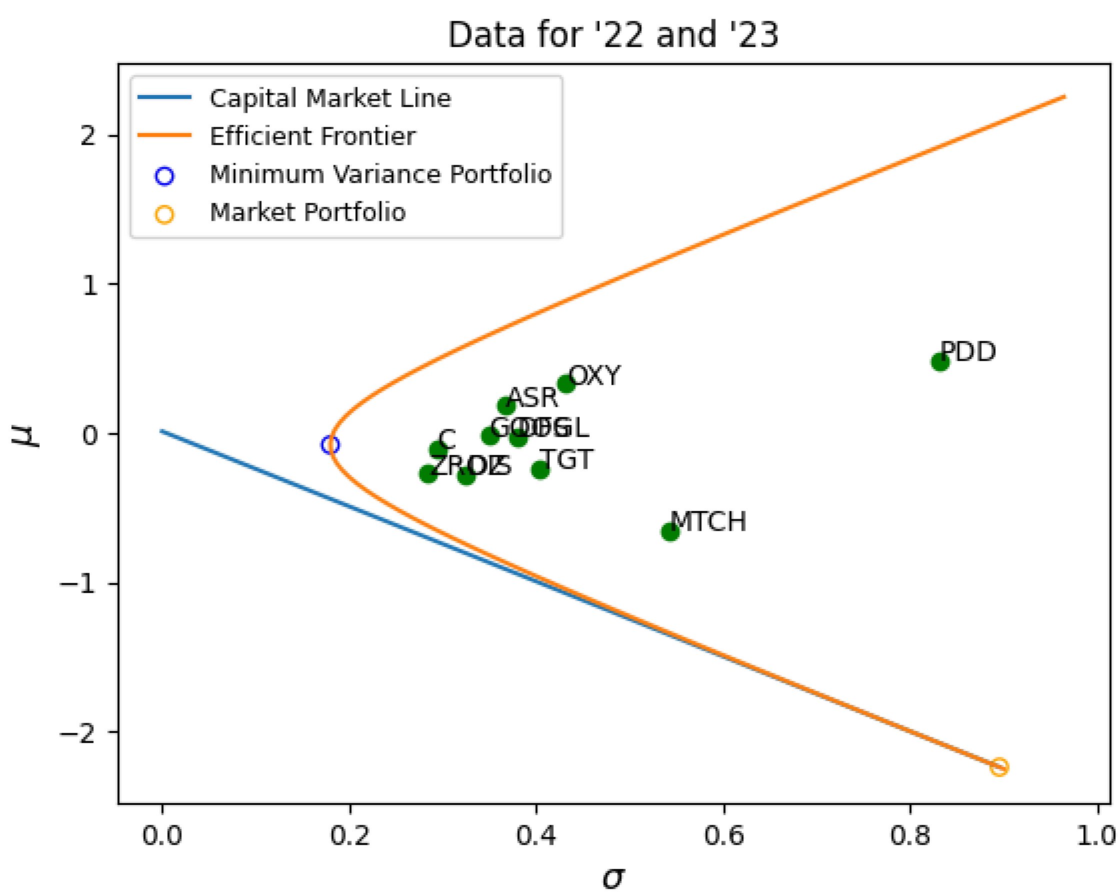
Markotwitz Portfolio Optimization for 10 Stocks

This project seeks to perform Markowitz portfolio optimization on a bucket of stocks labeled as "10 of the Best Stocks to Buy for 2024" by US news report (https://money.usnews.com/investing/articles/best-stocks-to-buy-this-year). The goal is to use market data through December 2023 to to find the maximum Sharpe Ratio portfolio, and evaluate the portfolio's performance for 2024 through May.

