Homework 5

Overall description, 30 points total Write a variable-stepsize MATLAB code for solving *stiff* ODE initial value problems using the variable-stepsize second order BDF method (with Newton iteration). Find the starting values required by the BDF method by using your theta method code, with $\theta = 0$, with a small stepsize. Explain on paper how you are doing the following, and highlight these elements in your code documentation:

1. Storing the past solution values
2. Estimating the error and accepting or rejecting the step
3. Choosing the next stepsize
4. Changing the stepsize
5. Deciding how to terminate the Newton iteration

Part 1, 10 points Implement the variable-stepsize method, but for now without the adaptivity. For this, you will need to find the starting values and store the past solution values. Use it to solve the problem

$$\begin{aligned} y_1' &= -y_1 \\ y_2' &= -1000(y_2 - t^2) + 2t \end{aligned}$$

for $0 \le t \le 1$, with initial value $y_1 = 1$, $y_2 = 2$, with stepsize $h = .01$. Plot the solutions obtained.

Part 2, 20 points Now add the variable-stepsize capability to your code. For this, you will need to be able to estimate the error and accept or reject each step, choose the next stepsize, and change the stepsize. Use your code to solve the following problems. Plot the solutions versus time, and the stepsize versus time, for error tolerances $\epsilon = 10^{-3}$ and $\epsilon = 10^{-6}$.

1. Predator-Prey Problem

$$\begin{aligned} y_1' &= .25y_1 - .01y_1y_2 \\ y_2' &= -y_2 + .01y_1y_2 \end{aligned}$$

for $0 \le t \le 100$ with initial values $y_1 = y_2 = 10$. Plot y_1 vs. t and y_2 vs. t, and y_1 vs. y_2.

2. Van der Pol's equation

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= \eta[(1 - y_1^2)y_2 - y_1] \end{aligned}$$

   for $0 \leq t \leq 11$ with initial values $y_1(0) = 2$, $y_2(0) = 0$. Plot y_1 vs. t and y_2 vs. t. Take $\eta = 2$.

3. Method of lines solution of a PDE

   Consider the method of lines applied to the diffusion equation in one space dimension, $u_t = au_{xx}$, with $a = 1$, boundary conditions $u = 0$ at $x = 0$ and at $x = 1$ for $t \geq 0$, and with initial values given by $u = 2x$ for $0 \leq x \leq 0.5$ and by $u = 2 - 2x$ for $0.5 \leq x \leq 1$. Formulate the method of lines using the central difference approximation to the derivative u_{xx}, as you did in Homework 2. Solve this problem with grid size $\Delta x = 0.05$, for $0 \leq t \leq 1$. Plot the solution as a function of x at $t = 0, 0.25, 0.5, 0.75, 1$ (on the same plot, in different colors), and plot the stepsize as a function of time.

Turn in your source code on GauchoSpace, and your explanations and results on paper in class.

DEBUGGING HINT: The stepsize should be able to both increase and decrease. It should be small where there is a lot going on in the problem, and larger where the solution is not changing as rapidly.

IT IS STRONGLY RECOMMENDED NOT TO WAIT UNTIL THE LAST MINUTE TO DO THIS HOMEWORK. GETTING AN ADAPTIVE MULTISTEP CODE TO WORK PROPERLY CAN BE TRICKY. THE DEVIL IS IN THE DETAILS WHICH REQUIRE A COMPLETE UNDERSTANDING OF ALL OF THE ISSUES ABOVE. PLAN IT OUT, AND LEAVE PLENTY OF TIME TO GO TO OFFICE HOURS IF YOU NEED TO.