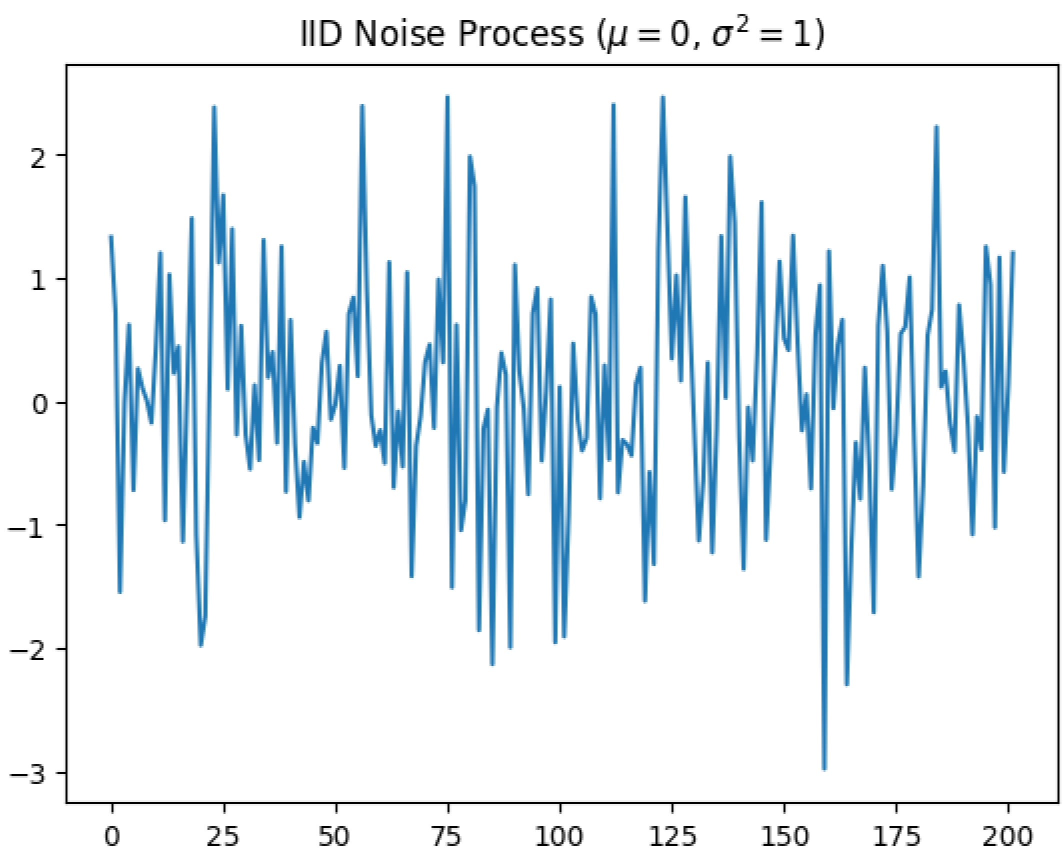
In [ ]:
```
import pandas as pd
import numpy as np
import scipy.stats as sp
import matplotlib.pyplot as plt
```