Preliminary: Data Choice

The prompt suggested to use data from 2022 through 2023 to perform the folowing anaylsis, but this produced undesired results. Since 2022 was a poor year for stock performance, as the Fed had started to raise interest rates, the minimum variance portfolio had a negative return. This led to complications with the capital market line and the conclusion that the portfolio should be 100% bonds. I chose to reject this result and proceed with only 2023 data to reach a more interesting conclusion. This is a valid approach since the market regime has changed significantly since 2022, and I believe there are better, or at least more interesting to explore, portfolios that can be found. Additionally, it is important to constantly rebalance the portfolio, so focusing on more recent data will give a more accurate estimate of the true Σ matrix. The results of 2022-2023 data are shown below:



Import Packages and Download Data

In [ ]:

```python
import yfinance as yf
from datetime import date
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

data = yf.download(
    "GOOGL DFS DIS PDD OXY MTCH ASR TGT ZROZ C",
    start = '2023-01-01',
    end = '2023-12-31'
)

data.index = data.index.to_series().apply(lambda x: x.date())
data = data['Close']
```

```
[*********************100%%***********************]  10 of 10 completed
```

a) Compute Daily Log Returns

In [ ]:

```python
# Take the ratio of the previous day's price
log_return = data / data.shift(1)

# Drop NA so log can be taken
log_return.dropna(inplace=True)

log_return = np.log(log_return)

log_return
```

Out[ ]:

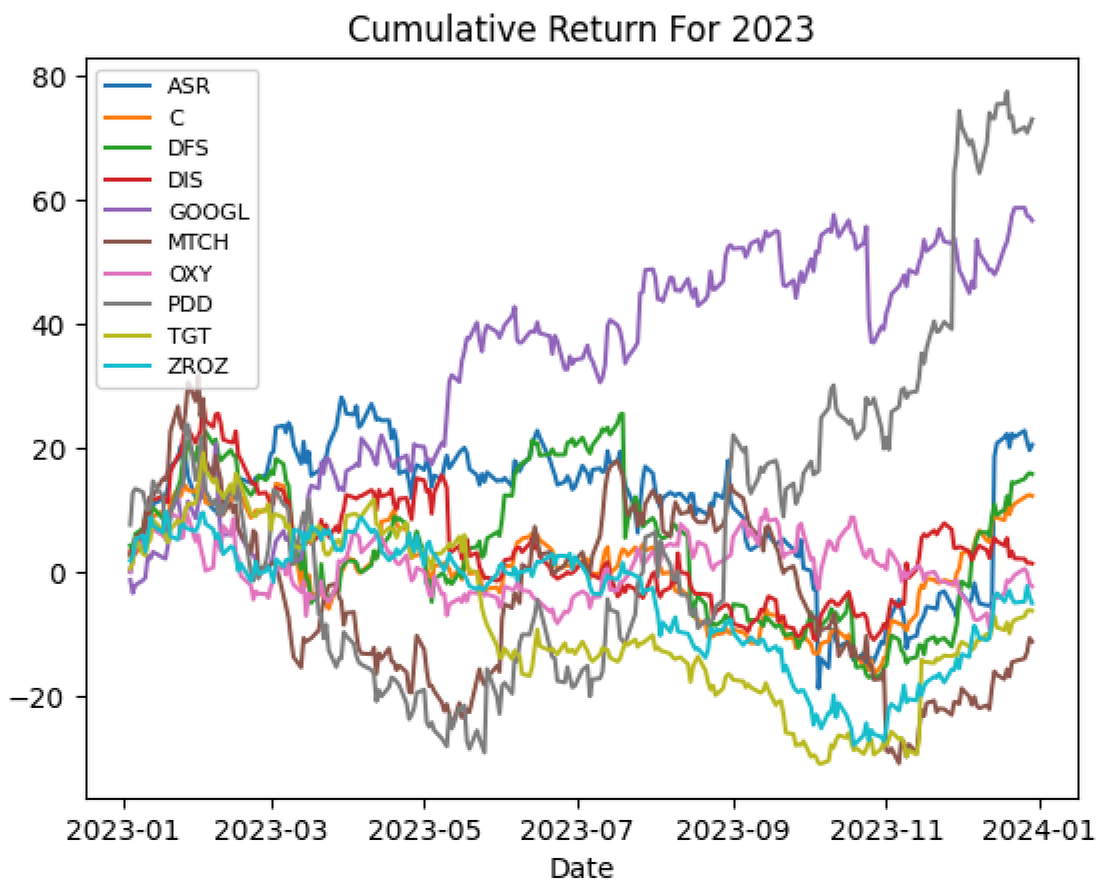
Ticker	ASR	C	DFS	DIS	GOOGL	MTCH	OXY	PDD	TG
Date									
2023-01-04	0.001475	0.025449	0.043182	0.033272	-0.011738	0.028807	0.000982	0.074412	0.00650
2023-01-05	0.023464	-0.004482	-0.017130	-0.000653	-0.021575	0.005663	0.018001	0.046874	0.01003
2023-01-06	0.026203	0.011907	0.034457	0.021525	0.013138	-0.003536	0.024293	0.005538	0.03747
2023-01-09	0.009343	0.004850	0.008119	0.009010	0.007756	0.035262	0.005163	-0.004596	-0.02401
2023-01-10	0.004620	0.013580	0.016517	0.008301	0.004534	0.024765	0.001092	-0.015084	-0.00243
...	...	...	...	...	...	...	...	...	..
2023-12-22	-0.004464	0.005125	-0.001531	-0.010927	0.007591	0.001706	0.006103	-0.015385	0.00593
2023-12-26	0.008442	0.009393	0.005929	-0.000769	0.000212	0.005101	0.010307	0.005109	0.00590
2023-12-27	-0.014922	0.002335	0.005270	-0.006287	-0.008159	0.012081	-0.006532	-0.005316	0.00952
2023-12-28	-0.011168	0.001165	0.002936	0.000221	-0.000998	0.025367	-0.017685	0.006211	0.00112
2023-12-29	0.007573	-0.001554	-0.001511	-0.001218	-0.003858	-0.006282	-0.004345	0.006583	-0.00084

