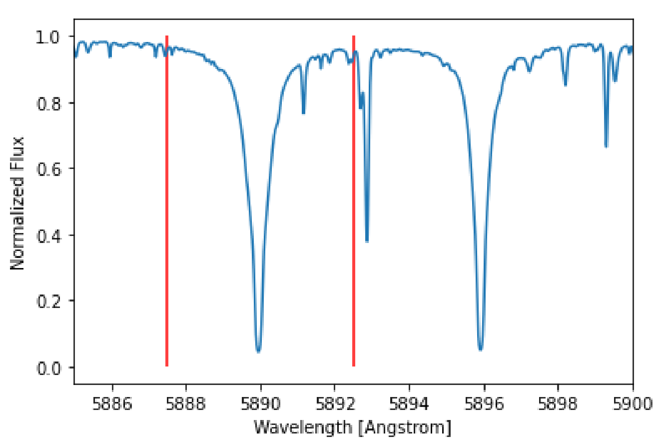
Measuring the Stellar Abundance of Sodium
Astronomy 5205 SP22
Payton Cassel, Shannon McKinney, Madison Englerth, Richard Kane, Bailee Wolfe

1. Introduction/Motivation

Measuring the elemental abundance of a star not only gives us a better understanding of stellar composition but also a possible clue as to the composition of its planets. Current research suggests that the stellar composition, or chemical structure of a star, has a great impact on the formation and structure of its orbiting planets. In particular, the metallicity of a host star reflects the composition of the core of its terrestrial planets (Schulze et al. 2020). The composition of a rocky planet is often defined by the relative abundance of its metals (most commonly Fe, Si, and Mg). Wang et al. 2019 shows that the bulk composition of Earth strongly reflects the photospheric abundances of the Sun. Understanding the elemental abundance of various host stars can allow us to approximately determine the chemical composition of rocky planets, which then allows us to determine if they are analogous or not to Earth. The composition of a planet and its core (along with other factors) is key in determining habitability. By calculating the abundance of Sodium in our sun, we are able to understand the process for calculating various abundances while also gaining insight into the distribution of Sodium in the Sun and planets of our solar system.

2. Methods

We first used the solar spectrum from BASS2000 to find the equivalent width of the Sodium doublet line at 5890 Angstroms as shown in Figure 1. Equivalent width (ew) is defined as the width of a rectangle with a height equal to that of continuum emission such that the area of the rectangle is equal to the area in the spectral line. Therefore, we defined the equivalent width of a line as the integrated area "above the curve" of an absorption line.



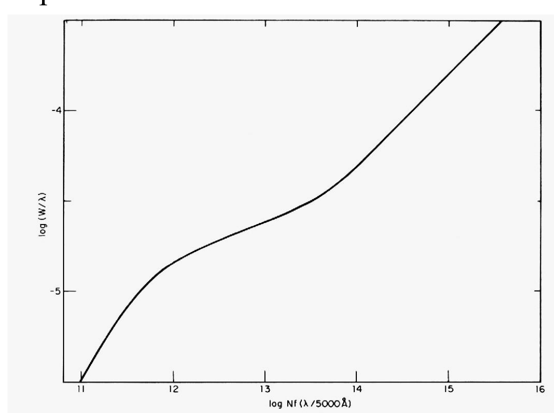


Figure 9.22 A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Figure 1

Figure 2

Next, we found the number density of sodium atoms based on a specific growth plot as shown by Figure 2. We plugged in the value for the equivalent width into the equation on the vertical axis (log[W/λ]) and used the number given to solve for the corresponding value on the horizontal axis. Once we had this value, we solved for N, which is the number density of sodium atoms, using the equation on the horizontal axis (logNf[λ/5000]) and assuming f = 0.65, where f is the oscillator strength for this transition at 5890 Angstroms.

Once we had the number density of sodium atoms we used the Boltzmann equation to estimate the ratio between ground and excited states. The Boltzmann equation is given as

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{E_2 - E_1}{kT}\right) \qquad \text{Eq (1)}$$

Where the subscripts 1 and 2 refer to the 3s and 3p states respectively, N is the number density, g is the number of separate states degenerate in energy, E is the energy, k is the Boltzmann constant, and T is the temperature.

We then used the Saha equation to estimate the ratio between neutral and ionized atoms. The Saha equation is given as

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e} \frac{Z_{II}}{Z_I} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi}{kT}\right) \qquad \text{Eq (2)}$$

Where m_e is the electron mass, Z_I and Z_{II} are partition functions equaling 2.4 and 1.0 respectively, P_e is electron pressure, and χ is the ionization energy equaling 5.1 eV.

