HOMEWORK 2
LINEAR ALGEBRA *

10-606 MATHEMATICAL FOUNDATIONS FOR MACHINE LEARNING

START HERE: Instructions

- **Collaboration Policy**: Please read the collaboration policy in the syllabus.
- **Late Submission Policy:** See the late submission policy in the syllabus.
- **Submitting your work:** You will use Gradescope to submit answers to all questions.
  - **Written:** For written problems such as short answer, multiple choice, derivations, proofs, or plots, please use the provided template. Submissions can be handwritten onto the template, but should be labeled and clearly legible. If your writing is not legible, you will not be awarded marks. Alternatively, submissions can be written in LaTeX. Each derivation/proof should be completed in the boxes provided. To receive full credit, you are responsible for ensuring that your submission contains exactly the same number of pages and the same alignment as our PDF template.
  - **Latex Template:** https://www.overleaf.com/read/pnkjfhgmqgvv#bfb35a

Question	Points
Vector Spaces	9
Basic Matrix Operations	10
Matrix Multiplication	5
Traces of a Matrix	3
Total:	27

*Compiled on Monday 25th August, 2025 at 02:36

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

- ● Matt Gormley
- ○ Marie Curie
- ○ Noam Chomsky

If you need to change your answer, you may cross out the previous answer and bubble in the new answer:

Select One: Who taught this course?

- ● Henry Chai
- ○ Marie Curie
- ⊗ Noam Chomsky

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

- ■ Stephen Hawking
- ■ Albert Einstein
- ■ Isaac Newton
- □ I don't know

Again, if you need to change your answer, you may cross out the previous answer(s) and bubble in the new answer(s):

Select all that apply: Which are scientists?

- ■ Stephen Hawking
- ■ Albert Einstein
- ■ Isaac Newton
- □ I don't know

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

10-606	10-6067

1 Vector Spaces (9 points)

1. (1 point) **Select one:** For any finite-dimensional vector space, the size of its basis is equal to its dimension.

   ◯ True

   ◯ False

2. (1 point) **Select one:** For any finite-dimensional vector space, there is always a unique basis.

   ◯ True

   ◯ False

3. (1 point) **Select one:** Every vector space contains a zero vector.

   ◯ True

   ◯ False

4. (1 point) **Select one:** A vector space can have more than one zero vector.

   ◯ True

   ◯ False

5. (1 point) **Select one:** Matrix multiplication is associative and commutative.

   ◯ True

   ◯ False

6. (2 points) Give an example of two vectors that form a basis for $\mathbb{R}^2$, neither of which is one of the standard basis vectors. Explain why they form a basis.

7. (2 points) Express the vector $a = (4, 2)$ in terms of your chosen basis.

2 Basic Matrix Operations (10 points)

Given two vectors, $\mathbf{x} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, and matrices $\mathbf{A} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & -3 \\ 2 & -1 \\ 3 & -2 \end{bmatrix}$

1. (2 points) **Numerical answer:** Compute the l_1-norm of $\mathbf{x}$.

2. (2 points) **Numerical answer:** Compute the l_2-norm of $\mathbf{y}$.

3. (2 points) **Numerical answer:** Compute sum of all elements in $\mathbf{x}^T\mathbf{y}$.

4. (2 points) **Numerical answer:** Compute sum of all elements in $\mathbf{x}\mathbf{y}^T$.

5. (2 points) **Numerical answer:** Compute the sum of all elements in $\mathbf{AB}$.

3 Matrix Multiplication (5 points)

1. (5 points) Suppose you have a vector $\mathbf{v} \in \mathbb{R}^m$ and matrix $\mathbf{A} \in \mathbb{R}^{mm}$. Write an expression for $\mathbf{v}^T e^{\mathbf{Av}}$ without using any linear algebra notation such as matrices or vectors. In other words, write $\mathbf{v}^T e^{\mathbf{Av}}$ in terms of the elements of $\mathbf{A}$, the elements of $\mathbf{v}$, and m, without including the full vector $\mathbf{v}$ or the full matrix $\mathbf{A}$ in your answer.

   Notation: The exponential function applied to a matrix is simply equal to applying the exponential function to each element of the matrix. For example, for a 2×2 matrix $\mathbf{B} = \begin{bmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{bmatrix}$,

$$e^{\mathbf{B}} = \begin{bmatrix} e^B_{1,1} & e^B_{1,2} \\ e^B_{2,1} & e^B_{2,2} \end{bmatrix}.$$

   You may write the element in the i-th position of $\mathbf{v}$ as v_i. You may write the element in the i-th row and j-th column of $\mathbf{A}$ as $A_{i,j}$. You can use summation and product notation, e.g., $\sum_{i=1}^{N}$ and $\prod_{i=1}^{N}$.

4 Traces of a Matrix (3 points)

The trace of a square matrix is the sum of its diagonal elements. For $\mathbf{A}, \mathbf{B}, \mathbf{C} \in \mathbb{R}^{n \times n}$,

1. (1 point) **Select one:** $tr\mathbf{A} = tr\mathbf{A}^T$.
   - ◯ True
   - ◯ False
2. (1 point) **Select one:** $tr(\mathbf{A} + \mathbf{B} + \mathbf{C}) = tr\mathbf{A} + tr\mathbf{B} + tr\mathbf{C}$.
   - ◯ True
   - ◯ False
3. (1 point) **Select one:** $tr(\mathbf{AB}) = tr(\mathbf{BA})$.
   - ◯ True
   - ◯ False

5 Collaboration Questions

After you have completed all other components of this assignment, report your answers to these questions regarding the collaboration policy. Details of the policy can be found in the syllabus.

1. Did you receive any help whatsoever from anyone in solving this assignment? If so, include full details.
2. Did you give any help whatsoever to anyone in solving this assignment? If so, include full details.
3. Did you find or come across code that implements any part of this assignment? If so, include full details.

Your Answer