Momentum-Aware Planning Synthesis for Dynamic Legged Locomotion

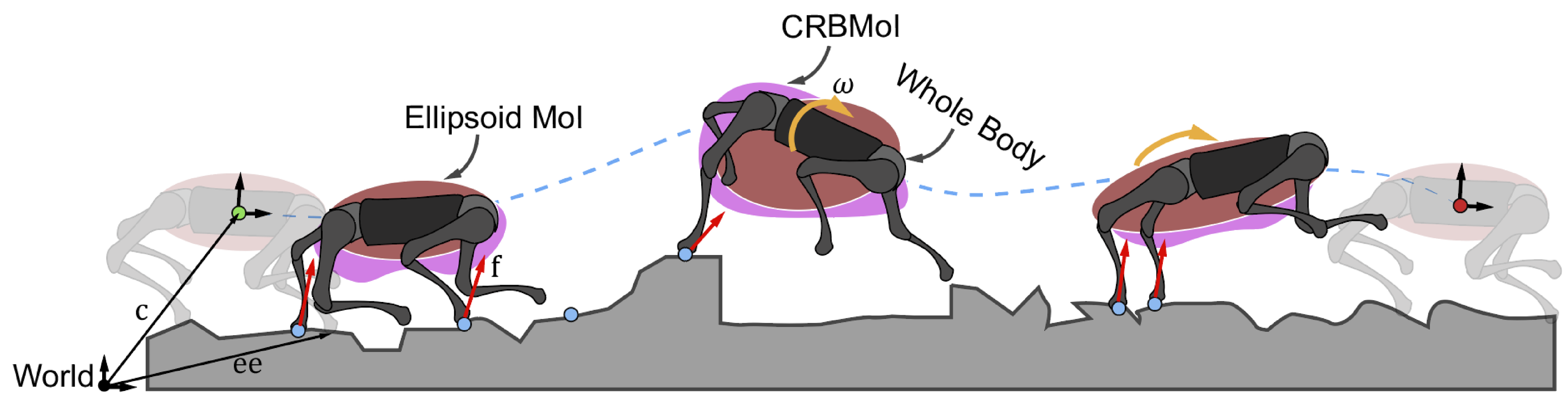
Ziyi Zhou, Bruce Wingo, Nathan Boyd, Seth Hutchinson, and Ye Zhao
Institute for Robotics and Intelligent Machines, Georgia Institute of Technology

Introduction and Objective

- Hierarchical gait-->centroidal-->whole-body pipelines reduce planning complexity, additional constraints on momentum and full-body kinematics enable more dynamically feasible solutions.
- **Design a centroidal optimization capable of discovering both contact sequences and angular momentum trajectories.**
- **Achieve a dynamic consensus between centroidal and whole-body models** using constrained ADMM.

Centroidal and Whole-Body Optimization

- Centroidal optimization utilizes a single rigid body model with equimomental-ellipsoid-based Moment of Inertia (MoI) [2, 3].
- Simultaneously **solve for footholds, contact forces, centroidal and momentum** trajectories.
- **Ellipsoid MoI tracks joint motion effects on Composite Rigid Body MoI** from whole-body model for accurate momentum generation.
- WBD tracks the consensus quantities from centroidal optimization, and then solved via Differential Dynamic Programming (DDP).



ADMM Constrained Trajectory Optimization

- The consensus [1] is enforced by adding equality **consistency constraints for Center of Mass (CoM) positions, momentum, footholds**. The MoI is directly computed from whole-body CRBMoI.

Sample for one iteration procedure:

Centroidal Primal Updates

$$\min_{\phi_{\text{cen}}} \sum_{k=1}^{K}\sum_{i=1}^{N_k} \mathcal{L}_{\text{cen}}(\phi_{\text{cen}}) + \sum_{i\in\mathcal{I}} \frac{\rho_i}{2}\|\mathbf{r}_i + \mathbf{w}_i^k\|_2^2$$

$$\text{s.t.} \quad \begin{bmatrix} m\ddot{\mathbf{c}} \\ \dot{\mathbf{h}} \end{bmatrix} = \begin{bmatrix} \sum_j \mathbf{f}_j + m\mathbf{g} \\ \sum_j \mathbf{f}_j \times (\mathbf{c} - \mathbf{ee}_j) \end{bmatrix}$$

$$\mathbf{h} = \mathbf{I}_{\text{ellip}}\boldsymbol{\omega}$$

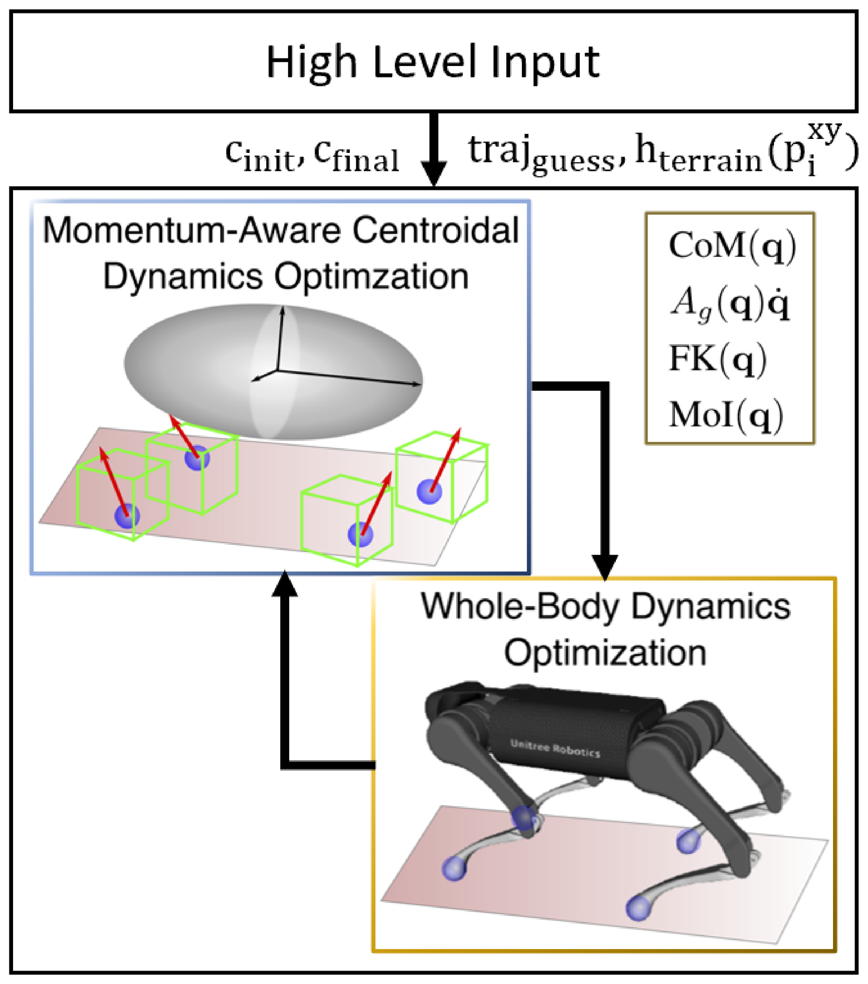
Force, terrain and RoM constraints

WBD Primal Updates

$$\min_{\phi_{\text{wbd}}} \sum_{k=1}^{K}\sum_{i=1}^{N_k} \mathcal{L}_{\text{wbd}}(\phi_{\text{wbd}}) + \sum_{i\in\mathcal{I}} \frac{\rho_i}{2}\|\mathbf{r}_i + \mathbf{w}_i^k\|_2^2$$

$$\text{s.t.} \quad \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{B}\mathbf{u} + \mathbf{J}_c^T\boldsymbol{\lambda}_c$$

$$\ddot{\mathbf{x}}_e = \mathbf{J}_c\ddot{\mathbf{q}} + \dot{\mathbf{J}}_c\dot{\mathbf{q}} = 0$$



$$\mathbf{w}_c^{k+1} = \mathbf{w}_c^k + \text{CoM}(\mathbf{q}^{k+1}) - \mathbf{c}^{k+1}$$