249 rows × 10 columns

b) Estimate Annualized Log-Return

In [ ]:

```
# Obtain cumulative returns
cumulative_return = (np.exp(log_return.cumsum()) - 1) * 100

cumulative_return.plot()
plt.title('Cumulative Return For 2023')
plt.legend(fontsize=8)
plt.show()
```



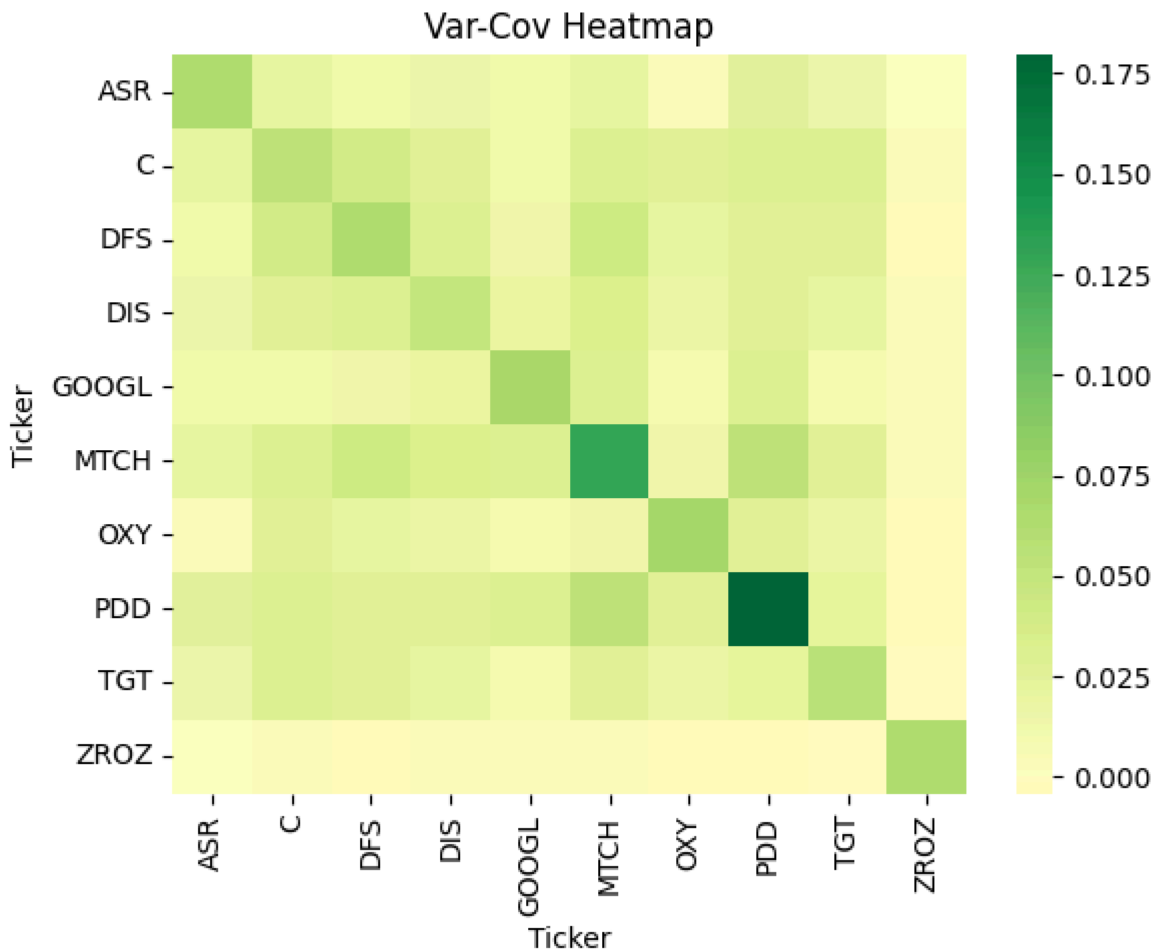
Estimate Annualized Variance-Covariance Log-Return

