Using limits to detect asymptotes (ULTDA)

Infinite Limits

$\lim_{x \to a} f(x) = \infty$ means the values of $f(x)$ grow arbitrarily large as x approaches a.

$\lim_{x \to a} f(x) = -\infty$ means the values of $|f(x)|$ grow arbitrarily large as x approaches a with $f(x)$ negative.

Limits at Infinity

$\lim_{x \to \infty} f(x) = L$ means the values of $f(x)$ becomes arbitrarily close to L by making x sufficiently large.

$\lim_{x \to -\infty} f(x) = L$ means the values of $f(x)$ becomes arbitrarily close to L by making $|x|$ sufficiently large with x negative.

Vertical and Horizontal Asymptotes:

A function f has a **vertical asymptote** at $x = a$ if at least one of the following conditions hold:

- $\lim_{x \to a} f(x) = \pm\infty$
- $\lim_{x \to a^+} f(x) = \pm\infty$
- $\lim_{x \to a^-} f(x) = \pm\infty$

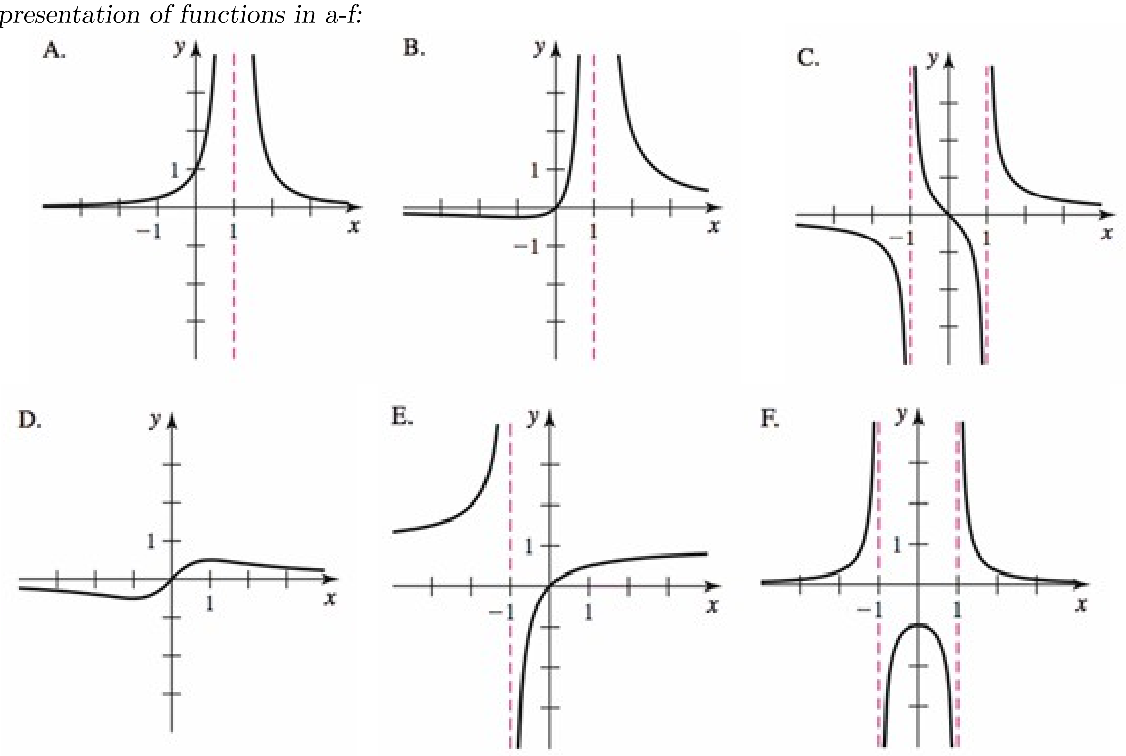
A function f has a **horizontal asymptote** at $y = L$ if at least one of the following conditions hold:

- $\lim_{x \to \infty} f(x) = L$
- $\lim_{x \to -\infty} f(x) = L$

Both vertical asymptotes and horizontal asymptotes are written as the equation of a line. Vertical asymptotes are given as the equation of a vertical line, and horizontal asymptotes are given as the equation of a horizontal line.

Recitation Questions

Problem 1 *Without using a graphing utility, match each graph of functions in A-F with the algebraic representation of functions in a-f:*



(a) *The function f defined by* $f(x) = \dfrac{x}{x^2+1}$.

(b) *The function g defined by* $g(x) = \dfrac{x}{x^2-1}$.

(c) *The function* h *defined by* $h(x) = \dfrac{1}{x^2 - 1}$.