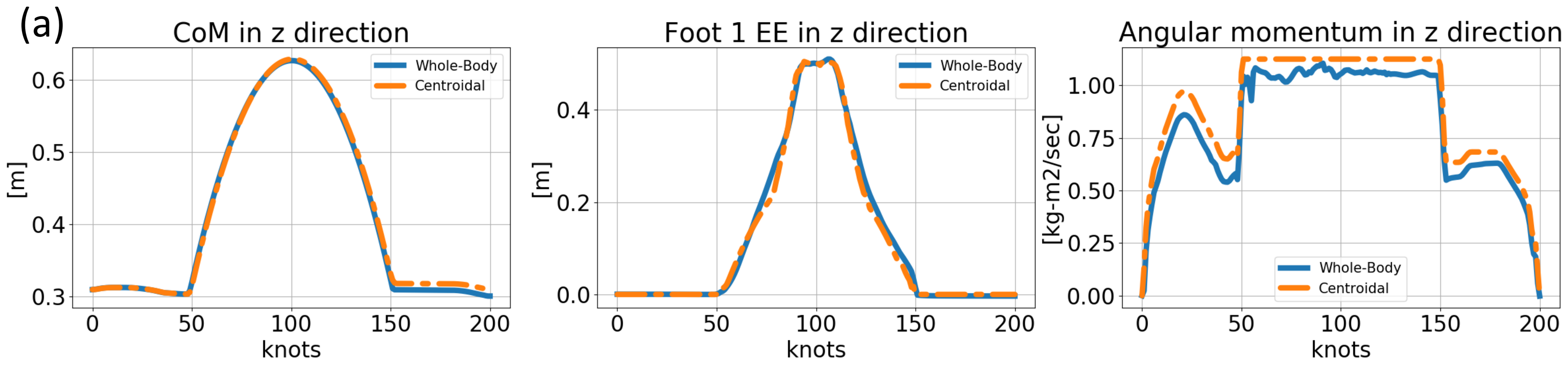
$$\mathbf{w}_h^{k+1} = \mathbf{w}_h^k + \boldsymbol{A}_g(\mathbf{q}^{k+1})\dot{\mathbf{q}}^{k+1} - \begin{bmatrix} m\dot{\mathbf{c}}^{k+1} \\ \mathbf{I}\dot{\boldsymbol{\theta}}^{k+1} \end{bmatrix}$$

$$\mathbf{w}_{ee}^{k+1} = \mathbf{w}_{ee}^k + \text{FK}(\mathbf{q}^{k+1}) - \mathbf{ee}^{k+1}$$

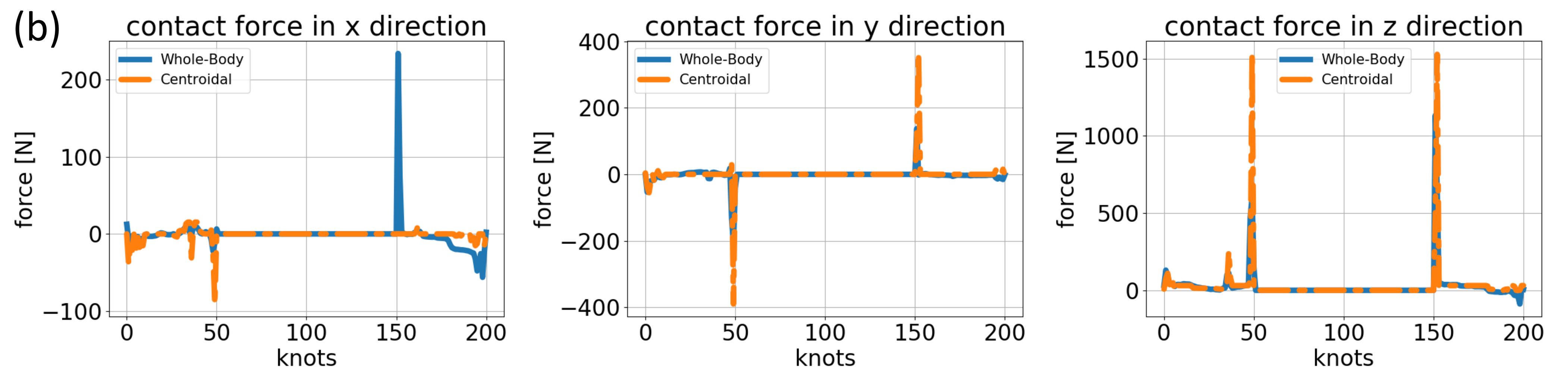
Dual Updates

Results

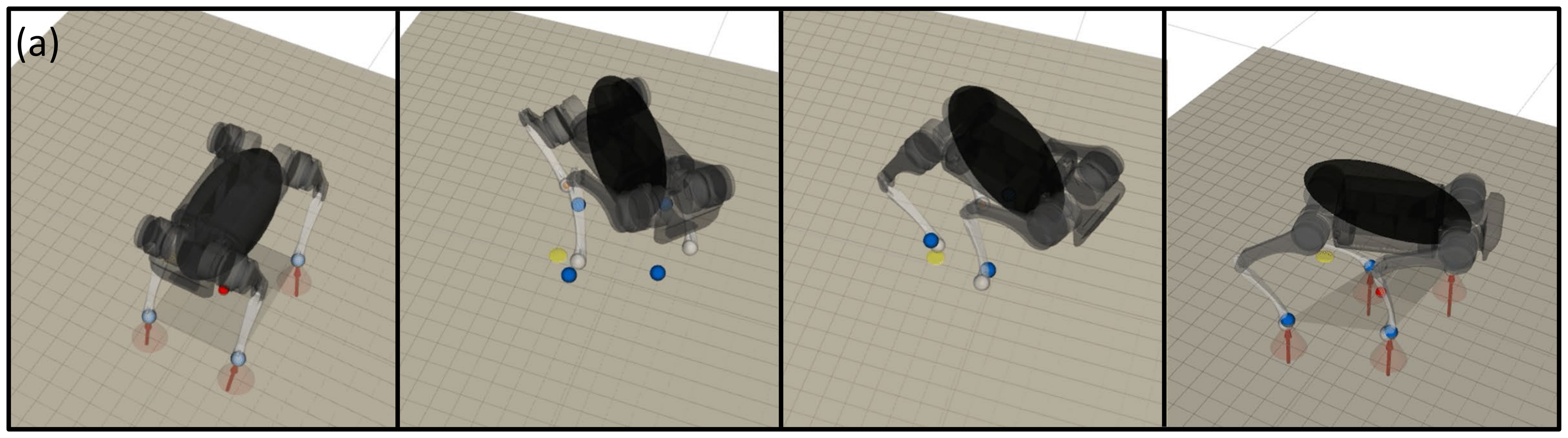
Quadruped Robot jump-twist and trotting examples.

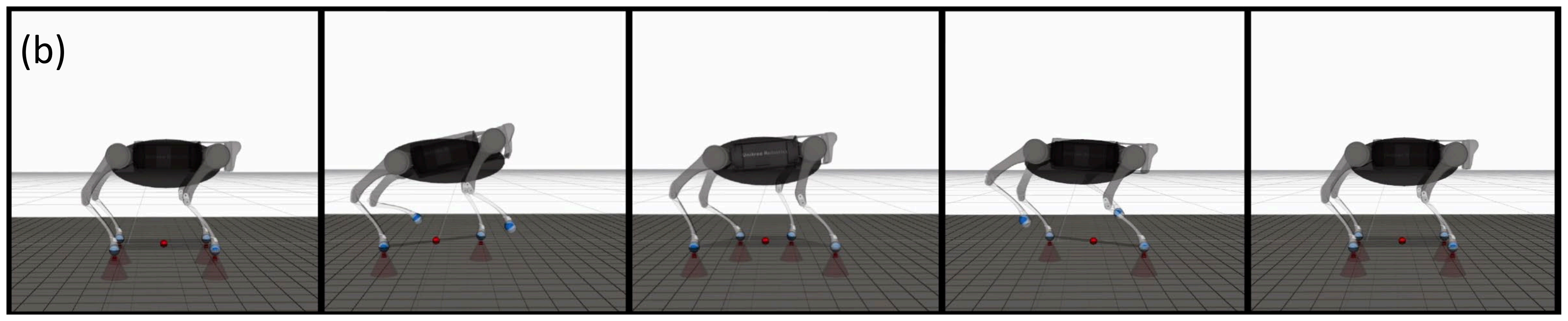


Dynamic consensus of the desired variables for a jump-twist maneuver between centroidal and whole-body models.