Problem 1

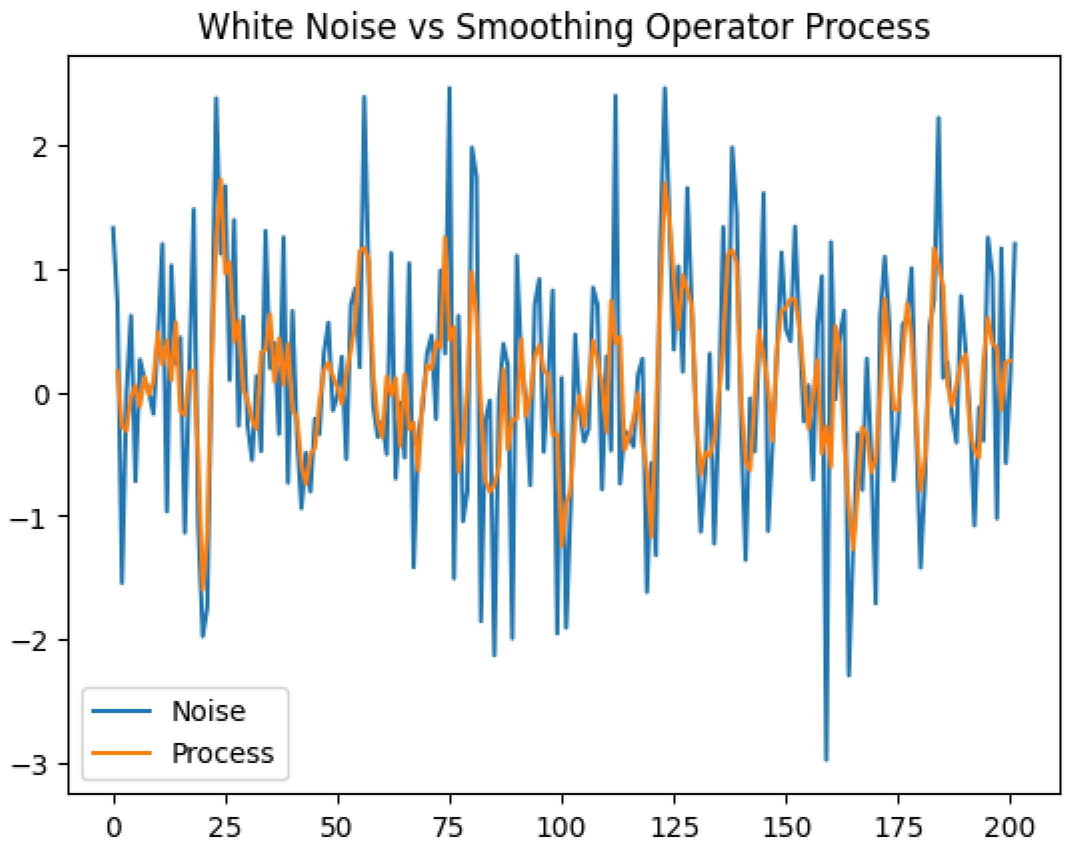
Problem 1.1

In [ ]:
```
rs = np.random.RandomState(seed=10)
mean = 0
std = 1

noise = []

t = range(202)

for i in t:
    noise.append(sp.norm.rvs(loc=mean,scale=std,random_state=rs))

plt.plot(t,noise)
plt.title('IID Noise Process ($\mu=0$, $\sigma^2=1$)')
plt.show()
```



Problem 1.2

In [ ]:
```
X_t = []
for i in range(1,201):
    X_t.append(1/3*(noise[i-1] + noise[i] + noise[i+1]))

plt.plot(t,noise,label='Noise')
plt.plot(t[1:-1],X_t,label='Process')
plt.title('White Noise vs Smoothing Operator Process')
plt.legend()
plt.show()
```



The process seems to have slightly less volatility.

