

# Concepts of graphing functions (COGF)

## Recitation Questions

**Problem 1** Fill in the following blanks with the correct choice of the words from this list:

Increasing, decreasing, positive, negative, concave up, concave down

(a) If you know  $f''(x) > 0$ , then you know  $f'(x)$  is \_\_\_\_\_ and  $f(x)$  is \_\_\_\_\_.

(b) If you know  $g'(x) < 0$  and decreasing, then you know  $g(x)$  is \_\_\_\_\_ and \_\_\_\_\_.

(c) If you know  $h(x)$  is positive, increasing, and concave down, then you know  $h'(x)$  is \_\_\_\_\_ and \_\_\_\_\_ and that  $h''(x)$  is \_\_\_\_\_.

**Problem 2** Sketch a graph of a function that is continuous on  $(-\infty, \infty)$  that has the following properties.

(a) Function  $f$  does not have a local maximum or minimum.  $f$  contains a point where  $f'(x) = 0$

(b)  $g'(x) < 0$  on  $(-\infty, -1)$ ;  $g'(x) > 0$  on  $(-1, 2)$ ;  $g'(x) < 0$  on  $(2, \infty)$ .

**Problem 3** Give an example or sketch of a function that is continuous on  $(-\infty, \infty)$  and satisfies given properties. If such a function does not exist, explain why.

(a) A function  $f$  is concave up and negative everywhere.

(b) A function  $f$  is decreasing and concave up everywhere.

(c) A function  $s$  has exactly 3 local extrema and four inflection points.

(d) A function  $f$  has exactly 2 zeros and one local extrema.

**Problem 4**

- (a) You are given that  $f''(x) > 0$  for all  $x$ . Which of the following must be true about  $f(x)$  on the region  $0 \leq x \leq 2$ ?
- (i) There is a critical point between 0 and 2.
  - (ii) There is a local maximum, but not enough information is given to determine where.
  - (iii)  $f$  need not have a local maximum.
- (b) You are told that  $f''(x) > 0$  for all  $x$ . Which of the following must be true about the graph of  $y = f(x)$ ?
- (i) The graph is a straight line.
  - (ii) The graph crosses the  $x$ -axis at most once.
  - (iii) The graph is concave down.
  - (iv) The graph crosses the  $y$ -axis more than once.
  - (v) The graph is concave up.
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**Problem 5** Suppose a function  $f$  satisfies the following conditions:

- (a)  $f(0) = 0$  and  $f'(-4) = f'(2) = f'(10) = 0$
- (b)  $\lim_{x \rightarrow 6} f(x) = -\infty$ , and  $\lim_{x \rightarrow +\infty} f(x) = 6$
- (c)  $f'(x) < 0$  on  $(-\infty, -4)$ ,  $(2, 6)$ , and  $(10, +\infty)$
- (d)  $f'(x) > 0$  on  $(-4, 2)$ , and  $(6, 10)$
- (e)  $f''(x) > 0$  on  $(-\infty, 0)$ , and  $(14, +\infty)$
- (f)  $f''(x) < 0$  on  $(0, 6)$ , and  $(6, 14)$

(a) List the **interval(s)** where the function  $f$  is **both increasing and concave UP**.

(b) List the **interval(s)** where the function  $f$  is **both increasing and concave DOWN**.

(c) List the **interval(s)** where the function  $f$  is **both decreasing and concave UP**.

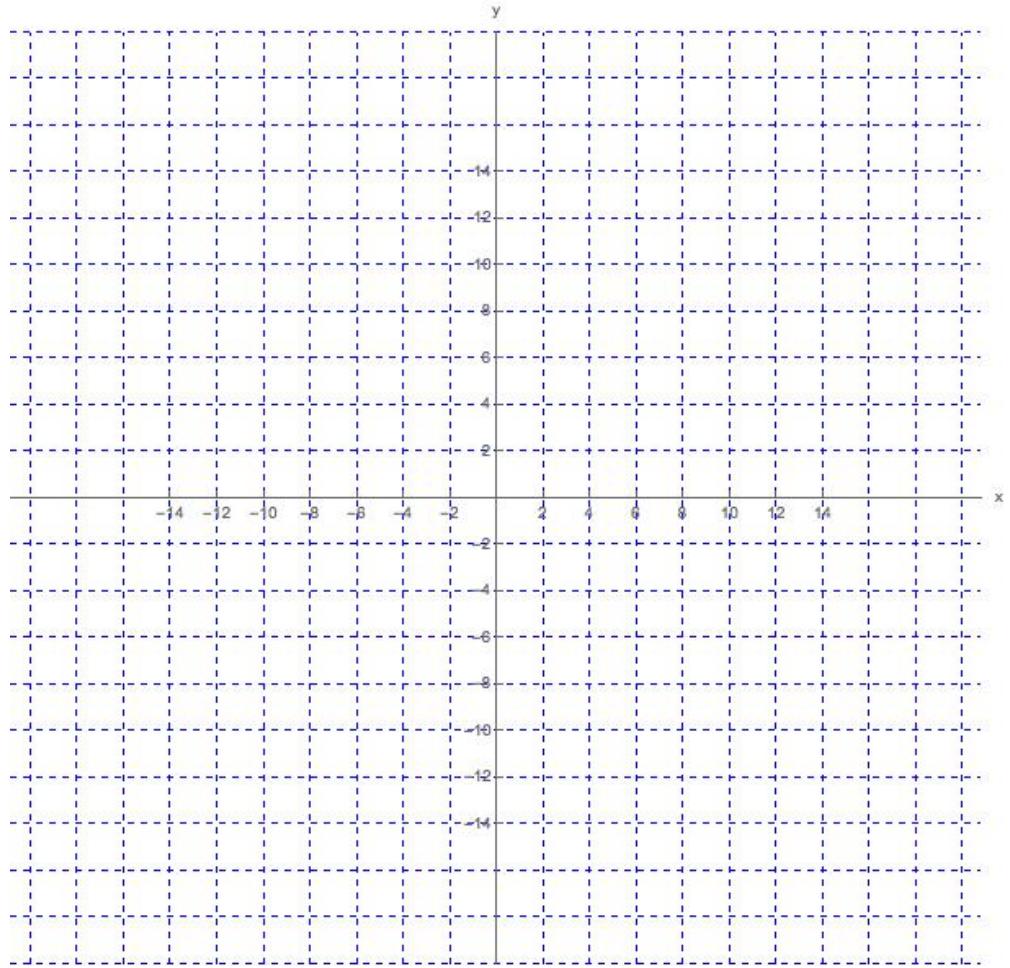
(d) List the **interval(s)** where the function  $f$  is **both decreasing and concave DOWN**.

(e) List the **x-coordinates** at which  $f$  has a **local minimum**. Write "none" if appropriate.

(f) List the **x-coordinates** at which  $f$  has a **local maximum**. Write "none" if appropriate.

(g) List the **x-coordinates** of all **inflection points** of  $f$ . Write "none" if appropriate.

(h) Sketch the graph of  $f$ .



**Problem 6** Sketch the graph of a function  $f$  satisfying all of the conditions:

(a)  $f$  is continuous and odd,  $f(0) = 0$ ,

(b)  $\lim_{x \rightarrow \infty} f(x) = -5$ ,

(c)  $f'(x) > 0$  on  $(6, \infty)$ ,

(d)  $f'(x) < 0$  on  $(0, 6)$ ,

(e)  $f''(x) > 0$  on  $(0, 12)$ , and

(f)  $f''(x) < 0$  on  $(12, \infty)$ .