Dynamic consensus for a trotting gait motion between centroidal and whole-body models.





Snapshots of an athletic jump-twist maneuver (a) and quadruped trotting gait (b) solved by SNOPT [4] and Crocoddyl [5] for centroidal and whole-body updates respectively.

Discussion and On-going Work

- Designed a centroidal optimization scheme for generating contact sequences and momentum.
- Dynamic **consensus between centroidal and full body** dynamic models.
- On-going work includes improving the angular momentum and inertia tracking. We are also exploring **real-time constrained MPC implementations**. This would require more improvements on the algorithm efficiency and scope for real applications.

References

[1] Z. Zhou and Y. Zhao (2020), [2] A. W. Winkler, C. D. Bellicoso, M. Hutter, and J. Buchli (2018), [3] V. Zordan, D. Brown, A. Macchietto, and K. Yin (2014), [4] P. Gill, W. Murray, and M. Saunders (2005), [5] C. Mastalli, R. Budhiraja, W. Merkt, G. Saurel, B. Hammoud, M. Naveau, J. Carpentier, L. Righetti, S. Vijayakumar and N. Mansard (2020)

Momentum-Aware Planning Synthesis for Dynamic Legged Locomotion

Georgia Tech | Laboratory for Intelligent Decision and Autonomous Robots

Ziyi Zhou, Bruce Wingo, Nathan Boyd, Seth Hutchinson, Ye Zhao

Introduction and Objective

- Hierarchical gait-->centroidal-->whole-body pipelines reduce complexity, but additional constraints are required to ensure feasible solutions.
- **Create a centroidal optimization capable of discovering both contact sequences and angular momentum trajectories.**
- **Achieve a dynamic consensus between centroidal and whole-body models**

Centroidal and Whole-Body Optimization

- Centroidal optimization utilizes phase-based gait [2] and equimomental-ellipsoid-based Moment of Inertia (MoI) [3].
- Simultaneously solve for footholds, contact forces, centroidal and momentum trajectories
- Ellipsoid MoI tracks joint motion effects on Composite Rigid Body MoI from whole-body model for accurate momentum generation
- Track the consensus quantities from centroidal optimization, and then solved via Differential Dynamic Programming (DDP).

$$\min_{\phi_{\text{cen}},\phi_{\text{wbd}}} \quad \sum_{k=1}^{K}\int_{t_k}^{t_k+\triangle t_k}\mathcal{L}_{\text{wbd}}(\phi_{\text{wbd}})dt + \int_{t_k}^{t_k+\triangle t_k}\mathcal{L}_{\text{cen}}(\phi_{\text{cen}})dt$$

subject to

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{B}\mathbf{u} + \mathbf{J}_c^T\boldsymbol{\lambda}_c \quad (2a)$$
$$m\ddot{\mathbf{r}} = \sum\nolimits_j \mathbf{f}_j - m\mathbf{g} \quad (2b)$$
$$\dot{\mathbf{L}} = \sum\nolimits_j (\mathbf{p}_j - \mathbf{r}_j)\times\mathbf{f}_j \quad (2c)$$
$$\dot{\phi} = W(\mathbf{I}^{-1}\mathbf{L}) \quad (2d)$$
$$\dot{\mathbf{e}} = C_e(t,u_e) \quad (2e)$$
$$\dot{\mathbf{v}} = C_v(t,u_v) \quad (2f)$$
$$[\mathbf{r},\boldsymbol{\theta}](t=0) = [\mathbf{r}_0,\theta_0] \quad (2g)$$
$$\mathbf{r}(t=T) = \mathbf{r}_g \quad (2h)$$

for every foot i :

$$\mathbf{p}_i(t) \in \mathcal{R}_i(\mathbf{r},\boldsymbol{\theta}) \quad (2i)$$

Update

if foot i in contact: (2j)

$$\dot{\mathbf{p}}_i(t\in\mathcal{C}_i) = 0 \quad (2k)$$
$$\mathbf{p}_i^z(t\in\mathcal{C}_i) = h_{terrain}(\mathbf{p}_i^{xy}) \quad (2l)$$
$$\mathbf{f}_i(t\in\mathcal{C}_i)\cdot\mathbf{n}(\mathbf{p}_i^{xy}) \geq 0 \quad (2m)$$
$$\mathbf{f}_i(t\in\mathcal{C}_i) \in \mathcal{F}(\mu,\mathbf{n},\mathbf{p}_i^{xy}) \quad (2n)$$

if foot i in air: (2o)

$$\mathbf{f}_i(t\notin\mathcal{C}_i) = 0 \quad (2p)$$
$$\sum_{j=1}^{2n_{s,i}}\triangle T_{i,j} = T \quad (2q)$$
$$\text{CoM}(\mathbf{q}) = \mathbf{r} \quad (2r)$$
$$FK(\mathbf{q}) = \mathbf{p} \quad (2s)$$
$$\boldsymbol{A}_g(\mathbf{q})\dot{\mathbf{q}} = \begin{bmatrix} m\dot{\mathbf{r}} \\ \mathbf{L}_g \end{bmatrix} \quad (2t)$$
$$\mathbf{I}_{cen}(\mathbf{q}) = \mathbf{I} \quad (2u)$$
$$\mathbf{s}\in\mathcal{S},\ \ \mathbf{u}\in\mathcal{U} \quad (2v)$$
$$\boldsymbol{\lambda}_j \in \mathcal{F}_j,\ \ \forall j \in \mathcal{I}_{\text{contact}} \quad (2w)$$

ADMM Constrained Trajectory Optimization

- Stage-wise Accelerated Alternating Direction Method of Multiplier (SWA-ADMM) framework [1] alternately updates optimization

Primal updates:

