HOMEWORK 1
LOGIC, PREDICATES, AND PROOFS *

10-607 COMPUTATIONAL FOUNDATIONS FOR MACHINE LEARNING

START HERE: Instructions

- **Collaboration Policy**: Please read the collaboration policy in the syllabus.
- **Late Submission Policy:** See the late submission policy in the syllabus.
- **Submitting your work:** You will use Gradescope to submit answers to all questions.
  - **Written:** For written problems such as short answer, multiple choice, derivations, proofs, or plots, please use the provided template. Submissions can be handwritten onto the template, but should be labeled and clearly legible. If your writing is not legible, you will not be awarded marks. Alternatively, submissions can be written in LaTeX. Each derivation/proof should be completed in the boxes provided. To receive full credit, you are responsible for ensuring that your submission contains exactly the same number of pages and the same alignment as our PDF template.
  - **Latex Template:** `https://www.overleaf.com/read/rzxxkbxrrknb#abfdd5`

Question	Points
Inferences	6
Propositional Logic	4
Predicates	5
Proofs	10
Total:	25

*Compiled on Monday 20th October, 2025 at 07:08

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

● Bryan Wilder

○ Marie Curie

○ Noam Chomsky

If you need to change your answer, you may cross out the previous answer and bubble in the new answer:

Select One: Who taught this course?

● Bryan Wilder

○ Marie Curie

~~●~~ Noam Chomsky

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton

□ I don't know

Again, if you need to change your answer, you may cross out the previous answer(s) and bubble in the new answer(s):

Select all that apply: Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton

□ I don't know

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

10-606	10-60~~6~~7

1 Inferences (6 points)

The following questions describe two supporting statements (1. and 2.), and a conclusion (3.). Please indicate whether the conclusion can be reasonably inferred to be true from the supporting statements, using the rules of propositional logic.

1. (1 point) **Pigs.**
   1. If pigs fly, then hell has frozen over.
   2. Pigs fly.
   3. Therefore, hell has frozen over.

   ◯ Can be Inferred. ◯ Cannot be Inferred.

2. (1 point) **Classes.**
   1. You took 10-606, or 10-607.
   2. It is not the case that you took 10-606.
   3. Therefore, you took 10-607.

   ◯ Can be Inferred. ◯ Cannot be Inferred.

3. (1 point) **Legally Blonde.**
   1. If Elle Woods goes to Harvard, then Elle Woods passes the bar exam.
   2. It is not the case that Elle Woods passes the bar exam.
   3. Therefore, it is not the case that Elle Woods went to Harvard

   ◯ Can be Inferred. ◯ Cannot be Inferred.

4. (1 point) **Creatures.**
   1. All smurfs are snorks.
   2. All ewoks are snorks.
   3. Therefore, all smurfs are ewoks

   ◯ Can be Inferred. ◯ Cannot be Inferred.

5. (1 point) **ML.**
   1. If you are a deep learning researcher, you are a machine learning researcher.
   2. You are a machine learning researcher.
   3. Therefore, you are a deep learning researcher.

   ◯ Can be Inferred. ◯ Cannot be Inferred.

6. (1 point) **ML Again.**
   1. If you are a machine learning researcher, you know statistics, and if you are a deep learning researcher, you know statistics.
   2. You are a deep learning researcher or a machine learning researcher.
   3. Therefore, you know statistics.

   ◯ Can be Inferred. ◯ Cannot be Inferred.

2 Propositional Logic (4 points)

1. (4 points) Make a truth table for the expression $((P \to Q) \wedge (Q \to R)) \to (P \to R)$. The table should have eight rows, one for each way to set P, Q, R to be T or F. And, the table should have eight columns, one each for the expressions P, Q, R, $(P \to Q)$, $(Q \to R)$, $(P \to R)$, $(P \to Q) \wedge (Q \to R)$, and $((P \to Q) \wedge (Q \to R)) \to (P \to R)$. Each cell should be T or F, depending on whether the column's expression is true when we assume the row's setting for P, Q, R.

Solution

3 Predicates (5 points)

1. (5 points) Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

   - A student in your class has a cat, a dog, and a ferret.
   - All students in your class have a cat, a dog, or a ferret.
   - Some student in your class has a cat and a ferret, but not a dog.
   - No student in your class has a cat, a dog, and a ferret.
   - For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

Solution

4 Proofs (10 points)

1. (5 points) Prove that if m and n are both perfect squares, then nm is also a perfect square. (An integer a is a perfect square if there is an integer b such that $a = b^2$.) In your proof, please use the following axioms regarding the integers. For every step in your proof, explain which axiom or assumption justifies that step.

   Definition 1: The system $\mathbb{Z}$ of integers can be characterized by the following definitions and axioms. The set $\mathbb{Z}$ has two binary operations, addition ($a + b$) and multiplication (ab), and two distinguished integers, 0 and 1, with $0 \neq 1$. $\mathbb{Z}$ is closed under addition and multiplication; that is, whenever $a, b \in \mathbb{Z}$ then the numbers $a + b$ and ab are also in $\mathbb{Z}$.

   Assume $a, b, c \in \mathbb{Z}$. Then the following axioms hold:

   1. Commutativity: $a + b = b + a,\ ab = ba$.
   2. Associativity: $a + (b + c) = (a + b) + c,\ a(bc) = (ab)c$.
   3. Distributivity: $a(b + c) = ab + ac$.
   4. Existence of Zero: $0 + a = a$.
   5. Existence of One: $1a = a$.
   6. Negation: For any $a \in \mathbb{Z}$, there exists a unique $b \in \mathbb{Z}$ such that $a + b = 0$.

   Each of these axioms means that we can use the inference rule of substitution of equal quantities with the stated equation, where a, b, c are any terms of type $\mathbb{Z}$. Note that you won't need all these facts but we include them for completeness.

   Solution

2. (5 points) Prove that if n is an odd integer, then n^2 is odd. An integer n is odd if there exists another integer k such that $n = 2k + 1$. As before, justify which axiom or assumption justifies each step in your proof.

Solution

5 Collaboration Questions

After you have completed all other components of this assignment, report your answers to these questions regarding the collaboration policy. Details of the policy can be found in the syllabus.

1. Did you receive any help whatsoever from anyone in solving this assignment? If so, include full details.
2. Did you give any help whatsoever to anyone in solving this assignment? If so, include full details.
3. Did you find or come across code that implements any part of this assignment? If so, include full details.

Your Answer