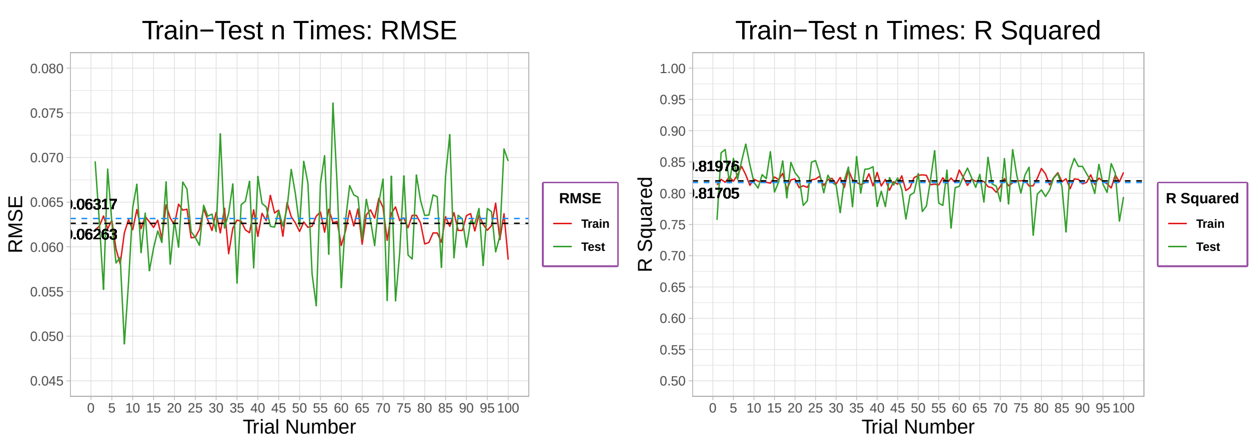
```
##
## Call:
## lm(formula = mpg ~ . + I(horsepower^2), data = data.transformed)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.175448 -0.043319 -0.004396  0.037993  0.245763
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      1.955e+00  5.870e-02  33.305  < 2e-16 ***
## cylinders6      -4.665e-02  1.429e-02  -3.264  0.00120 **
## cylinders8      -3.399e-02  2.503e-02  -1.358  0.17523
## displacement    -9.599e-05  1.535e-04  -0.625  0.53206
## horsepower      -4.513e-03  7.376e-04  -6.118 2.35e-09 ***
## weight          -4.304e-05  1.445e-05  -2.978  0.00309 **
## acceleration    -6.536e-03  2.111e-03  -3.095  0.00211 **
## originJapan      3.016e-02  1.076e-02   2.803  0.00532 **
## originUSA        5.084e-03  1.110e-02   0.458  0.64708
## I(horsepower^2)  9.702e-06  2.344e-06   4.139 4.30e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06376 on 382 degrees of freedom
## Multiple R-squared:  0.8179, Adjusted R-squared:  0.8136
## F-statistic: 190.6 on 9 and 382 DF,  p-value: < 2.2e-16
```

```
## [1] 1.158137
```

intercept	RMSE	Rsquared	MAE	RMSESD	RsquaredSD	MAESD
TRUE	0.0647524	0.8116947	0.0502064	0.0082721	0.0465688	0.0059165



Predictive model



	x
(Intercept)	1.9550754
cylinders6	-0.0466512
cylinders8	-0.0339915
displacement	-0.0000960
horsepower	-0.0045125
weight	-0.0000430
acceleration	-0.0065358
originJapan	0.0301615
originUSA	0.0050842
‘I(horsepower^2)‘	0.0000097

Cross-Fold validation

Conclusion

References

1. Transportation Technologies and Innovation. Union of Concerned Scientists. (n.d.). Retrieved November 12, 2021, from https://www.ucsusa.org/transportation/technologies.

2. McGregor, H. V., Gergis, J., Abram, N. J., & Phipps, S. J. (2016). The Industrial Revolution kick-started global warming much earlier than we realised.

3. Learning, U. C. I. M. (2017, July 2). Auto-mpg dataset. Kaggle. Retrieved November 12, 2021, from https://www.kaggle.com/uciml/autompg-dataset.