Centroidal Dynamics

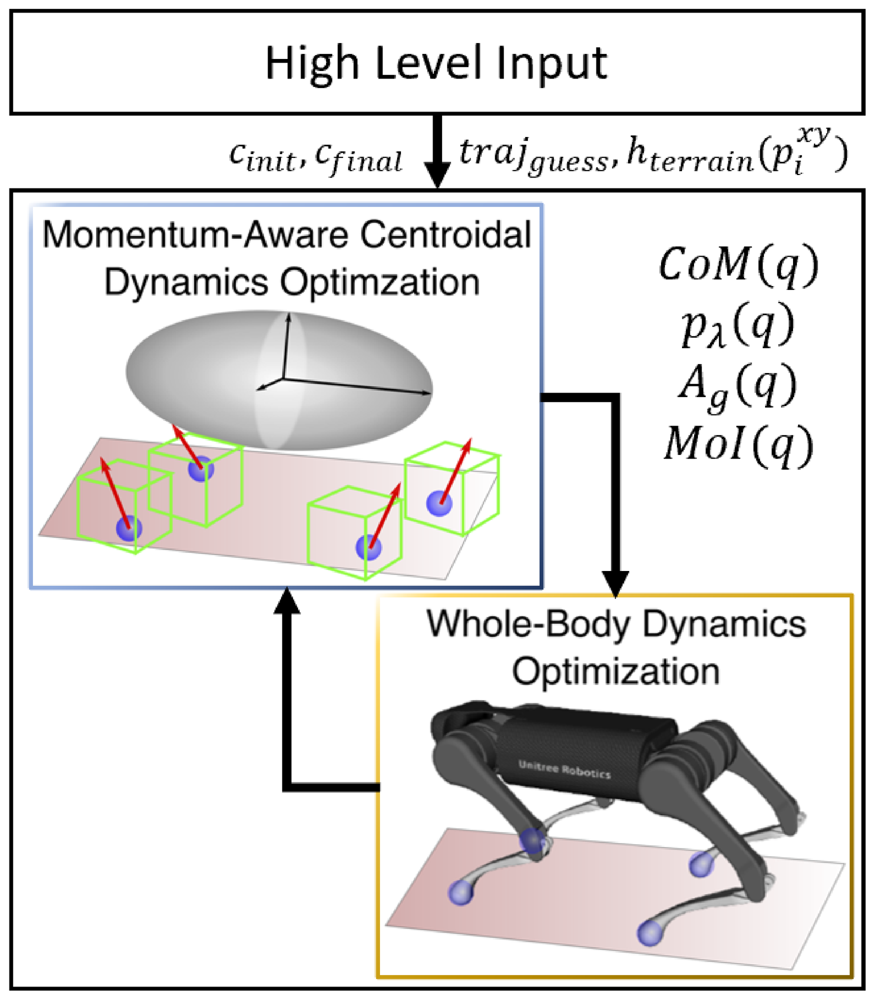
$$\phi_{\text{cen}}^{k+1} := \arg\min_{\phi_{\text{cen}}} \sum_{i=1}^{N}\mathcal{L}_{\text{cen}}(\phi_{\text{cen}}) \quad (32a)$$
$$+ \frac{\rho_c}{2}\|\text{CoM}(\mathbf{q}^k) - \mathbf{c} + \mathbf{w}_i^k\|_2^2 \quad (32b)$$
$$+ \frac{\rho_h}{2}\left\|\boldsymbol{A}_g(\mathbf{q}^k)\dot{\mathbf{q}}^k - \begin{bmatrix} m\dot{\mathbf{c}} \\ \mathbf{I}\dot{\boldsymbol{\theta}} \end{bmatrix} + \mathbf{w}_i^k\right\|_2^2 \quad (32c)$$
$$+ \frac{\rho_\lambda}{2}\|g_\lambda(\mathbf{q}^k,\dot{\mathbf{q}}^k,\mathbf{u}^k) - \boldsymbol{\lambda} + \mathbf{w}_i^k\|_2^2 \quad (32d)$$
$$+ \frac{\rho_{ee}}{2}\|FK(\mathbf{q}^k) - \mathbf{p} + \mathbf{w}_i^k\| \quad (32e)$$

Whole Body Dynamics

$$\phi_{\text{wbd}}^{k+1} := \arg\min_{\phi_{\text{wbd}}} \sum_{i=1}^{N}\mathcal{L}_{\text{wbd}}(\phi_{\text{wbd}}) \quad (32f)$$
$$+ \frac{\rho_c}{2}\|\text{CoM}(\mathbf{q}) - \mathbf{c}^{k+1} + \mathbf{w}_i^k\|_2^2 \quad (32g)$$
$$+ \frac{\rho_h}{2}\left\|\boldsymbol{A}_g(\mathbf{q})\dot{\mathbf{q}} - \begin{bmatrix} m\dot{\mathbf{c}}^{k+1} \\ \mathbf{I}\dot{\boldsymbol{\theta}}^{k+1} \end{bmatrix} + \mathbf{w}_i^k\right\|_2^2 \quad (32h)$$
$$+ \frac{\rho_\lambda}{2}\|g_\lambda(\mathbf{q},\dot{\mathbf{q}},\mathbf{u}) - \boldsymbol{\lambda}^{k+1} + \mathbf{w}_i^k\|_2^2 \quad (32i)$$
$$+ \frac{\rho_{ee}}{2}\|FK(\mathbf{q}) - \mathbf{p}^{k+1} + \mathbf{w}_i^k\| \quad (32j)$$

Dual updates:

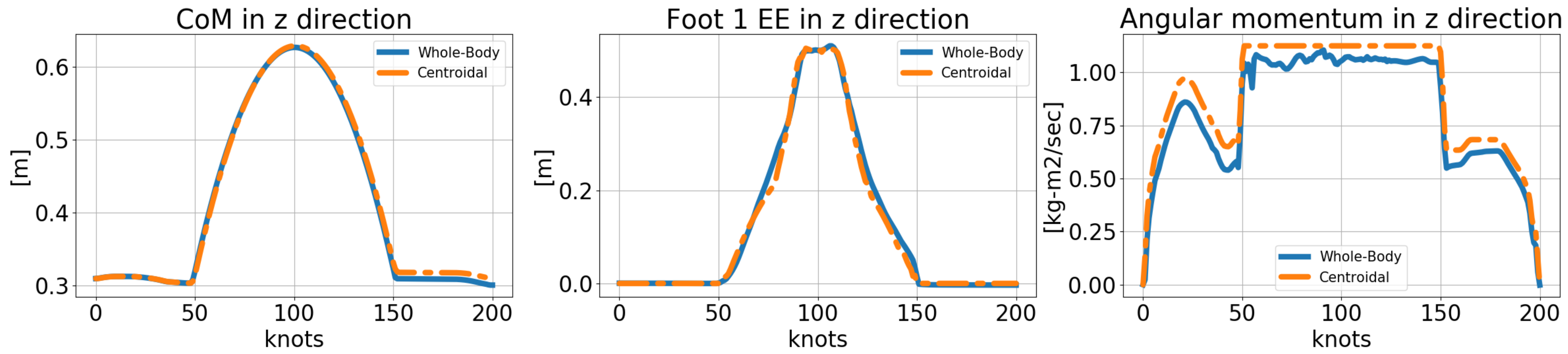
$$\mathbf{w}_c^{k+1} = \mathbf{w}_c^k + \text{CoM}(\mathbf{q}^{k+1}) - \mathbf{c}^{k+1} \quad (32k)$$
$$\mathbf{w}_h^{k+1} = \mathbf{w}_h^k + \boldsymbol{A}_g(\mathbf{q}^{k+1})\dot{\mathbf{q}}^{k+1} - \begin{bmatrix} m\dot{\mathbf{c}}^{k+1} \\ \mathbf{I}\dot{\boldsymbol{\theta}}^{k+1} \end{bmatrix} \quad (32l)$$
$$\mathbf{w}_\lambda^{k+1} = \mathbf{w}_\lambda^k + g_\lambda(\mathbf{q}^{k+1},\dot{\mathbf{q}}^{k+1},\mathbf{u}^{k+1}) - \boldsymbol{\lambda}^{k+1} \quad (32m)$$



