Sublinear-Space Approximations of Centrality in Evolving Massive Graphs

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Overview

1. Introduction & Background
2. Summary of Results
3. Pseudo-Asynchronous Communication for Distributed Algorithms
4. DegreeSketch and Local Triangle Count Heavy Hitters
5. Sublinear κ-Path Centrality

Overview

1. Introduction & Background
   1. Problem Overview
   2. Graph Primitives
   3. Streaming Data Model Background
   4. Sketching Definitions
2. Summary of Results
3. Pseudo-Asynchronous Communication for Distributed Algorithms
4. DegreeSketch and Local Triangle Count Heavy Hitters
5. Sublinear κ-Path Centrality

Motivation

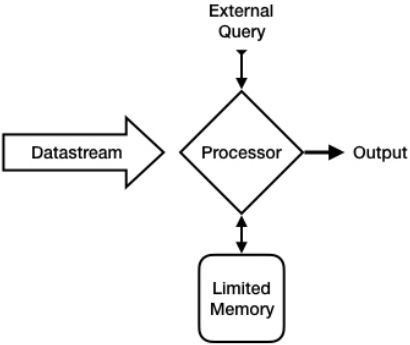
- Many modern computing problem focus on complex relational data
- Data are phrased as large graphs
  - e.g. the Internet, communication networks, transportation systems, protein networks, epidemiological models, social networks
- Often want to identify which vertices are "important"

Approach

Use data stream and distributed memory models

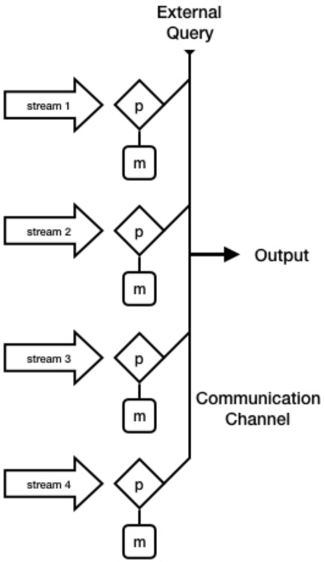
Overcoming Data Scale: Data Streaming

- Traditional RAM algorithms scale poorly
  - Awkward to store data in memory
  - Superlinear scaling unacceptable
- Data stream model to the rescue!
  - Sequential data access
  - Sublinear memory
  - Nearly linear amortized time
  - Constrained number of passes
  - Monte Carlo Approximations



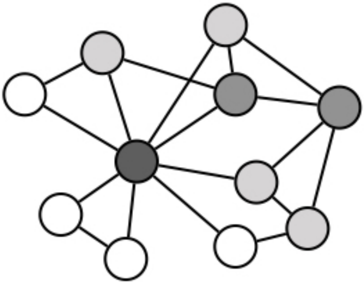
Overcoming Data Scale: Distributed Data Streaming

- Distributed memory model a staple of HPC
  - Divide computation
  - Immense scaling
- Why not distributed data streams!?!
  - **Sketches** - composable summaries
  - vertex-centric algorithms
  - Even greater scaling
  - Linear communication



Centrality Indices

- Assign scores to vertices
  - Higher score $\rightarrow$ more important
  - Depends on graph structure
  - Different indices in different domains
- Scores are not informative
  - Usually want top k vertices
- Relative order-preserving approximation is acceptable



Massive-Scale Graph Centrality

The Problem

- Memory overhead
- Compuational Overhead
- Communication Overhead
- Wasted effort
  - Generally only need top elements vis-á-vis a centrality index

Our Solution

- Sketch data structures
  - Utilize composable streaming summaries of vertex-local information
- Distributed memory
  - Partition graph and distribute sketches
  - Polyloglinear computation, memory, and communication

Streaming Background

Assume throughout that $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{w})$, where $|\mathcal{V}| = n$ and $|\mathcal{E}| = m$

- $\mathbf{w}_e$ is the weight of edge e if $e \in \mathcal{E}$ and zero otherwise
- $\mathcal{G}$ has adjacency matrix $A \in \mathbb{R}^{n \times n}$ so that $A_{x,y} = \mathbf{w}_{xy}$ for $xy \in \mathcal{E}$

$\mathcal{G}$ is given by a *stream* σ

- A list of edge insertions
- If deletions exist, say *turnstile stream*

An algorithm accumulating a data structure $\mathcal{S}$ and is said to be...

