In-class assignment # 2

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Note: Before you get started with writing code, read the instructions below and **make a prediction for the plot that you think you're going to see** Write that down in the file called `ANSWERS.md` and then commit and push those changes to the repository.

Instructions: Implement the two-point, three-point, and five-point formulae for numerical differentiation (Newman equations 5.90 and 5.107, and Table 5.1; alternately, Pang equations 3.4, 3.6, and 3.9) in a single function, using a function argument to decide which one you're going to use. Have the arguments to the function be (1) the point X where you are interested in calculating a derivative, (2) a function f(x) whose derivative you wish to calculate at point X (specifically, a software function that returns the value of a mathematical expression!), (3) the grid spacing Δx, and (4) a flag showing which of the three methods of estimating the derivative you wish to use. Invent a somewhat complex mathematical function of your own choosing that has a non-zero analytic first derivative that you can calculate, and use that to verify the correctness of your implementations.

Then, at a point X and initial grid spacing h_0 of your choosing, calculate the fractional error of each of these approximations for a variety of grid spacings $h(N) = h_0/10^N$, $N = 0, 1, 2, ..., 20$ (or even higher). Recall that the fractional error, ϵ, is defined as $\epsilon_N = \frac{|f'(X)_{num,N} - f'(X)_{an}|}{|f'(X)_{an}|}$, where $f'(X)_{an}$ is the analytic solution to the derivative at a point X and $f'(X)^N_{num}$ is the numerical solution to the derivative using an interval size $h(N)$. Plot the fractional error for each of the three numerical derivatives as a function of grid spacing. How do they behave? Is this what you expected? Make sure to describe this in `ANSWERS.md`.

Finally, compare the results to your predictions. Do they make sense? Why do you think that you see this outcome?

If you have time and are inclined to do so, try to duplicate the results from your own code using scipy.misc.derivative. Do you get the same answer?

What to turn in: Turn in `ANSWERS.md`, any source code you wrote, any plots you created (and the scripts you used to create them). **Do not** turn in object files or executables!