

1. Introduction/Motivation

The question the team aimed to answer is whether it is possible to calculate a planet's density and structure given its mass and stellar makeup. The goals behind this paper includes using ExoPlex to calculate a planet's possible density and structure, calculating a planet's structures from mass-proportions, and comparing them to stellar. The team then places the structure and composition in context of mineral proportions and orbital parameters. Once the mineral composition of the planet is found at various depths it can be compared to Earth. This is one potential factor in determining the habitability of a terrestrial planet. The planet the team chose Trappist 1e from the Trappist system. We used values sourced from NASA's Exoplanet Archive to find our initial values for metallicity and mass, and also as a way of determining the accuracy of our results.

Determining the structure and composition of a planet is key in understanding the possible conditions on the planet. While we cannot directly measure key factors such as mineral composition, core-mass fraction, etc for planets outside of our solar system, the elemental abundances (specifically metallicity) of the host star gives a lot of insight into the make up of surrounding planets. Not only does this affect the planets composition, but also radius, density, and core mass fraction for terrestrial planets. By calculating these values for various exoplanets, we are one step closer to finding an earth analog, and we are also able to extrapolate from these values to determine what the surface of these planets might be like (i.e., the presence of oceans, various rock formations, geological activity like plate tectonics, etc.).

2. Methods

In order to model the radius, density, and mineral composition of the planet, we used ExoPlex and changed input parameters such as the molar ratios of Fe/Mg and Si/Mg, the fraction of various elements found in the core versus the mantle, and the mass of the planet. We chose to focus on a tidally locked, terrestrial planet in the Trappist 1 system, Trappist 1e.

Before doing any calculations, we used Earth's stellar parameters and carefully tested the affect of changing certain input parameters on the planet's radius and core-mass fraction. Data from multiple test runs can be found in the additional document (step 4.) provided through the GitHub link. However, we were able to observe and generalize various trends associated with each parameter in ExoPlex. From our tests we found that changing the ratio of iron in the core to mantle and changing the core composition of Oxygen and Sulfur was directly proportional to the

change in radius. We also found that changing the FeMg core mass fraction and the core composition of Silicon was inversely proportional to the change in radius. We used this to guide our adjustments as we narrowed in on the radius of Trappist 1e.

Following this, we used the metallicity ($[\text{Fe}/\text{H}] = 0.040 \pm 0.080$) of Trappist 1 sourced from NEA (Gillon et. al 2017). Using this value we then used the relationship outlined in Griffith et. al, 2020 for the relationship between $[\text{Fe}/\text{H}]$ and $[\text{Mg}/\text{Fe}]$ based on location in the stellar bulge. We used Fig. 1 (shown below), and assumed Trappist 1 would fall along the division between High Ia and Low Ia regions (shown as a dashed line in the figure). Here we made broad estimates based on the plot to find an approximate value for $[\text{Mg}/\text{Fe}]$.

