Chain Rule (CR)

- **The Chain Rule**: If $f(x)$ and $g(x)$ are differentiable functions, then $\dfrac{d}{dx} f\left(g(x)\right) = f'\left(g(x)\right) g'(x)$.
- **The Derivatives of Trigonometric Functions**:

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\cot(x) = -\csc^2(x)$$
$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x) \qquad \frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

Recitation Questions

Problem 1 *For the following problems, the derivative is given. Determine which function was the original function.*

(a) *The derivative is* $f'(x) = \cos(x)e^{\sin(x)}$. *Which is the original function?*

(i) $f(x) = (\sin(x))(e^x)$
(ii) $f(x) = \sin(e^x)$
(iii) $f(x) = e^{\sin(x)}$
(iv) $f(x) = e^{x\sin(x)}$

(b) *The derivative is* $g'(x) = 4\left(\tan(x^4 - 5x)\right)^3 \sec^2(x^4 - 5x)(4x^3 - 5)$. *Which is the original function?*

(i) $g(x) = (\tan(x) - 5x)^4$
(ii) $g(x) = \tan^4(x) - 5x^4$
(iii) $g(x) = \tan(x^4 - 5x)$
(iv) $g(x) = \tan^4(x^4 - 5x)$

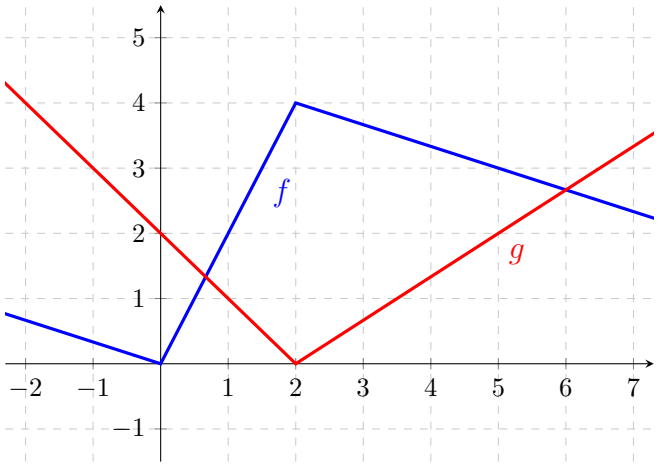
Problem 2 *A table of values for $f(x)$ and $f'(x)$ is shown below:*

x	$f(x)$	$f'(x)$
1	3	4
2	2	3
3	4	5
4	6	3

- *Evaluate the limit* $\lim_{x\to 2} \dfrac{f(x^2)-6}{x-2}$. ***EXPLAIN***.

- *Evaluate* $\dfrac{d}{dx}f(f(x))$ *at* $x=3$.
  (a) *6*
  (b) *25*
  (c) *5*
  (d) *15*
  (e) *DNE*
  (f) *None of the previous answers.*

Problem 3 *Given the following graphs of f and g (both piecewise linear functions), define new functions $u(x) = f(g(x))$ and $v(x) = f(x)g(x)$. Find:*



(a) $u'(1)$

(b) $v'(1)$

(c) $\lim_{x \to 1} \dfrac{\sqrt{g(x)} - 1}{x - 1}$

Problem 4 *Suppose the line tangent to the graph of f at $x = 1$ is $y = 6x - 7$. Find an equation of the line tangent to the following curves at $x = 1$:*

(a) $y = g(x) = 5(f(x))^4$

(b) $y = h(x) = x^2(f(x^3))$

Problem 5 *Differentiate each function (with respect to x)*

(a) $\cos\left(\sqrt{x+7}\right)$

(b) $\sqrt{\cos(x)+7}$

(c) $\sqrt{\cos(x)}+7$

(d) $\cos\left(\sqrt{x}+7\right)$

(e) $\cos(x)\cdot\left(\sqrt{x}+7\right)$

Problem 6 *Find the derivative of the following functions:*

(a) $f(x) = \sin(x)\cos(x)$

(b) $f(x) = \dfrac{e^x \tan(x)}{\sec(x) + 2}$

(c) $f(x) = e^{x \tan(x)}$

(d) $f(x) = \sin(x)\cos(x)e^{3x}$

(e) $f(x) = \dfrac{x+5}{7x^6 + \cot(x)}$

(f) $f(x) = \sin(2x)\sec^3\left(x^2 + 4x\right)$

Problem 7 *Find values for a, b, and c so that the following function is differentiable everywhere.*

$$f(x) = \begin{cases} a\sin(x) + b\cos(x) & \text{if } x < 0 \\ ax^2 + bx + c & \text{if } x \geq 0 \end{cases}$$