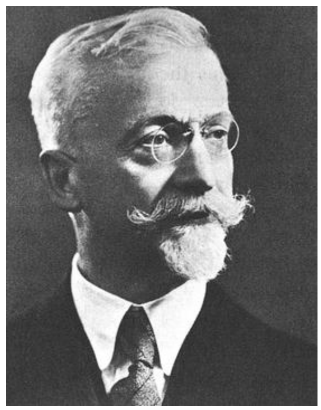
The derivative symbol $\frac{dy}{dx}$ is a *function*. Except when it isn't!

The French mathematician Élie Cartan helped to formalize the intuitive idea that dx and dy represent tiny changes in x and y. He called them *differentials* and constructed them using local linear approximation inspired by the definition of derivative.



Élie Cartan's magnificent moustache

Let $y = f(x)$ be a differentiable function with derivative $f'(x)$ (using Lagrange's notation). Let dx be a variable representing a nonzero real number. Define dy as:

$$dy = f'(x)dx.$$

Note that here, dy is a function of x **and** of dx since it depends on both of them. Defined in this way, dy and dx are called "differentials."

If you divide each side of the above equation by dx you get:

$$\frac{dy}{dx} = f'(x),$$

which is just the usual way we write the derivative! Thus, we can think of dy/dx the usual way—as a function of x—or, in differential form we can think separately of dy (as a function of both x and dx) and dx (as a number).