In [ ]:

```
from sklearn.covariance import MinCovDet
import seaborn as sns

# Robust Covariance Estimation using MCD method
mcd = MinCovDet()

mcd.fit(log_return)

robust_vcov = pd.DataFrame(mcd.covariance_, columns=data.columns, index = data.columns)
annual_robust_vcov = robust_vcov * 252 # annualize daily value

robust_mu = pd.DataFrame(mcd.location_, index=data.columns, columns = [''])
annual_robust_mu = robust_mu * 252 # annualize daily value

### Normal Covariance Estimation for testing purposes
# robust_vcov = pd.DataFrame(log_return.cov(), columns=data.columns, index = data.columns
# annual_robust_vcov = robust_vcov * 252

# robust_mu = pd.DataFrame(log_return.mean(), index=data.columns, columns = [''])
# annual_robust_mu = robust_mu * 252
```

```python
print('####### Robust Mu')
print(annual_robust_mu)
print('')

print("####### Robust Variance-Covariance Matrix")
print(annual_robust_vcov)

sns.heatmap(annual_robust_vcov, cmap='RdYlGn', center=0)
plt.title('Var-Cov Heatmap')
plt.show()
```

```
#######Robust Mu

Ticker
ASR      0.086148
C        0.247485
DFS      0.523170
DIS      0.027067
GOOGL    0.470387
MTCH    -0.089099
OXY      0.288498
PDD      0.045258
TGT     -0.196055
ZROZ     0.164641

#######Robust Variance-Covariance Matrix
Ticker       ASR         C       DFS       DIS     GOOGL      MTCH       OXY  \
Ticker
ASR     0.066095  0.020249  0.012265  0.015035  0.012951  0.020309  0.003832
C       0.020249  0.052071  0.038356  0.026889  0.011924  0.030869  0.027403
DFS     0.012265  0.038356  0.062314  0.029610  0.013339  0.043649  0.020633
DIS     0.015035  0.026889  0.029610  0.049231  0.020205  0.032799  0.016628
GOOGL   0.012951  0.011924  0.013339  0.020205  0.068506  0.029213  0.008884
MTCH    0.020309  0.030869  0.043649  0.032799  0.029213  0.129090  0.013732
OXY     0.003832  0.027403  0.020633  0.016628  0.008884  0.013732  0.069399
PDD     0.023844  0.028580  0.027731  0.027699  0.029834  0.052139  0.025607
TGT     0.015592  0.029010  0.025978  0.021467  0.010151  0.027747  0.016482
ZROZ    0.000370  0.002169 -0.004200  0.001957  0.001946  0.003409 -0.003696

Ticker       PDD       TGT      ZROZ
Ticker
ASR     0.023844  0.015592  0.000370
C       0.028580  0.029010  0.002169
DFS     0.027731  0.025978 -0.004200
DIS     0.027699  0.021467  0.001957
GOOGL   0.029834  0.010151  0.001946
MTCH    0.052139  0.027747  0.003409
OXY     0.025607  0.016482 -0.003696
PDD     0.179663  0.022861 -0.003973
TGT     0.022861  0.056029 -0.000903
ZROZ   -0.003973 -0.000903  0.064782
```



c) Plot the Minimum Variance Curve

Minimum Variance Portfolio

We seek to solve the following problem:

