Recitation #5

10-606

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Convexity and Optimization

1. Let $f(x) = x^2 + 3x + 5$. Is f convex?
2. Compute the Hessian of the function $f(x, y) = x^2 + y^2 + xy$. Can we conclude whether or not f is convex?
3. Show that the Hessian of $f(x) = x^\top Ax$ is $A + A^\top$.
4. Consider the loss function in linear regression $L(\beta) = \|y - X\beta\|^2$. Prove that this loss function is convex.
5. Perform two iterations of gradient descent for $f(x) = x^2 + 4x + 4$, starting at $x_0 = 3$, with a learning rate $\alpha = 0.1$.
6. Consider the function $f(x, y) = x^2 + y^2$. Starting at $(x_0, y_0) = (1, 2)$, perform one iteration of gradient descent with learning rate $\alpha = 0.1$.

Probability

1. You roll a fair six-sided die.
   (a) What is the sample space?
   (b) Define the event "rolling an even number." What is its probability?
   (c) Define the random variable $X =$ (outcome mod 2). Give the distribution of X.
2. A fair coin is flipped 3 times.
   (a) What is the probability of exactly two heads?
   (b) What is the probability of at least one head?
   (c) If the coin were biased with $P(H) = 0.7$, what changes?
3. Two fair dice are rolled. Let X=value of die 1, Y=value of die 2.
   (a) What is $P(X = 3, Y = 5)$?
   (b) What is $P(X = 3 \mid X + Y = 8)$?
   (c) Are X and Y independent?