$x = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \in \mathbb{R}^3$

Is xy a valid product?

$x^T y$? Yes: $(1 \times n) \times (n \times 1) \to 1$

$x y^T$ yes: $(n \times 1) \times (1 \times n) \to n$

$x^T y = 1 \cdot 2 + 0 \cdot 1 + 1 \cdot 3 = 5$

$$x y^T = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 0 \\ 2 & 1 & 3 \end{bmatrix}$$

$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Is AB valid? no: $(\underline{n} \times \underline{1}) \times (n \times n)$

BA? yes: $(n \times n) \times (n \times 1) \to (n \times 1)$

$A^T B$? yes: $(1 \times n) \times (n \times n)$

What is B^T? $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

What dimension is C^T? 3×2

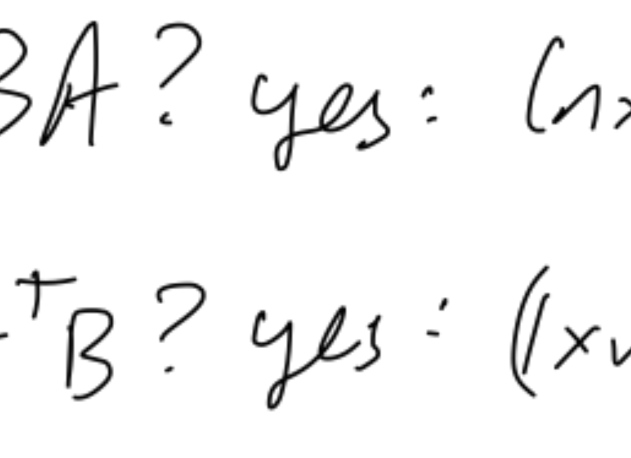
$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

L_p norm of vector:

$$\|v\|_p = \left(\sum_{i=1} |v_i|^p \right)^{\frac{1}{p}}$$

$x = \begin{bmatrix} 3 \\ -1 \\ 4 \\ 5 \end{bmatrix} \quad \|x\|_2 = \sqrt{9+1+16+25}$

$\|x\|_1 = (|3| + |-1| + |4| + |5|)^1 = 13$



L_2: Euclidean $\sqrt{x+y}$

L_1: "Manhattan" $|x| + |y|$

L_∞ norm: $\lim_{p \to \infty} \left(\sum_{i=1} |v_i|^p \right)^{\frac{1}{p}}$

$= \max_i |v_i|$

$x = \begin{bmatrix} -1 \\ 5 \\ -10 \end{bmatrix} \quad \|x\|_\infty = 10$

$A = \begin{bmatrix} -1 & 3 \\ 4 & 0 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\|Ax\|_1$? $\|Ax\|_2$?

$$Ax = \begin{bmatrix} [-1 \ \ 3]\, x \\ [4 \ \ 0]\, x \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$= 1 \cdot \begin{bmatrix} -1 \\ 4 \end{bmatrix} + 3 \cdot \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$\|Ax\|_1 = |5| + |4| = 9$

$\|Ax\|_2 = (|5|^2 + |4|^2)^{\frac{1}{2}} = \sqrt{34}$