Pre-class assignment discussion

1) Discuss 2 required reading + code assignments
   - Compare Qs you have about readings: what questions remain?
   - Compare code results: do you get the same answers?
     what's the answer to Q3 of the PCA?

2) What are the implications of floating-point arithmetic?

Differentiation: Taylor expansion!

$$f(x) = f(x_0) + (x-x_0)\,f'(x_0) + \frac{(x-x_0)^2}{2} f''(x_0) + \ldots + \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$

1st deriv: $$f'(x_i) = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

"two-point formula"

$x_{i+1} - x_i = \Delta x = h$, so $$f'_i = \frac{f_{i+1} - f_i}{h} + \underbrace{\mathcal{O}(h)}_{\text{error}}$$ 1st-order solution

"three-point formula"

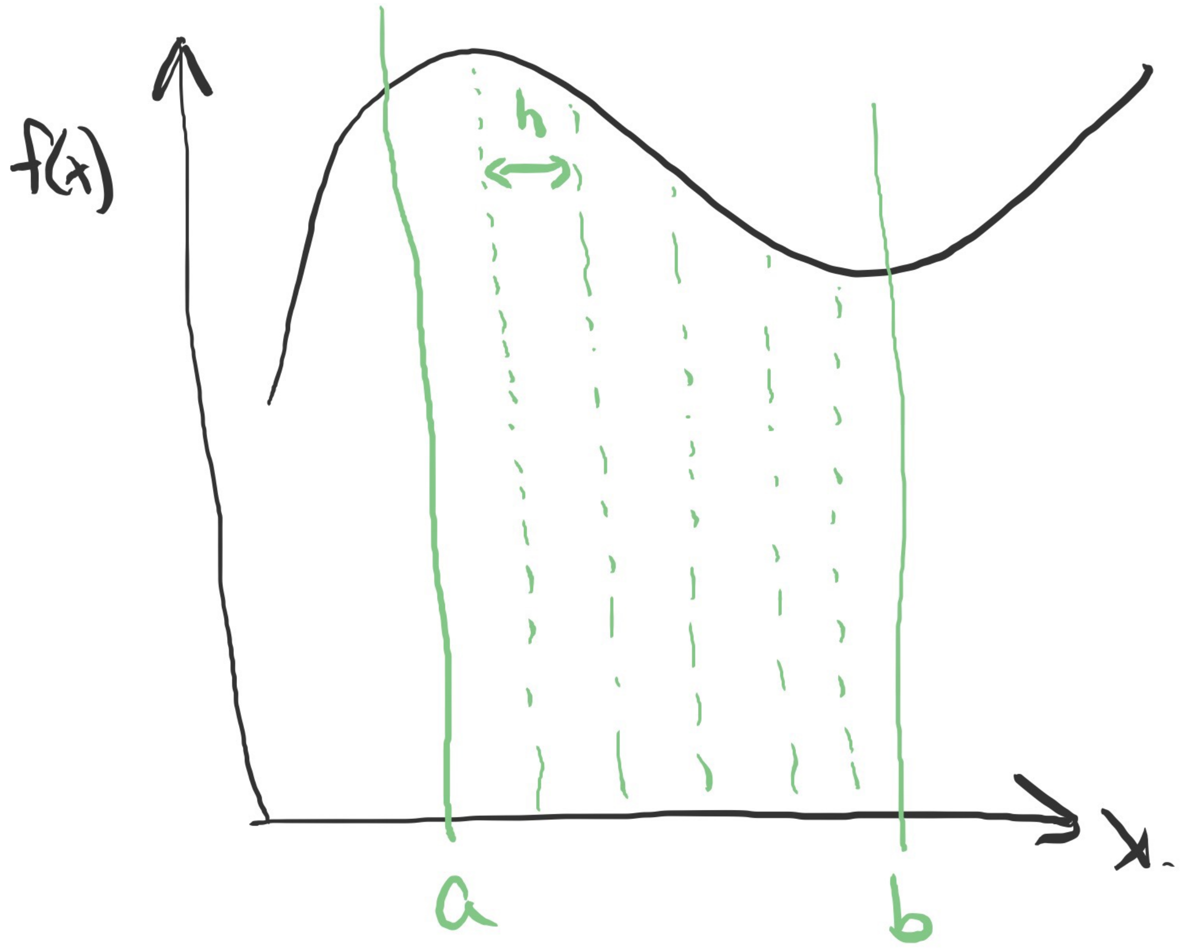
better solution: $$f'(x_i) = \frac{f(x_i+h) - f(x_i-h)}{2h}$$

$$f(x_i \pm h) \simeq f(x_i) \pm h f'(x_i) + \frac{h^2}{2} f''(x_i) \pm \frac{h^3}{6} f^{(3)}(x_i)$$

end up with: $$f'(x_i) \simeq \frac{f(x_i+h) - f(x-h)}{2h} + \underbrace{\mathcal{O}(h^2)}_{\text{error}}$$ 2nd order solution

Numerical integration

$$F = \int_a^b f(x)\,dx$$



Can approximate area over small intervals and sum:

for evenly-sized intervals:

rectangles: $\Theta(h)$ error

trapezoids: $\Theta(h^2)$ error

parabolas (Simpson's): $\Theta(h^4)$ error

Floating-point errors

IEEE standard:

sign	exponent	mantissa

32 bit: exponent: $+127$, -126 mantissa ~ 7 digits

64 bit: exponent: $+1023$, -1022 mantissa: ~16 digits

Example: -1.234 =

-	-3	1234

$+\pi$ =

+	-7	31415927

not exact; rounding

types of floating-point errors

truncation - truncating ∞ sum (represent by finite sum)

roundoff - difference between calculated approx. of a number and exact value (**quantization** error)

summation - accumulation of errors from computing sums (or generally from arithmetic operations)

representation - errors coming from numbers of diff. scales (e.g., 1.23×10^{17} and 0.11321197)

+ additional errors coming from compilers, "minimum ex", **hardware**, type conversion (float 64 → float 32, etc.)