Then we used both ratios to compute column density in the photosphere. The total number of sodium atoms is equal to the following equation:

$$N_1 \times \frac{N_2}{N_1} \times \frac{Na_{II}}{Na_I} \qquad \text{Eq (3)}$$

N_1 is measured from the growth curve and the ratios are the ratios of ground to excited states and neutral to ionized atoms.

Finally we found the sodium abundance relative to hydrogen. The column density of hydrogen is about 6.6x10^23 and the relative log abundance for hydrogen is set to 12. Therefore we can determine the sodium abundance relative to hydrogen using the following equation:

$$12 + log_{10}(N_{element}/N_H) \qquad \text{Eq (4)}$$

Where N is the number density.

3. Results

Our value for equivalent width was calculated to be 0.8347 Angstroms, which eventually gave us an N_1 value of 8.24x10^14. When doing the calculation for equation 1, the values in the equation were chosen for the following reasons: g_1 and g_2 are 6 and 2 respectively because they represent the first excited state (outer electron in 3s state) and ground state (electron in 3p state). The change in energy is calculated by setting it equal to h (planck's constant) times c (the speed of light) over the wavelength of our transition (5890 Angstrom). T was chosen to be 5780K since this is the temperature of the sun. Our N_2/N_1 ratio was calculated to be 200 ionized for each neutral.

The ratio between ionized and neutral atoms (N_{II}/N_I) was calculated to be 0.05. Therefore, when plugging the values for N_1, N_2/N_1, and N_{II}/N_I into equation 3 we get N_{total} = 2.14 x 10^18.

The astronomer's ratio of Na to H is calculated by comparing the abundance of Sodium in the star measured to the abundance measured in the Sun. This was calculated to be about 0.6. The physicist's ratio is just the mole ratio of Na to H and this was calculated to be 3.3 x 10^-7.

4. Discussion/Conclusions

We realized after finishing our project that we had misused some of our equations which led to some of our results being skewed. For example, Eq (3) should be N1 multiplied by each of the ratios plus one rather than just the ratios alone. This caused our total number of Sodium atoms to be off, which in turn caused physicist's ratio to be off by a factor of ten. We also, initially, used the Astronomer's ratio for galaxies rather than the Astronomer's stellar ratio. However, we tried to correct this. Our value for the Astronomer's ratio is reasonable as it is close to 0, which we expect. We expect the log of the ratio to be zero, since we are comparing our measurement of the Sodium in the Sun to the expected value of the Sun. However, our value is off by 0.6. There is definitely some error in our calculation, which leads back to our issue with calculating the total number of Sodium with Eq (3). The Sodium abundance in the Sun is dominated by N1 or ground state Sodium, which meant that despite using the wrong equation, our value came out to be close to the accepted value.

Measuring elemental abundance in stars is very important in understanding the composition and formation of planets. The composition of a planet's atmosphere can be measured similarly to that of stars by inspecting their spectra while they are transiting in front of their host star. However, this is easier said than done as many planets have very thin and/or cloudy atmospheres which means that we are not able to probe deeper into their atmosphere or get clear absorption lines for elements that are below the upper layers of the atmosphere. It has been predicted that hot gas giant planets (Hot Jupiters) have Sodium in their atmospheres, but it was not able to be confirmed until recently. Two planets were discovered to have little to no cloud cover and measured to have Sodium in their atmospheres. Sodium can only be clearly observed in planets without clouds in their atmosphere. Wasp-96b and Wasp-39b were observed to have a clear absorption line for Sodium, with abundance similar to that found in atmospheres in our Solar

System (Nikolov et al., 2018). Without being able to directly measure abundances of planets, we rely on the measurements gathered from their stars. This study only further confirms that the elemental abundance of a planet's host star directly correlates to its composition.

5. References

(Schulze et al. 2020) https://iopscience.iop.org/article/10.3847/PSJ/abcaa8#psjabcaa8s3
(Wang et al. 2019) https://www.sciencedirect.com/science/article/pii/S0019103518305670
(Nikolov et al. 2018)
https://www.nature.com/articles/s41586-018-0101-7.epdf?author_access_token=Crer6ZBIsz-psreNAfiHz9RgN0jAjWel9jnR3ZoTv0NxDidCwYU7JuQKe4oRrxtp5mDXHhVWhs0PzZsP5GEZMd6jxdpz09kaCIqQeXvHWN-rsTny3FmqAHHg1pf5LbB-8-IEAaUgIQRPFDmGpcYxqQ%3D%3D

6. Contributions

Payton Cassel - Coding and calculations
Shannon McKinney - Coding and calculations
Bailee Wolfe - Presentation, write-up
Maddie Englerth - Presentation, write-up
Richard Kane - Write-up