(In)determinate Forms (IF)

1 Some notes on notation for finding limits:

(a) **The first step in evaluating any limit is to find the form.** This is true, even if the question does not specifically ask for the form.

(b) To find the form of $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)}$, take the limits of the numerator and denominator SEPARATELY. Do not just "plug in $x = a$". "Plugging in" can not occur until after the function is noticed as continuous.

(c) When writing a form of a limit, we NEVER write $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{0}{0}$. It is the $=$ that makes this a mathematically incorrect statement. Instead, write $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{0}{0}$.

(d) When evaluating limits, do NOT drop the $\lim\limits_{x\to a}$ until you have evaluated the limit. For example, the following is INCORRECT:

$$\lim_{x\to 2} \frac{2x^2 - 4x}{x-2} = \frac{2x(x-2)}{x-2} = 2x = 4$$

Instead, we write:

$$\lim_{x\to 2} \frac{2x^2 - 4x}{x-2} = \lim_{x\to 2} \frac{2x(x-2)}{x-2} = \lim_{x\to 2} 2x = 4$$

Recitation Questions

Problem 1 *Which of the following limits are of the form* $\dfrac{0}{0}$*?*

(a) $\displaystyle\lim_{x\to 7} \frac{\sin(x-7)}{|x-7|}$

(b) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\sin(x)}{x}$

(c) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{|\cos(x)|}{x}$

(d) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\pi-2x}{\cos(x)}$

(e) $\displaystyle\lim_{x\to 2} \frac{x^2-4}{x-4}$

(f) $\displaystyle\lim_{x\to 4} \frac{x^2-4}{x-4}$

(g) $\displaystyle\lim_{x\to 4} \frac{x^2-16}{x-4}$

(h) $\displaystyle\lim_{x\to 3} \frac{x-4}{\sqrt{25-x^2}-4}$

(i) $\displaystyle\lim_{x\to 3} \frac{x-3}{\sqrt{25-x^2}-4}$

Problem 2 *State the form of the limit and then evaluate the limit.*

(a) $\lim_{x\to 6} \dfrac{4x^2 - 144}{x - 6}$

(b) $\lim_{x\to 6} \dfrac{x - 6}{\sqrt{2x - 8} - 2}$

(c) $\lim_{x\to 2} \dfrac{(3x - 2)^2 - 16}{x - 2}$

(d) $\lim_{x\to 6} \dfrac{\frac{x}{x-2} - \frac{3}{2}}{x - 6}$

(e) $\lim_{x\to 1} \dfrac{\sqrt{5x - 2} - \sqrt{3}}{x - 1}$

Problem 3 *Evaluate the following limits.*

(a) $\lim_{x \to \frac{\pi}{2}} \frac{\cos(x)}{x}$

(b) $\lim_{x \to 3} \frac{\sqrt{25 - x^2} - 4}{x - 4}$

(c) $\lim_{x \to 3} \frac{\sqrt{25 - x^2} - 4}{x - 3}$

Problem 4 *Two polynomials, h and g, are given*

$$h(x) = \frac{x^2 - 4}{4}$$

$$g(x) = x - 2$$

State the form of the limit, evaluate the limit, or state that it does not exist. Justify your answer.

(a) $\lim_{x \to 2} \frac{h(x)}{g(x)}$

(b) $\lim_{x \to 3} \frac{g(x) - g(3)}{x - 3}$

(c) $\lim_{x \to 4} \frac{h(x) - h(4)}{x - 4}$

Problem 5 *Evaluate each of the following limits. Possible answers include a number, $+\infty$, $-\infty$ and "Does Not Exist" (DNE). Make sure to state the form of the limit. Justify your answer.*

(a) $\displaystyle\lim_{x\to 3^-} \frac{x^2-3}{x^2-x-6}$

(b) $\displaystyle\lim_{x\to 5^+} \frac{x^2+6}{x^2-3x-10}$

(c) $\displaystyle\lim_{x\to 1} \frac{4-x}{x^2-2x+1}$

(d) $\lim_{x \to 2} \frac{-e^x}{(2-x)^3}$

(e) $\lim_{x \to 1} \frac{\sin x}{\sqrt{2-x^2}-1}$

Problem 6 *A piecewise defined function f is given by*

$$f(x) = \begin{cases} \dfrac{2x-3}{x-2} & \text{if } \quad x < 2 \\[2ex] \dfrac{x^2-5x+6}{x^2-4} & \text{if } \quad x > 2 \end{cases}$$

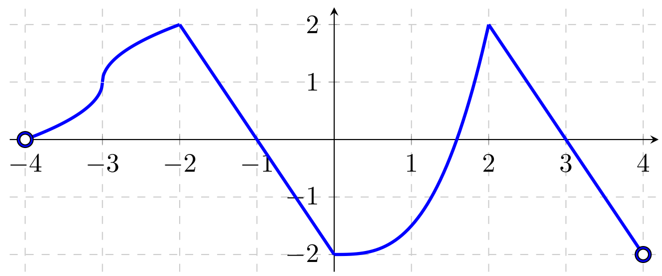
Determine the form of the limit, then find the limit.

(a) $\lim_{x \to 2^+} f(x)$

(b) $\lim_{x \to 2^-} f(x)$

(c) $\lim_{x \to 2} f(x)$

Problem 7 *The graph of a function $g(x)$ with domain $(-4, 4)$ is given in the figure. This portions of this graph in the intervals $(-2, 0)$ and $(2, 4)$ are straight lines. The portion on the interval $(0, 2)$ is not parabolic.*



For each of the limits below, give the form of the limit, then evaluate the limit.

(a) $\displaystyle\lim_{x\to-3} \frac{g(x)+\sin\left(\frac{\pi}{4}x\right)}{x\sqrt{x+4}}$.

(b) $\displaystyle\lim_{x\to0^+} \frac{e^x}{|g(x)+2|}$.

(c) $\displaystyle\lim_{x\to3} \frac{x^2+2x-15}{x+g(x)-3}$