Homework # 3

1. (2 points each part) Are the following initial value problems stiff? Explain why or why not. In case a problem is stiff in one part of the time interval and nonstiff in another part, identify the approximate time intervals over which it is stiff or nonstiff, respectively. Assume an error tolerance of 10^{-2}.

   (a) $y' = -10^6 y$, $t \in [0, 10^{-6}]$, $y_0 = 1$

   (b) $y' = -10^6(y - t^2) + t$, $t \in [0, 1]$, $y_0 = 1$

   (c) $y' = -10^6(y - \sin(10^6 t)) + \cos(10^6 t)$, $t \in [0, 1]$, $y_0 = 0$

2. (2 points each part) Given the differential equation

$$y' = -\lambda(y - g(t)) + g'(t),$$

   where $\lambda > 0$.

   (a) Write down the 'smooth solution' (the solution as $\lambda \to \infty$)

   (b) What controls the stability? If you wanted to make the problem 'stiffer', how would you do it? If you wanted to make it unstable, how would you do it?

3. Consider the θ-method:

$$y_{n+1} = y_n + \theta h f_n + (1 - \theta) h f_{n+1}$$

   where $0 \leq \theta \leq 1$ and $f_n = f(t_n, y_n)$.

   (a) (2 points) What is the local truncation error?

   (b) (1 point) What is the local error?

   (c) (1 point) Given that the method is 0-stable, what is the order of this method? (hint: this depends on θ)

   (d) (2 points) A method is called A-stable if its region of absolute stability contains the left half of the complex plane. For which values of θ is the θ-method A-stable? Justify your answer.