Computations for graphing functions (CFGF)

Recitation Questions

Problem 1 *Let f be a function given by': $f(x) = \left(\tan^{-1}(x)\right)^2$.*

(a) *State the domain of f. Write your answer in interval notation.*

(b) *Find the critical points of f. Show your work!*

(c) *Show that* $f''(x) = \dfrac{2 - 4x\tan^{-1}(x)}{(x^2+1)^2}$.

(d) *Use the **second derivative test** to determine whether critical point(s) of f correspond to local minimums, local maximums, or whether the test is inconclusive.*

Problem 2 *Determine the following information about the given function and then graph the function:*

$$f(x) = \frac{x^2 + x + 1}{x^2}$$

Domain
x, y-intercepts
Symmetry
Asympotes
Intervals of increasing/decreasing
Maxima/Minima
Intervals of concavity
Inflection points

Problem 3 *Given that:*

$$\lim_{x\to-\infty} f(x) = 0 \qquad \lim_{x\to-3^-} f(x) = \infty \qquad \lim_{x\to-3^+} f(x) = -\infty \qquad \lim_{x\to4^-} f(x) = \infty$$

$$\lim_{x\to4^+} f(x) = -\infty \qquad f(1) = 1 \qquad f(5) = -2 \qquad \lim_{x\to9} f(x) = 3$$

$$f'(1) \neq 0 \qquad f'(7) = 0 \qquad f'(x) = 2 \text{ for } x > 9 \qquad f''(1) = 0$$

the domain of f is: $(-\infty,-3)\cup(-3,4)\cup(4,9)\cup(9,\infty)$ and f is continuous on its domain. The following sign chart for the first and second derivatives of f:

find the following:

(a) *Critical points.*

(b) *Intervals where f is increasing/decreasing.*

(c) *Local extrema.*

(d) *Inflection points.*

(e) *Intervals of concavity.*

(f) *Sketch the graph of f.*

Problem 4 *Determine the following information about the given function and then graph the function:*

$$f(x) = 3\sin\left(\frac{x}{2}\right), [-2\pi, 2\pi]$$

Domain
x, y-intercepts
Symmetry
Asympotes
Intervals of increasing/decreasing
Maxima/Minima
Inflection points
Intervals of concavity
Sketch a graph of f

Problem 5 *Determine the following information about the given function and then graph the function:*

$$f(x) = x\ln(x)$$

Domain
x, y-intercepts
Symmetry
Intervals of increasing/decreasing
Maxima/Minima (including absolute)
Intervals of concavity
Inflection points
Given $\lim_{x\to 0^+} f(x) = 0$, *sketch a graph* f.

Problem 6 *Sketch a graph of the function f defined by*

$$f(x) = \begin{cases} 2x - 2\tan^{-1}(x) - x\ln(x^2+1) & \text{for } x \leq 1 \\ \sqrt{x-2} & \text{for } x \geq 2 \end{cases}$$

(a) *Find the domain of f.*

(b) *Show that $f'(x) = -\ln(x^2+1)$ on the interval $(-\infty, 1)$.*

(c) *Compute a formula for f' on the interval $(2, \infty)$.*

(d) *Compute a formula for f'' on the interval $(-\infty, 1)$.*

(e) *Compute a formula for f'' on the interval $(2, \infty)$.*

(f) *List all the asymptotes (with limit justifications, if appropriate). (If f does not have a particular asymptote write "NONE".)*

(i) *Vertical asymptotes:*

(ii) *Horizontal asymptotes:*

(g) *In the following sign chart for f', list the points (in the domain of f) where $f'(x) = 0$ or $f'(x)$ is undefined, indicate the sign of f' on each subinterval, and identify any local extrema (with "local max" or "local min".) In the following sign chart for f'', list the points (in the domain of f) where $f''(x) = 0$ or $f''(x)$ is undefined, indicate the sign of f'' on each subinterval, and identify any inflection points. Use this information to sketch a graph of f in the blank grid. (Be sure to indicate any x- or y-intercepts.)*