Pre-class assignment #7

PHY-905-005
Computational Astrophysics and Astrostatistics
Spring 2023

This assignment is due the evening of Wednesday, February 1, 2023. Turn in your code and all materials via the GitHub Classroom.

Reading:

1. Methods of solving initial value ODEs: Sections 8.1-8.5 of *Computational Physics*, by Newman. Note: you can skim Sections 8.5.2-8.5.6, but try to get the general idea of why these various methods exist!
2. Wikipedia article on the Euler-Cromer method
3. Wikipedia article on the midpoint method
4. (Optional) Methods of solving initial value ODEs: Sections 4.1-4.4 of *An Introduction to Computational Physics*, by T. Pang (PDF provided)

Your assignment: Consider the simple mass on a spring, acting in the absence of gravity so that the only force acting on it is a linear restoring force pointing toward its resting position:

$$F = -kx \tag{1}$$

where F is force, k is a spring constant, and x is position. The total energy of the spring is:

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \tag{2}$$

where m is the mass of the object at the end of the spring and v is its velocity. Assume that at $t = 0$ the spring is at its resting position ($x = 0$) with $v = 1$. The object on the spring as $m = 1$ and the spring constant is $k = 1$. Analytically, we can easily derive the position and velocity of the object on the spring as a function of time:

$$x(t) = x_m \sin(\omega t) \tag{3}$$

$$v(t) = x_m \omega \cos(\omega t) \tag{4}$$

with $\omega = (k/m)^{1/2} = 1$ and $x_m = 1$. Integrate the ODEs that describe this system from $t = 0$ to $t = 4\pi$ using two different methods – the Euler method and the leapfrog method (described in Sections 8.1 and 8.5.1 of Newman, respectively). Use functions as appropriate, and also use the class coding standard! Use fixed timesteps of $\Delta t = 0.1\pi$, 0.01π, and 0.001π. Using **one graph per method**, plot the position as a function of time for the analytic solution and each of the three time steps, using different colors or symbols so you can clearly differentiate the time steps and adding a legend as appropriate. Then, **on a single graph**, show how well energy is conserved by both methods for the three choices of time step between the starting time and $t = 4\pi$. You can do this by making a plot of relative change in energy, ϵ, between the beginning (e_0) and end (e_f) of the simulation:

$$\epsilon = \frac{|e_f - e_0|}{e_0} \tag{5}$$

Make sure to double-check that you're comparing the same times! If you don't take the same timesteps, and end up with the same actual time (due to accumulation of floating-point error) you might have slightly different answers.

Do you see a difference in behavior between the two methods? Record your observations, as well as any questions you have after doing the readings and this assignment, in the file `ANSWERS.md`.