

# Higher order derivatives and graphs (HODAG)

## Recitation Questions

**Problem 1** Fill in the following blanks with the correct choice of the words from this list:

Increasing, decreasing, positive, negative, concave up, concave down

- (a) If you know that  $f''(x) > 0$ , for all  $x$  in some open interval  $I$ , then you know  $f'(x)$  is \_\_\_\_\_ and  $f(x)$  is \_\_\_\_\_, for all  $x$  in  $I$ .
- (b) If you know  $g'(x) < 0$  and decreasing, for all  $x$  in some open interval  $I$ , then you know  $g(x)$  is \_\_\_\_\_ and \_\_\_\_\_, for all  $x$  in  $I$ .
- (c) If you know  $h(x)$  is positive, increasing, and concave down, for all  $x$  in some open interval  $I$ , then you know  $h'(x)$  is \_\_\_\_\_ and \_\_\_\_\_ and that  $h''(x)$  is \_\_\_\_\_, for all  $x$  in  $I$ .

**Problem 2** Suppose we know that function  $f$  is positive and increasing, and the function  $g$  is negative and decreasing. Determine whether the following functions are increasing or decreasing.

(a) The product function:  $f(x)g(x)$ .

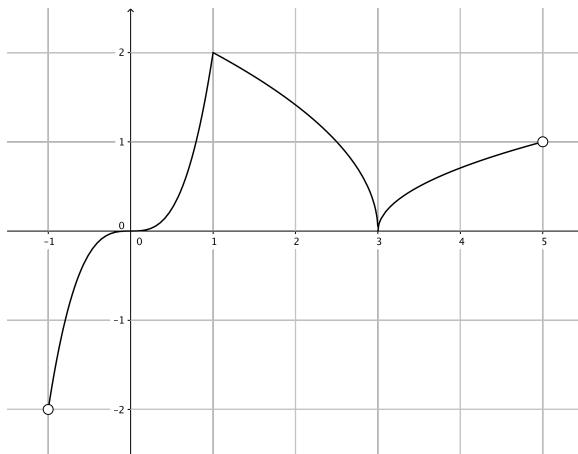
(b) The composite function:  $f(g(x))$ .

**Problem 3** Let  $f(x) = 2x^3 - 4x^2 + 1151$  and  $g(x) = x^3 \sin(x)$ . Compute  $f^{(4)}(x)$  and  $g^{(4)}(x)$ .

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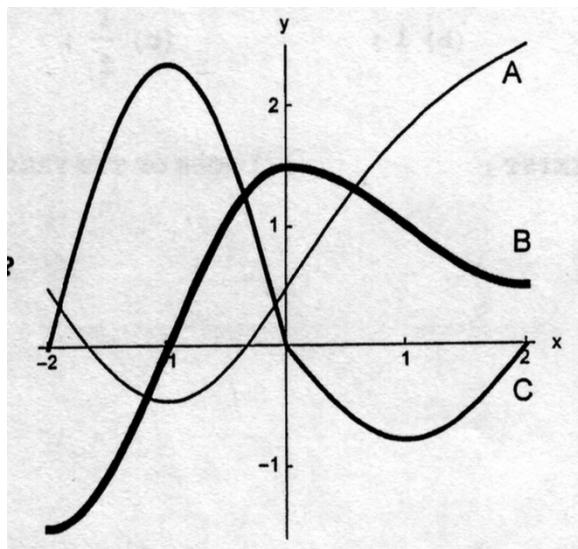
**Problem 4** Let  $f(x) = \sin(x)$  and  $g(x) = \cos(x)$ . Can you compute  $f^{(48)}(x)$ ,  $g^{(42)}(x)$ , and  $g^{(39)}(x)$  ?

**Problem 5** (a) The (entire) graph of a function  $f$  is shown in the figure below.



Find the interval (or intervals) on which the derivative of  $f$  is increasing.

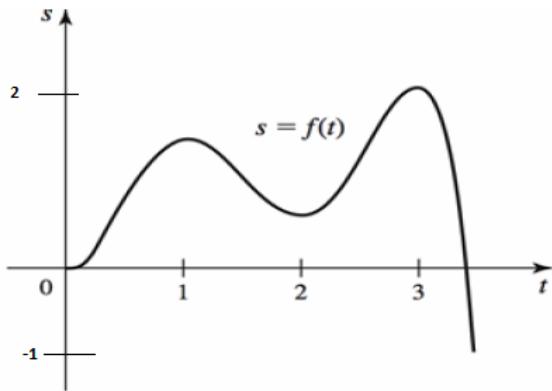
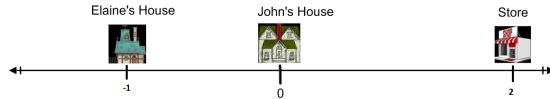
(b) The figure below shows the graphs of  $f$ ,  $f'$ , and  $f''$ . Which curve is which?



**Problem 6** Consider the parabola  $f(x) = ax^2 + bx + c$  where  $a, b, c$  are constants. For what values of  $a, b, c$  is  $f$  concave up? For what values of  $a, b, c$  is  $f$  concave down?

