Applications of Integrals (AOI)

SUMMARY: Velocity, Speed, Position, Displacement, Distance

Position, $s(t)$, of an object at time t:
$$s(t) = s(a) + \int_a^t v(z)dz$$

Displacement, $\Delta s = s(b) - s(a)$,
of an object over the time interval $[a, b]$:
$$s(b) - s(a) = \int_a^b v(t)dt$$

Distance traveled by an object
over the time interval $[a, b]$:
$$\int_a^b |v(t)|dt$$

SUMMARY: Rate of Accumulation, Amount, Change in the Amount

The amount, $A(t)$, of some
substance/population at the time t:
$$A(t) = A(a) + \int_a^t A'(z)dz$$

The amount, $A(b)$,
over the time interval $[a, b]$:
$$A(b) = A(a) + \int_a^b A'(t)dt$$

The change in the amount, $A(b) - A(a)$,
over the time interval $[a, b]$:
$$A(b) - A(a) = \int_a^b A'(t)dt$$

SUMMARY: Average Value, the Mean Value Theorem for Integrals

Average value $\bar{f}$,
of the function f on the interval $[a, b]$:
$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx$$

Mean Value Theorem for Integrals

Let f be continuous on $[a, b]$. There exists a value c in (a, b) such that

$$\frac{1}{b-a}\int_a^b f(x)dx = f(c)$$

Note 1: $f(c) = \bar{f}$, the average value of f on $[a, b]$.

Note 2: The net area of the region between the curve $y = f(x)$ and the $x-$axis is given by

$$\int_a^b f(x)dx = f(c)(b-a)$$

Recitation Questions

Problem 1 *Solve the following word problems:*

(a) *The velocity function for an object moving along a line east/west is given by* $v(t) = -t^2 + 4t - 3$ *feet per minute.*

(i) *Find the total displacement the object traveled from 2 minutes to 6 minutes (assume east is positive).*

(ii) *Find the total distance the object traveled from 2 minutes to 6 minutes.*

(iii) *Suppose that the object's position 2 minutes into the trip is 5 feet of a placement marker. What is its position (relative to the placement marker) at 6 minutes.*

(b) *Sammy the Snail sets up camp in the median of I-70 and, starting at noon and ending at 6pm, hikes back and forth along the highway. He starts his hike at his campsite. His velocity at time t hours (after noon) is given by $v(t) = (t-2)(t-5)$ inches per hour. Find the total distance Sammy travelled on his hike.*

Problem 2 *Suppose that $r(t) = r_0 e^{-kt}$ (with $k > 0$) is the rate at which a nation extracts oil. The current rate of extraction is $r(0) = 10^7$ barrels/yr. Also assume that the estimate of the total oil reserve (ie, the amount of oil remaining beneath the ground in this country) is 2×10^9 barrels.*

(a) *Find $A(t)$, the total amount of oil extracted by the nation after t years.*

(b) *Evaluate $\lim_{t\to\infty} A(t)$ and explain the meaning of this limit.*

(c) *Find the minimum constant k for which the total oil reserves will last forever.*

(d) *Suppose that the constant k is half the minimum value found in part (c). When will the nation deplete its oil reserve?*

Problem 3 *Assume that the rate of change (in dollars per day) of the price of shares of stock in the WeSaySo Company (with t in days) is modeled by the equation $r(t) = -3t^2 + 30t - 63$ (note that this is technically a discrete function, but prices change so often with stocks that modeling this with a continuous function makes sense). Assume also that the price of a share of stock on day 1 (i.e., $t = 1$) is \$51. Answer the following questions:*

(a) *Find the rate of change of price at $t = 5$.*

(b) *Find the price of a share of stock at $t = 5$.*

(c) *How fast is the rate of change of price changing at $t = 5$?*

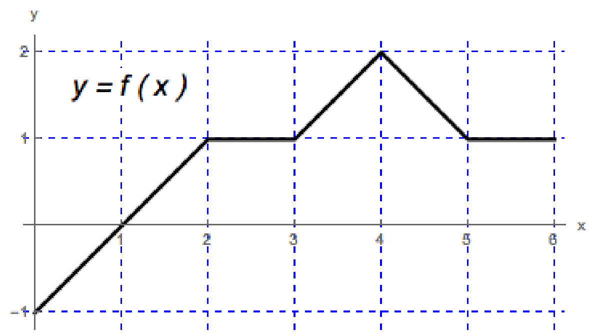
(d) *How much did the price of a share of stock change in the first* 6 *days (i.e., on* $[1, 6]$*)?*

(e) *What was the greatest rate of change of price during the first* 6 *days (i.e., on* $[1, 6]$*)?*

(f) *What was the greatest price of a share of stock during the first* 6 *days (i.e., on* $[1, 6]$*)?*

Problem 4 *A cup of coffee has temperature $20 + 75e^{-.02t}$ degrees (Celsius) t minutes after being poured into a cup. What is the average temperature of the coffee during the first half hour?*

Problem 5 *The graph of a function f defined on the interval $[0,6]$ is given in the figure.*



(a) *Compute the net area of the region between the graph of f and the x-axis, on the interval $[0,6]$.*

(b) *Draw a rectangle with base on the x-axis, $0 \leq x \leq 6$, whose area is equal to the net area in part (a).*

(c) *In the figure, mark a point c in $(0,6)$ such that $f(c)$ is the height of the rectangle from part (b).*

(d) *Using the figure and parts (a-c), what is the relationship between the rectangle from part (b), the net area from part (a), and the average value of f on $[0,6]$?*

Problem 6 *Find all points at which the given function equals its average value on the given interval.*

(a) $f(x) = e^x \qquad [0, 4]$

(b) $g(x) = \dfrac{\pi}{4}\sin(x) \qquad [0, \pi]$

Problem 7 *Find the average value of the function* $g(t) = 4te^{-t^2}$ *on the interval* $[0, 3]$.