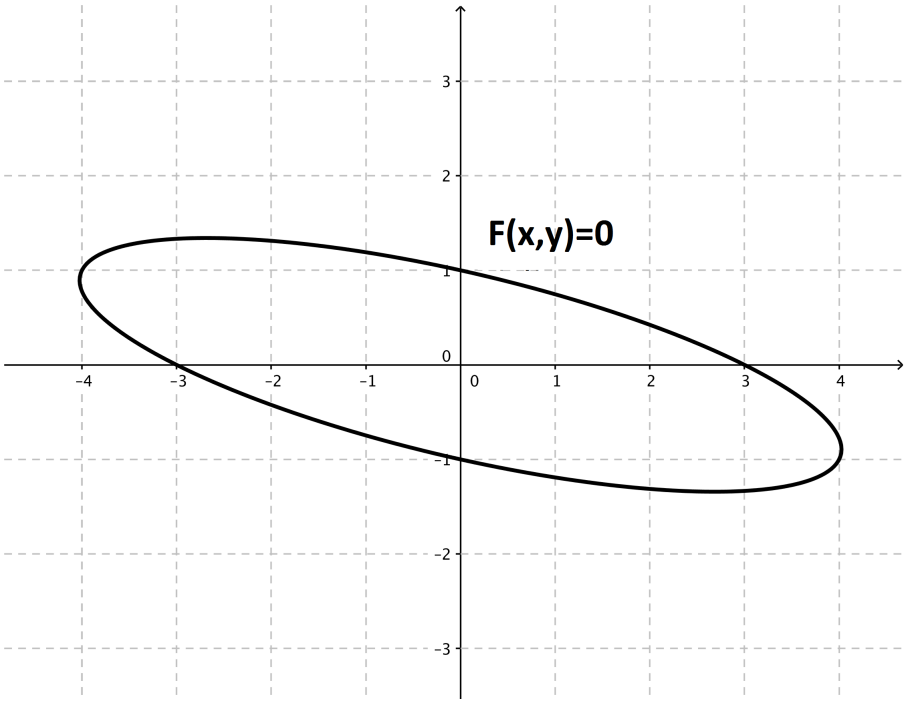
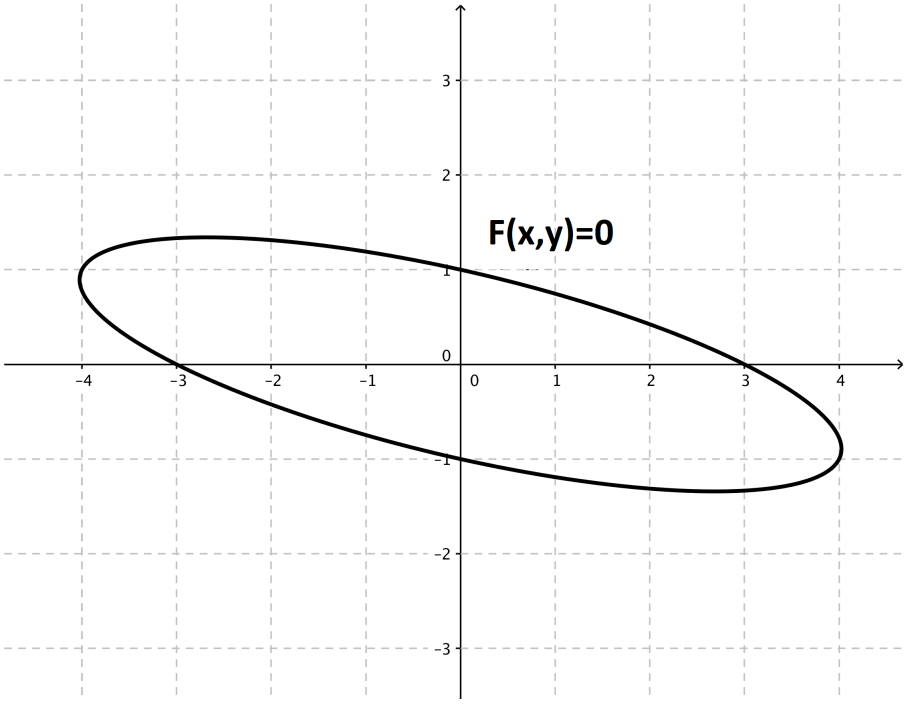
Implicit differentiation (ID)