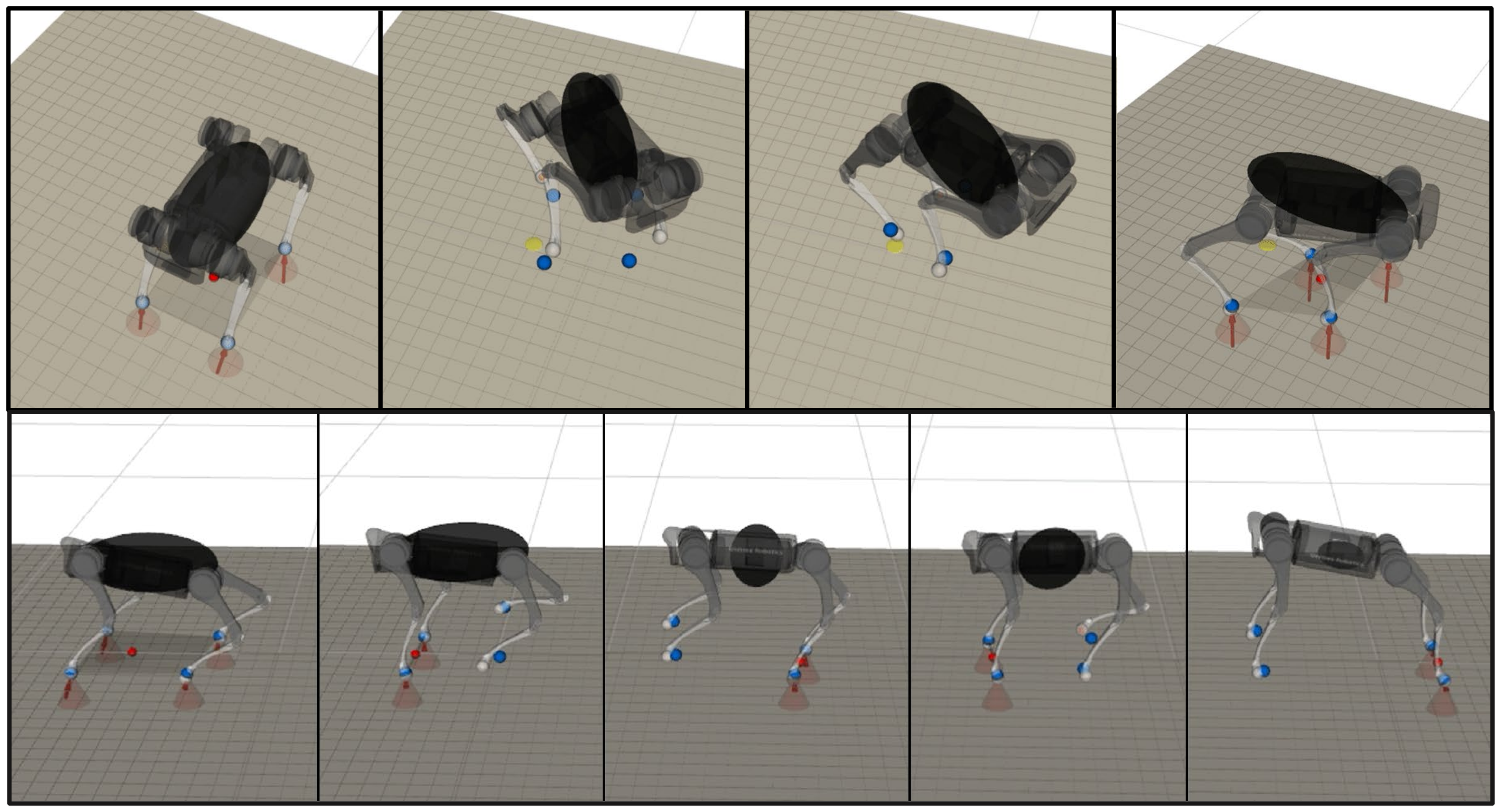
Results

- Quadruped Robot two block ADMM optimization problem

Dynamic Consensus
Center of Mass
Momentum
Foothold Position
Moment of Inertia



Dynamic consensus of a jump-twist maneuver between centroidal and whole-body model.



Snapshots of an athletic jump-twist maneuver (a) and quadruped bound gait (b) computed by the proposed approach.

Discussion and Future Work

1. Designed a centroidal optimization scheme for generating contact sequences and momentum
2. Dynamic consensus between template and full body dynamic models

Future work includes improving the angular momentum and inertia tracking. We are also considering exploring real-time constrained MPC implementations. This would require more multiple improvements on the algorithm efficiency and scope for real applications.

References

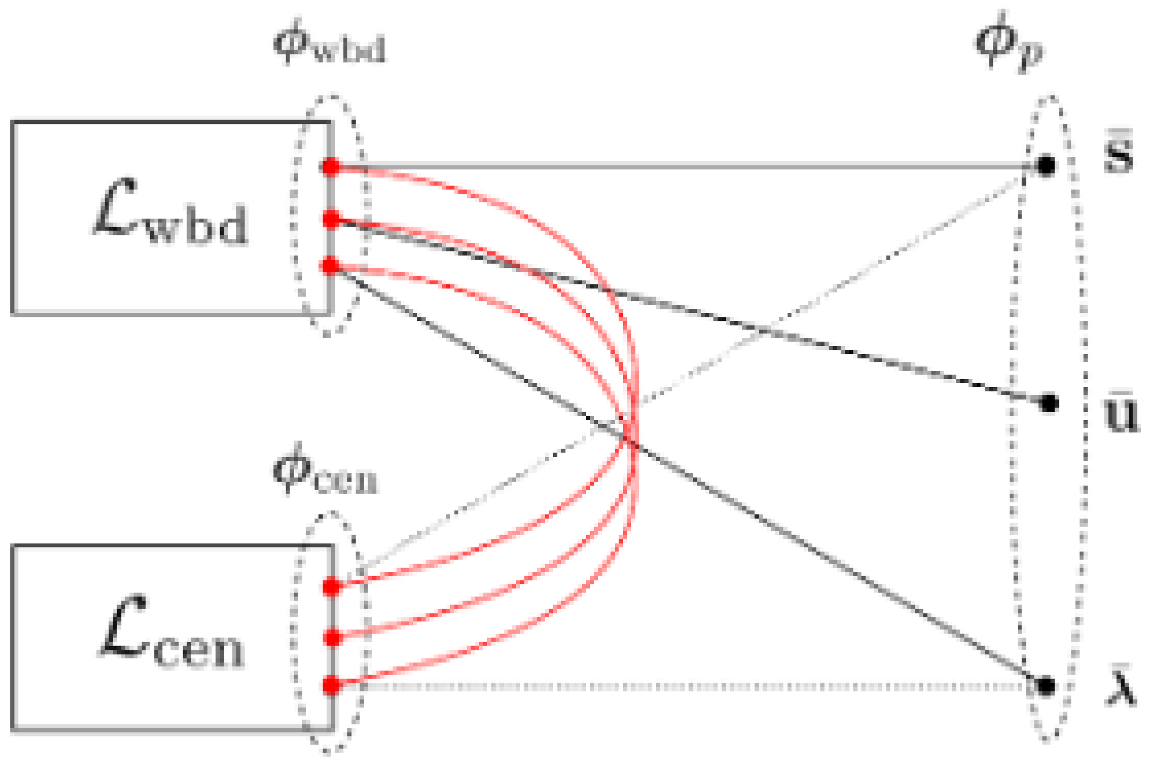
[1] Z. Zhou and Y. Zhao (2020), [2] A. W. Winkler, C. D. Bellicoso, M. Hutter, and J. Buchli (2018), [3] V. Zordan, D. Brown, A. Macchietto, and K. Yin (2014), [4] P. Gill, W. Murray, and M. Saunders (2005), [5] C. Mastalli, R. Budhiraja, W. Merkt, G. Saurel, B. Hammoud, M. Naveau, J. Carpentier, L. Righetti, S. Vijayakumar and N. Mansard (2020)

Momentum-Aware Planning Synthesis for Dynamic Legged Locomotion

Ziyi Zhao, Bruce Wingo, Nathan Boyd, Ye Zhao
Georgia Tech | Laboratory for Intelligent Decision and Autonomous Robots