Appendix: R code used

```
#global options
# keeps this here to remove comments from knitted output.
knitr::opts_chunk$set(comments=NA)
knitr::opts_chunk$set(echo=F)
knitr::opts_chunk$set(warning = FALSE, message = FALSE)
library(tidyverse)
library(knitr)
library(leaps)
library(GGally)
library(ggbiplot)
library(caret)
library(RColorBrewer)
library(dendextend)
library(cowplot)
library(kableExtra)
# data cleaning up
data <- read.csv('auto-mpg.csv')
#convert horsepower chr->dbl
data$horsepower <- as.numeric(data$horsepower)
#remove rows with missing values
data <- na.omit(data)
#translate origin numbers to country strings
data$origin <- ifelse(data$origin==1,"USA",ifelse(data$origin==2,"Europe","Japan"))
data$origin <- as.factor(data$origin)
#cylinders count for 3 and 5 low combine with 4 and 6 respectively
data$cylinders <- replace(data$cylinders,data$cylinders %in% c(3,5),c(4,6))
data$cylinders <- as.factor(data$cylinders)
```

```
#remove model.year, not interested in this feature
data <- data[-c(7)]
data$car.name <- word(data$car.name,1)
#car.name fix typos
data$car.name[160] <- "chevrolet"
data$car.name[330] <- "volkswagen"
data$car.name[82] <- "toyota"
h.clustering.complete <- hclust(dist(scale(data[-c(2,7,8)])),method="complete") %>% as.dendrogram() %>%
color.order <- as.numeric(data$origin)
colors.dendro <- color.order[order.dendrogram(h.clustering.complete)]
colors.dendro <- ifelse(colors.dendro==3,"green",ifelse(colors.dendro==2,"red","blue"))
labels_colors(h.clustering.complete) <- colors.dendro
h.clustering.complete <- h.clustering.complete %>% set("labels_col",colors.dendro)
plot(h.clustering.complete,
     main="Dendrogram of clustered cars colored by country",
     ylab="Height",
     xlab="Country")
legend(x=290,y=9,legend=c("USA","Europe","Japan"),fill=c("green","red","blue"))
hclust.data <- data.frame(data,cluster=cutree(h.clustering.complete,h=5))
hclust.data.clusters <- data.frame(data,cluster=cutree(h.clustering.complete,h=5)) %>% count(c('cluster
hclust.in.clusters <- data.frame(data,cluster=cutree(h.clustering.complete,h=5)) %>% count(c('cluster',

#cluster analysis
c.1 <- hclust.data %>% filter(cluster==1 ) %>% summarise(mean.mpg=mean(mpg),
                                                          mean.displacement=mean(displacement)
                                                          mean.horsepower=mean(horsepower),
                                                          mean.weight=mean(weight),
                                                          mean.acceleration=mean(acceleration)
                                                           )

c.2 <- hclust.data %>% filter(cluster==2 ) %>% summarise(mean.mpg=mean(mpg),
                                                          mean.displacement=mean(displacement)
                                                          mean.horsepower=mean(horsepower),
                                                          mean.weight=mean(weight),
                                                          mean.acceleration=mean(acceleration)
                                                           )

c.3 <- hclust.data %>% filter(cluster==3 ) %>% summarise(mean.mpg=mean(mpg),
                                                          mean.displacement=mean(displacement)
                                                          mean.horsepower=mean(horsepower),
                                                          mean.weight=mean(weight),
                                                          mean.acceleration=mean(acceleration)
                                                           )

clusters.data <- rbind(c.1,c.2,c.3)
hclust.data.clusters <- cbind(hclust.data.clusters,clusters.data)

# Within cluster analysis
US.1 <-  hclust.data %>% filter(cluster==1 & origin=="USA") %>% summarise(mean.mpg=mean(mpg),
