First and Second Fundamental Theorem of Calculus (FFTOC, SFTOC)

SUMMARY of the First Fundamental Theorem of Calculus:

Given a continuous function f on the interval $[a, b]$, we define an accumulation function, A by

$$A(x) = \int_a^x f(t)\,dt$$

The First Fundamental Theorem of Calculus (FFTOC) states that

$$A'(x) = f(x)$$

or, equivalently, that

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Note 1: $A(a) = 0$

Note 2: The function A is an antiderivative of f.
Therefore, any antiderivative, G of f can be written as $G(x) = A(x) + C$, or, equivalently,

$$G(x) = \int_a^x f(t)\,dt + C.$$

Since $G(a) = A(a) + C = 0 + C = C$, it follows that $G(x) = A(x) + G(a)$,
or, equivalently, that

$$G(x) = G(a) + \int_a^x f(t)\,dt$$

Note 3: Let f', the derivative of f, be continuous on $[a, b]$. Since f is its antiderivative, we have

$$f(x) = f(a) + \int_a^x f'(t)\,dt$$

SUMMARY of the Second Fundamental Theorem of Calculus:

FFTOC implies that, for $x = b$, $A(b) = \displaystyle\int_a^b f(t)\,dt$. Since $A(a) = 0$, we can write

$$\int_a^b f(t)\,dt = A(b) - A(a)$$

Note 4: This is the Second Fundamental Theorem of Calculus (SFTOC).

Note 5: Equivalently, using the result in Note 3, for $x = b$, $f(b) = f(a) + \displaystyle\int_a^b f'(t)\,dt$. So $\displaystyle\int_a^b f'(t)\,dt = f(b) - f(a)$, or equivalently,

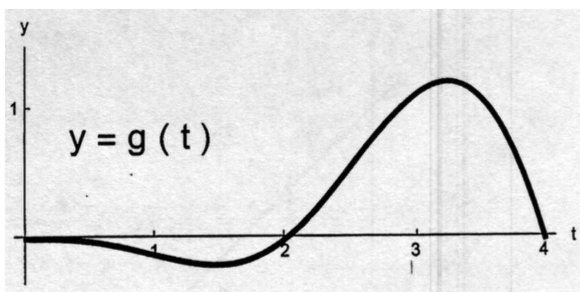
$$\int_a^b \frac{d}{dx} f(x)\,dx = f(b) - f(a).$$

Recitation Questions

Problem 1 *True of False: If f is continuous on the closed interval $[a, b]$, then*

$$\frac{d}{dx}\left(\int_a^b f(t)\,dt\right) = f(x)$$

Problem 2 *The graph of g, a continuous function on $[0, 4]$, is shown in the figure. Let $A(x) = \int_0^x g(t)\,dt$, for $0 \le x \le 4$.*



(a) *Circle the correct statement about $A(2)$.*

(i) $A(2) = 0$;
(ii) $A(2) > 0$;
(iii) $A(2) < 0$;
(iv) *none of the previous answers.*

(b) *Circle the correct statement about $A(3.8)$.*

(i) $A(3.8) = 0$;
(ii) $A(3.8) > 0$;
(iii) $A(3.8) < 0$;
(iv) *none of the previous answers.*

(c) *Circle the correct statement about* $A'(3.8)$.

(i) $A'(3.8) = 0$;
(ii) $A'(3.8) > 0$;
(iii) $A'(3.8) < 0$;
(iv) *none of the previous answers.*

(d) *Find the solution of the following initiual value problem:* $y'(x) = g(x)$ *and* $y(0) = 2$.

(i) $y(x) = g(x)$;
(ii) $y(x) = g(x) + 2$;
(iii) $y(x) = A(x)$;
(iv) $y(x) = A(x) + 2$;
(v) $y(x) = g'(x)$;
(vi) $y(x) = g'(x) + 2$;
(vii) *none of the previous answers.*

(e) *Find the expression for* $\int_0^4 |g(t)|\, dt$.

(i) $A(4)$;
(ii) $A(2) - A(4)$;
(iii) $A(4) - A(2)$;
(iv) $A(4) - 2A(2)$;
(v) *none of the previous answers.*

(f) *Find the midpoint Riemann sum for the function* g *on the interval* $[0, 4]$ *with* $n = 2$ *(the number of subintervals).*

(i) $g(1) + g(3)$;
(ii) $g(0) + g(2)$;
(iii) $g(2) + g(4)$;
(iv) $2g(1) + 2g(3)$;
(v) $2g(0) + 2g(2)$;
(vi) $2g(2) + 2g(4)$;
(vii) *none of the previous answers.*

Problem 3 *The limit of Riemann sums for a function f on the interval $[1,5]$ is given by*

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(2x_k^* + \frac{1}{x_k^*}\right)\Delta x \text{ on } [1,5].$$

