Recitation Material: Linear Algebra

TA

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System of Equations: Word Problems

Problem 1: Unique Solution

A company produces two types of products: gadgets and widgets. The company earns \$300 per gadget and \$150 per widget. On a particular day, they produced 10 total items and earned \$2250. How many gadgets and widgets did the company produce?

Problem 2: No Solution

Two friends decide to meet at a café. One friend is traveling by car, and the other is walking. The car travels at 60 km/h, and the walker moves at 5 km/h. They both leave at the same time and plan to meet at the café, which is 30 km away. However, the car takes 20 minutes longer than expected to leave. Can they still meet at the same time?

Problem 3: Infinite Solutions

A contractor is hired to build houses and garages. Each house requires 8 tons of material, and each garage requires 2 tons of material. The contractor has 40 tons of material and builds 5 buildings in total. How many houses and garages can the contractor build?

Matrix Applications and Properties

Problem 1: System of Equations in Matrix Form (3x3)

Consider the following system of equations:

$$2x + y - z = 8$$

$$3x - 2y + 4z = -2$$

$$x + y + z = 4$$

Write this system in matrix form $A\mathbf{x} = \mathbf{b}$ and discuss how the solution can be found using the inverse of A.

Problem 2: Matrix Multiplication (3x3)

Let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}.$$

Compute AB and BA. What do you observe?

Problem 3: Associative Property of Matrix Multiplication (3x3)

Verify the associative property of matrix multiplication by calculating $(AB)C$ and $A(BC)$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}.$$

Problem 4: Distributive Property of Matrix Multiplication (3x3)

Let

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Verify the distributive property by calculating $A(B + C)$ and $AB + AC$.

Norms and Their Applications

Problem 1: Manhattan Norm ($\|\mathbf{x}\|_1$)

Consider the vector

$$\mathbf{x} = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}.$$

Calculate the $\|\mathbf{x}\|_1$ (Manhattan norm) of the vector.

Problem 2: Euclidean Norm ($\|\mathbf{x}\|_2$)

Given the vector

$$\mathbf{y} = \begin{bmatrix} 6 \\ 8 \\ -2 \end{bmatrix},$$

calculate the Euclidean norm $\|\mathbf{y}\|_2$.

Problem 3: Infinity Norm ($\|\mathbf{x}\|_\infty$)

For the vector

$$\mathbf{z} = \begin{bmatrix} -10 \\ 5 \\ 7 \end{bmatrix},$$

calculate the infinity norm $\|\mathbf{z}\|_\infty$.

Problem 4: General p-Norm

Given the vector

$$\mathbf{w} = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$$

and $p = 3$, calculate the $\|\mathbf{w}\|_3$.

Problem 5: Application in Machine Learning - Frobenius Norm

Consider the matrix

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & -9 \end{bmatrix}.$$

Compute the Frobenius norm $\|A\|_F$.