- *streaming* if $\mathcal{S}$ uses $\widetilde{O}(1) = O(\log n)$ [1] memory
- *semi-streaming* if $\mathcal{S}$ uses $\widetilde{O}(n) = O(n \operatorname{polylog} n)$ [2] memory

Want to minimize the number of passes over σ

- 1 pass ideal
- Constant or logarithmic passes sometimes acceptable

[1] $\widetilde{O}()$ notation suppresses polylogarithmic factors

[2] sometimes $\widetilde{O}\left(n^{1+\alpha}\right)$ for $\alpha \in (0, 1/2]$

Sketching

Definition (Sketch)

A *Sketch* is a streaming data structure $\mathcal{S}$ that admits a merge operator $\oplus$. If $\circ$ is the stream concatenation operator, then for any streams σ_1 and σ_2,

$$\mathcal{S}(\sigma_1) \oplus \mathcal{S}(\sigma_2) = \mathcal{S}(\sigma_1 \circ \sigma_2).$$

Definition (Linear Sketch)

A *Linear Sketch* $\mathcal{S}$ is a linear projection of $\mathbf{f}$ to a lower dimension. For any streaming frequency vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ and scalars a and b,

$$a\mathcal{S}(\mathbf{f}_1) + b\mathcal{S}(\mathbf{f}_2) = \mathcal{S}(a\mathbf{f}_1 + b\mathbf{f}_2).$$

Sketches are useful for stream summarization when *comparisons between streams* are important

Overview

1. Introduction & Background
2. Summary of Results
   1. Streaming Degree Centrality
   2. $O(1)$-Pass Semi-Streaming Closeness Centrality
   3. 2-Pass Semi-Streaming Triangle Count Heavy Hitters
   4. Distributed Sublinear κ-Path Centrality
3. Pseudo-Asynchronous Communication for Distributed Algorithms
4. DegreeSketch and Local Triangle Count Heavy Hitters
5. Sublinear κ-Path Centrality

Summary of Results: Serial Algorithms

Degree Centrality

$$\mathcal{C}^{\mathrm{DEG}}(x) = |\{(u, v) \in E \mid x \in \{u, v\}\}| = \|A_{x,:}\|_1 = \|A_{:,x}\|_1$$

- Naïve online $O(n)$-space and -time algorithm exists

Summary of Results: Serial Algorithms

Degree Centrality

We show $\widetilde{O}(1)$-space distributable streaming algorithms

- Naïve online $O(n)$-space and -time algorithm exists

Summary of Results: Serial Algorithms

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Closeness Centrality

$$\mathcal{C}^{\text{CLOSE}}(x) = \frac{1}{\sum_{y \in V} d(x,y)}$$

- Online exact $O(n^2)$-space $O(nm)$-time algorithm [WC14]
- Batch Approximate $O(n^2)$-space and almost-linear time algorithm [CDPW14]

Summary of Results: Serial Algorithms

Degree Centrality

We show $\widetilde{O}(1)$-space distributable streaming algorithms

- Naïve online $O(n)$-space and -time algorithm exists

Closeness Centrality

$$\mathcal{C}^{\mathrm{CLOSE}}(x) = \frac{1}{\sum_{y} d(x, y)}$$

We show constant-pass semi-streaming algorithm

- Onl
- Batch Approximate $O(n^2)$-space and almost-linear time algorithm [CDPW14]

Summary of Results: Distributed Streaming Algorithms

Triangle Count Centrality

$$\mathcal{C}^{\mathrm{TRI}}(x) = |\{yz \in \mathcal{E} \mid xy, yz, xz \in \mathcal{E}\}| \qquad \text{(vertex-local)}$$

$$\mathcal{C}^{\mathrm{TRI}}(xy) = |\{z \in \mathcal{E} \mid xy, yz, xz \in \mathcal{E}\}| \qquad \text{(edge-local)}$$

- Exact $O(m)$-space, $O\left(m^{\frac{3}{2}}\right)$-time serial and distributed algorithms [AKM13, Pea17] [a]
- Streaming sampling sublinear-space algorithms [LK15, SERU17]
    - Including distributed generalizations [SHL+18, SLO+18]

[a] Roger Pearce, "Triangle counting for scale-free graphs at scale in distributed memory," HPEC 2017

Summary of Results: Distributed Streaming Algorithms

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We show 2-pass, semi-streaming, distributed sketch-based query algorithms for estimating heavy hitters [PPS18, PPS19] [a] [b]

[a] Benjamin W. Priest, Roger Pearce and Geoffrey Sanders, "Estimating Edge-Local Triangle Count Heavy Hitters in Edge-Linear Time and Almost-Vertex-Linear Space," HPEC 2018

[b] Benjamin W. Priest, Roger Pearce and Geoffrey Sanders, "DegreeSketch: Distributed Cardinality Sketches on Graphs, with Applications to Counting Triangles," In preparation for SigKDD 2019

