Going Beyond Ideal Point Points: Modeling Measurement Model Measurement Error

AP Retreat

Bertrand Wilden

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Overview

- Latent variables (e.g. political ideology) are often measured using statistical models
- When using these variables in downstream analysis, bias can occur if measurement uncertainty is ignored
- I develop a method for fixing this

Measurement Models

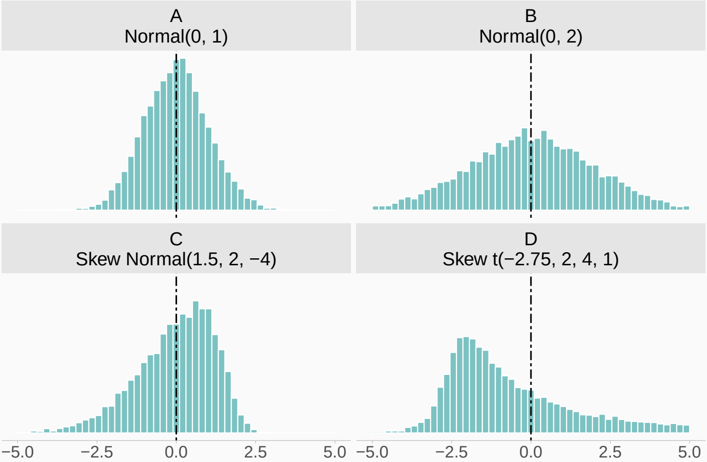
- Used to measure latent variables: *ideology, corruption, democracy, racial resentment*
- Example model: item response theory (IRT) ideal point model

Theory-Testing Models

- Testing a causal theory: *being an ideologically extreme candidate hurts one's reelection chances*
- Example model: regression with controls

Bayesian Measurement Models

- Output is a full posterior distribution of values for the latent variable according to their relative plausibility
- In practice, only a point estimate such as mean or median is used is subsequent analyses



Four different measurement model posterior distributions with mean zero

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3. Possibility of backdoor confounding through the measurement process - ***Coefficients are biased in unpredictable ways***

Solution: Joint Bayesian Measurement Error Model

$$y_i \sim f(\theta_i)$$
$$y_i^* \sim g(\theta_i)$$

- y, outcome in theory-testing model
- θ, latent variable
- y^*, training data in measurement model
- $f(\cdot)$, theory-testing model
- $g(\cdot)$, measurement model

Simplifications

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- The real $g(\cdot)$ can take a lot of computation time
- My solution is to use an approximation of $g(\cdot)$ that fits the measurement error posterior distribution

Joint IRT Ideal Point Measurement Error Model

$$y_i \sim \text{Normal}(\beta_0 + \beta_1 \theta_i, \sigma^2)$$
$$y^*_{ij} \sim \text{Bernoulli}[\Phi(\gamma_j \theta_i + \xi_i)]$$

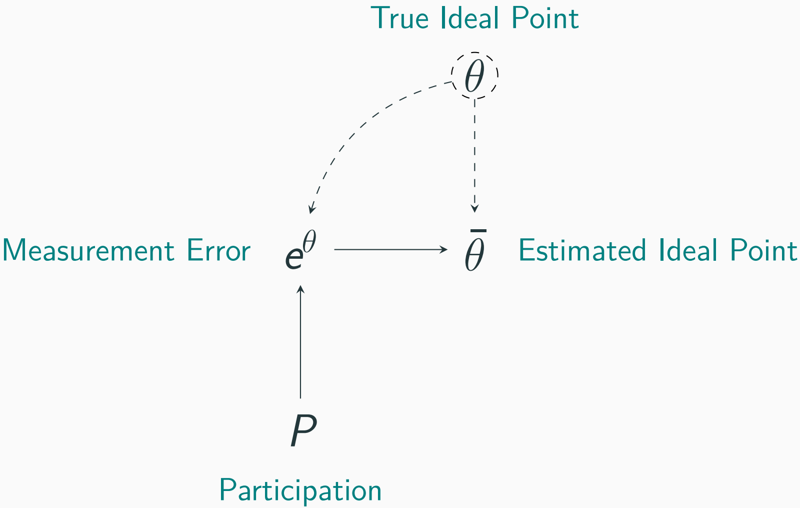
Is simplified as:

$$y_i \sim \text{Normal}(\beta_0 + \beta_1 \theta_i, \sigma^2)$$
$$\bar{\theta}_i \sim \text{Normal}(\theta_i, \sigma^2_{\theta[i]})$$

- $\bar{\theta}_i$, posterior mean from measurement model
- $\sigma^2_{\theta[i]}$, posterior variance from measurement model

Measurement Error and Attenuation Bias

Measurement Model



Theory-testing Model

$$\begin{tikzcd}
& \theta \arrow[dl, dashed, bend right] \arrow[d, dashed] & \\
e^{\theta} \arrow[r] & \bar{\theta} \arrow[r] & \overset{\text{Outcome}}{Y} \\
P \arrow[u] & &
\end{tikzcd}$$