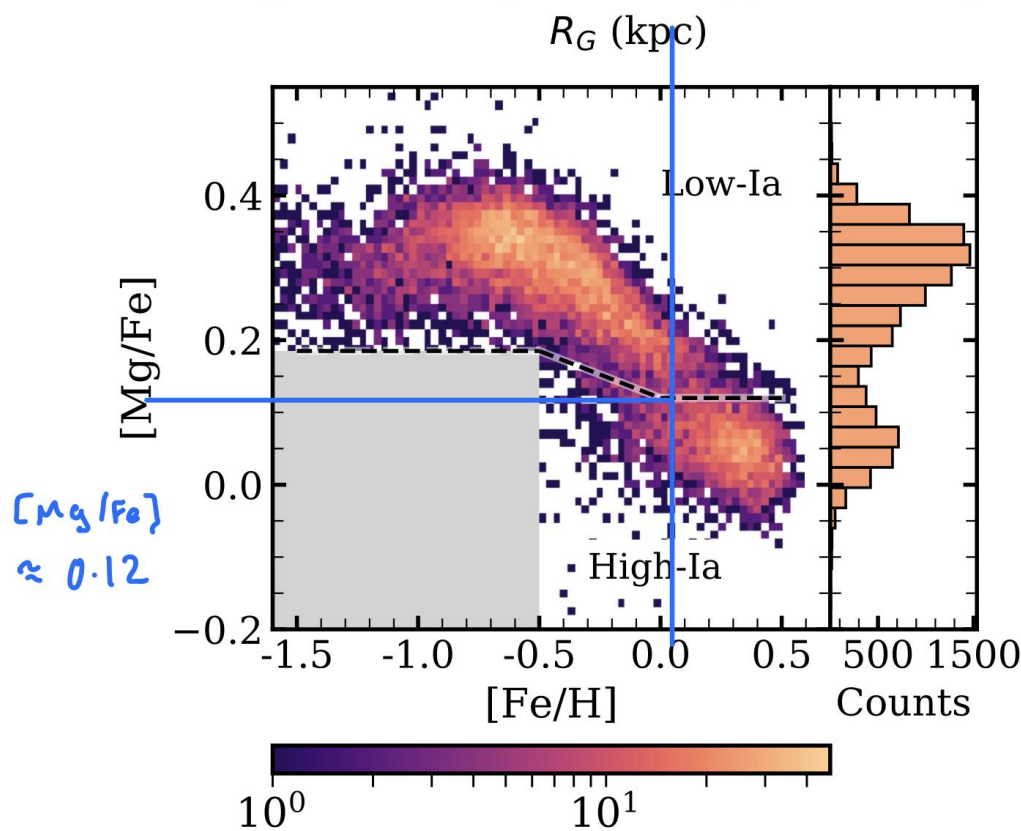


Fig. 1 (Griffith et al., 2020) This figure shows the density of stars in the bulge (excluding the region in grey). The dashed line indicates the division between high-Ia and low-Ia regions. Here, we used $[\text{Fe}/\text{H}] \sim 0.040$ and then found the correlated $[\text{Mg}/\text{Fe}]$ along the dashed line to be approximately 0.12.

This of course introduces some error, as the Gillon et al., 2017 calculation for the $[\text{Fe}/\text{H}]$ has error ± 0.080 , and we were also doing this visually. However, from our estimated $[\text{Mg}/\text{Fe}]$ value we used equations [1] and [2] to find the molar ratio Fe/Mg ($=0.68$) and also the log relation $[\text{Fe}/\text{Mg}] (= -0.12)$ for Trappist 1.

$$\frac{N}{M} = \frac{N_{\odot}}{M_{\odot}} * 10^{[N/M]}$$

Equation 1. Relationship between log mole definition for elemental abundance and molar ratio.

| Physicist | Astronomers working galaxies | Astronomers working on stars |
|----------------------------|---|---|
| Mole ratio between O and H | Log of mole ratio between O and H and add an offset of 12 | $\log \left(\frac{N_O/N_H}{(N_O/N_H)_{\odot}} \right)$ |
| N_O/N_H | $12 + \log(O/H)$ | $[O/H]$ |
| 5.3×10^{-4} | 8.73 | 0.0 |

Equation 2. Found in upper right cell of the figure. It shows the definition for the stellar astronomer's preferred definition of elemental abundance or ratio. This is a logarithmic definition relating the molar ratio of the given star and the accepted molar ratio of these elements in our Sun.

From Tables 1 and 2 in Griffith et al., 2020 we were able to find molar ratios Mg/H and then our desired value for Si/Mg. For this we also had to make some assumptions, as our calculated value for [Fe/Mg] was around -0.12, however, the closest value listed in the table was -13.4. Due to the major error associated with our previous assumptions we chose to adopt the value of [Fe/Mg]=-13.4 in order to find molar ratio Si/Mg=0.86. (These calculations were done by hand and can also be found in the document “hand calculations” found in the Github repository.)

We then used the calculated values for molar ratios Fe/Mg and Si/Mg along with the mass of Trappist 1e (mass= 0.772 Earth masses sourced from Grimm et al., 2018) as parameters in ExoPlex to model the composition of the planet. On the first run, this produced a result that was much higher than the anticipated radius (radius= 0.910 Earth radii from Grimm et al., 2018). We adjusted various parameters to attempt to lower the radius, but we eventually found that the largest change came from raising the stellar ratio Fe/Mg.

