Pre-class assignment # 2

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This assignment is due the evening of Wednesday, Jan. 11, 2023.

Instructions: Read the materials below and follow the instructions/answer the questions at the end. Turn in your code and all materials via the GitHub Classroom - in the directory where this assignment exists (i.e., the directory corresponding to the Git repository that you have cloned), add whatever new files you have created to the repository by typing "git add mynewfile" (you can use wild cards as well). Then, commit the new files and any changed files by typing "git commit -a" (the "-a" means "commit every change and new thing" – if you typed "git commit mynewfile" it would just commit that file). Finally, push your changes to the repository by typing "git push".

What to turn in: Turn in plots (and the Python scripts or Jupyter notebooks used to generate them), text files, etc. **Do not** turn in object files, binary files, or very large files of any kind unless you are explicitly asked to do so!

Reading:

1. Chapter 5 of *Computational Physics*, by M. Newman
2. "What every computer scientist should know about floating-point arithmetic," by D. Goldberg (doi:10.1145/103162.103163; PDF provided)
3. Sections 3.1 and 3.2 of *An Introduction to Computational Physics*, by T. Pang (optional reading; PDF provided)
4. Wikipedia article on floating-point numbers (optional reading)
5. Wikipedia article on numerical differentiation (optional reading)
6. Wikipedia article on numerical integration (optional reading)

Questions:

1. Consider the function $f(x) = e^x \sin(x)$. Calculate the derivative $f'(x) = \frac{df}{dx}$ at $x = 2$ using the three-point formula given in Equation 5.102 of Newman (Equation 3.6 of Pang), with $h(N) = 0.5/10^N$ and N=1,2,3,4,5,6. Note: you should do this in a general way: write a function that takes as arguments f(x), X (the point where you wish to evaluate $f'(x)$), and $h(N)$, and returns the value of $f'(X)$. Create a plot that shows $f'(X)$ as calculated at $X = 2$ for the values of N shown above.
2. Using the trapezoidal rule (Equation 5.3 in Newman or Equation 3.23 in Pang) and the same function f(x) as above, calculate $F = \int_0^{10} f(x)dx$ using 10^N equally-sized steps in the interval $x = 0$ to $x = 10$, with N=1,2,3,4,5,6. Note: write a function that takes as arguments f(x), the starting and ending points of the interval over which you wish to integrate, and the number of equal-sized steps you wish to take over the interval, and returns the integrated quantity F. Create a plot that shows F as calculated over the given interval for the values of N shown above.

3. We define $f'(X)_{an}$ to be the analytic solution to the derivative described in the first question, and $f'(X)^N_{num}$ to be the numerical solution to the integral using an interval size $h(N)$. The fractional error, ϵ, is defined as $\epsilon_N = \frac{|f'(X)_{num,N} - f'(X)_{an}|}{|f'(X)_{an}|}$. Create a **log-log plot** that shows ϵ_N as a function of $h(N)$ for the values of N shown. If you fit a power law to it, approximately what is the exponent of the best-fit power law? (Note: I'm asking you to eyeball it, not calculate the fit precisely.)

4. After you have done the readings and completed the assignment above, please put at least one question that you have about each part of the reading (Chapter 5 of Newman and the Goldberg article) in the file called `ANSWERS.md`. Please also make a note in that text file that says how long it took you to do this assignment!