Antiderivatives (A)

SUMMARY of Antiderivatives:

A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$, for all x in I.

If F is an antiderivative of f on an interval I, then the function f has a whole **family of antiderivatives**.
If a function G is an antiderivative of f on I, then $G(x) = F(x) + C$, for all x in I, for some constant C.
The family of of *all* antiderivatives of f is denoted by $\int f(x)\,dx$ and called **indefinite integral of** f.
Therefore, if F is an antiderivative of f, $\quad \int f(x)\,dx = F(x) + C.$

Basic Indefinite Integrals

- $\int k\,dx = kx + C$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$
- $\int e^x\,dx = e^x + C$
- $\int a^x\,dx = \frac{a^x}{\ln(a)} + C$
- $\int \cos(x)\,dx = \sin(x) + C$
- $\int \sin(x)\,dx = -\cos(x) + C$
- $\int \sec^2(x)\,dx = \tan(x) + C$
- $\int \csc^2(x)\,dx = -\cot(x) + C$
- $\int \sec(x)\tan(x)\,dx = \sec(x) + C$
- $\int \csc(x)\cot(x)\,dx = -\csc(x) + C$
- $\int \frac{1}{x^2+1}\,dx = \arctan(x) + C$
- $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$

Recitation Questions

Problem 1 *Find the most general antiderivative of the function*

$$g(t) = e^{-2t} - 5 + 6\sqrt{t} - \frac{7}{t} + \frac{5}{1+t^2}$$

Problem 2 *Determine the following indefinite integrals.*

(a)

$$\int \left(\sec^2(x) + 5\right)\, dx$$

(b)

$$\int \left(\sec^2(x) - \sec(x)\tan(x)\right)\, dx$$

(c)

$$\int \frac{x + 3x^5}{x^3}\, dx$$

(d)

$$\int \frac{1 + 2x}{1 + x^2}\, dx$$

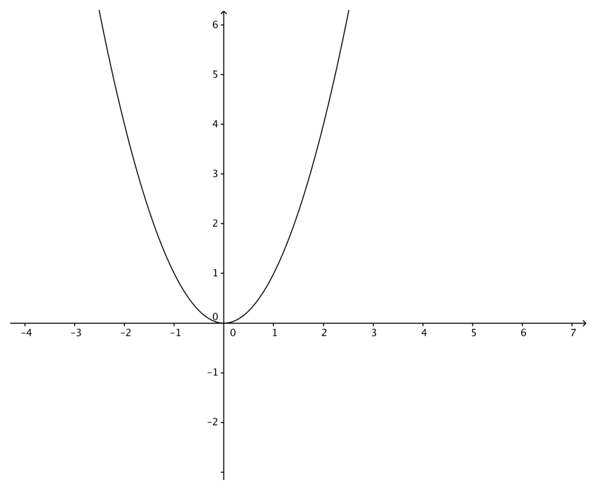
(HINT: Think about the derivative of $\ln(f(x))$*).*

(e)

$$\int \frac{2 + x^2}{1 + x^2}\, dx$$

Problem 3 *Assume that* $f'(t) = 4t^3 + 2t$ *and* $f(3) = 5$. *Find* $f(t)$.

Problem 4 *The graph of a function f is the parabola given in the figure below.*



(a) *Find a formula for $f(x)$.*

(b) *Suppose F_1 is an anti-derivatives of f satisfying $F_1(0) = 2$. Sketch and label the graph of F_1. Sketch the graphs of three more antiderivatives of f.*

(c) *Using the expression you found in part (a) and your sketches in part (b), find the algebraic representation of F_1.*

(d) *Suppose we're given $h(x) = (1/3)x^3 + 17$, what is $h'(x)$?*

(e) *What is the relationship between f, F_1, h, and h'?*

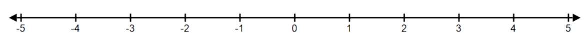
Problem 5 *Given the acceleration function, the initial velocity and initial position of an object moving along a line, find the **position function**.*