Problem 1 *On the graph below, sketch the tangent lines at $x = 0$. Then, explain why both the x-coordinate and the y-coordinate are generally needed to find the slope of the tangent line at a point on the graph of an equation of the form $F(x, y) = 0$*



Problem 2 *Consider the equation $x^2+4xy+9y^2 = 9$. Note: This equation is equivalent to $x^2+4xy+9y^2-9 = 0$. Therefore it has a form $F(x, y) = 0$*

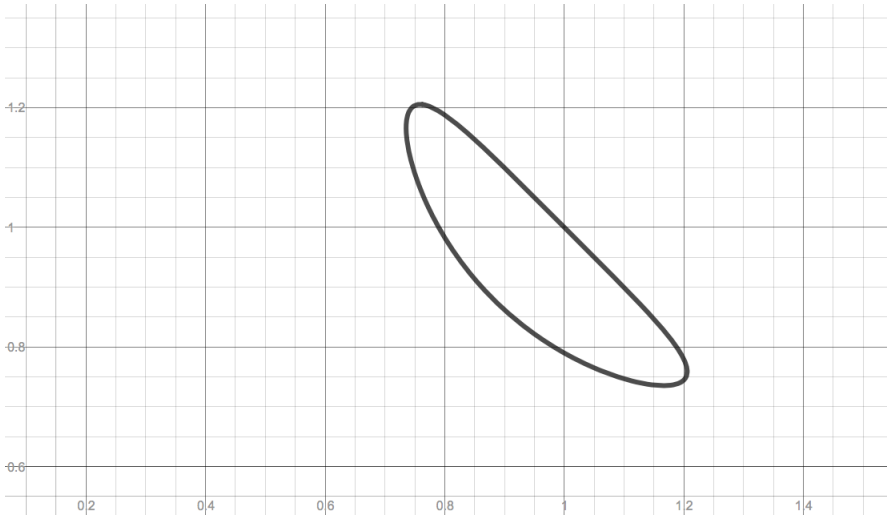


(a) *Find* $\dfrac{dy}{dx}$.

(b) *Find the equation(s) of the tangent line(s) when* $x = 0$. *Draw the tangent line(s) on the above picture.*

(c) *Find the point(s) where the tangent line is horizontal. Draw the point(s) and line(s) on the above picture.*

Problem 3 *A part of the curve with equation* $\cos(\pi xy) + x + y = 1$ *is sketched below.*



(a) *Use the implicit differentiation to find the derivative* dy/dx.

(b) *Consider the point* $(1,1)$. *Show (algebraically) that this point lies on the curve.*

(c) *Find the equation of the line tangent to the curve at* $(1,1)$. *Draw this line in the figure above.*

Problem 4 *For each of the curves given by the following equations, find a formula for the slope of the tangent line at a point* (x,y).

(a) $e^{x^2y^3} - 5x + 7y = 36$

(b) $7 = 22\tan(y) + \dfrac{4}{x} - \dfrac{7}{y}$

(c) $\cos(xy) - x^3 = 5y^3$

Problem 5 *The volume of a doughnut with an inner radius of* a *and an outer radius of* b *is*

$$V = \pi^2 \frac{(b+a)(b-a)^2}{4}.$$

Find db/da *if the volume of a doughnut is* $64\pi^2$ *and does not change.*

Problem 6 *The curve is given by the equation* $x^{1/3} + y^{2/3} = 2$. *Find* $\dfrac{d^2y}{dx^2}$.

Problem 7 *Sketch both the curve* $y = \ln x$ *and the tangent line to the curve at the point where* $x = 1$. *Then, write an equation of the tangent line to the curve* $y = \ln x$ *at the point where* $x = 1$.

Problem 8 (a) *Let* f *be a positive differentiable function, defined on an open interval* I. *Find the formula for the derivative of the function* $\ln(f(x))$.

(b) *Using the formula obtained in part (a), compute the derivatives of the following functions.*

(i) $f(x) = \ln(x^2 + x + 1)$

(ii) $f(x) = \ln(\sec x + \tan x)$

(iii) $f(x) = \ln(\ln x)$

Problem 9 *Compute* $f'(x)$.

(a) $f(x) = x \ln x$

(b) $f(x) = \sin x \Big(\ln(\sec x + 1)\Big)$

(c) $f(x) = 2^x \sqrt{\ln(5x+7)}$
