Homework 1 due Thursday April 18 11:59pm PST

PSTAT 176/276, Spring 2024

1. Consider a Binomial Tree for the stock price process $\{S_n, n = 0, 1, 2, 3\}$ with $S_0 = 16$, $u = 1.25$, $d = 0.75$, and $N = 3$ periods (and so, there are a total of $8(= 2^3)$ scenarios). Interest rate is $r = 0.1$. We are interested in an *Asian*-type option that pays the *positive* part

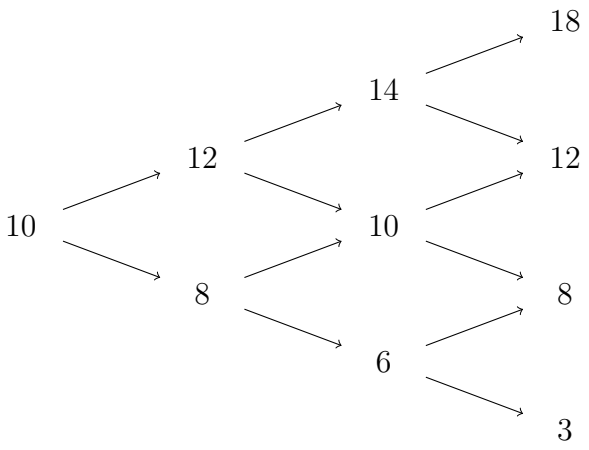
$$\left(\frac{1}{3}Y_3 - S_3\right)_+$$

of difference between the historical average $A_3 = Y_3/3$ and the spot price S_3 at time 3, where $Y_3 = S_1 + S_2 + S_3$ is the sum of prices at time $1, 2, 3$. Thus, to determine the payoff we compare the average asset price A_3 to the terminal stock price S_3 and collect the difference.

(A) Determine the no-arbitrage price of this contingent claim today: V_0 (time-0 price).

(B) Also determine the replicating portfolios for each time step $t = 0, 1, 2$ along the scenario $\omega = THT$.

2. Consider the following Binomial Tree with $N = 3$ periods:

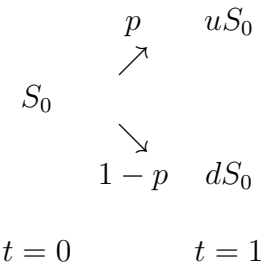


Note that in this tree the movements u and d vary throughout. We assume interest rate of $r = 0.05$ per period. Our goal is to price the *Mountain Altiplano Call* with strike price $K = 11$ in the above model.

Mountain Altiplano contracts are those "in which a vanilla option [here a Call] is combined with a compensatory coupon payment if the underlying security never reaches its strike price during a given period." It was developed by Société Générale in 1990's.

We shall assume that the compensatory coupon is worth \$3. Using above parameters, find the no-arbitrage price of the Call at $t = 0$. As part of your solution, clearly explain the payoff formula of this contract for a scenario ω.

3. {For **PSTAT 276** students ONLY} Consider the one-period binomial model



with $d < 1 + r < u$ where r is the (simple) interest rate for 1 period. We also assume $d < 1, u > 1$. We wish to price *at-the-money* Call option on S_1 with payoff $V_1 = (S_1 - S_0)_+$.

(a) Calculate the no-arbitrage price V_0 of the option. Your answer should be an algebraic expression involving S_0, p, r, u, d (some of the above may not be needed).

(b) Calculate the discounted expected payoff π of the option under the physical probability measure $\mathbb{P}$, i.e., p and $1-p$. Your answer should be an algebraic expression involving S_0, p, r, u, d (some of the above may not be needed).

(c) (Sensitivity Analysis) How does π change as up-probability p increases? How does V_0 change as p increases? Hint: compute partial derivative of π (or P) as a function of p and check its sign; If the derivative is positive, it is increasing.

(d) How does π change as interest rate R increases? How does P change as R increases?

(e) Discuss how the above answers differ for π vs V_0.