Continuity and the Intermediate Value Theorem (CATIVT)

Recitation Questions

Problem 1 (a) *Let* $f(x) = \dfrac{x-1}{x^2-5x}$. *Then* $f(2) = -\dfrac{1}{6}$ *and* $f(6) = \dfrac{5}{6}$, *but there is no value of* c *between* 2 *and* 6 *for which* $f(c) = 0$. *Does this fact violate the Intermediate Value Theorem?*

(b) *True or False: At some time since you were born your weight in pounds exactly equaled your height in inches.*

Problem 2 *For the following function g defined by*

$$g(t) = \begin{cases} 5t+7 & \text{if } t < -3 \\ \dfrac{(t-1)(t+2)}{t+2} & \text{if } -3 \leq t < 1 \text{ and } t \neq -2 \\ 4\ln t & \text{if } t \geq 1 \end{cases}$$

*find the **Intervals of Continuity**.*
*(**Important Note:** Write your answer as a list of intervals, with each interval separated by a comma.)*

Problem 3 *Determine the value of a constant b for which f is continuous at 0.* ***EXPLAIN***.

$$f(x) = \begin{cases} \dfrac{2x+b}{x-5} & \text{if } x < 0 \\[2ex] \dfrac{x+16}{x^2-16} & \text{if } x \geq 0 \end{cases}$$

Problem 4 *Use the Intermediate value theorem to find an interval in which you can guarantee that there is a solution to the equation* $x^3 = x + \sin(x) + 1$. ***EXPLAIN***. *(Do not use a graphing device or calculator to solve this problem!)*

Problem 5 (a) *True or False: If* f *and* g *are two functions defined on* $(-1,1)$, *and if* $\lim_{x\to 0} g(x) = 0$, *then it must be true that* $\lim_{x\to 0}(f(x)\cdot g(x)) = 0$.

(b) *True or False: If* f *is continuous on* $(-1,1)$, *and if* $f(0) = 10$ *and* $\lim_{x\to 0} g(x) = 2$, *then*

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = 5.$$

(c) *True or False: If* f *is continuous on* $[1,3]$, *and if* $f(1) = 0$ *and* $f(3) = 4$, *then the equation* $f(x) = \pi$ *has a solution in* $(1,3)$.

(d) *True or False: Let* f *be a positive function with vertical asymptote* $x = 5$. *Then*

$$\lim_{x\to 5} f(x) = \infty.$$

Problem 6 *Let*

$$h(u) = \begin{cases} \dfrac{u^2 - 5u + 4}{u - 4} & \text{if } u < 4 \\ \\ \dfrac{-\sqrt{u+4}}{u-6} & \text{if } u \geq 4, u \neq 6. \end{cases}$$

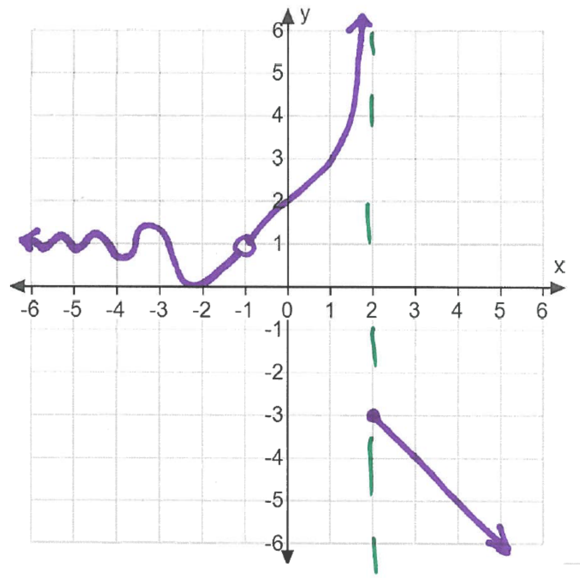
(a) *What is the domain of* h*?*

(b) *Find all vertical asymptotes of* h*.* ***EXPLAIN****.*

(c) *Find all horizontal asymptotes of* h*.* ***EXPLAIN****.*

(d) *List the* ***Intervals of Continuity*** *for the function* h*.*

Problem 7 *Use the graph of f to answer the questions below.*



(a) *State the domain of f.*

(b) *Find the following values or state "does not exist":*

(i) $\lim_{x\to 2^-} f(x) =$

(ii) $\lim_{x\to 2^+} f(x) =$

(iii) $\lim_{x\to 2} f(x) =$

(iv) $\lim_{x\to -1} f(x) =$

(v) $\lim_{x\to 3} f(x) =$

(vi) $\lim_{x\to -\infty} f(x) =$

(vii) $f(-1) =$

(c) *State the equation of any vertical asymptotes.*

(d) *State the equation of any horizontal asymptotes.*

(e) *Find the Intervals of Continuity.*

Problem 8 *Suppose a taxi ride costs \$7.50 for the first mile (or any part of the first mile), plus an additional \$1.00 for each additional mile (or any part of a mile).*

(a) *Graph the function $c = f(t)$ that gives the cost of a taxi ride for t miles, for $0 \leq t \leq 5$.*

(b) *Evaluate* $\lim_{t \to 2.9} f(t)$

(c) *Evaluate* $\lim_{t \to 3^-} f(t)$ *and* $\lim_{t \to 3^+} f(t)$

(d) *Interpret the meaning of the limits in part (c).*

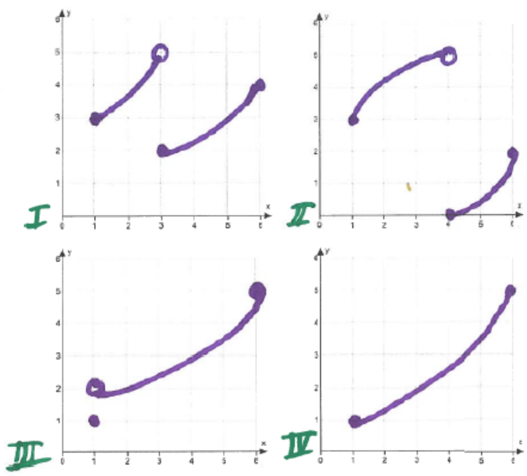
(e) *On what intervals is the function c continuous? Explain.*

Problem 9 (a) *Given function f on an interval $[a, b]$:*

(i) *What are the conditions of the Intermediate Value Theorem?*

(ii) *What is the conclusion of the Intermediate Value Theorem?*

(b) *Given the four functions on the interval $[1, 6]$, answer the questions below.*



(i) *For each of the functions I through IV, indicate $f(1)$ and $f(6)$. Then mark the interval of all numbers strictly between $f(1)$ and $f(6)$, on the y-axis.*

(ii) *For each of the functions I through IV, write an interval of all numbers strictly between $f(1)$ and $f(6)$*

(iii) *List the functions that satisfy the conditions of the Intermediate Value Theorem on $[1, 6]$*

(iv) *For which of the functions is the following statement true: For any number L strictly between $f(1)$ and $f(6)$, there exists a number c in $(1, 6)$ satisfying $f(c) = L$.*

(v) *Does the function III satisfy the conclusion of the Intermediate Value Theorem? Why or why not?*