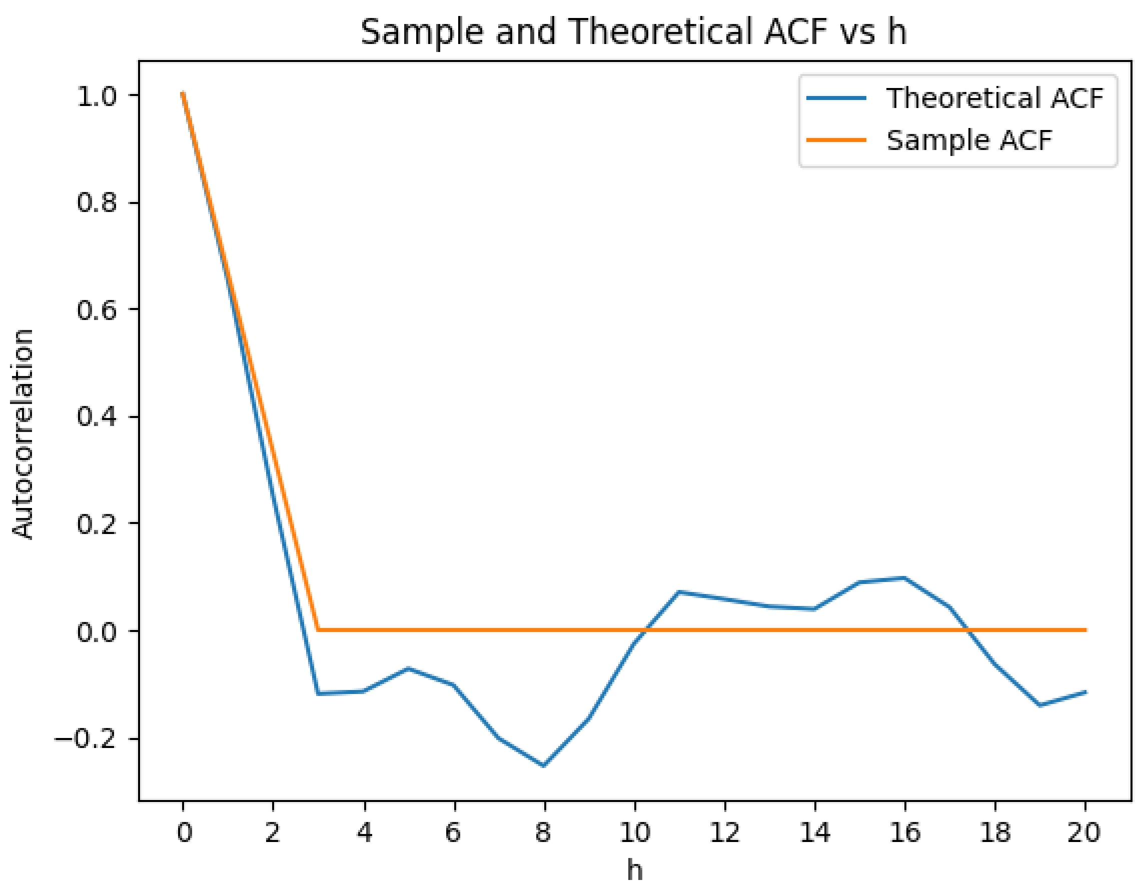
Problem 1.3

See PDF

Problem 1.4

In [ ]:
```
n = len(X_t)

def sample_acvf_fxn(h,X_t):
    sample_mean = np.mean(X_t)
    n = len(X_t)
    acvf = 0
    for t in range(n-abs(h)):
        acvf += (X_t[t+abs(h)]-sample_mean) * (X_t[t]-sample_mean)
    acvf = acvf/n
    return acvf

sample_acf = []
```

```
for h in range(21):
    sample_acf.append(sample_acvf_fxn(h,X_t) / sample_acvf_fxn(0,X_t))

theoretical_acf = [1,2/3,1/3] + [0] * (21-3)

plt.plot(range(21),sample_acf,label='Theoretical ACF')
plt.plot(range(21),theoretical_acf, label='Sample ACF')
plt.title('Sample and Theoretical ACF vs h')
plt.xlabel('h')
plt.xticks(np.arange(0,21,2))
plt.ylabel('Autocorrelation')
plt.legend()
plt.show()
```