Summary of Results: Distributed Streaming Algorithms

κ-Path Centrality

$$\mathcal{C}^{\kappa}(x) = \Pr_{p:|p|\leq\kappa}[x \in p \wedge p \text{ a simple path }]$$

- $O(m)$-space $O(n^{1+\alpha}\log^2 n)$-time approximation algorithm [WC14]
- Empirical proxy for betweenness centrality heavy hitters
  - Online exact and approximate $O(n^2)$- and $O(m)$-space algorithms exist [GMB12, WC14, KMB15, BMS14]

Summary of Results: Distributed Streaming Algorithms

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- We show distributed sublinear vertex-centric random walk and simple path sampling algorithms
- Yields sublinear distributed κ-path centrality approximation algorithm

Overview

Motivation: Vertex-Centric Algorithms

The Problem:

- Most distributed graph algorithms are vertex-centric
  - Partition local vertex information across processors
  - Processors communicate as in rounds [MAB$^+$10]
- Scale-free graphs common in applications
  - High degree vertices cause computation "hotspots"
  - Moves at the speed of the slowest processor

Solutions:

- Asynchronous Communication
  - Processors communicate point-to-point as needed
  - Increased implementation complexity
- Vertex delegation [PGA14] [3]
  - Cut hubs between processors

[3] Roger Pearce, Maya Gokhale and Nancy M. Amato, "Faster parallel traversal of scale free graphs at extreme scale with vertex delegates," SC 2014

Approach: Pseudo-Asynchronous Communication Protocol

The Idea

Aggregate and route messages "asynchronously", allowing processors to drop out of communication exchanges when finished

- Partion processor set $\mathcal{P}$ into *local* and *remote* exchanges
  - Takes advantage of hybrid distributed memory
- Mailbox abstraction
  - Aggregate messages at intermediaries
  - Route destination node traffic traffic through same remote channel
- Three protocols:
  - Node Local
  - Node Remote
  - Node Local Node Remote (NLNR)