Introduction

- dafdf



Algorithm 1

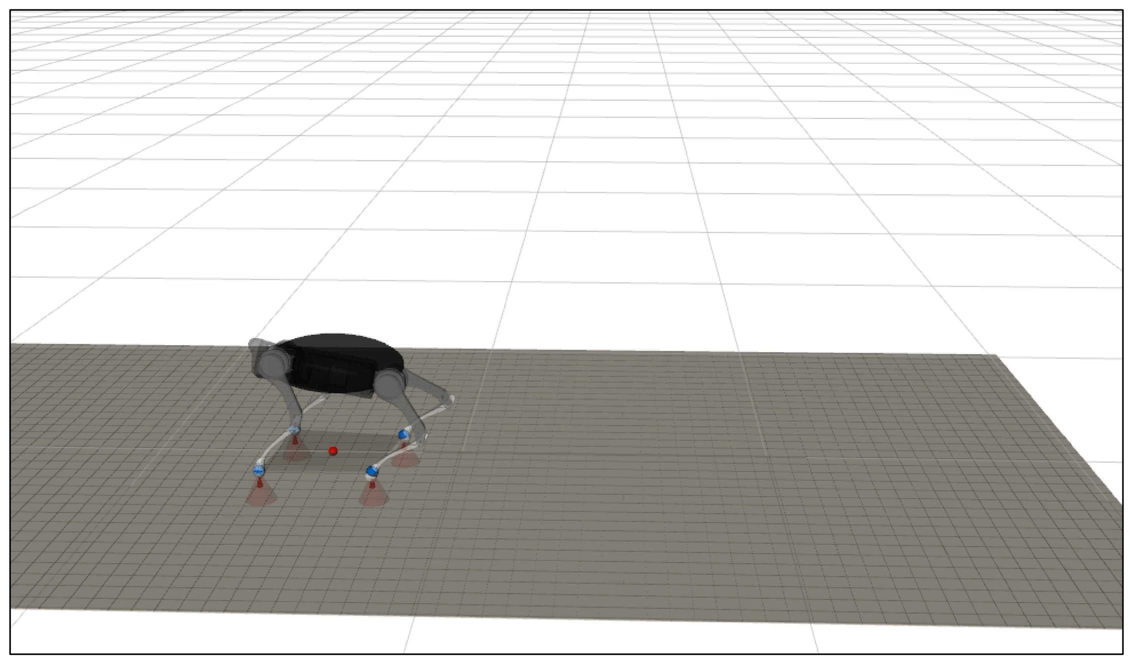
- $\phi_{\text{cen}} \leftarrow \phi^0_{\text{cen}}, \phi_{\text{wbd}} \leftarrow \phi^0_{\text{wbd}}, \phi_p \leftarrow \phi^0_p$
- $\mathbf{w}_i \leftarrow \mathbf{w}^0_i, \rho_i \leftarrow \rho^0_i, i = \mathcal{I}$
- **repeat**
  - $\phi_{\text{cen}} \leftarrow$ DDP-centroidal$(\phi_{\text{cen}}, \phi_{\text{wbd}}, \phi_p, \mathbf{w}_i, \rho_i)$ $i = \{c, h, \lambda\}$
  - $\phi_{\text{wbd}} \leftarrow$ DDP-wholebody$(\phi_{\text{wbd}}, \phi_{\text{cen}}, \phi_p, \mathbf{w}_i, \rho_i)$ $i = \mathcal{I}$
  - $\phi_p \leftarrow$ Projection$(\phi_{\text{wbd}}, \phi_{\text{cen}}, \mathbf{w}_i, \rho_i)$ $i = \{j, t, f\}$
  - $\mathbf{w}_i \leftarrow \mathbf{w}_i + \mathbf{r}_i$ $i = \mathcal{I}$
- **until** stopping criterion is satisfied

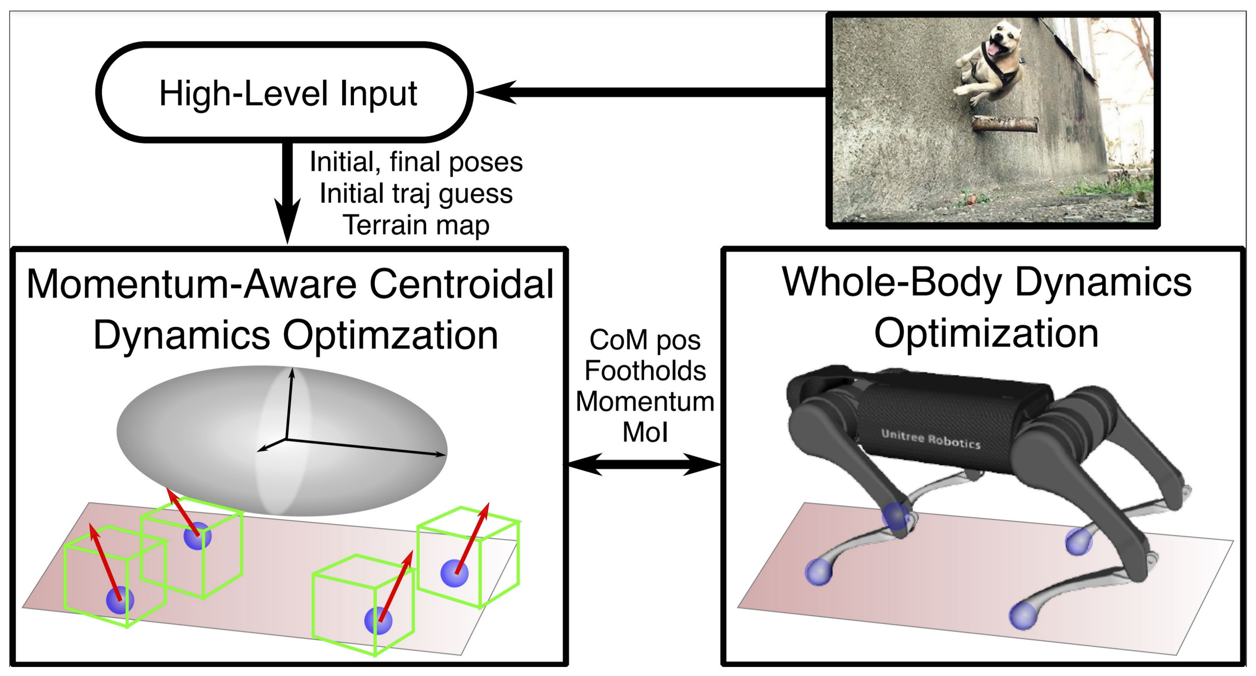
NLP

-

Algorithm 3 Stage-wise Accelerated ADMM solver

- $\phi_{\text{cen}} \leftarrow \phi^0_{\text{cen}}, \phi_{\text{wbd}} \leftarrow \phi^0_{\text{wbd}}, \phi_p \leftarrow \phi^0_p$
- $\mathbf{w}_i \leftarrow \mathbf{w}^0_i, \rho_i \leftarrow \rho^0_i, i = \mathcal{I}$
- **repeat**
  - $\phi_{\text{cen}} \leftarrow$ DDP-centroidal$(\phi_{\text{cen}}, \phi_{\text{wbd}}, \phi_p, \mathbf{w}_i, \rho_i)$ $i = \{c, h, \lambda\}$
  - $\phi_{\text{wbd}} \leftarrow$ DDP-wholebody$(\phi_{\text{wbd}}, \phi_{\text{cen}}, \phi_p, \mathbf{w}_i, \rho_i)$ $i = \mathcal{I}$
  - $\phi'_p \leftarrow \alpha(\mathbf{s}, \mathbf{u}, \boldsymbol{\lambda}) + (1-\alpha)(\bar{\mathbf{s}}, \bar{\mathbf{u}}, \bar{\boldsymbol{\lambda}})$
  - $\phi_p \leftarrow$ Projection$(\phi'_p, \mathbf{w}_i, \rho_i)$ $i = \{j, t, f\}$
  - $\mathbf{w}_i \leftarrow \mathbf{w}_i + \mathbf{r}_i$ $i = \mathcal{I}$
  - **if** current iteration $> k_{\text{sw}}$ **then**
    - **if** $\|\mathbf{r}_i\|_2^2 > \mu\|\mathbf{d}_i\|_2^2$ **then**
      - $\rho_i = \tau^{\text{incr}}\rho_i$
    - **else if** $\|\mathbf{d}_i\|_2^2 > \mu\|\mathbf{r}_i\|_2^2$ **then**
      - $\rho_i = \rho_i/\tau^{\text{decr}}$
    - **end if**
  - **end if**
- **until** stopping criterion is satisfied