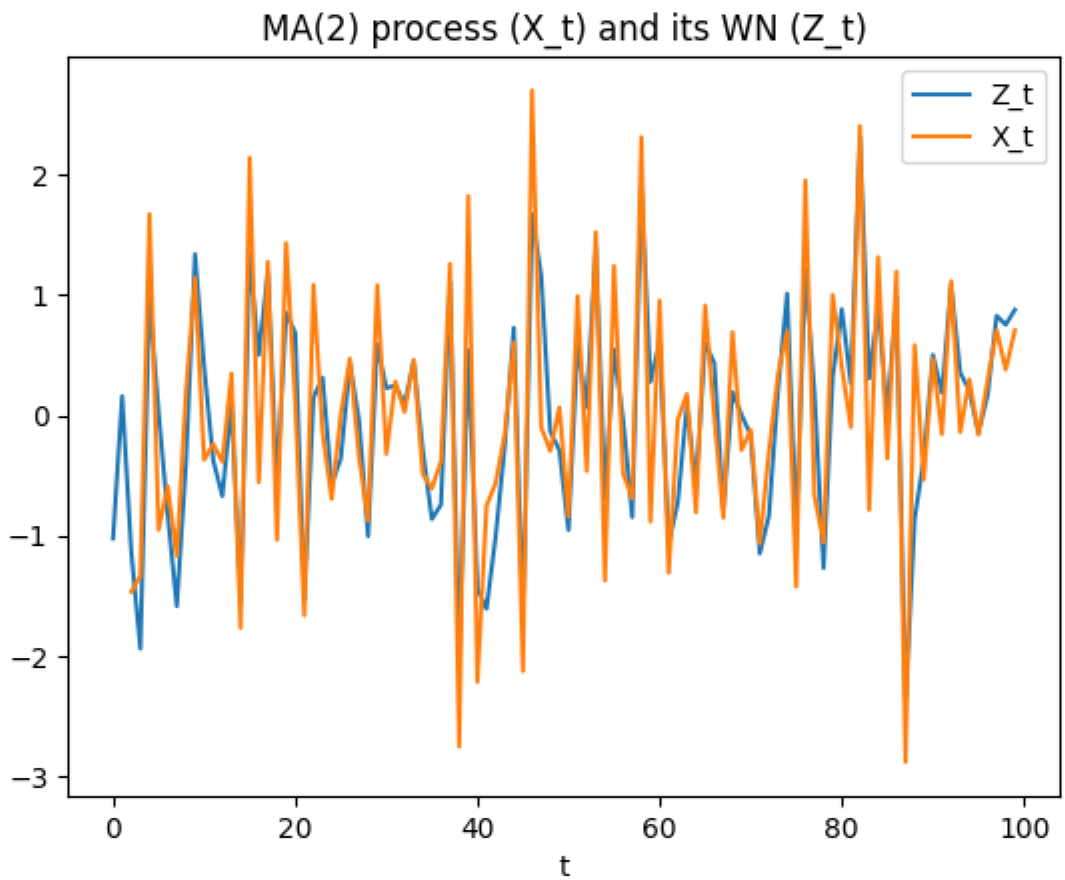
The relationship is complicated but they imply a similar result: a larger h is shown/thought to have a lower ACF, close to zero. Also, we can see that for h = 0,1,2, the values match almost identically. However, when the theoretical ACF is zero (h>2), the sample ACF fluctuates moderately around 0.

Problem 2

Problem 2.1

In [ ]:
```
Z_t, X_t = ([],[])

for t in range(100):
    Z_t.append(sp.norm.rvs(loc=0,scale=1, random_state = rs))

    if (t > 1):
```