                                                          mean.displacement=mean(displacement)
                                                          mean.horsepower=mean(horsepower),
                                                          mean.weight=mean(weight),
                                                          mean.acceleration=mean(acceleration)
```

```
                                                                      )
EU.2 <-  hclust.data %>% filter(cluster==2 & origin=="Europe") %>% summarise(mean.mpg=mean(mpg),
                                                                   mean.displacement=mean(displacement)
                                                                   mean.horsepower=mean(horsepower),
                                                                   mean.weight=mean(weight),
                                                                   mean.acceleration=mean(acceleration)
                                                                    )
JN.2 <-  hclust.data %>% filter(cluster==2 & origin=="Japan") %>% summarise(mean.mpg=mean(mpg),
                                                                   mean.displacement=mean(displacement)
                                                                   mean.horsepower=mean(horsepower),
                                                                   mean.weight=mean(weight),
                                                                   mean.acceleration=mean(acceleration)
                                                                    )
US.2 <-  hclust.data %>% filter(cluster==2 & origin=="USA") %>% summarise(mean.mpg=mean(mpg),
                                                                   mean.displacement=mean(displacement)
                                                                   mean.horsepower=mean(horsepower),
                                                                   mean.weight=mean(weight),
                                                                   mean.acceleration=mean(acceleration)
                                                                    )
EU.3 <-  hclust.data %>% filter(cluster==3 & origin=="Europe") %>% summarise(mean.mpg=mean(mpg),
                                                                   mean.displacement=mean(displacement)
                                                                   mean.horsepower=mean(horsepower),
                                                                   mean.weight=mean(weight),
                                                                   mean.acceleration=mean(acceleration)
                                                                    )
JN.3 <-  hclust.data %>% filter(cluster==3 & origin=="Japan") %>% summarise(mean.mpg=mean(mpg),
                                                                   mean.displacement=mean(displacement)
                                                                   mean.horsepower=mean(horsepower),
                                                                   mean.weight=mean(weight),
                                                                   mean.acceleration=mean(acceleration)
                                                                    )
US.3 <-  hclust.data %>% filter(cluster==3 & origin=="USA") %>% summarise(mean.mpg=mean(mpg),
                                                                   mean.displacement=mean(displacement)
                                                                   mean.horsepower=mean(horsepower),
                                                                   mean.weight=mean(weight),
                                                                   mean.acceleration=mean(acceleration)
                                                                    )
in.clusters.data <- rbind(US.1,EU.2,JN.2,US.2,EU.3,JN.3,US.3)
