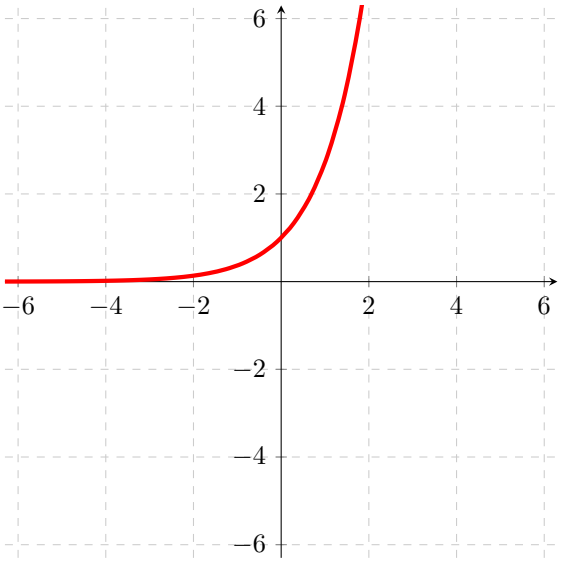
Review of famous functions (ROFF)

SUMMARY: Polynomial, rational, exponential, logarithmic, trigonometric and inverse trigonometric functions

- Know the graphs and properties of "famous" functions
- Understand the definition of a polynomial function
- Understand the definition of a rational function
- Know and use the properties of exponential and logarithmic functions
- Understand the relationship between exponential and logarithmic functions
- Evaluate expressions and solve equations involving exponential and logarithmic functions
- Understand the properties of trigonometric functions
- Evaluate expressions and solve equations involving trigonometric functions and inverse trigonometric functions
- Use sign-charts to solve inequalities involving famous functions.

Recitation Questions

Problem 1 *The graph of $g(x) = e^x$ is given below.*



(a) *Find the domain and range of g.*

(b) *Find the values of $g(1), g(0), g(-1)$ and plot the points $(1, g(1)), (0, g(0))$, and $(-1, g(-1))$ on the graph of $g(x) = e^x$.*

(c) *Graph $h(x) = \ln(x)$ on the same axis as $g(x) = e^x$.*

(d) *Find the domain and range of h.*

(e) *Find the values of $h(1), h(0), h(-1), h(e), h\left(\frac{1}{e}\right)$, or say x not in the domain.*

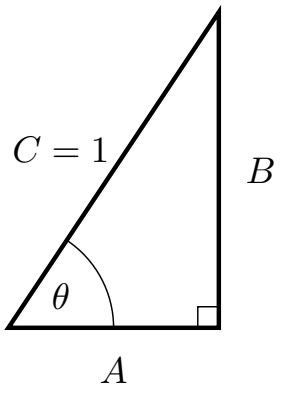
Problem 2 *Solve the following inequalities:*

(a) $\dfrac{x^3 (2x-5)^2}{x+1} > 0.$

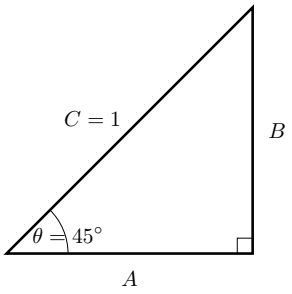
(b) $\dfrac{2 \cdot 5^x - 10^x}{x^2 - 5} \leq 0.$

Problem 3

(a) *Suppose we're given the right triangle below. Express* $\sin(\theta)$ *and* $\cos(\theta)$ *in terms of the sides of the triangle.*



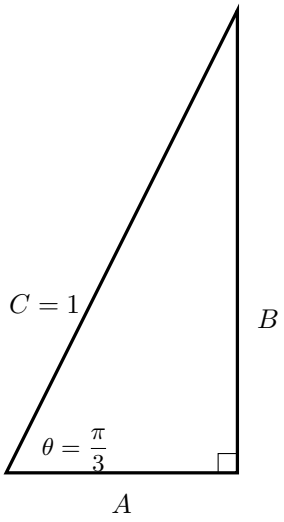
(b) *Suppose we are given the triangle below.*



(i) *Find the length of the sides A and B.*

(ii) *Express* $\sin\left(\frac{\pi}{4}\right)$ *and* $\cos\left(\frac{\pi}{4}\right)$ *in terms of the sides of the triangle.*

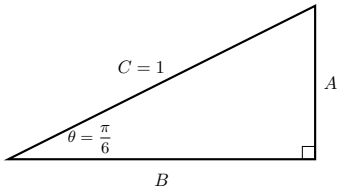
(c) *Suppose we are given the triangle below.*