(a) *Identify f and express the limit as a definite integral.*

(b) *Evaluate the limit of Riemann sums.*

Problem 4 *Compute the following integrals:*

(a) $\displaystyle\int_0^1 e^{5x}\, dx$

(b) $\displaystyle\int\limits_{-2}^{-1} \frac{1}{x^3}\, dx$

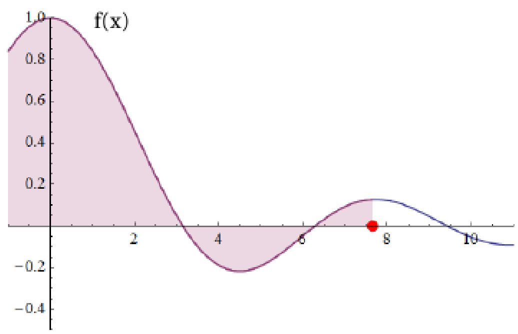
(c) $\displaystyle\int_0^4 \left(3x - 5 + 7\sqrt{16 - x^2}\right)\, dx$

Problem 5 *Find the derivative of the following functions:*

(a) $F(x) = \displaystyle\int_{\sqrt{x}}^{1} \frac{t^2}{2+3t^4}\, dt$

(b) $G(x) = \displaystyle\int_{x}^{x^3} \sin(7t^2)\, dt$

Problem 6 *Given the following graph of $y = f(x)$, let $g(x) = \int_{-1}^{x} f(t)\,dt$.*



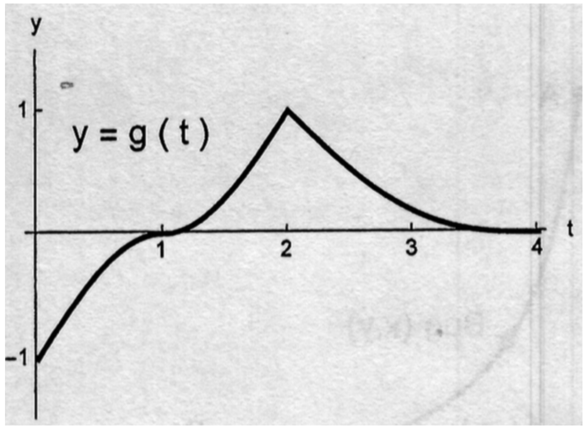
(a) *Is g continuous? Why or why not?*

(b) *Is g differentiable? Why or why not?*

(c) *Where does g achieve its absolute maximum and minimum values? Where does g achieve any local extreme values? Assume the domain of g is $[-1, 7.6]$.*

(d) *Where is the graph of g concave up? Concave down?*

Problem 7 *The graph of g, a continuous function $[0,4]$, is shown in the figure. Let $A(x) = \int_0^x g(t)\,dt$ for $0 \le x \le 4$.*



(a) *Circle the correct statement about $A(1)$.*

- (i) $A(1) = 0$;
- (ii) $A(1) < 0$;
- (iii) $A(1) > 0$;
- (iv) *none of the previous answers.*

(b) *Circle the correct statement about $A(1.5)$.*

- (i) $A(1.5) = 0$;
- (ii) $A(1.5) < 0$;
- (iii) $A(1.5) > 0$;
- (iv) *none of the previous answers.*

(c) *Circle the correct statement about $A'(1.5)$.*

- (i) $A'(1.5) = 0$;
- (ii) $A'(1.5) < 0$;
- (iii) $A'(1.5) > 0$;
- (iv) *none of the previous answers.*

(d) *Circle the correct expression for* $\int_1^4 (g(t)+1)\,dt$.

(i) $A(4)+1$;

(ii) $A(4)-A(1)$;

(iii) $A(4)-A(1)+1$;

(iv) $A(4)+3$;

(v) $A(4)-A(1)+3$;

(vi) *none of the previous answers.*

(e) *Circle the correct statement about* $A(0)$.

(i) $A(0)=0$;

(ii) $A(0)=1$;

(iii) $0<A(0)<1$;

(iv) $A(0)>1$;

(v) $A(0)<0$;

(vi) *none of the previous answers.*

(f) *Circle the interval (or intervals) where the function* A *is DECREASING.*

(i) $(0,1)$;

(ii) $(1,2)$;

(iii) $(2,4)$;

(iv) *none of the previous answers.*

(g) *Circle the value (or values) where the function A attains its MAXIMUM.*

(i) $x = 0$;
(ii) $x = 1$;
(iii) $x = 2$;
(iv) $x = 3$;
(v) $x = 4$;
(vi) *none of the previous answers.*

(h) *Circle the value (or values) where the function A attains its MINIMUM.*

(i) $x = 0$;
(ii) $x = 1$;
(iii) $x = 2$;
(iv) $x = 3$;
(v) $x = 4$;
(vi) *none of the previous answers.*

(i) *Circle the interval (or intervals) where the function A is CONCAVE DOWN.*

(i) $(0, 1)$;
(ii) $(1, 2)$;
(iii) $(2, 4)$;
(iv) *none of the previous answers.*

(j) *Circle the value (or values) of x where the function A has an inflection point.*

(i) $x = 1$;
(ii) $x = 2$;
(iii) $x = 3$;
(iv) *none of the previous answers.*

(k) *Sketch the graph of A, based on items (e)-(j)*