Node Local and Node Remote

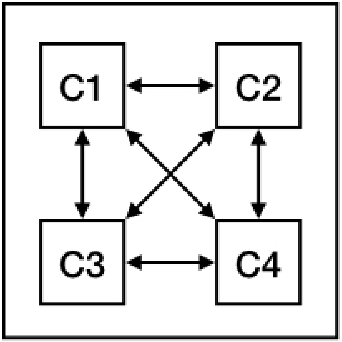
Node Local

1. Exchange locally
2. Forward remotely

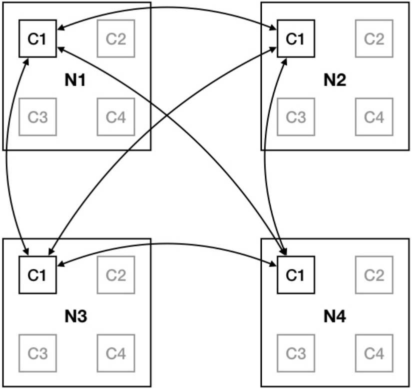
Node Remote ✓

1. Exchange remotely
2. Forward locally

Local Route	**Remote Route**



Node



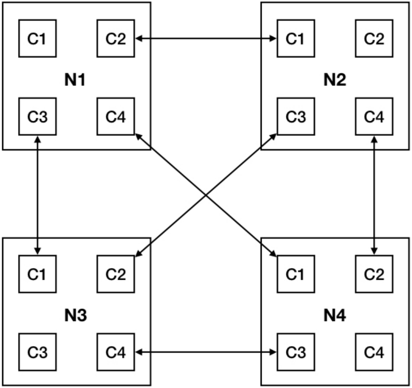
Node Local Node Remote

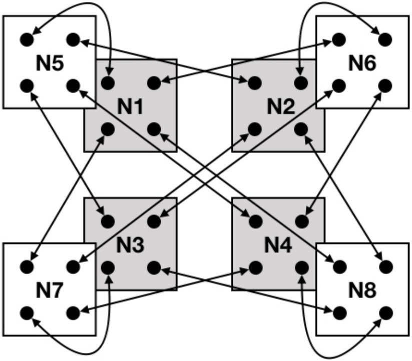
Node Local Node Remote

- Further partition $\mathcal{P}$ by *layers*
    - Set of (# cores) nodes

1. Exchange locally
2. Forward remotely
3. Forward locally

Intra-Layer Remote Route	Inter-Layer Remote Route

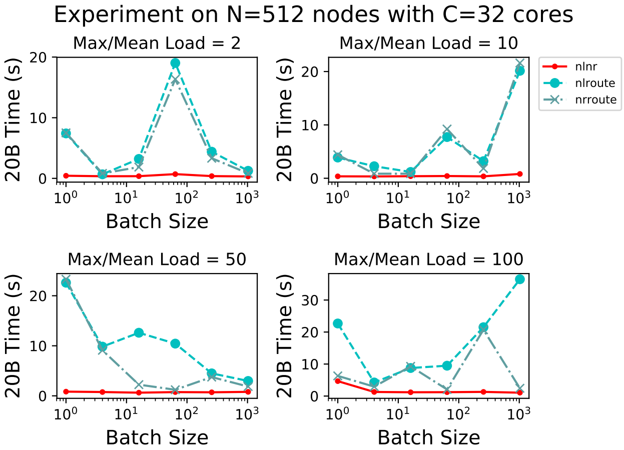




Validation of Claims

Experiment

- 20B message exchange
  - Destination sampled from Pareto distribution
  - Batch size is maximum $|\mathcal{S}[P]|$
- Run on quartz @ LLNL



NLNR exhibits best scaling, NR best with fewer nodes

Implementation Details

YGM **C++/MPI Library**

- Authored by myself, Trevor Steil (UMN), and Roger Pearce (LLNL)
- Simple API for handling pseudo-asynchronous communication
  - Clients need only specify receive behavior
- Supports LLNL Projects
  - HavoqGT (graph challenge & pattern matching)
  - Sierra 42 - largest scale to date graph 500 ($\sim$ 70T edges)
  - Possibly more in the future
    - Eccentricity
- Improvements over legacy HavoqGT routing
  - Flow control via pseudo-asynchronicity
  - NLNR more scalable
  - Variable length messages

YGM to be open sourced, published in IPDPS 2019 workshop

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   1. Local Triangle Counting
   2. HyperLogLog Cardinality Sketches
   3. DegreeSketch and Triangle Counting
   4. Verification & Implementation Details
5. Sublinear κ-Path Centrality

Motivation: Local Triangle Counting

The Problem:

- Local triangle counting a common big data analytic
  - Exact computation expensive $O\left(m^{\frac{3}{2}}\right)$!
- Recall

$$\mathcal{C}^{\mathrm{TRI}}(x) = |\{yz \in \mathcal{E} \mid xy, yz, xz \in \mathcal{E}\}| \qquad \text{(vertex-local)}$$
$$\mathcal{C}^{\mathrm{TRI}}(xy) = |\{z \in \mathcal{E} \mid xy, yz, xz \in \mathcal{E}\}| \qquad \text{(edge-local)}$$

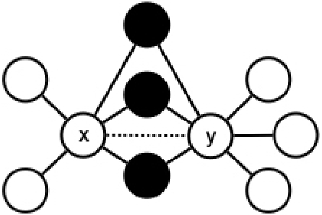
Existing Solutions:

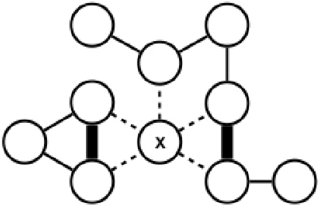
- Many exact distributed algorithms [AKM13, Pea17]
- Many approximate streaming algorithms via sampling [LK15, SERU17]
- ... and some utilizing both models [SHL+18, SLO+18]

Approach: Sublinear Intersection Method

Idea: Intersection method, but using cardinality sketches

- Cardinality sketches summarize set size
- Support union operation, and some support limited intersection operation
  - High variance if intersection is small
  - Likely best performance on heavy hitters
- Affords edge- and vertex-local triangle count estimation
- Outputs only reliable if *triangle density* is nontrivial
  - Triangle density $= \frac{\#\text{ triangles}}{\#\text{ possible triangles}}$





HyperLogLog Cardinality Sketches

HLL cardinality sketches

Maintain $r = 2^p$ 6-bit registers M and a 64-bit hash function h

- Insert x: let $i = \langle x_1, \ldots, x_p \rangle$ and $w = \langle x_{p+1}, \ldots, x_{64} \rangle$
- $\rho(w)$ = initial zero bits of w plus 1
- $M_i = \max\{M_i, \rho(w)\}$
- Estimator derives from harmonic mean of M

HyperLogLog Cardinality Sketches

HLL cardinality sketches

Maintain $r = 2^p$ 6-bit registers M and a 64-bit hash function h

- Insert x:
- $\rho(w) =$ in
- $M_i = \max\{M_i, \rho(w)\}$
- Estimator derives from harmonic mean of M

Outputs $\widetilde{C}$ such that for cardinality C,
w.h.p. $|C - \widetilde{C}| \leq \frac{1.04}{\sqrt{m}} C$ [FFGM07]