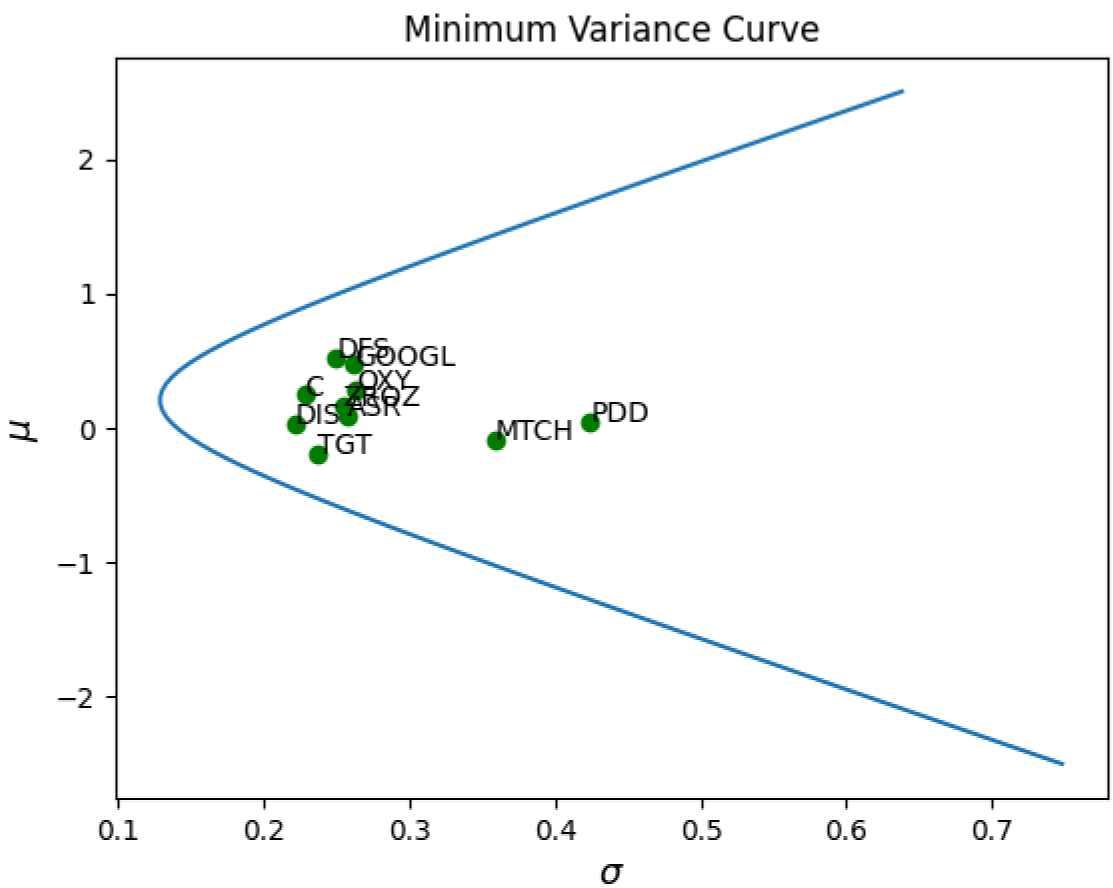
$$\min_w w^T \Sigma w \quad \text{subject to} \quad w^T 1 = 1$$

which has the solution:

$$w^\star = (1^T \Sigma 1)^{-1} \Sigma^{-1} 1 \quad \text{with mean} \quad m^T w^\star = (1^T \Sigma 1)^{-1} m^T \Sigma^{-1} 1$$