Once we were able to get the radius as small as possible, we reviewed the data output by ExoPlex that described the mineral composition. We plotted the abundance of each mineral as a function of depth for both Trappist 1e and the Earth for side by side comparison. This can be seen in Figures 2 and 3 below.

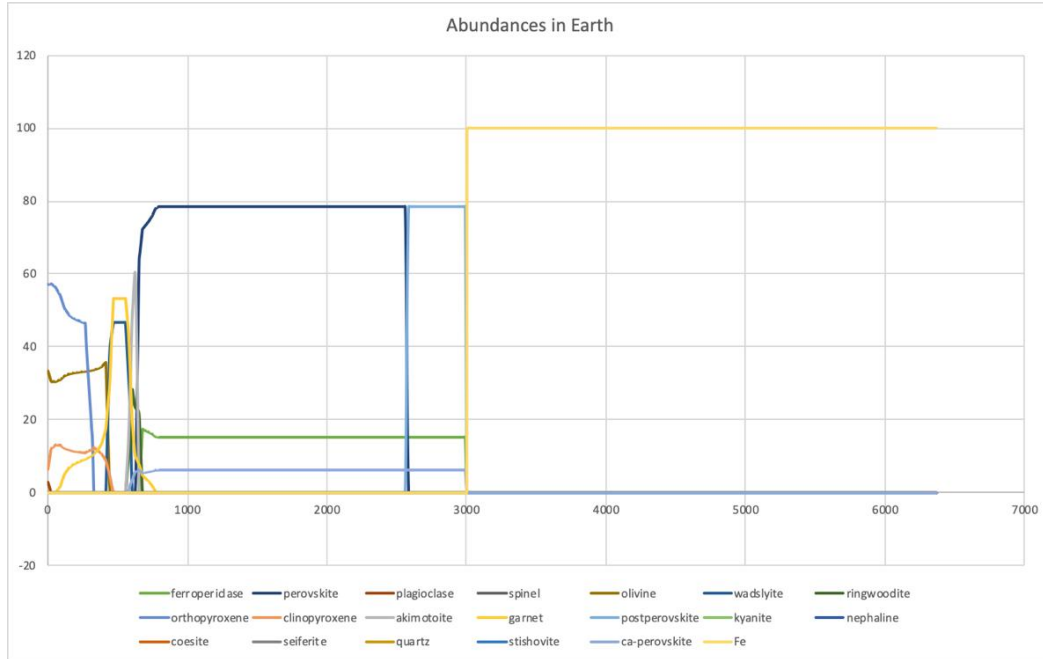


Figure 2, Mineral abundances in Earth as a function of depth

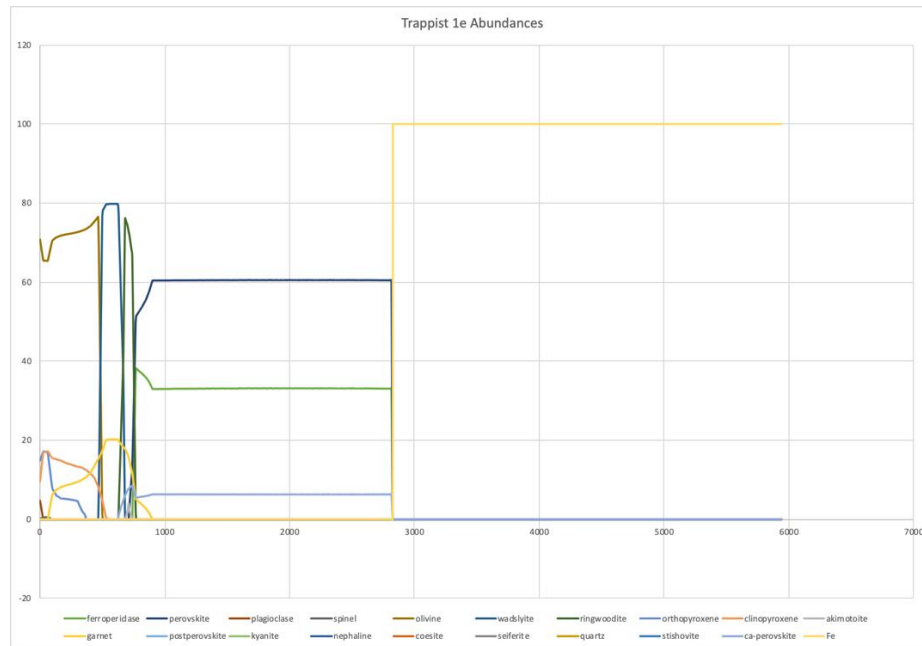


Figure 3, Mineral abundances in Trappist 1e as a function of depth

3. Results

Using a mass of 0.772 Earth mass and molar ratios $\text{Si/Mg}=0.68$ and $\text{Fe/Mg}=0.75$, the ExoPlex model gave us a planet with radius 0.932 Earth radii. This is not quite the 0.91 Earth radii that we used as reference from Grimm et al., 2018, however with the amount of assumptions made and

error in our calculation we were able to accept this error. Other sources calculated radii close to 0.92 Earth radii, so we were off by between 0.012 and 0.022 Earth radii. We chose to only input those variables (mass, Fe/Mg, and Si/Mg) as we found that since we needed to decrease our radius, any other variables that we changed made very limited changes to the radius. We determined that the only thing that will dramatically lower our radius to the 0.91 that we need is increasing the Fe/Mg ratio, and we think that it is unrealistic for Trappist 1e to have an Fe/Mg ratio higher than 0.75.

Further, we used that radius to calculate an approximate density. By assuming uniform density (which is not accurate, but in this case proved to be adequate), we calculated 5.06g/cm^3 (see hand calculation). This is close, but not quite the same as the density (5.646g/cm^3) calculated in Grimm et al., 2018. Given our assumptions, we determined that our calculation for the density was reasonable, but could be improved had we been able to get a more accurate value for the radius and not assumed uniform density.

