Homework # 4

1. (15 points) Consider again the θ-method:

$$y_{n+1} = y_n + \theta h f_n + (1-\theta) h f_{n+1}$$

where $0 \leq \theta \leq 1$ and $f_n = f(t_n, y_n)$. Write a MATLAB code to implement the theta method for systems of ODEs. Describe the structure of your code (a flowchart or diagram will suffice).

For $\theta = 0, 0.5, 1$, use your code for solving:

- The following problem:

$$\begin{aligned} y_1' &= -y_1 \\ y_2' &= -100(y_2 - \sin(t)) + \cos(t) \end{aligned}$$

for $0 \leq t \leq 1$, with initial value $y_1 = 1$, $y_2 = 2$. Try this for stepsizes $h = .01$ and $h = .05$.

- The predator-prey problem

$$\begin{aligned} y_1' &= .25y_1 - .01y_1y_2 \\ y_2' &= -y_2 + .01y_1y_2 \end{aligned}$$

for $0 \leq t \leq 100$ with stepsizes $h = 0.1$ and $h = 0.001$, and initial values $y_1 = y_2 = 10$. Plot y_1 vs. t and y_2 vs. t, and y_1 vs. y_2.

Explain your results for both problems.

Note: You should write your own Newton iteration, but there is no need to write your own linear system solver. Use the function provided in Matlab for that. Provide a subroutine for each problem that computes the Jacobian matrix. Do not use finite difference approximation or automatic differentiation to compute the Jacobian. DO NOT USE THE MATRIX INVERSE. IF YOU USE THE MATRIX INVERSE, YOU WILL NOT GET CREDIT FOR THIS PROBLEM!

Turn in your source code on GauchoSpace, and your plots and explanations on paper.