Substitution (TIOS, WWS)

Recitation Questions

Problem **1** *Compute the following indefinite integrals.*

(a) $\int 2t \sin(t^2) \, dt.$

(b) $\int \cos(x) \sqrt{\sin(x)} \, dx.$

(c) $\int \left(3t^2 - 4 + \dfrac{1}{t} \right) e^{t^3 - 4t + \ln(t) - 9} \, dt.$

Problem   2   *Evaluate the following definite integrals:*

(a) $\int_{-2}^{1} t^2 \sin(t^3) \, dt$

(b) $\int_{0}^{1/2} \frac{13e^u}{3e^u - 5} \, du$

(c) $\int_{0}^{\pi/2} \frac{\cos(x)}{4 + \sin(x)} \, dx$

(d) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{3\sqrt{x}}\,dx$

(e) $\displaystyle\int_{\pi/3}^{\pi/2} \sin(x)\sec^2(\cos(x))\,dx$

Problem   3   *Suppose that* $\int_1^3 f(x)\,dx = 4$

(a) *Evaluate the following integrals.*

(i) $\int_0^2 f(x+1)\,dx$

(ii) $\int_1^9 3\dfrac{f(\sqrt{x})}{\sqrt{x}}\,dx$
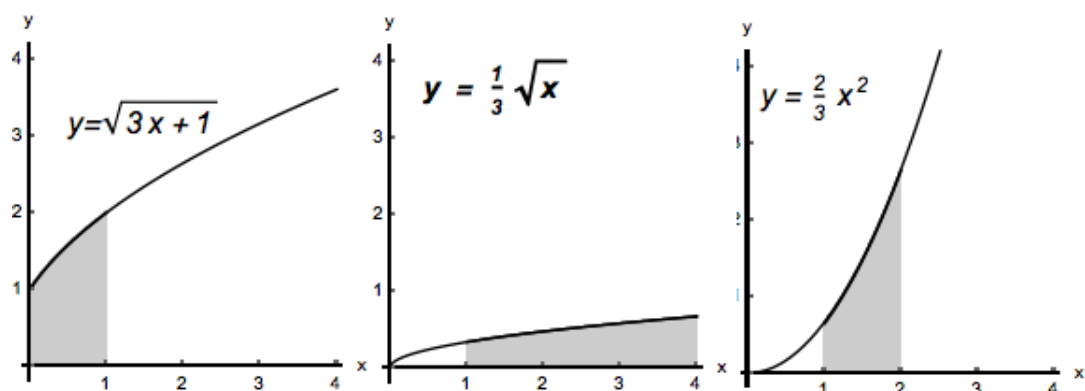
(iii) $\int_{\frac{1}{3}}^1 f(3x)\,dx$

(iv) $\int_2^4 3f(x-1)\,dx$

(v) $\int_0^{\sqrt{2}} 3xf(x^2+1)\,dx$

(b) *Assume that f is odd. Evaluate* $\displaystyle\int_{-3}^{-1} f(x)\,dx$

(c) *Assume that f is even. Evaluate* $\displaystyle\int_{-3}^{-1} f(x)\,dx$

Problem 4 *Show that the areas of the three shaded regions in the figure below are equal. Find this area.*

$$y=\sqrt{3x+1}$$

$$y = \frac{1}{3}\sqrt{x}$$

$$y = \frac{2}{3}x^2$$

Problem   5   *Find the error in the following "solution":*

Find $\int_{-2}^{2} \frac{1}{x^8 - 1} \, dx.$

let $u = x^4$
then $du = 4x^3 dx$

$\frac{du}{4x^3} = dx$

and $u^{1/4} = x$

$u(2) = 2^4 = 16$

$u(-2) = (-2)^4 = 16$

$= \int_{16}^{16} \frac{1}{(u^2 - 1)} \cdot \frac{du}{4u^{3/4}}$

but an integral $\int_a^a f(x) dx = 0$

so $\int_{-2}^{2} \frac{1}{x^8 - 1} dx = 0$

Problem 6 *Find the error in the following "solution", correct the error (or errors) and evaluate the integral.*

Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\sqrt{1 - \cos^2(x)}\, dx$.

"Solution":

Since $\sin(x) = \sqrt{1 - \cos^2(x)}$, let $u = \sin(x)$. Then $du = \cos(x)dx$;

when $x = -\dfrac{\pi}{2}$, $u = \sin\left(-\dfrac{\pi}{2}\right) = -1$; when $x = \dfrac{\pi}{2}$, $u = \sin\left(\dfrac{\pi}{2}\right) = 1$. Therefore

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\sqrt{1 - \cos^2(x)}\, dx = \int_{-1}^{1} u \cdot du = 0,$$

due to symmetry (integrating an odd function over a symmetric interval, $[-1, 1]$).

Problem   **7**   *Compute the integral:* $\int \dfrac{1 + 3x}{4 + 4x^2}\, dx.$

Problem   **8**   *Evaluate the integral* $\int \dfrac{x^2}{1 + x^2}\, dx.$

Problem 9 *Compute the integral:* $\int \sec^2(x) \tan(x)\, dx.$

Problem **10** *What are two substitutions that can be used to evaluate the integral*

$$\int x\sqrt{x+8}\,dx$$

Problem   11   *Compute the integral:* $\displaystyle\int \frac{x}{\sqrt{x-4}}\, dx.$

Problem 12 *Evaluate the following indefinite integrals:*

(a) $\int \dfrac{13x^7}{\sqrt{3x^4 - 5}} \, dx$

(b) $\int \dfrac{u^3}{u^2 - 3} \, du$

(c) $\int e^{t^2 + \ln(t)} \, dt$