In class assignment day 12

Ref. Toro Riemann Solver Ed. 3

Implement the Lax-Friedrich scheme in conservative form, see Toro p185

Simple algebraic manipulations of this expression lead to a conservative version of the Lax–Friedrichs scheme for systems

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n + \frac{\Delta t}{\Delta x}\left[\mathbf{F}_{i-\frac{1}{2}} - \mathbf{F}_{i+\frac{1}{2}}\right] , \tag{5.76}$$

with the Lax–Friedrichs intercell flux given by

$$\mathbf{F}_{i+\frac{1}{2}}^{\mathrm{LF}} = \frac{1}{2}(\mathbf{F}_i^n + \mathbf{F}_{i+1}^n) + \frac{1}{2}\frac{\Delta x}{\Delta t}(\mathbf{U}_i^n - \mathbf{U}_{i+1}^n) . \tag{5.77}$$

Start with your existing code and make sure to write generic code, i.e., implement a function for F^LF_i+/-1/2 and one function for the F^n_i. Before starting to code, discuss with your group how array indices/slices are linked to the equations.

Now solve the inviscid Burgers equation, see below, plot and describe your results. Please submit your solution in an jupyter notebook.

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5.5.2 The Inviscid Burgers Equation

Our Test 3 consists of the inviscid Burgers equation

$$u_t + f(u)_x = 0 , \quad f(u) = \frac{1}{2}u^2 \tag{5.112}$$

in the domain $[0, \frac{3}{2}]$ with initial conditions

$$u(x,0) = \begin{cases} -\frac{1}{2} & \text{if } x \le \frac{1}{2} , \\ 1 & \text{if } \frac{1}{2} \le x \le 1 , \\ 0 & \text{if } x \ge 1 . \end{cases} \tag{5.113}$$

We solve this problem numerically on a domain of length $L = 1.5$ discretised by $M = 75$ equally spaced cells of width $\Delta x = 0.02$; the CFL coefficient used is 0.8. Fig. 5.15 shows computed results (symbol) along with the exact (line) solution, for the Godunov and Lax–Friedrichs schemes at time $t = 0.5$ units (32 time steps). Two new features are now present in solving non–linear PDEs.