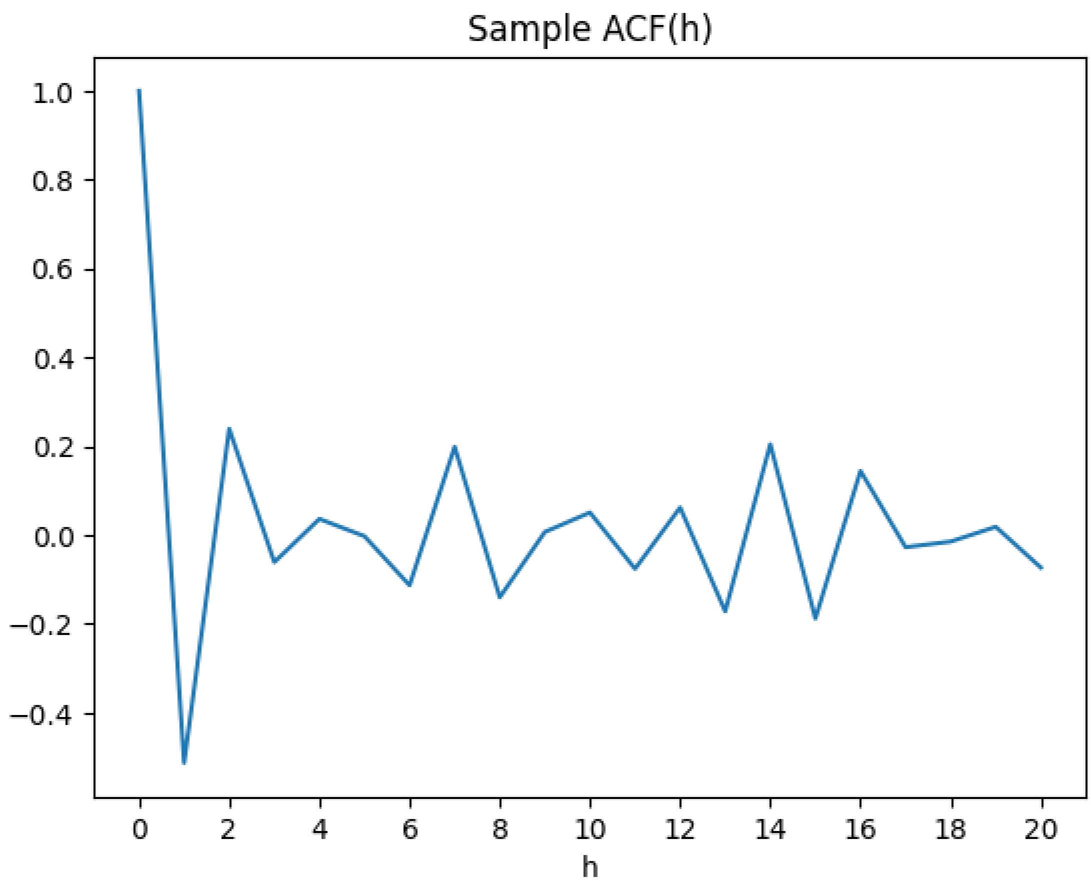
```
        X_t.append(Z_t[t] - .5*Z_t[t-1] + .25*Z_t[t-2])

plt.plot(range(100), Z_t, label='Z_t')
plt.plot(range(2,100), X_t, label='X_t')
plt.legend()
plt.xlabel('t')
plt.title('MA(2) process (X_t) and its WN (Z_t)')
plt.show()
```



Problem 2.2

In [ ]:

```
sample_mean = np.mean(X_t)
sample_ACVF = []

n = len(X_t)

for h in range(21):
    sample_ACVF.append(
        sample_acvf_fxn(h,X_t) / sample_acvf_fxn(0,X_t)
        )

plt.plot(range(21), sample_ACVF)
plt.title('Sample ACF(h)')
plt.xlabel('h')
plt.xticks(np.arange(0,21,2))
plt.show()
```



