Quiz 2 Practice Problems

10-606

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1 Matrix Inverses

1. True/False: If $AB = I$, then $BA = I$.
2. Prove or disprove: Every square diagonal matrix is invertible.
3. Compute the inverse of $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$.
4. Suppose A is invertible. Show that $(A^T)^{-1} = (A^{-1})^T$.
5. Explain, without resorting to calculation, why a matrix with two identical rows cannot be invertible.
6. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. Is A invertible?
7. If A is invertible, does $Ax = b$ has a unique solution for all b?
8. If $Ax = b$ has no solution, show that $ABx = b$ has no solution for any matrix B.
9. A set of vectors $\{v_1, \ldots, v_n\}$ in $\mathbb{R}^n$ are *orthonormal* if $v_i^\intercal v_j = 1$ if $i = j$ and 0 otherwise. A matrix $Q \in \mathbb{R}^{n \times n}$ is orthonormal if its column and row vectors are orthonormal. If Q is orthonormal, show that $Q^\intercal = Q^{-1}$.

Linear Systems

1. Can the following system be solved? $\{x + y = 2,\ 2x + 2y = 5\}$.
2. How many solutions are there to $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} x = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$.

3. Row-reduce $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$.

4. For which a is $\begin{bmatrix} 1 & a \\ 2 & 4 \end{bmatrix}$ invertible?

5. Solve the system: $\{x + y + z = 6,\ x - y + z = 2,\ 2x + z = 5\}$.

6. True/False: If the equation $Ax = 0$ has only the trivial solution $x = 0$, then the columns of A form a basis for $\mathbb{R}^n$.

7. Solve $Ax = b$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, b = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$.

8. Construct 3 equations in 3 unknowns with no solutions.

Eigenvalues and Eigenvectors

1. Find the characteristic polynomial of $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ for any $a \in \Re$ (i.e., compute $\det(A - \lambda I) = 0$).

2. What sort of geometric action does the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ correspond to?

3. What sort of geometric action does the matrix $\begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$ correspond to?

4. Prove that an orthogonal matrix (see Section 1) always has eigenvalues of magnitude 1.

5. If 0 is an eigenvalue of A, what does this tell you about the rank of A?

6. Let A have eigenvectors v_1, v_2 with eigenvalues λ_1, λ_2. Suppose $x = 3v_1 - 2v_2$. What is Ax in terms of v_1, v_2? Explain geometrically what happened to x.