Results

- Dfa;hdjljadf

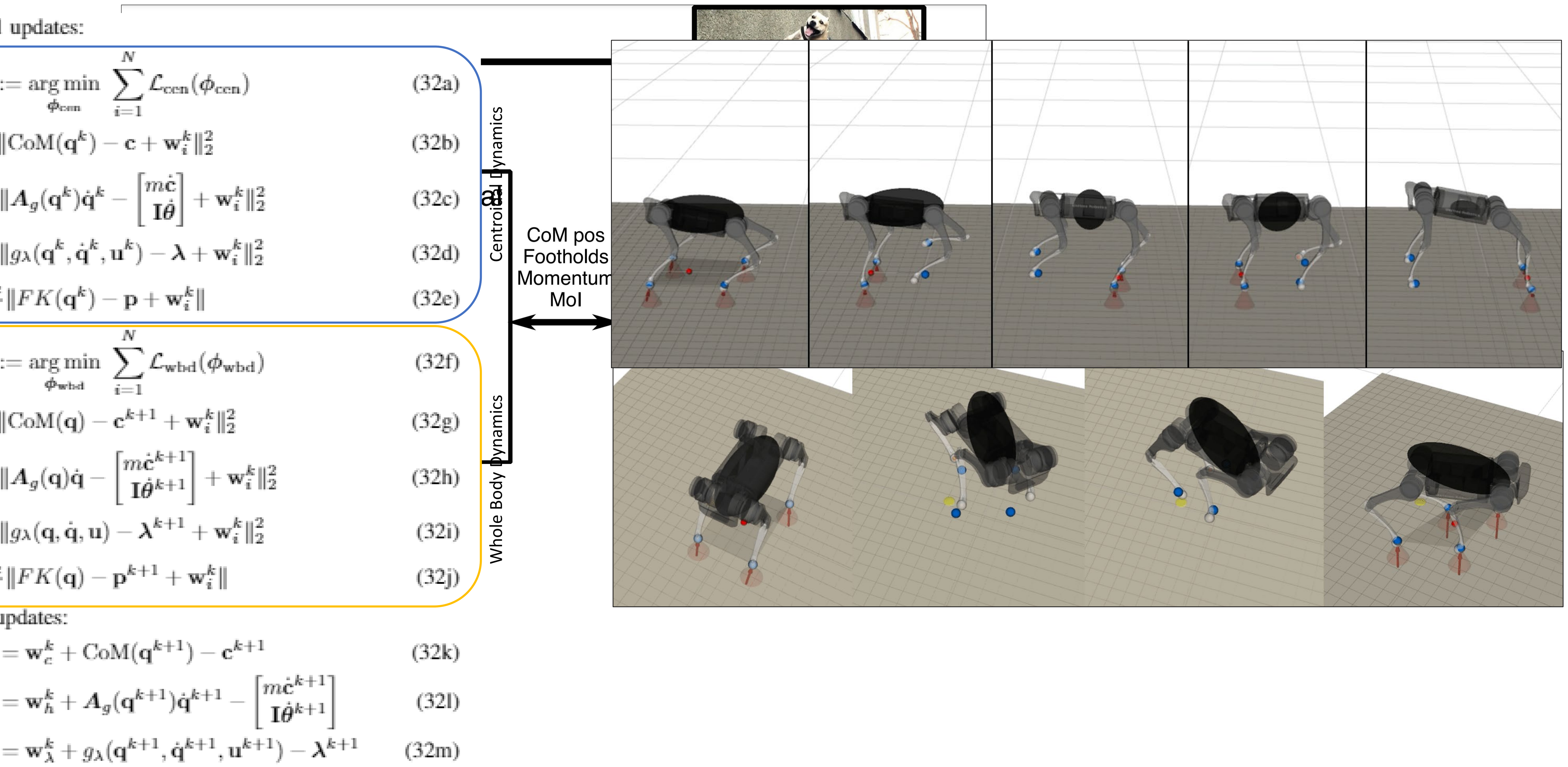
$$
\begin{aligned}
\min_{\phi_{\text{cen}};\phi_{\text{wbd}}} \quad & \sum_{k=1}^{K} \int_{t_k}^{t_k+\Delta t_k} \mathcal{L}_{\text{wbd}}(\phi_{\text{wbd}})dt + \int_{t_k}^{t_k+\Delta t_k} \mathcal{L}_{\text{cen}}(\phi_{\text{cen}})dt \\
\text{subject to} \quad & \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{B}\mathbf{u} + \mathbf{J}_c^T\boldsymbol{\lambda}_c && \text{(2a)} \\
& m\ddot{\mathbf{r}} = \sum\nolimits_j \mathbf{f}_j - m\mathbf{g} && \text{(2b)} \\
& \dot{\mathbf{L}} = \sum\nolimits_j (\mathbf{p}_j - \mathbf{r}_j) \times \mathbf{f}_j && \text{(2c)} \\
& \dot{\boldsymbol{\phi}} = W(\mathbf{I}^{-1}\mathbf{L}) && \text{(2d)} \\
& \dot{\mathbf{e}} = C_e(t, u_e) && \text{(2e)} \\
& \dot{\mathbf{v}} = C_v(t, u_v) && \text{(2f)} \\
& [\mathbf{r}, \boldsymbol{\theta}](t=0) = [\mathbf{r}_0, \theta_0] && \text{(2g)} \\
& \mathbf{r}(t=T) = \mathbf{r}_g && \text{(2h)} \\
\text{for every foot } i: \quad & \\
& \mathbf{p}_i(t) \in \mathcal{R}_i(\mathbf{r}, \boldsymbol{\theta}) && \text{(2i)} \\
& \text{if foot } i \text{ in contact:} && \text{(2j)} \\
& \quad \dot{\mathbf{p}}_i(t \in \mathcal{C}_i) = 0 && \text{(2k)} \\
& \quad \mathbf{p}_i^z(t \in \mathcal{C}_i) = h_{terrain}(\mathbf{p}_i^{xy}) && \text{(2l)} \\
& \quad \mathbf{f}_i(t \in \mathcal{C}_i) \cdot \mathbf{n}(\mathbf{p}_i^{xy}) \geq 0 && \text{(2m)} \\
& \quad \mathbf{f}_i(t \in \mathcal{C}_i) \in \mathcal{F}(\mu, \mathbf{n}, \mathbf{p}_i^{xy}) && \text{(2n)} \\
& \text{if foot } i \text{ in air:} && \text{(2o)} \\
& \quad \mathbf{f}_i(t \notin \mathcal{C}_i) = 0 && \text{(2p)} \\
& \sum_{j=1}^{2n_{s,i}} \Delta T_{i,j} = T && \text{(2q)} \\
& \mathrm{CoM}(\mathbf{q}) = \mathbf{r} && \text{(2r)} \\
& FK(\mathbf{q}) = \mathbf{p} && \text{(2s)} \\
& \mathbf{A}_g(\mathbf{q})\dot{\mathbf{q}} = \begin{bmatrix} m\dot{\mathbf{r}} \\ \mathbf{L}_g \end{bmatrix} && \text{(2t)} \\
& \mathbf{I}_{cen}(\mathbf{q}) = \mathbf{I} && \text{(2u)} \\
& \mathbf{s} \in \mathcal{S}, \;\; \mathbf{u} \in \mathcal{U} && \text{(2v)} \\
& \boldsymbol{\lambda}_j \in \mathcal{F}_j, \;\; \forall j \in \mathcal{I}_{\text{contact}} && \text{(2w)}
\end{aligned}
$$