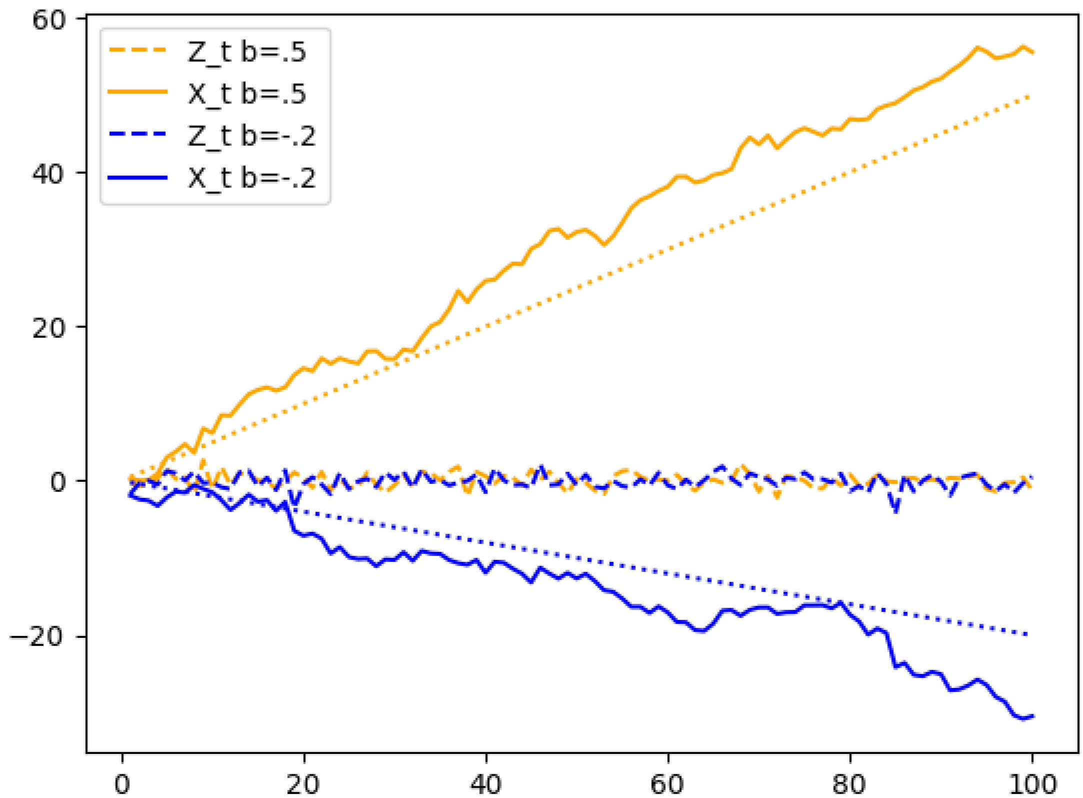
Problem 2.3

It seems that for $h > 2$ the sample ACF becomes close to zero. Thus, I would assume that the order is 2nd order, however, that is not extremely certain since fluctuations to the same value as h=2 occur later in the series.

Problem 3

In [ ]:
```
Z_t, X_t = ([],[])

b = .5

for i in range(1,101):
    Z_t.append(
        sp.norm.rvs(loc=0, scale=1, random_state = rs)
    )

    X_t.append(
        b*i + np.sum(Z_t)
    )


plt.plot(range(1,101), Z_t, label='Z_t b=.5', color='orange',ls='--')
plt.plot(range(1,101), X_t, label='X_t b=.5', color='orange')

Z_t, X_t = ([],[])

b = -.2
```

```python
for i in range(1,101):
    Z_t.append(
        sp.norm.rvs(loc=0, scale=1)
    )

    X_t.append(
        b*i + np.sum(Z_t)
    )

plt.plot(range(1,101), Z_t, label='Z_t b=-.2', color='blue', ls='--')
plt.plot(range(1,101), X_t, label='X_t b=-.2', color='blue')

plt.plot(range(1,101), [.5*r for r in range(1,101)],ls='dotted', color='orange')
plt.plot(range(1,101), [-.2*r for r in range(1,101)],ls='dotted', color='blue')
plt.legend()
plt.legend()
plt.show()
```



I can see that the random walk process follows the function of its drift process (dotted), which makes sense since Z_t has mean 0, but not perfectly, which reflects the variance of Z_t.