hclust.in.clusters <- cbind(hclust.in.clusters,in.clusters.data)
kable(hclust.data.clusters,format="latex",booktabs=T,longtable=T,caption = "Table 1: Cluster Analysis")
#hclust.data.clusters
#hclust.in.clusters
kable(hclust.in.clusters,format="latex",booktabs=T,longtable=T,caption="Table 2: Within Cluster Analysi
pcs.out <- prcomp(data[-c(2,7,8)],scale.=T)
pcs.dat <- data.frame(rownames(pcs.out$rotation),pcs.out$rotation)
colnames(pcs.dat)[1] <- "Features"
pcs.importance <- data.frame(summary(pcs.out)[6])
pcs.importance <- cbind(c("Standard deviation","Proportion of Variance","Cumulative Proportion"),pcs.im
colnames(pcs.importance) <- c("Metrics","PC1","PC2","PC3","PC4","PC5")
cols <- brewer.pal(3, "Dark2")
PCA.interp <- data.frame(Position=c("Top Right","Top Left","Bottom Right","Bottom Left"),
           MPG=c("Larger","Lower","Larger","Lower"),
           Displacement=c("Lower","Larger","Lower","Larger"),
```

```
          Horsepower=c("Lower","Larger","Lower","Larger"),
          Weight=c("Lower","Larger","Lower","Larger"),
          Acceleration=c("Lower","Larger","Lower","Larger"))
PCA.interp

ggbiplot(pcs.out,labels = data$car.name,groups=data$origin,obs.scale = 1,labels.size = 2.3)+
  geom_hline(yintercept = 0,col="hotpink3")+
  geom_vline(xintercept = 0,col="hotpink3")+
  ylim(0,1.8)+
  xlim(-5,0)+
  theme_light()+
  theme(plot.title=element_text(hjust=.5,size=20),
        axis.text = element_text(size=15)
        )+
  labs(title="Top left",
       )+
  scale_color_manual(values=cols)+ theme(legend.box.background = element_rect(linetype="solid", colour =
        legend.title = element_text(face="bold", hjust = .5),
        legend.text = element_text(face="bold"))+
  guides(colour=guide_legend("Country"))



ggbiplot(pcs.out,labels = data$car.name,groups=data$origin,obs.scale = 1,labels.size = 2.3)+
  geom_hline(yintercept = 0,col="hotpink3")+
  geom_vline(xintercept = 0,col="hotpink3")+
  ylim(0,1.8)+
  xlim(0,2.8)+
  theme_light()+
  theme(plot.title=element_text(hjust=.5,size=20),
        axis.text = element_text(size=15)
        )+
  labs(title="Top Right",
       )+
  scale_color_manual(values=cols)+ theme(legend.box.background = element_rect(linetype="solid", colour =
        legend.title = element_text(face="bold", hjust = .5),
        legend.text = element_text(face="bold"))+
  guides(colour=guide_legend("Country"))


ggbiplot(pcs.out,labels = data$car.name,groups=data$origin,obs.scale = 1,labels.size = 2.3)+
  geom_hline(yintercept = 0,col="hotpink3")+
  geom_vline(xintercept = 0,col="hotpink3")+
  ylim(-2.5,0)+
  xlim(-4.5,0)+
  theme_light()+
  theme(plot.title=element_text(hjust=.5,size=20),
        axis.text = element_text(size=15)
        )+
  labs(title="Bottom Left",
       )+
    scale_color_manual(values=cols)+
  theme(legend.box.background = element_rect(linetype="solid", colour ="hotpink3", size=1.25),
```

```
        legend.title = element_text(face="bold", hjust = .5),
        legend.text = element_text(face="bold"))+
  guides(colour=guide_legend("Country"))


ggbiplot(pcs.out,labels = data$car.name,groups=data$origin,obs.scale = 1,labels.size = 2.3)+
  geom_hline(yintercept = 0,col="hotpink3")+
  geom_vline(xintercept = 0,col="hotpink3")+
  ylim(-3,0)+
  xlim(0,2.8)+
  theme_light()+
  theme(plot.title=element_text(hjust=.5,size=20),
        axis.text = element_text(size=15)
        )+
  labs(title="Bottom Right",
       )+
    scale_color_manual(values=cols)+
   theme(legend.box.background = element_rect(linetype="solid", colour ="hotpink3", size=1.25),
        legend.title = element_text(face="bold", hjust = .5),
        legend.text = element_text(face="bold"))+
  guides(colour=guide_legend("Country"))

data.transformed <- data
data.transformed$mpg <- log(data.transformed$mpg,base=10)
data.transformed <- data.transformed[-c(8)]
ggpairs(data[-c(2,7,8)],aes(color=data$origin))+
  theme_bw()+
  theme(panel.grid=element_blank(),
