Homework 1

Reading: Scholarpedia article on method of lines (from the beginning, through the section on ODE Integration within the MOL). You may be asked to answer questions about this article in class.

Review: If you are not fluent in Matlab, use this week and next to learn/review Matlab. There are numerous Matlab tutorials on the Web, including some on the Mathworks website. The next homework includes an assignment to program the forward Euler method.

1. (2 points) Convert to standard form ODE system $y' = f(t, y)$:

$$\begin{aligned} u'' + v'' &= 0 \\ u'' + sin(u) - v' &= 0. \end{aligned}$$

2. (2 points) Are the following initial value problems (an initial value problem means the differential equation together with its initial condition) well-posed? (i.e. do they satisfy the assumptions of the existence and uniqueness theorem) Why or why not?

$$\begin{aligned} y' &= y^2, \quad y(0) = 1 \\ y' &= \frac{1}{1-y}, \quad y(0) = 0 \end{aligned}$$

3. (3 points) Write down the ODE system corresponding to the method of lines discretization of the following PDE. Use the centered difference approximation for u_{xx} (i.e. $u_{xx}(x_j) \approx \frac{u(x_{j+1}) - 2u(x_j) + u(x_{j-1})}{\Delta x^2}$)

$$\begin{aligned} u_t &= u_{xx} + u(1-u) \\ u(0,x) &= 1 - x \\ u(t,0) &= 1 \\ u(t,1) &= 0 \end{aligned}$$

4. (3 points) Write down the ODE system corresponding to the method of lines discretization of the following PDE for $0 \leq x \leq 2$. Use the

first order backward difference approximation for u_x, (i.e. $u_x(x_j) \approx \frac{u(x_j)-u(x_{j-1})}{\Delta x}$)

$$\begin{aligned} u_t + xu_x &= 0 \\ u(0,x) &= \begin{cases} 1, & \text{if } 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases} \\ u(t,0) &= 0, \quad (t > 0) \end{aligned}$$