

Pre-class assignment discussion

- 1) Discuss 2 required reading + code assignments
 - Compare Qs you have about readings: what questions remain?
 - Compare code results: do ya get the same answers?
What's the answer to Q3 of the PCA?
- 2) What are the implications of floating-point arithmetic?

Differentiation : Taylor expansion!

$$f(x) = f(x_0) + (x-x_0) f'(x_0) + \frac{(x-x_0)^2}{2} f''(x_0) + \dots + \frac{(x-x_0)^n}{n!} f^{(n)}(x_0)$$

1st deriv: $f'(x_i) = \lim_{\Delta x \rightarrow 0} \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$

"two-point formula"

$$x_{i+1} - x_i = \Delta x = h, \text{ so } f'_i = \frac{f_{i+1} - f_i}{h} + \underbrace{\mathcal{O}(h)}_{\text{error}} \quad \left. \begin{array}{l} \text{1st-order} \\ \text{solution} \end{array} \right\}$$

"three-point formula"

better solution: $f'(x_i) = \frac{f(x_i+h) - f(x_i-h)}{2h}$

$$f(x_i \pm h) \approx f(x_i) \pm h f'(x_i) + \frac{h^2}{2} f''(x_i) \pm \frac{h^3}{6} f^{(3)}(x_i)$$

end up with: $f'(x_i) \approx \frac{f(x_i+h) - f(x_i-h)}{2h} + \underbrace{\mathcal{O}(h^2)}_{\text{error}} \quad \left. \begin{array}{l} \text{2nd} \\ \text{order} \\ \text{solution} \end{array} \right\}$

Numerical integration

$$F = \int_a^b f(x) dx$$

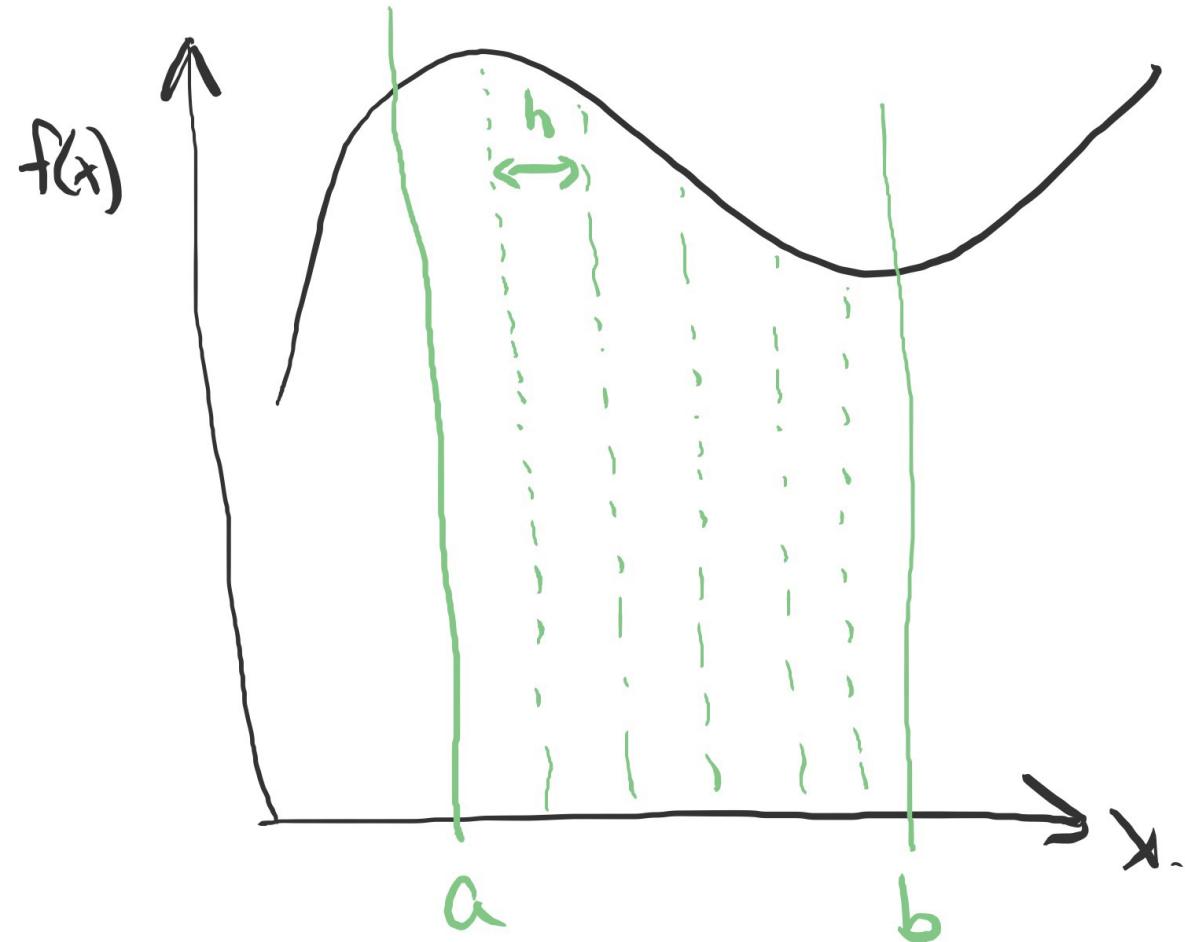
Can approximate area over small intervals and sum:

for evenly-sized intervals:

rectangles: $\mathcal{O}(h)$ error

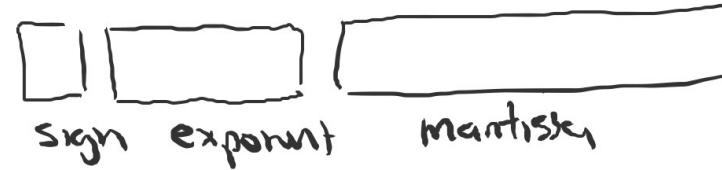
trapezoids: $\mathcal{O}(h^2)$ error

parabolas: $\mathcal{O}(h^4)$ error
(Simpson's)



Floating-point errors

IEEE standard:



32 bit: exponent: +127 mantissa \sim 7 digits
-126

64 bit: exponent: +1023 mantissa: \sim 16 digits
-1022

Example: $-1.234 =$

A diagram showing the binary representation of -1.234. It consists of three boxes: a small box with a red minus sign, a medium box with a red -3, and a large box with a red 1234. The boxes are aligned horizontally.

$+\pi =$

A diagram showing the binary representation of π . It consists of three boxes: a small box with a red plus sign, a medium box with a red -7, and a large box with a red 31415927. The boxes are aligned horizontally. The digit 7 in the large box is circled in blue, and the text "not exact; rounding" is written in blue to the right of the boxes.

types of floating-point errors

truncation - truncating ∞ sum (represent by finite sum)

roundoff - difference between calculated approx. of a number and exact value (quantization error)

summation - accumulation of errors from computing sums
(or generally from arithmetic operations)

representation - errors coming from numbers of diff. scales
(e.g., 1.23×10^{17} and 0.11321197)

+ additional errors coming from compilers, "minimum ex.",
hardware, type conversion (float 64 \rightarrow float 32, etc.)