In [ ]:
```
# Minimum Variance Curve
def min_std_curve(mu, means, vcov):
    "Takes a mu (y) value and returns the corresponding minimum std (x)"
    iC = np.linalg.inv(vcov)
    ones = np.ones(len(means.values))
    meanstil = np.hstack((ones.reshape((-1,1)), means.values))
    mutil = np.array([1., mu]).reshape((-1,1))

    x = np.sqrt(
        mutil.T @ np.linalg.inv(meanstil.T @ iC @ meanstil) @ mutil
    )
    return x[0][0]

mus = np.linspace(-2.5, 2.5, 100)

x = [min_std_curve(mu, annual_robust_mu, annual_robust_vcov) for mu in mus]
```

```
plt.plot(x, mus)

plt.scatter(np.sqrt(np.diag(annual_robust_vcov)), annual_robust_mu, color='g')
plt.xlabel('$\sigma$', fontsize=13)
plt.ylabel('$\mu$', fontsize=13)
for i in range(10):
    plt.text(np.sqrt(np.diag(annual_robust_vcov))[i], annual_robust_mu.values[i], data.c
plt.title("Minimum Variance Curve")
plt.show()
```

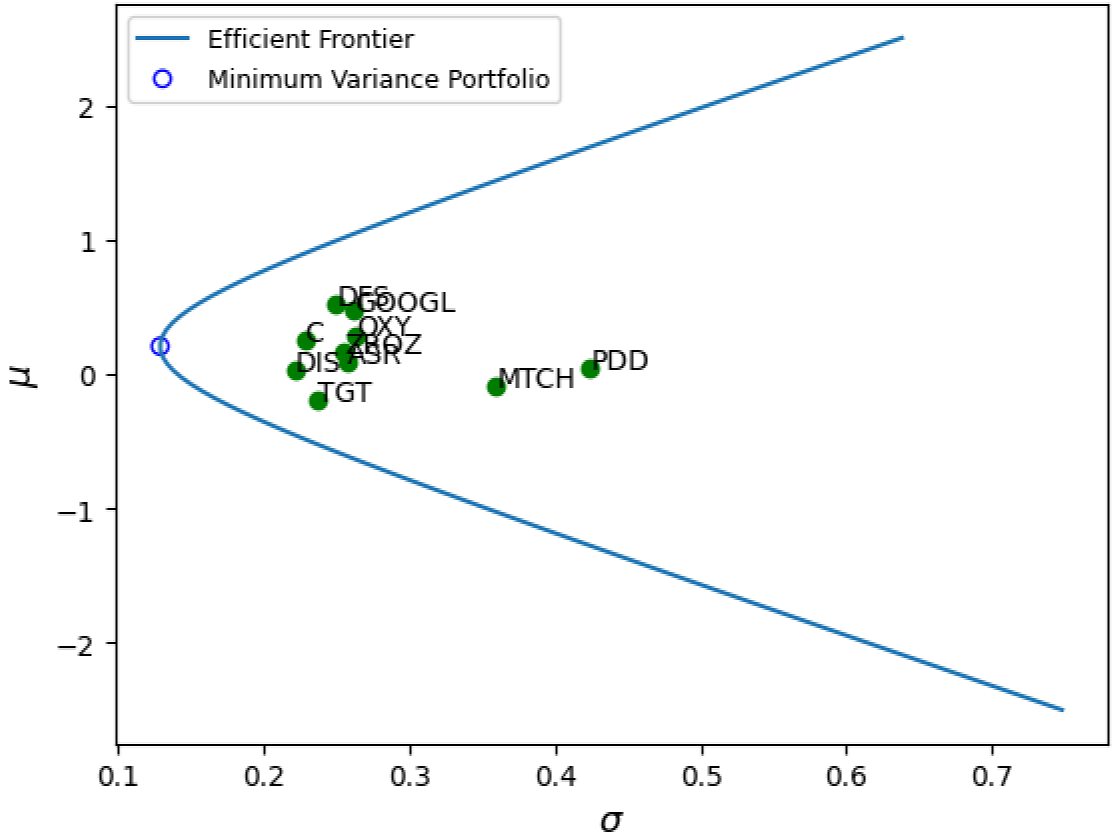


d) Minimum Variance Portfolio

In [ ]:

```
ones = np.ones(len(annual_robust_mu.values))
iC = np.linalg.inv(annual_robust_vcov)
w_star = (np.dot(ones.T , np.dot(iC, ones)))**-1 * np.dot(iC, ones)

mu_star = np.dot(w_star.T, annual_robust_mu)
v_star = np.dot(w_star.T, np.dot(annual_robust_vcov, w_star))

print(f"sigma_star: {round(np.sqrt(v_star),3)}, mu_star: {round(mu_star[0],3)}")

plt.plot(x, mus, label='Efficient Frontier')
plt.scatter(np.sqrt(v_star), mu_star, facecolors='none', edgecolors='b', label='Minimum

plt.scatter(np.sqrt(np.diag(annual_robust_vcov)), annual_robust_mu, color='g')
plt.xlabel('$\sigma$', fontsize=13)
plt.ylabel('$\mu$', fontsize=13)
for i in range(10):
    plt.text(np.sqrt(np.diag(annual_robust_vcov))[i], annual_robust_mu.values[i], data.c
plt.legend(fontsize=9)
plt.show()
```

```
sigma_star: 0.129, mu_star: 0.205
```