        plot.title=element_text(hjust=.5,size=20)) +
  labs(title="Pairplot of numerical features")+
  scale_color_manual(values=brewer.pal(3,"Set1"))

ggpairs(data.transformed[-c(2,7,8)],aes(color=data$origin))+
  theme_bw()+
  theme(panel.grid=element_blank(),
        plot.title=element_text(hjust=.5,size=20),
        plot.subtitle =element_text(hjust=.5,size=15)) +
  labs(title=" Pairplot of numerical features",
       subtitle = "Log MPG transformed")+
  scale_color_manual(values=brewer.pal(3,"Set1"))
lr.data <- lm(mpg~.,data=data[-c(8)])
summary(lr.data)
#residuals vs fitted plot
p1 <- ggplot(lr.data)+
  theme_light()+
  labs(title = "Residuals Fitted Values",x="Fitted Values",y="Residuals")+
  geom_point(aes(x=lr.data$fitted.values,y=lr.data$residuals),col="darkslateblue",pch=21,fill="turquois
  geom_hline(yintercept = 0)+
  theme(axis.title = element_text(size=15),
        axis.text = element_text(size=10),
        plot.title = element_text(hjust = .5, size = 15))

p2 <- ggplot(lr.data,aes(sample=lr.data$residuals))+
```

```r
  labs(title = "QQ Normality Plot",x="Theoretical Quantile",y="Residuals")+
  theme_light()+
  stat_qq(col="darkslateblue",pch=21,fill="turquoise3",alpha=.75,size=2.5,stroke=0.5)+
  geom_qq_line()+
  theme(axis.title = element_text(size=15),
        axis.text = element_text(size=10),
        plot.title = element_text(hjust = .5, size = 20))

log.lr.data <- lm(mpg~.+I(horsepower^2),data=data.transformed)
summary(log.lr.data)
#residuals vs fitted plot
p1a <- ggplot(log.lr.data)+
  labs(title = "Log Transformed Residuals", x = "Log Fitted Values", y = "Log Residuals")+
  theme_light()+
  geom_point(aes(x=log.lr.data$fitted.values,y=log.lr.data$residuals),col="navyblue",pch=21,fill="viole
  geom_hline(yintercept = 0)+
  theme(axis.title = element_text(size=15),
        axis.text = element_text(size=10),
        plot.title = element_text(hjust = .5, size = 15))


p2a <- ggplot(log.lr.data,aes(sample=log.lr.data$residuals))+
  labs(title = "QQ Normality Plot",x="Theoretical Quantile",y="Log Residuals")+
  theme_light()+
  stat_qq(col="navyblue",pch=21,fill="violetred1",alpha=.75,size=2.5,stroke=0.5)+
  geom_qq_line()+
  theme(axis.title = element_text(size=15),
        axis.text = element_text(size=10),
        plot.title = element_text(hjust = .5, size = 20))

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plot_grid(p1, p2, p1a, p2a)
library(MASS)
step.model <- stepAIC(log.lr.data, direction = "both",
trace = FALSE)
summary(step.model)

train.test <- function(data,split.size){
  #randomize the data
  randomized.rows <- sample(nrow(data))
  randomized.data <- data[randomized.rows,]
  #split based on desired size
  split <- round(nrow(randomized.data)*split.size)
  train <- randomized.data[1:split,]
  test <- randomized.data[(split+1):nrow(randomized.data),]
  return(list(train,test))
}

#computes the Rsquared and MSE
model.metrics <- function(predicted,actual,data){
  SSE <- sum((predicted-actual)^2)
  SSTO <- sum((actual-mean(actual))^2)
  R.squared <- 1-(SSE/SSTO)
```

```r
  R.MSE <- sqrt(SSE/nrow(data))
  results <- c(R.MSE,R.squared)
  names(results) <- c("RMSE","R.squared")
  return(results)
}

#From the full model:mpg~.+I(horsepower^2), specify what features to remove
build.model.features <- function(data,feats="None"){
  if(sum(!feats%in%"None")!=0) {
  #input validation
  if(sum(!feats %in% colnames(data))!=0){
    return("Error: No Such feature(s)")
  }
  features <- as.formula(paste("mpg~.+I(horsepower^2)-",paste(feats,collapse= "-")))
  return(features)

  }
  else return(as.formula(paste("mpg~.+I(horsepower^2)")))
}

# Combines usage of build.model.features and model.metrics to simulate a train-test split evaluation
build.and.evaluate <- function(data,split.size,feats="None"){
  #train-test split
  train <- train.test(data,split.size)[[1]]
  test <- train.test(data,split.size)[[2]]