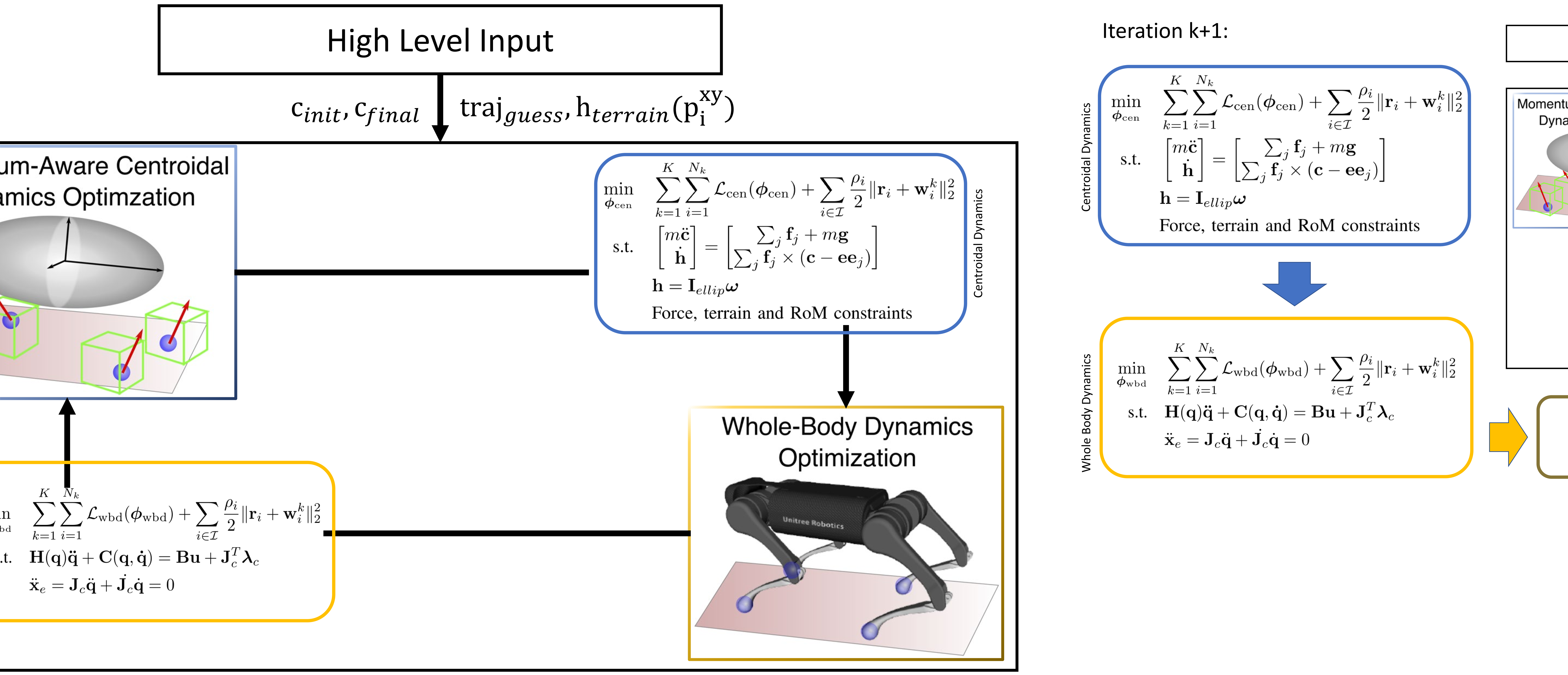
Future Work

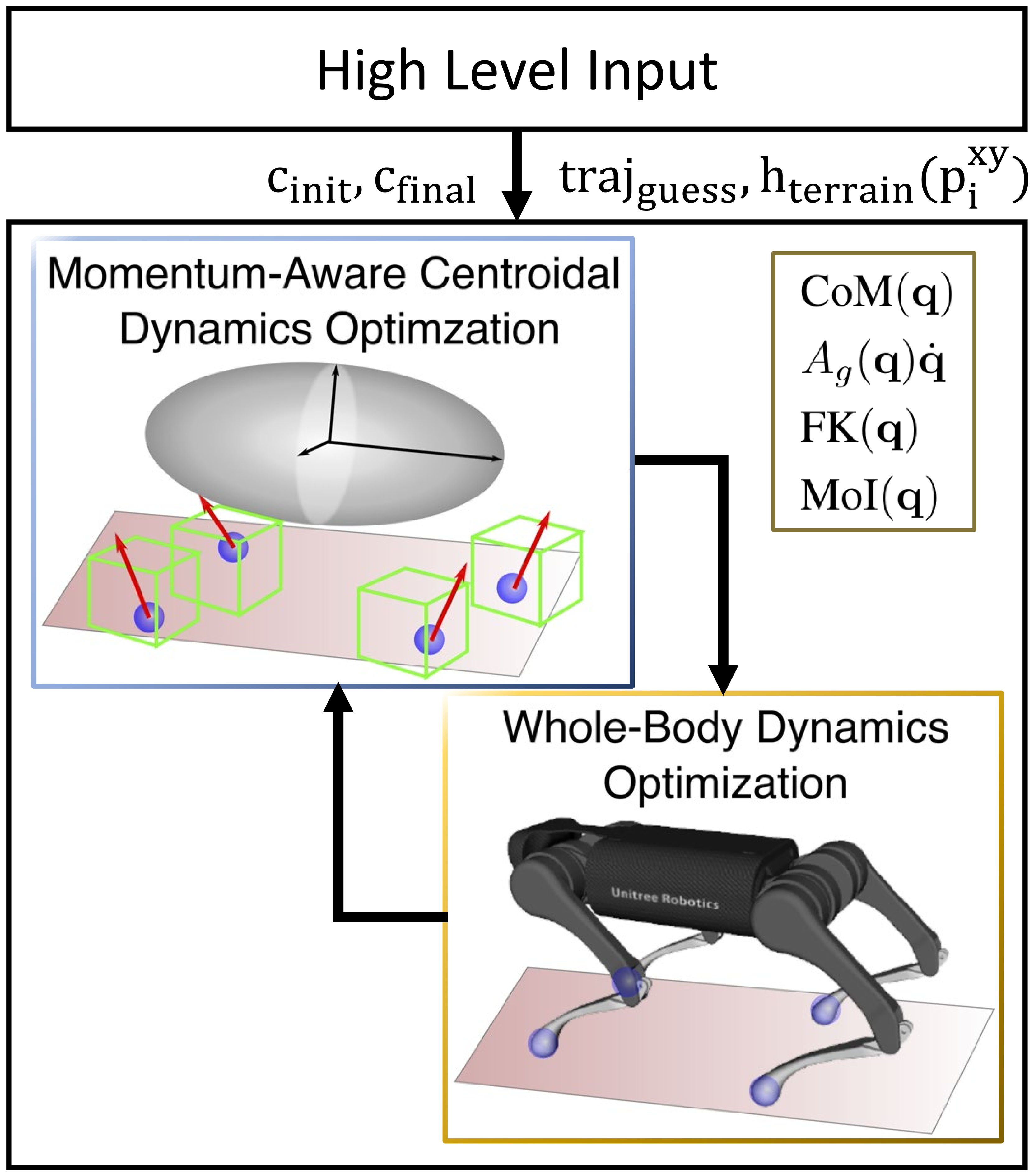
- dfasfdadf

References

dafdafadf


updates:
Centroidal Dynamics
Whole Body Dynamics
CoM pos
Footholds
Momentum
MoI
updates:


High Level Input
c_{init}, c_{final}
$traj_{guess}, h_{terrain}(p_i^{xy})$
um-Aware Centroidal
amics Optimization
Centroidal Dynamics
Force, terrain and RoM constraints
Whole-Body Dynamics
Optimization
Unitree Robotics
Iteration k+1:
Centroidal Dynamics
Force, terrain and RoM constraints
Whole Body Dynamics



High Level Input
c_{init}, c_{final}
$traj_{guess}, h_{terrain}(p_i^{xy})$
Momentum-Aware Centroidal Dynamics Optimzation
$\mathrm{CoM}(\mathbf{q})$
$A_g(\mathbf{q})\dot{\mathbf{q}}$
$\mathrm{FK}(\mathbf{q})$
$\mathrm{MoI}(\mathbf{q})$
Whole-Body Dynamics Optimization
Unitree Robotics