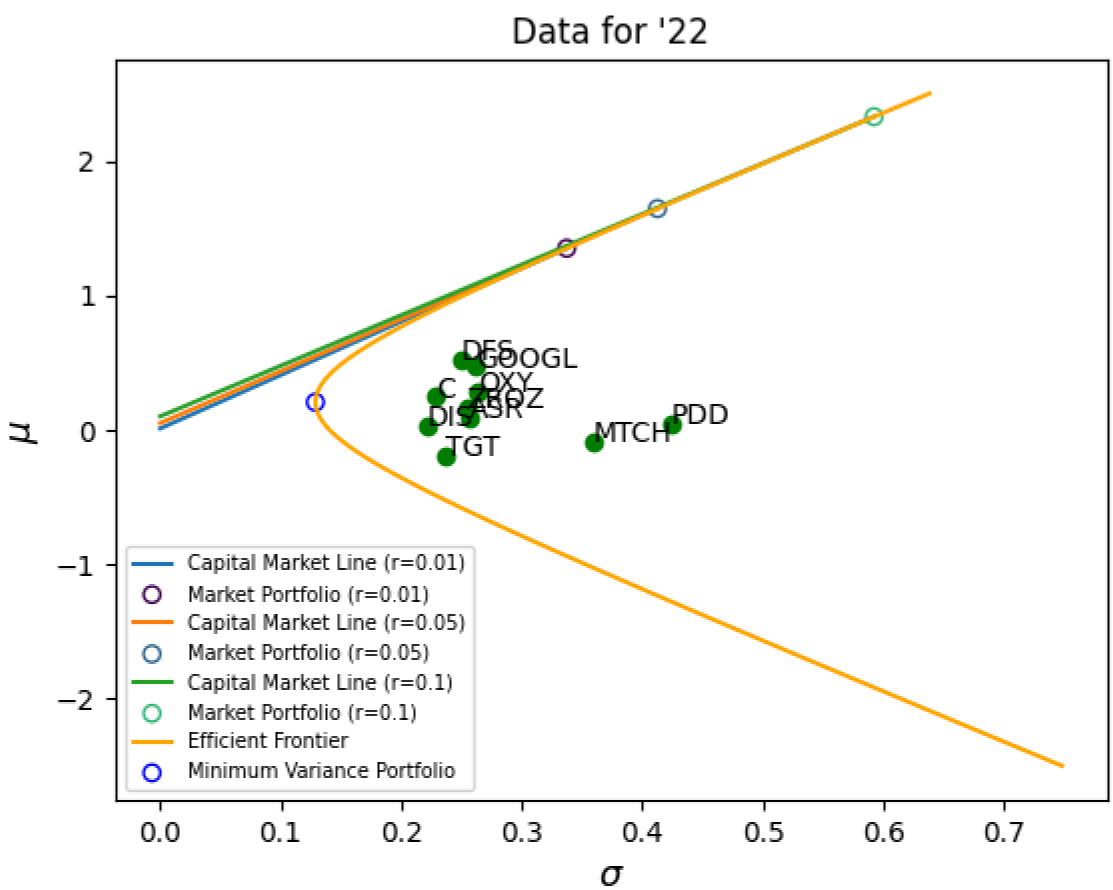
e) Capital Market Line

In [ ]:

```
import matplotlib.cm as cm

# Market Portfolio
def market_portfolio(mus, vcov, r):
    """
    Determine the mean and variance of the tangent (market) portfolio
    given a risk free rate r
    """

    iC = np.linalg.inv(vcov)
    mus = np.array(mus).reshape(-1,1)
    ones = np.ones(len(mus)).reshape(-1,1)

    w_m = (iC @ (mus - r*ones) / (ones.T @ iC @ (mus - r*ones))).flatten()
    mu_m = (np.dot(w_m.T, mus))[0]
    v_m = (np.dot(w_m.T, np.dot(vcov, w_m)))

    return w_m.flatten(), mu_m, np.sqrt(v_m)


rs = np.array([.01, .05, .1])
w_m = np.zeros((len(annual_robust_mu.values), len(rs)))
mu_m = np.zeros(len(rs))
sigma_m = np.zeros(len(rs))

for i, r in enumerate(rs):
    w_m[:, i], mu_m[i], sigma_m[i] = market_portfolio(annual_robust_mu, annual_robust_vc


for i in range(len(rs)):
    # Print Results of different market portfolios
    print(f"Market portfolio for r={rs[i]}: sigma_m={round(sigma_m[i],2)}, mu_m={round(m
```

```
    # Plot CAPM lines at diferent risk free rates
    sigs = np.linspace(0,sigma_m[i],100)
    cap_market_line = [rs[i] + (mu_m[i] - rs[i]) * sig / sigma_m[i] for sig in sigs]
    plt.plot(sigs, cap_market_line, label=f'Capital Market Line (r={rs[i]})')

    # Plot market portfolio
    plt.scatter(sigma_m[i], mu_m[i], facecolors='none', edgecolors=cm.viridis(i / len(rs


# Plot Results
plt.plot(x, mus, label='Efficient Frontier', color='orange')
plt.scatter(np.sqrt(v_star), mu_star, facecolors='none', edgecolors='b', label='Minimum


plt.scatter(np.sqrt(np.diag(annual_robust_vcov)), annual_robust_mu, color='g')
plt.xlabel('$\sigma$', fontsize=13)
plt.ylabel('$\mu$', fontsize=13)
for i in range(10):
    plt.text(np.sqrt(np.diag(annual_robust_vcov))[i], annual_robust_mu.values[i], data.c
plt.legend(fontsize=7)