Simulation Study

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1. Generate latent variable distributions from true θ_i
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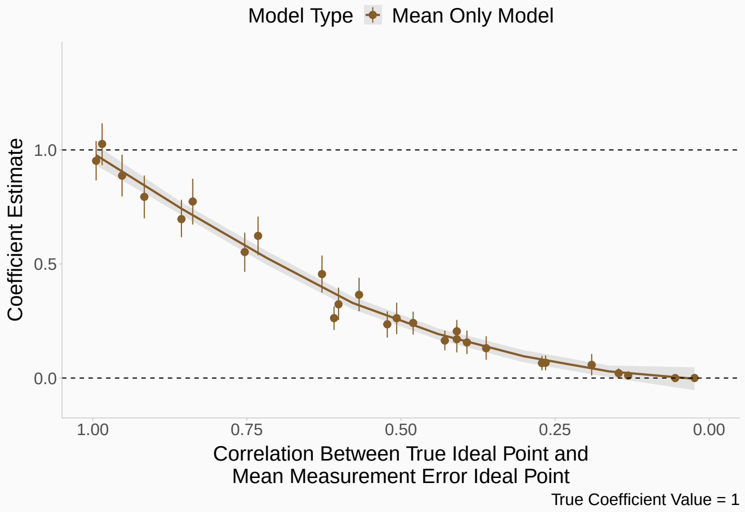
3. Use maximum likelihood to estimate distributional parameters (mean, variance, skew) from step 1 for use in simplified measurement model
4. Compare parameter recovery using a mean-only model and a joint measurement error model

Mean-Only Model

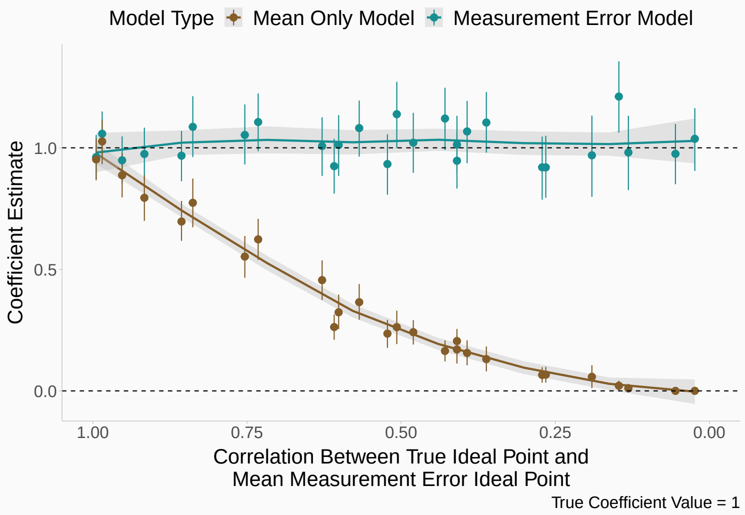
$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma^2) \\
\mu_i &= \beta_0 + \beta_1 \bar{\theta}_i \\
\beta_0, \beta_1 &\sim \text{Normal}(0, 2) \\
\sigma &\sim \text{Half Student t}(3, 0, 2)
\end{aligned}
$$

Joint Measurement Error Model

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma^2) \\
\mu_i &= \beta_0 + \beta_1 \theta_i \\
\bar{\theta}_i &\sim \text{Normal}(\theta_i, \sigma^2_{\theta[i]}) \\
\theta_i &\sim \text{Normal}(\pi, \tau) \\
\beta_0, \beta_1 &\sim \text{Normal}(0, 2) \\
\sigma &\sim \text{Half Student t}(3, 0, 2) \\
\tau &\sim \text{Half Student t}(3, 0, 2) \\
\pi &\sim \text{Normal}(0, 1)
\end{aligned}
$$



Model Type
Mean Only Model
Coefficient Estimate
1.0
0.5
0.0
1.00
0.75
0.50
0.25
0.00
Correlation Between True Ideal Point and
Mean Measurement Error Ideal Point
True Coefficient Value = 1



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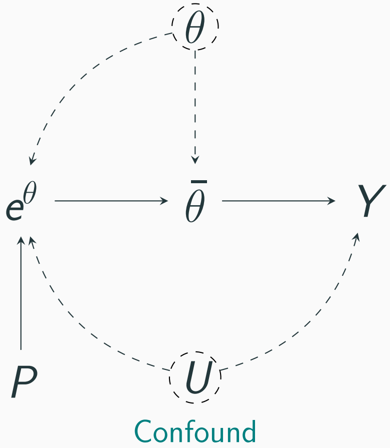
Measurement Error and Confounding Bias

