

# Logarithmic differentiation (LD)

**Basic properties of the natural logarithm:**

- $\ln(yz) = \ln(y) + \ln(z)$
- $\ln\left(\frac{y}{z}\right) = \ln(y) - \ln(z)$
- $\ln(y^z) = z \ln(y)$
- For any function  $g$ :  $\ln(e^{g(x)}) = g(x)$
- $\frac{d}{dx} (\ln(f(x))) = \frac{f'(x)}{f(x)}$ .

## Recitation Questions

**Problem 1** (a) Write as an exponential with base 5.  $7^{3x}$ .

(b) Write in terms of the natural logarithm.  $\log_3(4)$ .

(c) Expand the following:  $\log_{1/2} \left( \frac{6x^5(2 + \tan(x))^x}{\sqrt[5]{e^{4x} + 1}} \right)$ .

**Problem 2** Find all real numbers  $x$  which satisfy each of the following equations.

(a)  $\log_x(25) = 2$ .

(b)  $7^x = 15$

(c)  $\ln(x) + 1 = 0$ .

**Problem 3** True or False:

(1) If  $f(x) = (x - 2)^x$ , then  $f'(x) = x(x - 2)^{x-1}$ .

(2) If  $f(x) = (3x)^x$ , then  $f'(x) = (3x)^x \ln(3x)$ .

**Problem 4** Find the derivatives of the following functions:

(a)  $f(x) = x^{e^x} + 7x$

(b)  $g(x) = (\ln(x) + 9)^{\sec(x^4)}$

$$(c) \ f(x) = \frac{(x+1)^5(\sin(x)+5)^4}{(x^2+5)\sqrt{x-3}}$$

$$(d) \ h(x) = \frac{(x^2-7)^5}{\cos^7(x^2-5)}$$