Recitation 1: Logic & Basic Proofs

1 Truth tables and equivalences

Decide if the following statements are equivalent. You can use truth tables or logical basic identities.

a) $p \to q$ vs. $\neg p \vee q$.

b) $p \wedge (q \vee r)$ vs. $(p \wedge q) \vee (p \wedge r)$.

c) $p \leftrightarrow q$ vs. $(p \wedge q) \vee (\neg p \wedge \neg q)$.

d) $\neg(p \wedge q)$ vs. $\neg p \vee \neg q$.

e) $(p \to r) \wedge (q \to r)$ vs. $(p \vee q) \to r$.

Decide whether each is a tautology / contradiction / contingent:

f) $(p \to q) \vee (q \to p)$

g) $(p \wedge (p \to q)) \to q$

h) $(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow \neg q)$

Prove using equivalence laws (no truth tables):

i) $p \to (q \to r) \equiv (p \wedge q) \to r$.

j) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$.

2 From English to Logic

Convert the following sentences to propositional logic.

a) "Alice will meet Bob unless Bob is seeing Claire."

b) "You cannot access the dataset if you lack credentials unless the PI has approved you."

c) "The server restarts iff the watchdog fails or the temperature exceeds the threshold, but not both."

d) "If the model overfits, then either the dataset is too small or the regularizer is disabled."

3 Inference rules & short derivations

a) From $A \vee B$, $\neg A \to C$, $\neg B \to C$, is C entailed?

b) Show: from $p \to (q \vee r)$ and $\neg q$ infer $p \to r$.

4 Basic Proofs

Here $a \bmod b = c$ is the same as $a \equiv c \bmod b$.

1. Show: If $a \mid b$ and $a \mid c$ then $a \mid (mb + nc)$ for any $m, n \in \mathbb{Z}$.
2. Show that $a^2 \bmod 4 = 0$ or $a^2 \bmod 4 = 1$ for all $a \in \mathbb{Z}$.
3. Show: For all $x, y \in \mathbb{Z}$, if $x \equiv y \pmod{m}$ then $x^2 \equiv y^2 \bmod m$.
4. Prove the triangle inequality. That is, show that for all $x, y \in \mathbb{R}$, $|x + y| \leq |x| + |y|$.