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**Problem 7** John is walking along a straight path. His position at the time  $t > 0$  is given by  $s = f(t)$ . He starts at  $t = 0$  from his house ( $f(0) = 0$ ) and the graph of  $f$  is given below.



- (a) Describe the motion of John as precisely as you can.
- (b) When is John's velocity zero? What is happening to John at those times?
- (c) When is John moving in the positive direction? When is John the furthest in the positive direction and the furthest in the negative direction?
- (d) When is John's velocity increasing? And when is it decreasing? When is John going at maximum velocity?

**Problem 8** Suppose that a stone is thrown vertically upward from a cliff on Mars with an initial velocity of 24 ft/s from a height of 192 ft. The height  $s$  of the stone above the ground after  $t$  seconds is given by  $s(t) = -6t^2 + 24t + 192$ .

- (a) Determine the velocity and acceleration of the stone after  $t$  seconds.

(b) What is the greatest height of the stone and when does it occur? What are the velocity and acceleration at that time?

(c) When does the stone hit the ground? What are the velocity and acceleration at that time?

**Problem 9** Suppose the function  $f$  has derivative given by  $f'(x) = \frac{2x}{4+x^2}$ .

(a) Find a formula for  $f''(x)$ .

(b) Construct a sign-chart for  $f'$  and  $f''$ .

(c) On what interval(s) is  $f$  increasing? On what interval(s) is  $f$  concave down?

**Problem 10** (a) True or False: *If the acceleration of an object is constant, then its velocity is constant.*

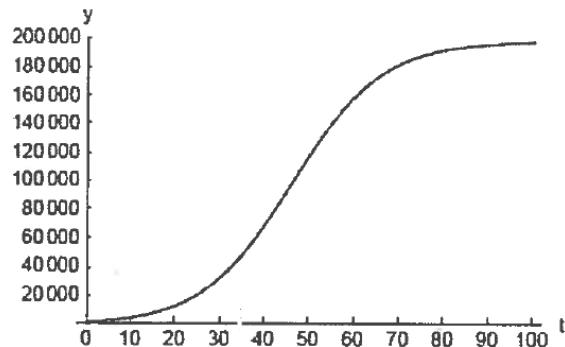
(b) True or False: *A moving object can have negative acceleration and increasing speed.*

**Problem 11** The total number of people,  $N$  who have contracted a common cold by a time  $t$  days after its outbreak on an island is given by  $N = N(t) = \frac{200000}{1 + 100e^{-0.1t}}, t \geq 0$ .

(a) Evaluate and interpret the limit  $\lim_{t \rightarrow \infty} N(t)$

(b) How long will it take for the number of people who have contracted the cold to reach 40,000?

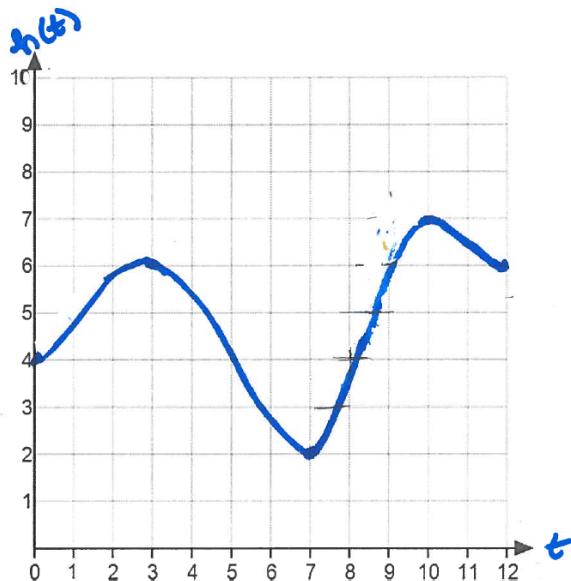
(c) The graph of the function  $N$  on the interval  $[0, 100]$  is given below. Sketch (as best you can) the graph of its derivative,  $N'(t)$ .



(d) Calculate  $N'(t)$ . What does  $N'(t)$  represent?

- (e) Evaluate and interpret the limit  $\lim_{t \rightarrow \infty} N'(t)$
- (f) Find the average growth rate of the number of people who have contracted the disease during the time interval  $[5, 6]$  (or during the sixth day after the outbreak).
- (g) Find the instantaneous growth rate of the number of people who have contracted the disease for  $t = 5$  (round to a whole number).

**Problem 12** The graph of  $h(t)$  represents the height in feet of water in a pool at time  $t$  minutes.



- During what time intervals is the height of water in the pool increasing? Decreasing?
- Assume the rate of change of the water in the pool is 0 when  $t = 0$ . For what other values of  $t$  is the rate of change of the water in the pool 0?
- If  $h(t)$  represents the height of the feet in water at time  $t$  minutes. What function represents the rate of change of the height over time?
- Sketch a graph of the rate of change of the height over time,  $h'(t)$ .