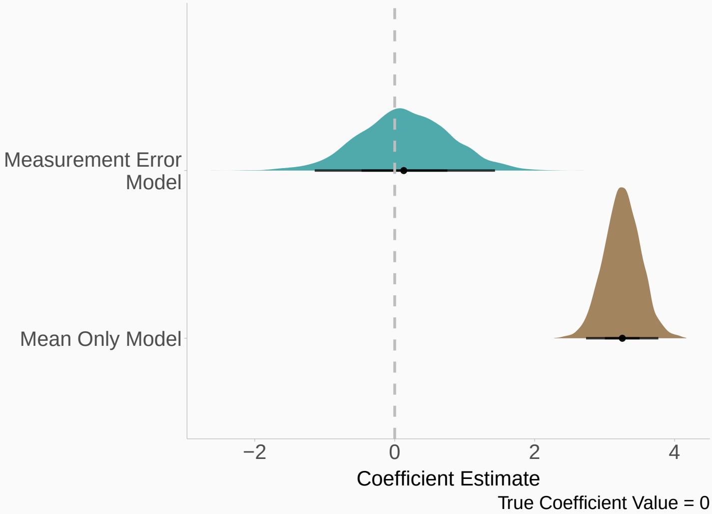
Theory-testing Model



Skew-Normal Joint Measurement Error Model

$$
\begin{aligned}
y_i &\sim \text{Normal}(\mu_i, \sigma^2) \\
\mu_i &= \beta_0 + \beta_1 \theta_i \\
\bar{\theta}_i &\sim \text{Skew Normal}(\theta_i, \omega_{\theta[i]}, \alpha_{\theta[i]}) \\
\theta_i &\sim \text{Normal}(\pi, \tau) \\
\beta_0, \beta_1 &\sim \text{Normal}(0, 2) \\
\sigma &\sim \text{Half Student t}(3, 0, 2) \\
\tau &\sim \text{Half Student t}(3, 0, 2) \\
\pi &\sim \text{Normal}(0, 1)
\end{aligned}
$$

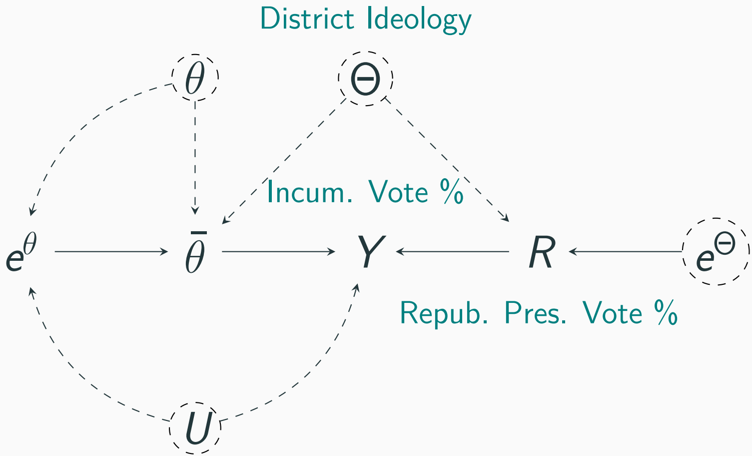
Skew-Normal Measurement Error Model Comparison



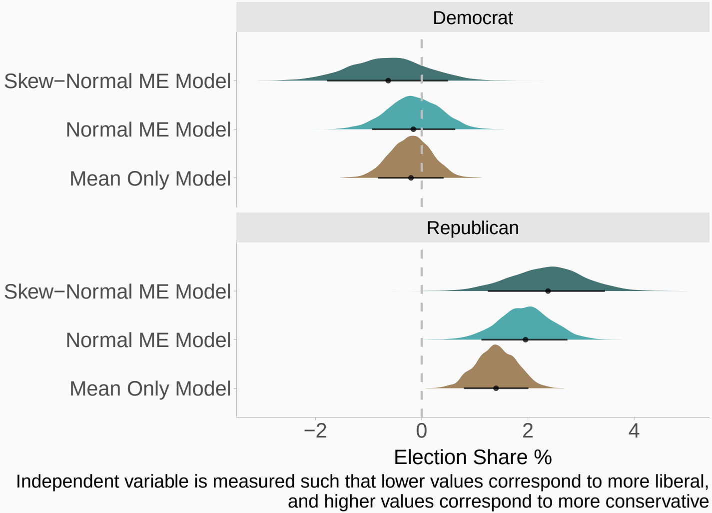
Case Study: Ideological Extremism and Electoral Success

Research Question: are ideologically extreme US House incumbents punished electorally?

- DV: General election vote share
- IV: Ideology estimated from previous Congress roll-call votes
- Data from 1990 to 2016

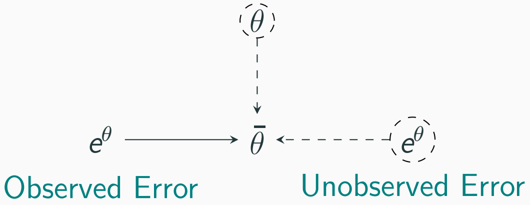

District Ideology
θ
Θ
Incum. Vote %
e^{θ}
$\bar{\theta}$
Y
R
e^{Θ}
Repub. Pres. Vote %
U

Effect of Incumbent Ideology on General Election Vote Share



Concluding Thoughts

- Uncertainty estimation matters
  - Better to use measurement models that provide uncertainty estimates than those that don't (eg NOMINATE)
  - Better to use state-of-the-art posterior sampling methods such as Hamiltonian Monte Carlo, rather than traditional Gibbs samplers for measurement models



Thank you!