PSTAT 174/274 Winter 2024 – Lab Assignment 2

Due Wednesday, January 31, 2024

Question 1: A Random Walk

Consider a Random Walk given by:

$$X_t = \delta + X_{t-1} + Z_t,\ Z_t \overset{iid}{\sim} WN(0,1)$$
$$X_0 = 0$$

a. Show that this process can be rewritten as the cumulative sum of white noise terms, $X_t = \delta t + \sum_{j=1}^{t} Z_j$

b. Simulate 200 observations **each** of two random walks; the first with $\delta = 0.6$ and the second with $\delta = 0.4$. Plot both realizations on the same plot using different colors.

c. Describe the plot. Does this Random Walk appear stationary?

d. Prove that a Random Walk is not Weakly Stationary, even if $\delta = 0$.

Question 2

Let's simulate and compare some AR and MA processes. Lets consider both AR(2) and MA(2) processes given by:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t \quad \varepsilon_t \overset{iid}{\sim} \mathcal{N}(0,1)$$
$$Y_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} \quad Z_t \overset{iid}{\sim} \mathcal{N}(0,1)$$

a. Recall that for an AR(1) process, we merely required that $|\phi| < 1$ for causality. What is the similar condition for an AR(2) process?

   *(**Hint**: An ARMA process is Causal if and only if the roots of the AR polynomial lie outside the unit circle)*

b. Let $\phi_1 = 0.35$ and $\phi_2 = 0.45$. Simulate the AR(2) process for 200 samples and take the sample ACF and PACF. Use 20 lags.

   *(**Hint**: use the `arima.sim` command.)*

c. For the Moving Average process: let $\theta_1 = 0.45$ and $\theta_2 = 0.55$. For the moving average model only, the theoretical ACF is given by:

$$\rho_Y(h) = \begin{cases} 1 & \text{if } h = 0 \\ \frac{\theta_1 + \theta_1\theta_2}{1+\theta_1^2+\theta_2^2} & \text{if } |h| = 1 \\ \frac{\theta_2}{1+\theta_1^2+\theta_2^2} & \text{if } |h| = 2 \\ 0 & \text{otherwise} \end{cases}$$

   Simulate this MA(2) process. On one plot, plot both the theoretical and sample ACF. Then plot the sample PACF. For both, use 20 lags.

d. Compare the Sample ACF of the AR(2) and MA(2) processes. Do the same for the PACF of both processes

Loading a Data Set

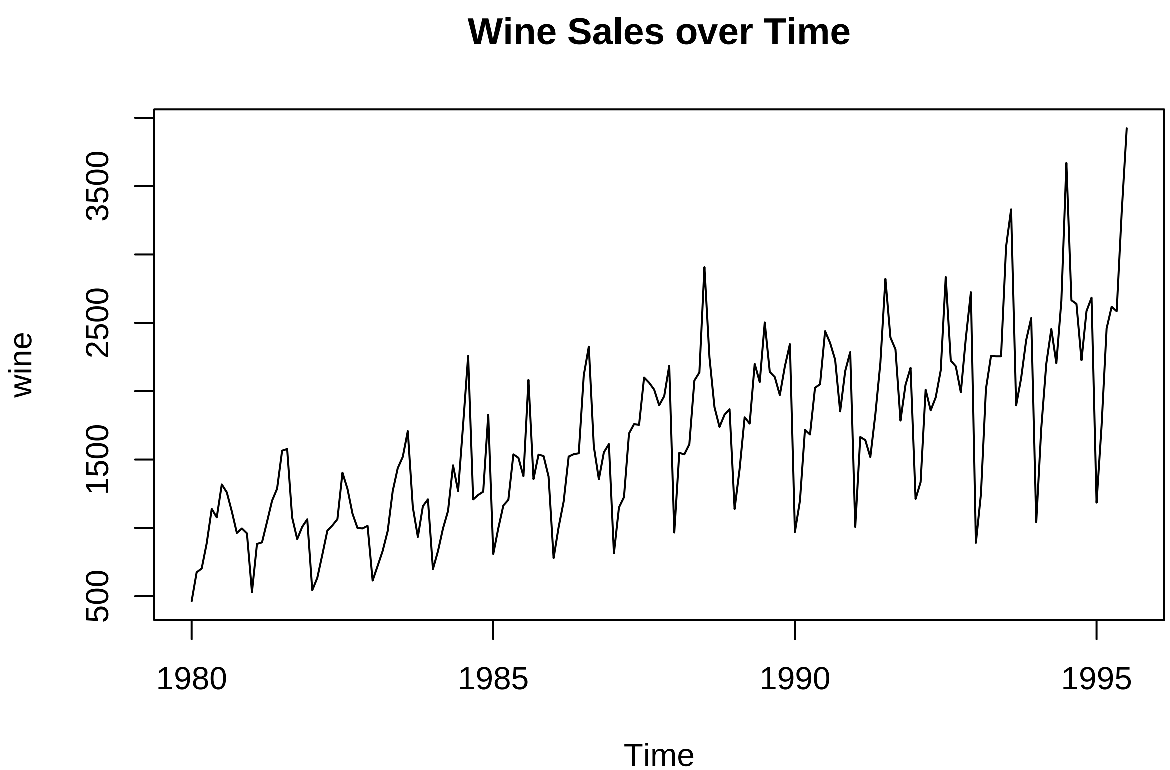
On Canvas, I have uploaded a file containing monthly wine sales in Australia.

To load that into R, do the following:

```
setwd("~/Downloads")  #You will need to set this according to your own filepath.
# You can use the R Import tool if you need to
wine.csv = read.table("monthly-australian-wine-sales-th.csv",
    sep = ",", header = FALSE, skip = 1, nrows = 187)
colnames(wine.csv) <- c("Month", "Sales")
```

Let's now create a time-series object and plot it:

```
wine = ts(wine.csv$Sales, start = c(1980, 1), frequency = 12)
ts.plot(wine, main = "Wine Sales over Time")
```



From this, we can see:

- This data appears to be increasing
- The data variability changes over time
- There appears to be seasonality

Let's explore how to deal with this.

Question 3

a. One thing we could try is to transform our data, as we often do in typical linear regressions. Doing so may or may not yield a way for us to make our data stationary. Try a few transformations of the response variable and see if that removes any of the trends. You can even try Box-Cox if you want. This won't be our focus, so don't look at too many transformations.

b. Let's try a simple differencing, and see what that gives us. If X_t represents our wine sales, differencing a series creates a new time series $Y_t = \nabla X_t = X_t - X_{t-1}$. Create a difference series, plot that, and plot a line through 0 so we can see if we're close to de-trending.

*(**Hint**: use the `diff` command in R. Plotting a horizontal line can be done with `abline` in R as well.)*

c. You should notice that we have removed our trend, but we still have some cyclic behavior. Plot the sample ACF and PACF of the differenced series, with 60 lags. What do they tell us? Why is there still a repeating pattern appearing?

d. You have reason to believe that there is a 12-month cycle to sales. Difference the series (the one we already differenced) again, but for 12 steps this time. Plot the time-series, the ACF and the PACF.