(i) *Find the length of the sides A and B.*

(ii) *Express* $\sin\left(\frac{\pi}{3}\right)$ *and* $\cos\left(\frac{\pi}{3}\right)$ *in terms of the sides of the triangle.*

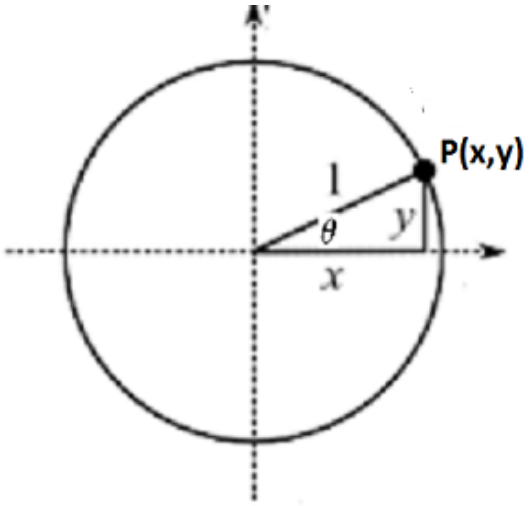
(d) *Suppose we are given the triangle below.*



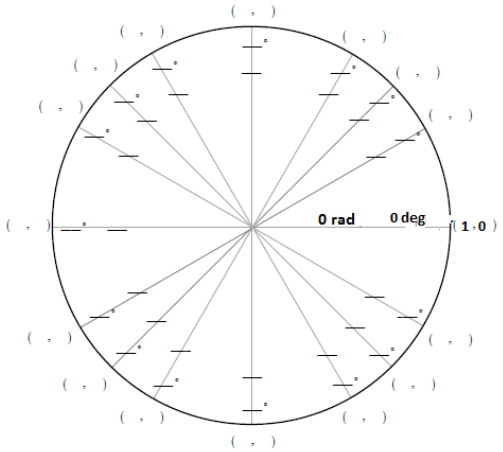
(i) *Find the length of the sides A and B.*

(ii) *Write* $\sin\left(\frac{\pi}{6}\right)$ *and* $\cos\left(\frac{\pi}{6}\right)$ *in terms of the sides of the triangle.*

(e) *For any point P(x,y) on the unit circle, we can express its coordinates in terms of* $\sin(\theta)$ *and* $\cos(\theta)$. *Here* θ *is the radian measure of the angle in standard position whose terminal side is the line through the origin and the point* $P(x, y)$.



(f) *Use all of the above information to label the given points on the unit circle. That is, for each point on the unit circle, provide the angle measure in radians and degrees, and give the (x,y) coordinate for the point.*



Problem 4 *Find all real numbers which satisfy each of the equations. In the previous problem, we used θ to denote the radian measure of the angle. However, we can use any variable to represent the angle measure. For example, in part a, x is the variable representing the radian measure of the angle.*

(a) $\cos(x) = 1$

(b) $\sin(3\theta) = \sqrt{3}/2$ *for* $0 \leq \theta \leq 2\pi$

Problem 5

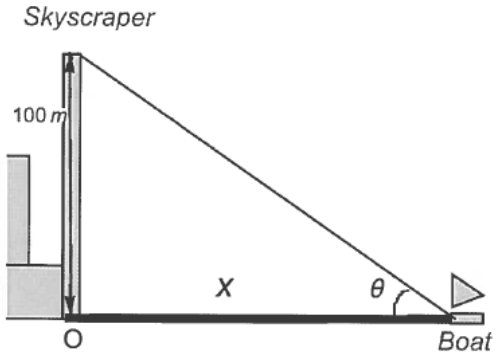
Graph $f(\theta) = \sin(\theta)$ *and* $g(\theta) = \cos(\theta)$ *from* $[-\frac{\pi}{8}, 2\pi + \frac{\pi}{8}]$

Problem 6 *Without using a calculator, determine if the statement*

$$\cos^{-1}\big(\cos(7\pi/6)\big) = 7\pi/6$$

is true or false.

Problem 7 *A boat sails directly toward a 100-meter skyscraper that stands on the edge of a harbor. The angular size θ of the building is the angle formed by lines from the top and bottom of the building to the observer on the boat (see figure below).*



(a) *Express the angle θ as the function of x, the distance of the boat from the building.*

(b) *Find the angular size, θ, when the boat is $x = 100\sqrt{3}$m from the building.*

Problem 8 ***True or False:*** $\sin^{-1}(0) = \pi$.

Problem 9 *Simplify each of the following expressions.*

(a) $\cos^{-1}\big(\sin(\pi/2)\big)$

(b) $\tan\big(\sin^{-1}(x/4)\big)$