Math 104C Homework #1

Due on Apr 10 by 11:59 PM

Topics: (04/02 – 04/09) Evaluation of polynomials, Floating point arithmetic, Cubic splines

Video: How to submit homework on Gradescope or copy and paste (`https://youtu.be/quBWbQ5opT0`)

Comment on submission and quiz coverage:

- Problems that **may** not be covered before the deadline: None
- If some problems are not covered before the deadline, they will be excluded from HW grading and the up-coming quiz. But they, only the excluded ones, can be asked for the next quiz.
- You can check whether these problems are covered by briefly skimming through the notes.

I. For presentation

1. (Exploration) Answer the following.

   (a) (Computation) Write a code for a polynomial evaluation. (You should really try your own code.) Use it to evaluate $P(x) = 1 + x + \cdots + x^{50}$ at $x = 1.00001$. Find the error of the computation by comparing with the equivalent expression $Q(x) = \left(x^{51} - 1\right)/(x - 1)$.

   (b) Convert the repeating binary number $0.1\overline{000111}_{(2)}$ to (i) a base 10 fraction (*Hint: Use the same trick as in middle school for converting a repeated decimal fraction to a rational number.*) and (ii) to a base 16 fraction.

2. (Exploration) Determine the double-precision floating point number fl(20.1) and find its machine number representation.

3. (Exploration) (a) Come up with a different expression that computes $y = x - \sin x$ without suffering from loss of significance. (*Hint: Use Taylor polynomial of an appropriate degree for* $\sin(x)$ *and use a similar idea to nested multiplication*) (b) Write and test a subroutine that accepts a machine numbers x (i.e., powers of 2) and returns the value $y = x - \sin x$ using the devised formula. (c) Explain why you believe that does not involve severe loss of significance.

4. (Exploration) Prove the following. Let f'' be continuous in $[a, b]$ and let $a = t_0 < t_1 < \cdots < t_n = b$. If S is the natural cubic spline interpolating f at the knots t_i for $0 \le i \le n$, then

$$\int_a^b [S''(x)]^2\, dx \le \int_a^b [f''(x)]^2\, dx.$$

(*Hint: Let* $g \equiv f - S$ *and examine* $\int_a^b (f'')^2\, dx$. *If you come across the quantity* $\int_a^b S''g''dx$, *consider using integration by parts. Natural spline conditions and* $S''' \equiv$ *(const) will be used.*)

II. Not for presentation

The following problems are for your own study, but not for presentation. They have been already discussed during lectures.

5. (Formation) Answer the following.

   (a) Give the number of multiplications and additions when Horner's algorithm evaluate a polynomial of degree $d \in \mathbb{N}$.

   (b) Give the definition of the machine epsilon.

   (c) We cannot carry out computations with smaller number than machine epsilon. (True/False)

   (d) Give the machine epsilon of IEEE 754 double precision. How is it determined?

   (e) Give the smallest number that can be represented by IEEE 754 double precision. How is it determined? How is it determined?

   (f) $11.3 - 11 - 0.3$ can be nonzero on the computer if IEEE Floating Point system is used. (True/False)

   (g) Give a situation where significant digits can be lost. And discuss whether it can be avoided or it can never be avoided.

End of homework