

Concepts of graphing functions (COGF)

Recitation Questions

Problem 1 Fill in the following blanks with the correct choice of the words from this list:

Increasing, decreasing, positive, negative, concave up, concave down

(a) If you know $f''(x) > 0$, then you know $f'(x)$ is _____ and $f(x)$ is _____.

(b) If you know $g'(x) < 0$ and decreasing, then you know $g(x)$ is _____ and _____.

(c) If you know $h(x)$ is positive, increasing, and concave down, then you know $h'(x)$ is _____ and _____ and that $h''(x)$ is _____.

Problem 2 Sketch a graph of a function that is continuous on $(-\infty, \infty)$ that has the following properties.

(a) Function f does not have a local maximum or minimum. f contains a point where $f'(x) = 0$

(b) $g'(x) < 0$ on $(-\infty, -1)$; $g'(x) > 0$ on $(-1, 2)$; $g'(x) < 0$ on $(2, \infty)$.



Problem 3 Give an example or sketch of a function that is continuous on $(-\infty, \infty)$ and satisfies given properties. If such a function does not exist, explain why.

(a) A function f is concave up and negative everywhere.

(b) A function f is decreasing and concave up everywhere.

(c) A function s has exactly 3 local extrema and four inflection points.

(d) A function f has exactly 2 zeros and one local extrema.

Problem 4

- (a) You are given that $f''(x) > 0$ for all x . Which of the following must be true about $f(x)$ on the region $0 \leq x \leq 2$?
- (i) *There is a critical point between 0 and 2.*
 - (ii) *There is a local maximum, but not enough information is given to determine where.*
 - (iii) *f need not have a local maximum.*
- (b) You are told that $f''(x) > 0$ for all x . Which of the following must be true about the graph of $y = f(x)$?
- (i) *The graph is a straight line.*
 - (ii) *The graph crosses the x -axis at most once.*
 - (iii) *The graph is concave down.*
 - (iv) *The graph crosses the y -axis more than once.*
 - (v) *The graph is concave up.*

Problem 5 Suppose a function f satisfies the following conditions:

(a) $f(0) = 0$ and $f'(-4) = f'(2) = f'(10) = 0$

(b) $\lim_{x \rightarrow 6} f(x) = -\infty$, and $\lim_{x \rightarrow +\infty} f(x) = 6$

(c) $f'(x) < 0$ on $(-\infty, -4)$, $(2, 6)$, and $(10, +\infty)$

(d) $f'(x) > 0$ on $(-4, 2)$, and $(6, 10)$

(e) $f''(x) > 0$ on $(-\infty, 0)$, and $(14, +\infty)$

(f) $f''(x) < 0$ on $(0, 6)$, and $(6, 14)$

(a) List the **interval(s)** where the function f is **both increasing** and **concave UP**.

(b) List the **interval(s)** where the function f is **both increasing** and **concave DOWN**.

(c) List the **interval(s)** where the function f is **both decreasing** and **concave UP**.

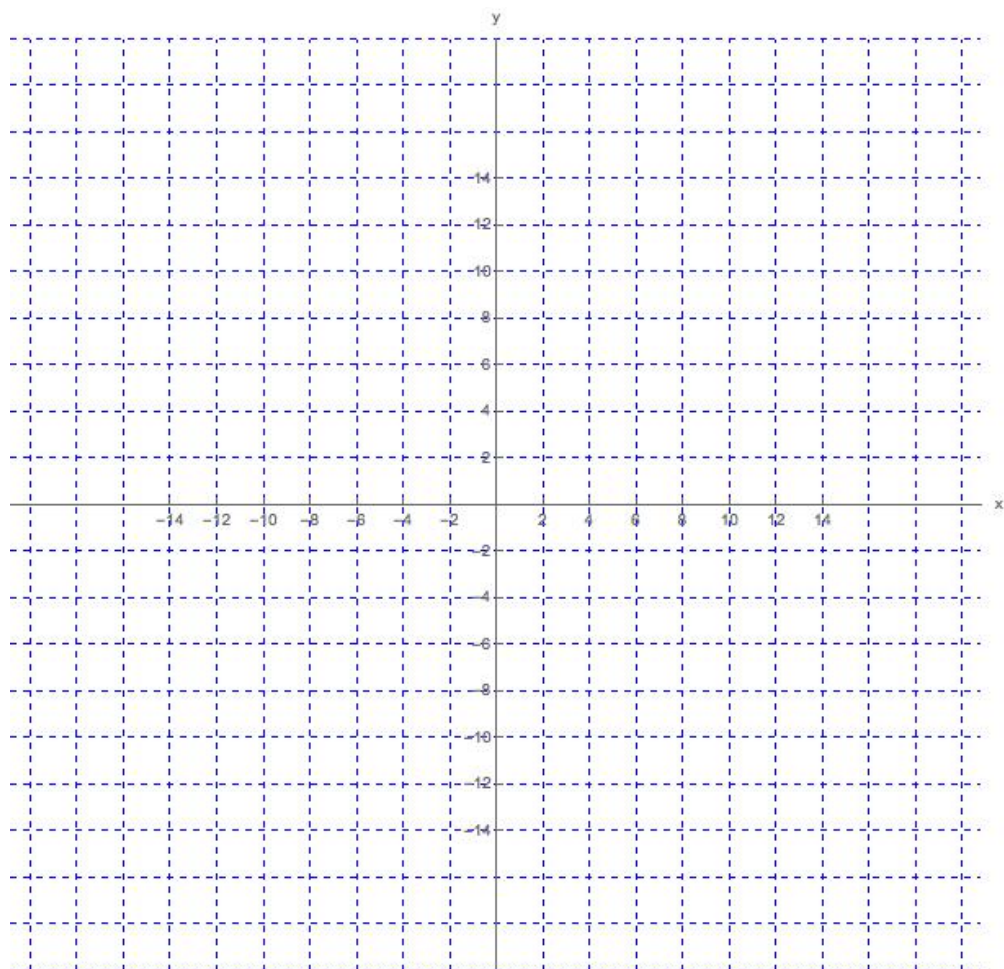
(d) List the **interval(s)** where the function f is **both decreasing** and **concave DOWN**.

(e) List the **x-coordinates** at which f has a **local minimum**. Write "none" if appropriate.

(f) List the **x-coordinates** at which f has a **local maximum**. Write "none" if appropriate.

(g) List the **x-coordinates** of all **inflection points** of f . Write "none" if appropriate.

(h) Sketch the graph of f .



Problem 6 Sketch the graph of a function f satisfying all of the conditions:

(a) f is continuous and odd, $f(0) = 0$,

(b) $\lim_{x \rightarrow \infty} f(x) = -5$,

(c) $f'(x) > 0$ on $(6, \infty)$,

(d) $f'(x) < 0$ on $(0, 6)$,

(e) $f''(x) > 0$ on $(0, 12)$, and

(f) $f''(x) < 0$ on $(12, \infty)$.