HyperLogLog Cardinality Sketches

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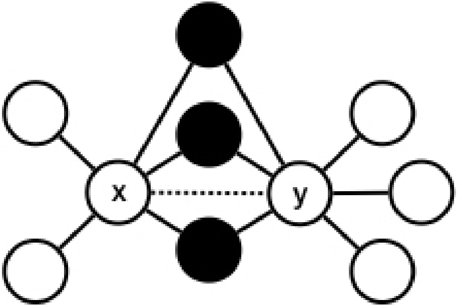
Useful results

- Native intersection operator (elementwise maximum)
- Various improved harmonic [HNH13, QKT16] and maximum likelihood estimators [XZC17, Lan17, Ert17]
- Sparsification for low cardinality sets [HNH13]
- Compression to 4 and 3 bit registers [XZC17]
- Intersection estimators [Tin16, CKY17, Ert17]

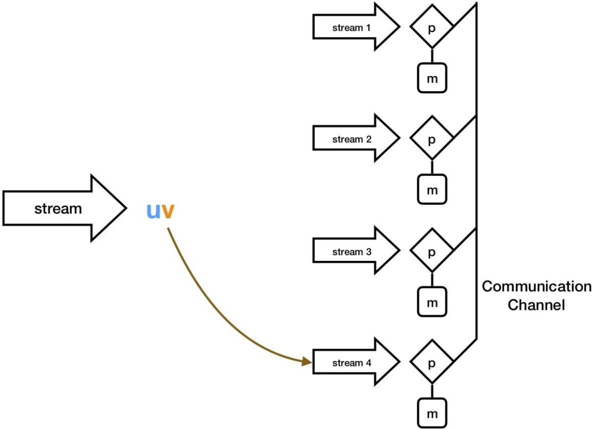
Set operation estimation with HLLs

Streaming sets X and Y with HLLs S_X and S_Y

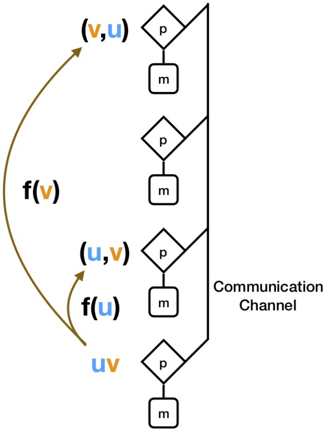
- $S_X \approx |X|$
- $S_Y \approx |Y|$
- $S_X \widetilde{\cup} S_Y \approx |X \cup Y|$
  - Same error guarantees
- $S_X \widetilde{\cap} S_Y \approx |X \cap Y|$
  - High variance if $|X \cap Y|$ small



DegreeSketch Accumulation

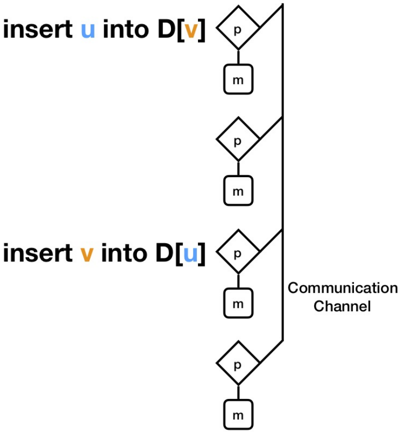


Partition stream across $\mathcal{P}$



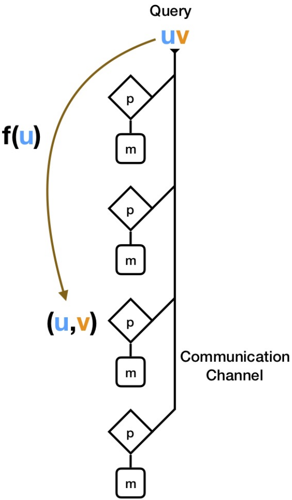
Distribute edges to endpoint owners

DegreeSketch Accumulation



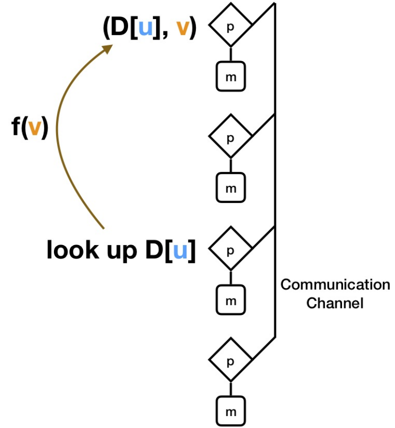
Insert into $\mathcal{D}$ for each vertex

