Homework # 3

PHY-905-003, Computational Astrophysics and Astrostatistics
Spring 2017

This assignment is due on Sunday April 2, 2017. Turn in all materials via GitHub. Include your code, plots, and answers to any questions asked in your assignment. Your code must (1) be easily readable, with good use of whitespace, clear variable names, and adequate commenting (which documents design and purpose, not mechanics) and (2) use functions to break up code into logical components. The solutions to individual problems should be saved in separate, clearly-named source files or Jupyter notebooks. Plots should have easily readable and logical axis labels and titles, and the source code and data used to generate the plots should be included. Questions should be answered in the file `ANSWERS.md` or in a LaTeX-created PDF document of a similar name (e.g., `ANSWERS.pdf`).

Please note that **solutions containing loops rather than Numpy array operations are not going to be graded!** The only exception is the `while` time loop.

Hints: Read the relevant sections in Toro carefully and consider how the equations translate into code. This assignment should not require a great deal of additional code beyond your working 1D hydro code. As a rule of thumb, one equation in Toro should correspond to one line of code in Python when you use Numpy array operations rather than loops.

Part 1: In the two classes immediately before Spring Break you implemented the HLL Riemann solver within a 1-D MUSCL-Hancock framework to solve the compressible Euler equations. Now, use your previous code to implement the HLLC Riemann solver. The analytic background is presented in Section 10.4 in Toro. A practical summary of how the pieces fit together is given in Section 10.6. Then, do/answer the following:

1. Plot the primitive quantities and compare the results at $t = 0.2$ for the shock-tube problem (discussed in class with $x_0 = 0.2$, $\rho_L = 1$, $u_L = 0.75$, $p_L = 1$, $\rho_R = 0.125$, $u_R = 0$, and $p_R = 0.1$,) between using the HLL and HLLC Riemann solver. What did you expect to see and how does it compare to what you see? How can you explain the differences, if your expectation and result are not in agreement?

2. Plot the primitive quantities and compare the results at $t = 1$ between HLL and HLLC Riemann solver for $x_0 = 0.5$, $\rho_L = 1.4$, $u_L = 0$, $p_L = 1$, $\rho_R = 1$, $u_R = 0$, and $p_R = 1$. What kind of problem/situation does this set of initial conditions correspond to? How do the results compare to the shock-tube problem?

3. Plot the primitive quantities and compare the results at $t = 1$ between HLL and HLLC Riemann solver for $x_0 = 0.5$, $\rho_L = 1.4$, $u_L = 0.1$, $p_L = 1$, $\rho_R = 1$, $u_R = 0.1$, and $p_R = 1$. What kind of problem/situation does these initial conditions correspond to? How do the results compare to the previous two problems?

4. With the insights obtained from Question 3 how can you explain the results of Question 1? Can you prove your explanation? Provide appropriate initial conditions and plot(s) to support your statement.

Part 2: Use a separate Jupyter notebook or text file to extend your 1-D implementation to 2-D. A general introduction to multi-dimensional extension is given in Chapter 16 of Toro. Specifically look at Sections 16.4.1 and 16.5 (p. 561) for the most important conceptual steps regarding the MUSCL-Hancock scheme in 2-D.

Hint: Remember to update your boundary conditions and timestep calculation (see 16.3.2) for 2-D. You also might need to use lower CFL values, e.g. 0.6, than in 1-D for stable simulations.

Then, do/answer the following:

1. In order to verify your framework, implement the initial conditions for the shock-tube problem in 2-D for

   - a plane shock in the x-direction and
   - a plane shock in the y-direction.

   Plot and compare the solutions for the density across the domain along the corresponding shock direction and at different positions (for both HLL and HLLC Riemann solver). How do the results compare to each other (with respect to shock direction) and to the 1-D results?

2. Implement the initial conditions for a 2-D spherical explosion (the Sedov-Taylor blast wave; see Toro Sections 17.1 and 17.2), i.e.

   - a 2-D domain $[0, 2] \times [0, 2]$ with Nx = Ny = 101, with an
   - inner sphere centered at $[1, 1]$ with radius $R_s = 0.4$ with $\rho_s = 1$, $u_x = u_y = 0$ and $p_s = 1$, and an
   - ambient medium outside the sphere with $\rho_a = 0.125$, $u_x = u_y = 0$ and $p_a = 0.1$.

   Plot and compare the primitive quantities at $t = 0.25$ across the center of the domain in x-, y-, and both diagonal directions as a function of distance from the center of the domain. How do these results differ from the plane shock, and why? Also, you can create the inner high-pressure sphere using a variety of radii and means of smoothing the edges (i.e., mapping a circle/sphere onto a Cartesian grid in a way that minimizes the 'stair-step' appearance of the initial conditions). Does changing how you initialize the inner region affect your results either qualitatively or quantitatively? Try varying the radius and, separately, smoothing the edges of the over-pressured region to see what happens and describe what you see.