plt.title("Data for '22")
plt.show()
```

```
Market portfolio for r=0.01: sigma_m=0.34, mu_m=1.35
Market portfolio for r=0.05: sigma_m=0.41, mu_m=1.64
Market portfolio for r=0.1: sigma_m=0.59, mu_m=2.33
```



Interpretation of the Capital Market Line

The capital market line is a tangent line between the risk free rate (μ=r, σ=0) and the efficient portfolio frontier. The intersection of the CAPM line and the effficient frontier is known as the "market portfolio," which represents the highest Sharpe Ratio portfolio available at that risk free rate. Although the efficient frontier reprsents all portfolios with the lowest variance for a given mean, this does not consider the available risk free asset. Thus, the market portfolio has the highest available Sharpe Ratio (also the slope), but so does any point on this tangent line. It follows that an investor can obtain the maximum Sharpe Ratio and also lower their risk by taking a portfolio somewhere on the capital market line. A portfolio on this line has $w_1^\star$ of the risk free asset and $w_2^\star$ of the market portfolio, where $w_1^\star$ + $w_2^\star$ = 1. To determine the overall weights of the individual stocks, we take $w_2^\star$ * w_m where $w_m = [w_1, \ldots, w_n]^T$.

My final portfolio recommendation depends on the risk free rate. If r=.01, I would recommend a full allocation to the market portfolio; if r=.05, I would recommend 25% of the risk free asset; if r=.1 I would recommend 50% of the risk free asset. This is based on the fact that 2023 was a great year and although the market portfolio suggests a much higher return, a 10% return on the risk free asset is about the average annual market return with no risk, and thus too good to turn down.

For Final Project:

- Add equations
- Add nice table with list of stocks at the start