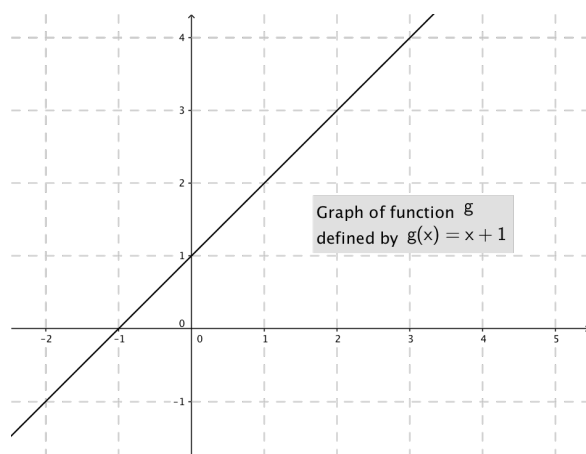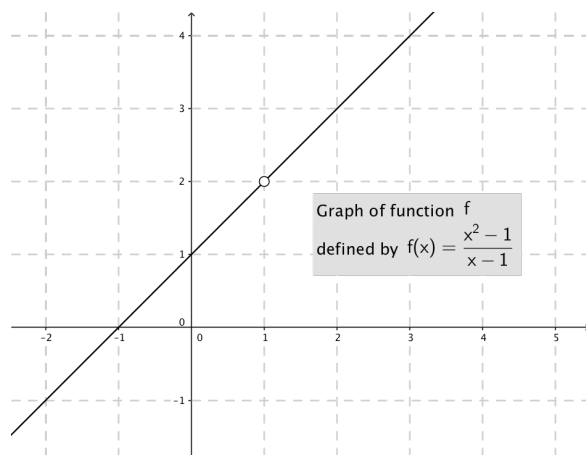# What is a Limit?

Recitation Questions

Problem 1

(a) *True or False: To find* $\lim\limits_{x \to 2} f(x)$, *it's enough to know the values of* $f(2.1)$, $f(2.01)$, $f(2.001)$, *and so on.*

(b) *True or False: If we know the value of* $f(2)$, *then can we conclude that* $\lim\limits_{x \to 2} f(x) = f(2)$?

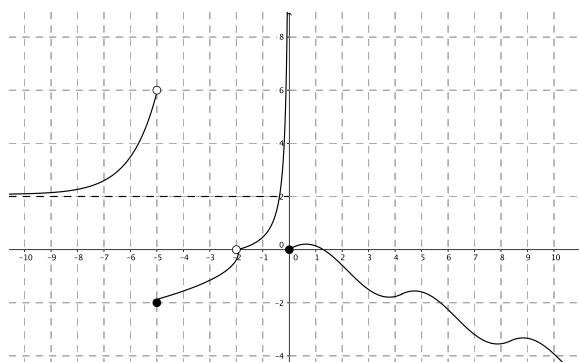**Problem 2** *Use the graphs and the given definitions of the following two functions to answer the questions below.*

Graph of function f

defined by $f(x) = \dfrac{x^2 - 1}{x - 1}$

Graph of function g

defined by $g(x) = x + 1$

(a) *Find the domain of f and the domain of g.*

(b) *Is $f = g$? (Why or why not?)*

(c) *Looking at the graphs, find $\lim\limits_{x \to 1} f(x)$ and $\lim\limits_{x \to 1} g(x)$.*

Problem 3 *The graph of a function f is given below. Use this graph to answer the following questions.*

(a) *Find the domain of f.*

(b) *Find the range of f.*

(c) *Find the following values.*

   (i) $\lim_{x \to -2} f(x) =$

   (ii) $f(-2) =$

   (iii) $f(-5) =$

   (iv) $\lim_{x \to 0^+} f(x) =$

   (v) $\lim_{x \to 0} f(x) =$

Problem 4 *Sketch a possible graph of a function that satisfies all of the given properties. (You do not need to find a formula for the function.)*

Domain: $[-4, 3) \cup (3, 5)$

$f(1) = 3$

$f(-1) = 1$

$f(-4) = 1$

$\lim_{x \to -1^-} f(x) = -2$

$\lim_{x \to -1^+} f(x) = 1$

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 3^+} f(x) = -1$

$\lim_{x \to 3^-} f(x) = 1$

$\lim_{x \to -4^+} f(x) = 1$

$\lim_{x \to 5^-} f(x) = 1$

Problem 5 *True/False: Give an explanation or counterexample. Assume a and L are finite numbers.*

(a) *If* $\lim\limits_{x \to a} f(x) = L$, *then* $f(a) = L$.

(b) *If* $\lim\limits_{x \to a^-} f(x) = L$, *then* $\lim\limits_{x \to a^+} f(x) = L$.

(c) *If* $\lim\limits_{x \to a} f(x) = L$ *and* $\lim\limits_{x \to a} g(x) = L$, *then* $f(a) = g(a)$.

(d) $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$ *does not exist if* $g(a) = 0$.