Pre-class assignment # 10

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This assignment is due the evening of Monday Feb. 13, 2023. Turn in all materials via GitHub.

Reading:

1. Chapter 5 ("Second- (and Higher-) Order Advection") of Mike Zingale's Computational Hydrodynamics Tutorial, which you can find in the pre-class assignment for Day 9. Focus on Sections 5.1-5.4, but make sure to read Section 5.5 as well and get a general sense of the motivations and tradeoffs with higher-order methods.

2. Wikipedia page on Flux Limiters

Your assignment:

1. Based on your linear wave advection code from the previous class, implement the 2^{nd} order finite-volume advection in one dimension that is discussed in Sections 5.1 and 5.2 of Zingale's lecture notes. Don't implement the minmod slope limiter (described in Section 5.2.1), but think about how you would do so since we'll do it in class! Unlike in the pre-class assignment from the previous class, **make sure to include ghost zones in your NumPy arrays so you can set arbitrary boundary conditions.** To do so, add the capability to your code to add an arbitrary number of ghost zones on each end of the arrays of values (1 or 2 is generally what we'll need), and also add a function to take care of boundary conditions. Have options for periodic and Dirichlet (constant-value) boundary conditions. **Make sure that you test both sets of boundary conditions, as well as left- and right-traveling waves** (i.e., $u > 0$ and $u < 0$). Periodic boundary conditions should behave as they did before; with the Dirichlet boundary condition, waves should disappear when they reach the edges of the domain.

2. Examine the behavior of both the square wave and the Gaussian wave after 1 and 10 periods (i.e., loops through the domain), and compare it to the 1^{st} order upwind method you implemented prior to the last class. Vary both the CFL condition and number of grid cells for both the finite-volume method and the upwind method, and describe how the behavior differs for the two. Write a function that calculates and returns the L2 and infinity norms between the initial conditions and the end state (described in Section 1.2.4 of Zingale's notes), and use it to compared the two different methods for $C = 1$ as a function of grid size from $N = 16$ to $N = 1{,}024$ – what do you see? (Note: make sure that you're going the correct number of time steps - after one period, the centers of the pulses should be exactly on top of each other, or every norm measurement will have a huge offset!)

3. Think about how you might implement a slope limiter into your code for the 2^{nd} order finite-volume method, but don't actually implement it. In `ANSWERS.md`, describe how this implementation differs from the previous question.

4. What are at least three questions that you have from the readings that you'd like us to address in class? Put those questions in `ANSWERS.md`.

Handing it in: Include your code, your plots, and your answers to the questions about (in the file `ANSWERS.md`) in your assignment.