DegreeSketch Query



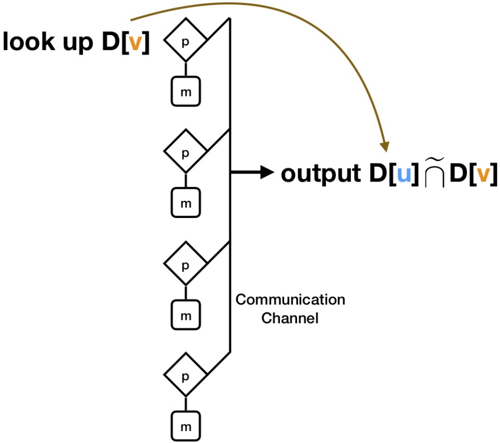
Query routes to one endpoint owner

DegreeSketch Query



Owner sends cardinality sketch to other endpoint owner

DegreeSketch Query



Final owner outputs intersection estimation

$\textsc{DegreeSketch}$ and Triangle Counting

Assume a partition $f : \mathcal{V} \to \mathcal{P}$, and let $\mathcal{V}_P = \{v \in \mathcal{V} \mid f(v) = P\}$

- Distribute $\textsc{DegreeSketch}$ $\mathcal{D}$ across $\mathcal{P}$
  - $\mathcal{D}[v]$ holds a HLL for adjacency set of $v \in \mathcal{V}$
  - P holds $\mathcal{D}[v]$ for $v \in \mathcal{V}_P$
- Accumulate $\mathcal{D}$ in one pass over σ
  - Assume $P \in \mathcal{P}$ gets substream σ_P
  - P sends $xy \in \sigma_P$ to $f(x)$ and $f(y)$
  - When P gets $xy : x \in \mathcal{V}_P$, insert y into $\mathcal{D}[x]$
    - $\mathcal{D}[x]$ starts sparse and eventually saturates
- $\mathcal{D}$ can be queried after estimation, e.g.
  - Estimate $\widetilde{\mathcal{C}}^{\mathrm{DEG}}(v) = \textsc{Estimate}(\mathcal{D}[v])$
  - Estimate $\widetilde{\mathcal{C}}^{\mathrm{TRI}}(uv) = \mathcal{D}[u] \widetilde{\cap} \mathcal{D}[v]$
    - Involves communication if $f(u) \neq f(v)$
  - Estimate $\widetilde{\mathcal{C}}^{\mathrm{TRI}}(v) = \frac{\sum_{uv \in \mathcal{E}} \widetilde{\mathcal{C}}^{\mathrm{TRI}}(uv)}{2}$
    - Requires second pass in general

$\widetilde{O}(m)$ time and communication and $\widetilde{O}(\varepsilon^{-2} n)$ space!

Algorithm 1 Edge-Local Triangle Count Heavy Hitters

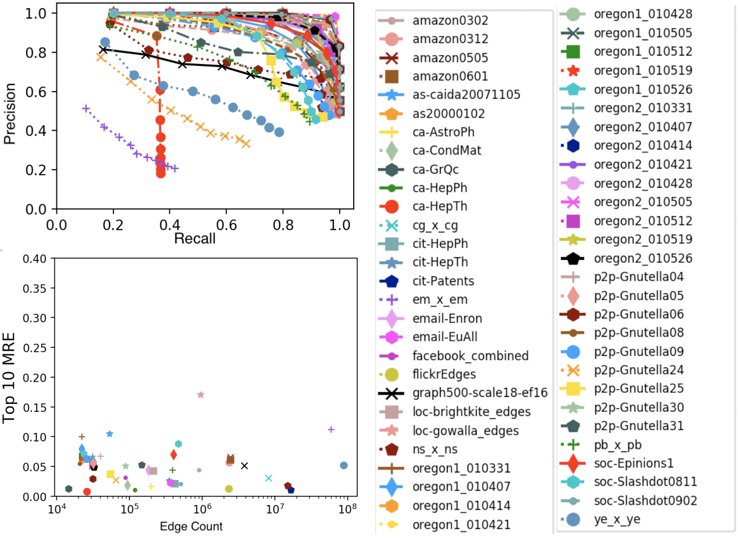
```
Accumulate $\mathcal{D}$ in distributed pass over $\sigma$
$H_k \leftarrow$ empty $k$-heap
$T \leftarrow 0$
parallel for $xy \in \sigma_P$ do        // second pass
    Send $(\mathrm{E}, xy)$ to $f(x)$
    for $(\mathrm{E}, xy) \in \mathcal{R}[P]$ do
        Send $(\mathrm{S}, xy, \mathcal{D}[x])$ to $f(y)$
    for $(\mathrm{S}, xy, \mathcal{D}[x]) \in \mathcal{R}[P]$ do
        Insert $(xy, \mathcal{D}[x] \widetilde{\cap} \mathcal{D}[y])$ into $H_k$
        $T \leftarrow T + \mathcal{D}[x] \widetilde{\cap} \mathcal{D}[y]$
$T \leftarrow T/2$
Global accounting of $T$, $H_k$
return $H_k$
```