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OH: Th: 10-12

Quiz: weeks 3, 7, 9

Lab 1

1.3

$Z_t \overset{iid}{\sim} N(0,1)$

$\gamma(X_t, X_s) = cov[X_t, X_s]$

$$\sigma(X_t, X_s) = \frac{cov[X_t, X_s]}{\sigma_{X_t} \ \sigma_{X_s}}$$

$$\sigma^2_{X_t} = Var[\tfrac{1}{3}(Z_{t-1} + Z_t + Z_{t+1})] = \tfrac{1}{9} Var[Z_{t-1} + Z_t + Z_{t+1}]$$

$$= \tfrac{1}{9}\left(\sigma^2_{Z_{t-1}} + \sigma^2_{Z_t} + \sigma^2_{Z_{t+1}} + \underbrace{2 \textstyle\sum cov(\ldots)}_{0}\right)$$

$$= \tfrac{1}{9}(3) = \tfrac{1}{3}$$

similarly, $\sigma^2_X = \frac{1}{3}$

$$\sigma(X_t, X_s) = \frac{cov[X_t, X_s]}{\sigma_{X_s} \sigma_{X_t}} = \frac{cov[X_t, X_s]}{\sqrt{\frac{1}{3} \cdot \frac{1}{3}}}$$

Step 1:

$E[X_t] = E[\tfrac{1}{3}(Z_{t-1} + Z_t + Z_{t-1})]$

$$= \frac{1}{3}\left(E[z_{t-1}] + E[z_t] + E[z_{t+1}]\right) = 0$$

$$\text{cov}[X_t, X_s] = E[X_t \cdot X_s] + \underset{0}{E[X_s]}\,\underset{0}{E[X_t]}$$

idpt

$$E[X_t \cdot X_s] = E\left[\frac{1}{3}(z_{t-1} + z_t + z_{t+1}) \cdot \frac{1}{3}(z_{s-1} + z_s + z_{s+1})\right]$$

$$= \frac{1}{9} E\left[z_{t-1} \cdot z_{s-1} + z_{t-1} \cdot z_s + z_{t-1} z_{s+1} + z_t \cdot z_{s-1} + z_t \cdot z_s + z_t \cdot z_{s+1} + z_{t+1} \cdot z_{s-1} + z_{t+1} \cdot z_s + z_{t+1} \cdot z_{s+1})\right]$$

0, idpt

$$\text{cov}[z_t, z_s] = E[z_t \cdot z_s] + E[z_t]E[z_s]$$

$$= \begin{cases} 0 & t \neq s \\ \sigma_{z_t} = \sigma_{z_s} = 1 & t = s \end{cases}$$

$h = 0$

Case 1: $t = s$ $\Rightarrow \quad \frac{1}{9} \cdot 3 = \frac{1}{3}$

$$E[X_t, X_s] = \frac{1}{9} E\begin{bmatrix} z_{t-1} z_{t-1} + 0 + 0 + \\ 0 + z_t z_t + 0 + \\ 0 + 0 + z_{t+1} z_{t+1} \end{bmatrix} = \frac{1}{9}(1 + 1 + 1)$$

$$= \frac{1}{3} = \text{cov}[X_t, X_s]$$

$$\sigma[X_t, X_s] = \frac{1/3}{1/3} = \boxed{1}$$

$h = \pm 1$

Case 2: $|t-s|=1$ $\qquad$ 2/9

$$E[X_t \cdot X_s] = \frac{1}{9} E\begin{bmatrix} 0 + Z_{t-1}\cdot Z_{t-1} + 0 + \\ 0 + 0 + Z_t \cdot Z_t + \\ 0 + 0 + 0 \end{bmatrix} = \frac{1}{9}(1+1)$$

$$= \frac{2}{9} = cov[X_t, X_s]$$

$$\sigma[X_t, X_s] = \frac{2/9}{1/3} = \boxed{\frac{2}{3}}$$

$h = \pm 2$

Case 3: $|t-s|=2$

$$E[X_t \cdot X_s] = \frac{1}{9} E\begin{bmatrix} 0 & 0 & Z_t \cdot Z_s \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{1}{9} \cdot 1 = \frac{1}{9} = cov[X_t, X_s]$$

$$\sigma[X_s, X_t] = \frac{1/9}{1/3} = \boxed{\frac{1}{3}}$$