(d) *The function* a *defined by* $a(x) = \dfrac{x}{(x-1)^2}$.

(e) *The function* s *defined by* $s(x) = \dfrac{1}{(x-1)^2}$.

(f) *The function* r *defined by* $r(x) = \dfrac{x}{x+1}$.

Problem 2 *Sketch a possible graph of a function g that satisfies the following conditions:*

Domain:$[-5,-2)\cup(-2,3)\cup(3,5)$	$\lim_{x\to -2} g(x) = 3$
$g(1) = 1$	$\lim_{x\to 1^+} g(x) = -\infty$
$\lim_{x\to 3} g(x) = \infty$	$\lim_{x\to 5^-} g(x) = \infty$
$\lim_{x\to 1^-} g(x) = 1$	$g(-5) = -1.8$

Problem 3 *Let f be a function given by $f(x) = \ln(1 + x)$.*

(a) *Find the domain of f. Write your answer in interval notation.*

(b) *Find the vertical asymptotes of f and **EXPLAIN** and justify your answer.*

(c) *Sketch a graph of f*

Problem 4 *Let* $f(x) = \dfrac{\ln(x)}{x-2}$.

(a) *Evaluate the limit.*

$$\lim_{x \to 2^-} \frac{\ln(x)}{x-2}$$

(b) *Find the vertical asymptotes of* f. ***EXPLAIN*** *and justify your answer.*

Problem 5 *Select the meaning of* $\lim_{x\to\infty} f(x) = 6$. *Support your explanation graphically.*

(a) *As* x *becomes arbitrarily negatively large,* $f(x)$ *approaches* 6.

(b) *As* x *becomes arbitrarily positively large,* $f(x)$ *approaches* 6.

(c) *As* x *approaches 6,* $f(x)$ *becomes arbitrarily negatively large.*

(d) *As* x *approaches 6,* $f(x)$ *becomes arbitrarily positively large.*

Problem 6 *Evaluate the following limits.*

(a) $\lim_{x\to\infty} \dfrac{\sqrt[3]{x^9+5}}{3x^3+\sqrt{4x^6+1}}$

(b) $\lim_{x\to\infty} \dfrac{\sin(9x)}{5x}$

Problem 7 *The function f is defined by* $f(x) = \dfrac{6e^x + 1}{3e^x + 5}$.

(a) *Find all vertical asymptotes of f.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

(b) *Find all horizontal asymptotes of f.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

Problem 8 *The function f is defined by* $f(x) = \dfrac{\sqrt{2x^2+1}}{3x-5}$.

(a) *Find all vertical asymptotes of f.* ***EXPLAIN*** *and ustify your answer by using appropriate limits.*

(b) *Find all horizontal asymptotes of f.* ***EXPLAIN*** *and justify your answer by using appropriate limits..*

Problem 9 *A piecewise defined function f is given by*

$$f(x) = \begin{cases} \dfrac{2x-3}{x-2} & \text{if } \quad x < 2 \\[2ex] \dfrac{x^2-5x+6}{x^2-4} & \text{if } \quad x > 2 \end{cases}$$

(a) *Find all vertical asymptotes.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

(b) *Find all horizontal asymptotes.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

Problem 10 *For the piecewise function f defined by*

$$f(x) = \begin{cases} \sin(x) & \text{if} \quad x \le 0 \\ \dfrac{x^2}{x^2-4} & \text{if} \quad 0 < x < 2 \\ \dfrac{x^2}{x^2+4} & \text{if} \quad 2 \le x \end{cases}$$

(a) *Find all vertical asymptotes.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

(b) *Find all horizontal asymptotes.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

Problem 11 *For the function g defined by*

$$g(t) = \frac{t^2 + 7t + 11}{t - 3}$$

(a) *Find all vertical asymptotes.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

(b) *Find all horizontal asymptotes.* ***EXPLAIN*** *and justify your answer by using appropriate limits.*

Problem 12 *Sketch a possible graph of a function that satisfies all of the given properties. (You* do not *need to find a formula for the function.)*

$\lim_{x\to -2^-} f(x) = \infty$	$f(-2) = -5$	$f(1) = 2$
$\lim_{x\to \infty} f(x) = -\infty$	$\lim_{x\to -\infty} f(x) = 4$	$\lim_{x\to 5} f(x) = \infty$
$\lim_{x\to 3} f(x) = 3$	$f(3) = 1$	
$\lim_{x\to -2^+} f(x) = -5$	$f(4)$ *is undefined*	$\lim_{x\to 4} f(x) = 3$