Validation of Claims

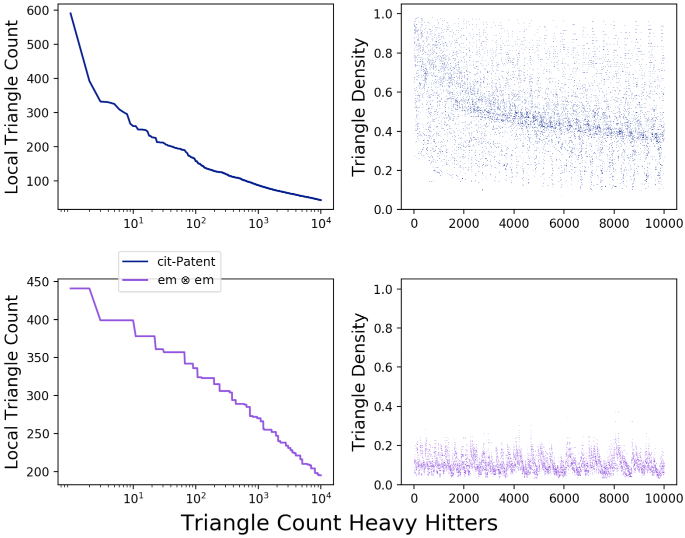
HPEC graph challenge results

Good performance on top 10 heavy hitters of *most* graphs

Why do some graphs perform worse?



High vs Low Triangle Density



Low triangle density → high variance

Implementation Details

DegreeSketch **C++/MPI Library**

- Authored by myself
- Utilizes YGM for communication
- Accumulation and query API for DegreeSketch
- Supports sparse and compressed registers
- Implementations for edge- and vertex-local triangle count heavy hitter estimation
- Supports more exotic queries
  - e.g. 2nd Neighborhood size

DegreeSketch to be open sourced

Overview

Motivation: Betweenness Centrality Heavy Hitters

The Problem:

- Computing Betweenness centrality exactly amounts to computing AllSourcesAllShortestPaths
  - Expensive $O(mn)$!

Existing Solutions:

- Approximate via a logarithmic number of SingleSourceAllShortestPaths [GMB12, BMS14, Yos14, KMB15, RK16]
  - Difficult to distribute
  - Unclear if possible in $o(m)$ memory

Approach: Sublinearize κ-Path Centrality

Idea: "Come at the problem sideways"

- High κ-path centrality empirically correlates with high betweenness centrality [KAS$^+$13]
- Algorithm amounts to sampling random simple paths
  - Use ℓ_p sampling sketches to sublinearize
- Sublinear approximation of κ-path centrality $\rightarrow$ emprical recovery of high betweenness centrality vertices?

κ-path centrality

$$\mathrm{PC}(x, \kappa) = \Pr_{p:|p| \leq \kappa}[x \in p \wedge p \text{ a simple path}]$$

"simple path" $=$ non-self-intersecting path

ℓ_p Sampling Sketches

ℓ_p sampling sketches

Sample from frequency vector $\mathbf{f}$ with probability relative to ℓ_p norm

- Sample $t_i \sim_R (0,1)\ \forall i \in [n]$
- Rescale updates to $\mathbf{f}_i$ by $1/t_i^{1/p}$
- Accumulate $\textsc{Tug-of-War}$, $\textsc{CountSketch}$, and ℓ_p norm sketches
- Use sketches to output $\textsc{CountSketch}$ argmax or FAIL

ℓ_p Sampling Sketches

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Sample from frequency vector **f** with probability relative to ℓ_p norm

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- Rescale updates
- Accumulate TUG ... nd ℓ_p norm sketches
- Use sketches to output COUNTSKETCH argmax or FAIL

Outputs (i, P) w.p. $1 - \delta$, where $i \in [n]$ is sampled w.p. $P = (1 \pm \varepsilon)\frac{|v_i|^p}{\|v\|_p^p}$ [MW10]

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where $i \in [n]$ is sampled w.p.
$P = (1 \pm \varepsilon)\frac{|v_i|^p}{\|v\|_p^p}$ [MW10]

Useful results [JST11, Vu18]

