

In-class assignment #13

PHY-905-005
Computational Astrophysics and Astrostatistics
Spring 2023

Instructions: In today's class we are going to experiment with the backward-Euler implicit discretization of the 1D diffusion equation with constant constant of diffusivity in a *finite-volume grid* and test it using the same Gaussian pulse as in the pre-class assignment (i.e., Equation 10.14 in Zingale with constants as in exercise 10.2). The equation we're going to solve is:

$$\frac{\partial \phi}{\partial t} = k \frac{\partial^2 \phi}{\partial^2 x} \quad (1)$$

Do the following:

1. Analytically calculate the matrix coefficients for a backward-Euler implicit discretization assuming (i) Dirichlet boundary conditions with $\phi_L = \phi_R = 1$ and (ii) homogeneous Neumann boundary conditions at both ends of the grid. Compare with your group members to make sure you've done it correctly!
2. Implement the backward-Euler solution for the Dirichlet boundary condition version using the matrix method for a grid of arbitrary size N_{grid} and Courant factor C . (Hint: can you recycle any code from a previous pre-class or in-class assignment?)
3. Test your solution for a variety of values of the Courant factor (C) and grid resolution (N_{grid}), from $C < 1$ to $C \gg 1$ and grid sizes where the initial pulse is poorly resolved to extremely well resolved ($N_{grid} \simeq 10$ to $N_{grid} \gg 100$). What combinations of N_{grid} and C produce good results? Poor results? How large can C be and still reproduce the expected analytic solution for a given N_{grid} ?

If you still have time after you're done with this, think about how you might implement this problem using the Crank-Nicholson method rather than backward-Euler. How much would you have to modify your code to do this?

As per usual, submit your code, plots, etc. via GitHub!