  #build model
  model <- lm(build.model.features(data,feats),train)
  print(build.model.features(data,feats))
  #predict on test set
  p.train <- predict(model,train)
  p.test <- predict(model,test)
  #evaluate model
  metric.results <- c(model.metrics(p.train,train$mpg,train),
                      model.metrics(p.test,test$mpg,test))
  names(metric.results) <- c("Train.RMSE","Train.R.Squared","Test.RMSE","Test.R.Squared")
  return(metric.results)
}

# Runs build and evaluate n times and returns a dataframe of the results
n.build.and.evaluate <- function(n,data,split.size,feats="None"){
  df <- data.frame(matrix(ncol=4,nrow = 0))
  for(i in 1:n){
    metric.results <- build.and.evaluate(data,split.size,feats)
    df <- rbind(df,metric.results)
  }
  df <- cbind(1:n,df)
  colnames(df) <- c("Trial.number","Train.RMSE","Train.R.Squared","Test.RMSE","Test.R.Squared")
  return(df)
}

b <- n.build.and.evaluate(100,data.transformed,.8)
avg.b.RMSE <- round(mean(b$Test.RMSE),5)
avg.b.Rsq <- round(mean(b$Test.R.Squared),5)
```

```r
avg.tr.RMSE <- round(mean(b$Train.RMSE),5)
avg.tr.Rsq <- round(mean(b$Train.R.Squared),5)
ggplot(data=b,aes(x=Trial.number))+
  labs(title="Train-Test n Times: RMSE", x="Trial Number", y="RMSE")+
  theme_light()+
  geom_line(aes(y=Train.RMSE,col="Train.RMSE"))+
  geom_line(aes(y=Test.RMSE,col="Test.RMSE"))+
  coord_cartesian(xlim=c(0,100),ylim=c(0.045,.08))+
  scale_x_continuous(breaks=seq(0,100,5))+
  scale_y_continuous(breaks=seq(0.045,0.08,0.005))+
  scale_color_manual(values = c(Train.RMSE="#E31A1C",Test.RMSE="#33A02C"), labels = c("Train", "Test"))+
  theme(legend.box.background = element_rect(linetype="solid", colour ="#984EA3", size=1.25),
        legend.title = element_text(face="bold", hjust = .5),
        legend.text = element_text(face="bold"),
        panel.grid.minor.x = element_blank(),
        axis.title = element_text(size=15),
        axis.text = element_text(size=10),
        plot.title = element_text(hjust = .5, size = 20))+
  guides(colour=guide_legend("RMSE"))+
  geom_hline(yintercept = avg.b.RMSE,col='dodgerblue',linetype='dashed')+
  geom_text(aes(0,avg.b.RMSE,label = avg.b.RMSE, vjust = -.9))+
  geom_hline(yintercept = avg.tr.RMSE,col='black',linetype='dashed')+
  geom_text(aes(0,avg.tr.RMSE,label = avg.tr.RMSE, vjust = 1.5))

ggplot(data=b,aes(x=Trial.number))+
  labs(title="Train-Test n Times: R Squared", x="Trial Number", y="R Squared")+
  theme_light()+
  geom_line(aes(y=Train.R.Squared,col="Train.R.Squared"))+
  geom_line(aes(y=Test.R.Squared,col="Test.R.Squared"))+
  scale_color_manual(values = c(Train.R.Squared="#E31A1C",Test.R.Squared="#33A02C"), labels = c("Train"
  coord_cartesian(xlim=c(0,100),ylim=c(.5,1))+
  scale_x_continuous(breaks=seq(0,100,5))+
  scale_y_continuous(breaks=seq(.5,1,0.05))+
  theme(legend.position="right",
    legend.box.background = element_rect(linetype="solid", colour ="#984EA3", size=1.25),
        legend.title = element_text(face="bold", hjust = .5),
        legend.text = element_text(face="bold"),
        panel.grid.minor.x = element_blank(),
        axis.title = element_text(size=15),
        axis.text = element_text(size=10),
        plot.title = element_text(hjust = .5, size = 20))+
  guides(colour=guide_legend("R Squared"))+
  geom_hline(yintercept = avg.b.Rsq,col='dodgerblue',linetype='dashed')+
  geom_text(aes(0,avg.b.Rsq,label = avg.b.Rsq, vjust = 1.5))+
  geom_hline(yintercept = avg.tr.Rsq,col='black',linetype='dashed')+
  geom_text(aes(0,avg.tr.Rsq,label = avg.tr.Rsq, vjust = -.9))

model <- train(
  build.model.features(data.transformed),
  data.transformed,
  method = "lm",
  trControl = trainControl(
    method = "repeatedcv",
```

```r
    number = 10,
    repeats = 10,
    verboseIter = TRUE
  )
)
knitr::kable(model$results) %>% kable_styling(font_size = 7)
kable(model$finalModel$coefficients) %>% kable_styling(font_size = 6)
```