$a(t) = 4\cos t,\ v(0) = 2,\ s(0) = 6.$

Problem 6 *Consider an object moving along a line with velocity $v(t) = \pi \sin(\pi t)$ on $[0, 2]$ and initial position $s(0) = 0$. Time is measured in seconds and velocity in m/s.*

(a) *Determine the position function, $s(t)$, on $[0, 2]$.*

(b) *Mark the position of the object at the time $t = 1$ on the line below.*

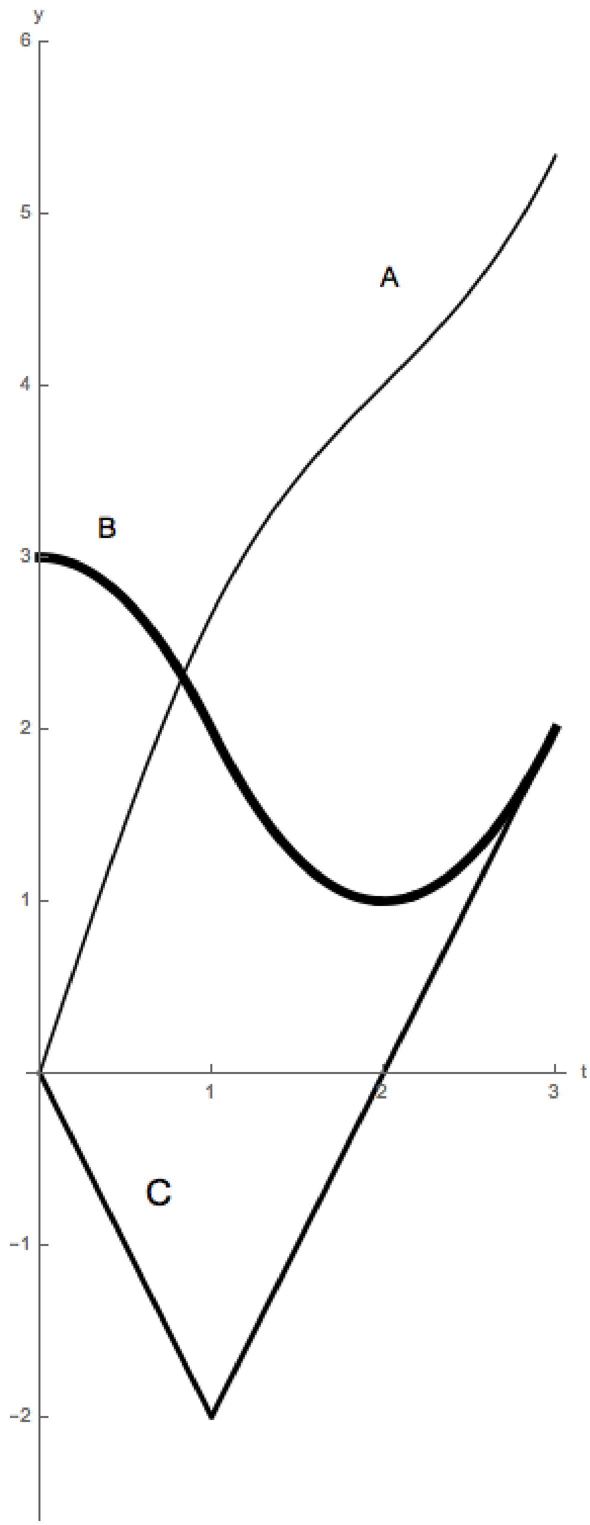


(c) *Determine the average velocity, v_{av}, of the object during the interval $[0, 2]$.*

(d) *Determine when the motion is in the positive direction.*

(e) *At what time (or times) is the object farthest from the origin?*

Problem 7 *Consider an object moving along a straight line. The graphs of acceleration function (in m/s^2), the velocity function and the position function of the object are given in the figure below.*



Find the initial velocity, $v(0)$, and initial position, $s(0)$, of the object. (HINT: Start by determining which graph corresponds to each of $s(t)$, $v(t)$, and $a(t)$.)