The mineral composition of Trappist 1e appears to be similar to that of Earth, with a largely Iron core and some mix of Perovskite, Postperovskite, and what appears to be Kyanite or Ferroperidase in the mantle (this is debatable based only on our plots due to an issue with repeated colors in the key). One key difference is that the section in between the mantle and core in Earth that is entirely Postperovskite (seen in light blue on Fig. 2 and 3), is not present in Trappist 1e, rather it is mixed in. This could be due to an issue with our model as it is not entirely accurate and our core mass fraction assumptions were not clear, however, this may be an area that would be worth further investigation. The mineralogy suggests that Trappist 1e may have a similar core and mantle composition to Earth.

4. Discussion/Conclusions

After reviewing the results, the team found that Trappist 1e is a terrestrial planet and has similar mineral composition to the Earth. This could be due to the fact that the metallicity and elemental abundance of a planet's star has a huge impact on the abundance of its surrounding planets. Both the Sun and Trappist 1 have a similar makeup which leads to a similar makeup of the planets that surround these two stars. The team also found that the molar ratio of Fe/Mg, among other factors we can put into Exoplex, greatly affects the planet's radius and core/mantle ratio. The team found this when experimenting with the "warm-ups" of the project which can be found in the other document shared in this Github.

Accurately predicting the mineral composition of terrestrial planets is important in the search for habitable planets. While our assumptions resulted in a great potential for error, and the model has

flaws (such as possible issues with mineral abundance), this process allows us to get a better understanding of the makeup of other terrestrial planets.

5. References

Fulton et al., 2017
Seager et al., 2007
Chen and Kipping, 2016
Griffith et al., 2020
Gillon et al., 2017
Grimm et al., 2018
<https://exoplanets.nasa.gov/trappist1/>

6. Contributions

Payton Cassel - coding and calculations
Shannon McKinney - Powerpoint, presentation, and write-up
Bailee Wolfe - Coding and calculations, write-up
Maddie Englerth - Powerpoint and write-up
Richard Kane - Powerpoint, presentation, and write-up