Recitation 2: Proof Techniques

10-607

1. Prove that there is no smallest positive rational number.
2. Let $n \in \mathbb{Z}$. Prove that if $3n + 2$ is even, then n is even.
3. Prove that there is no integer x such that $x^2 \equiv 2 \pmod 3$.
4. Prove that if ab is even (with $a, b \in \mathbb{Z}$), then a is even or b is even.
5. Prove that for all $n \geq 1$, $n! \geq 2^{n-1}$.
6. Prove that if $r \in \mathbb{Q}$ and $s \notin \mathbb{Q}$, then $r + s \notin \mathbb{Q}$.
7. Prove that for all $n \geq 1$, $10^n - 1$ is divisible by 9, i.e., that $9|(10^n - 1)$.
8. For all $r \in \mathbb{R}$, $r \neq 1$, prove that, for all $n \geq 0$,

$$\sum_{j=0}^{n} r^j = \frac{r^{n+1} - 1}{r - 1}. \tag{1}$$