PSTAT 174/274 Winter 2024 – Lab Assignment 1

Due on January 24, 2024

Problem 1. Consider a **IID noise** process $\{Z_t\} \sim IID(0, \sigma_Z^2 = 1)$ and a **smoothing operator** of the form

$$X_t = \frac{1}{3}(Z_{t-1} + Z_t + Z_{t+1}).$$

1. Simulate and plot the **IID noise** process $\{Z_t\}$, for $t = 0, 1, 2, \ldots, 201$.
2. Plot X_t in the same diagram of $\{Z_t\}$ with different colors. What can you say about the **volatility** of the two trajectories?
3. Compute the **theoretical** autocorrelation function (ACF) of $\{X_t\}$.
4. Let $x_1, \ldots, x_n$ be the n observations/realization of a time series, given the following formulas

   - **sample mean:** $\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$;
   - **sample autocovariance function:**

   $$\hat{\gamma}_X(h) = \frac{1}{n}\sum_{t=1}^{n-|h|}(x_{t+|h|} - \bar{x})(x_t - \bar{x}), \quad \text{for} |h| \leq n-1$$

   - **sample autocorrelation function:** $\hat{\rho}_X(h) = \dfrac{\hat{\gamma}_X(h)}{\hat{\gamma}_X(0)}$.

   Then plot both the sample ACF (you may want to use `acf` function in R) and the theoretical ACF of $\{X_t\}$ for $h = 0, 1, 2, \ldots, 20$. What is the relationship between the theoretical and sample ACF?

Problem 2. Consider the second-order moving average process MA(2) given by

$$X_t = Z_t - \frac{1}{2}Z_{t-1} + \frac{1}{4}Z_{t-2}, \tag{1}$$

where $\{Z_t\} \sim WN(0, \sigma_Z^2 = 1)$.

1. Plot X_t and Z_t in the same diagram with $n = 100$ (total observation of time series X_t) using different colors.
2. Plot the **sample** ACF $\hat{\rho}_X(h)$ for $h = 0, 1, \ldots, 20$ based on the realization $\{x_t\}$.
3. Without calculating and plotting the theoretical ACF, and do not refer to the definition (1). Based on the diagram of sample ACF obtained in (2) only, can you tell the order of the moving average process? Explain you answer.

Problem 3. Consider a random walk process with a drift term:

$$X_t = bt + \sum_{i=1}^{t} Z_t, \quad X_0 = 0, \tag{2}$$

where $\{Z_t\} \sim IID(0, 1)$.

- Simulate $n = 100$ observations of a random walk with $b = 0.5$ and $b = -0.2$ respectively and plot both realizations.
- Plot $y = 0.5t$ and $y = -0.2t$ on the same diagram, what do you observe?