- ℓ_0 sketch requires $\widetilde{O}(\log(1/\delta))$ memory and update time
  - Useful for unweighted random hops
- ℓ_1 sketch requires $\widetilde{O}(\varepsilon^{-1}\log(1/\delta))$ memory and $\widetilde{O}(\log(1/\delta))$ update time
  - Useful for weighted random hops
- s parallel ℓ_p sketches can be accumulated in time independent of s

Distributed Accumulation ℓ_p Sampling Sketches

- $P \in \mathcal{P}$ accumulates adjacency set $\mathcal{A}[v]$ for each $v \in \mathcal{V}_P$
  - When $\mathcal{A}[v]$ too large, replace it with s ℓ_0 sampling sketches
  - Write current state and all subsequent updates to disk memory
- Queries to $\mathcal{A}[v]$ return a sampled neighbor of v
  - If $\mathcal{A}[v]$ is a set of sketches, one is consumed
    - If FAIL, repeat
  - Once $\mathcal{A}[v]$ sketches are exhausted, P takes another pass over v's substream in disk memory

Avoids vertex cuts, exchanging communication for I/O

Sublinear Random Walk and Simple Path Sampling

Random Walk Simulation

- Sample t vertices $\{v_{1,1}, \ldots, v_{t,1}\}$ and:
  - Sample $v_{i,j+1}$ from $\mathcal{A}[v_{i,j}]$
  - Communicate $(v_{i,1}, \ldots, v_{i,j+1})$ to $f(v_{i,j+1})$

Random Simple Path Simulation

- Similar to random walks, except:
  - Do not accumulate sketches ahead of time
  - Sample $v_{i,j+1}$ from $\mathcal{A}[v_{i,j}] \setminus \{v_{i,1}, \ldots, v_{i,j-1}\}$
    - If $\mathcal{A}[v_{i,j}]$ is not in memory, accumulate a sketch ignoring edges to any of $\{v_{i,1}, \ldots, v_{i,j-1}\}$
  - Communicate $(v_{i,1}, \ldots, v_{i,j+1})$ to $f(v_{i,j+1})$

Sublinear distributed storage of graph by sketching high degree vertices

Sublinear κ-Path Centrality

κ-Path Centrality Approximation Algorithm ([KAS+13]):

1. Simulate $T = 2\kappa^2 n^{1-2\alpha} \ln n$ $(\leq \kappa)$-length simple paths over $\mathcal{G}$
   - maintain $count[x]$ for each $x \in \mathcal{V}$
2. $\widetilde{\mathcal{C}}^{\kappa}(x) \leftarrow \frac{count[x]}{2\kappa n^{-2\alpha} \ln n}$

- Given $\alpha \in [-1/2, 1/2]$, for each $x \in \mathcal{V}$, $\left|\widetilde{\mathcal{C}}^{\kappa}(x) - \mathcal{C}^{\kappa}(x)\right| \leq n^{1/2+\alpha}$ w.h.p.
- Easy to distribute in vertex-centric model

Sublinear κ-Path Centrality

Algorithm 2 Sublinear κ-Path Centrality

for $i \in \{1, \ldots, T\}$ **do**
    $p_i \leftarrow$ empty path
    $p_{i,1} \leftarrow$ uniform sample from $\mathcal{V}$
    $l_i \leftarrow$ uniform sample from $\{1, 2, \ldots, \kappa\}$
for $x \in \mathcal{V}$ **do** $c_x \leftarrow 0$
parallel for $j \in \{1, 2, \ldots, \kappa - 1\}$ **do**
    parallel for $i \in \{1, 2, \ldots, T\}$ **do**
        if $j < l_i$ **then**
            $p_{i,j+1} \leftarrow$ sample from $\mathcal{A}[p_{i,j}]$
            if $p_{i,j+1} = \emptyset$ **then** discard
        else if $j = l_i$ **then**
            $c_{p_{i,k}} \leftarrow c_{p_{i,k}} + 1$ for $k \in \{1, \ldots, j\}$
return $c_x / 2\kappa n^{-2\alpha} \ln n$ for $x \in V$

Summary of Results

- The Goal: distributed sublinear approximations of centrality indices
- Engineering Results
  - YGM: Pseudo-Asynchronous Communication Handler
- Algorithmic Results
  - A streaming degree centrality approximation and heavy hitter recovery algorithms
  - A O(1)-pass semi-streaming closeness centrality approximation algorithm
  - 2-pass distributed semi-streaming edge- and vertex-local triangle count heavy hitter estimation algorithms using DegreeSketch
  - Distributed sublinear semi-streaming random walk and random simple path sampling algorithms
  - Distributed sublinear semi-streaming κ-path centrality estimation algorithm
- Future Work
  - Applications for DegreeSketch
  - Sublinear random walk and random simple path implementation

Questions?

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