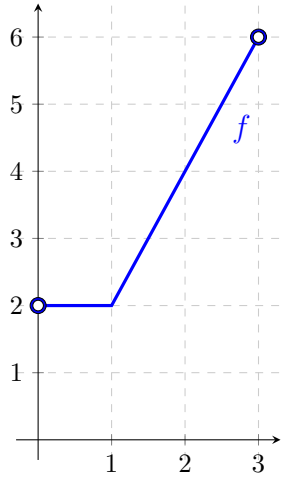
Understanding functions (UF)

SUMMARY: What we need to know about functions

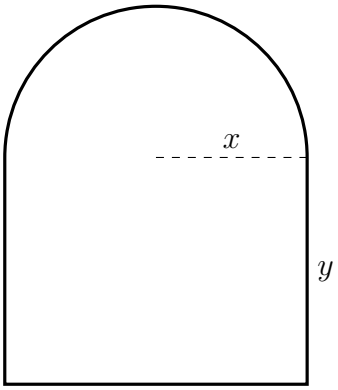
- State the definition of a function.
- Find the domain and range of a function.
- Distinguish between functions by considering their domains.
- Recognize different representations of the same function.
- Determine where a function is positive or negative.
- Determine algebraically whether a function is even, odd, or neither.
- Use symmetry when graphing even or odd functions.
- Plot basic functions.
- Apply appropriate transformations to graphs of basic functions
  (vertical and horizontal shifts, vertical and horizontal stretching and reflecting)
- Perform basic operations and compositions on functions.
- Work with piecewise defined functions.
- Determine if a function is one-to-one.
- Define and work with inverse functions.
- Plot inverses of basic functions.
- Find inverse functions (algebraically and graphically).
- Find the largest interval containing a given point where the function is invertible.
- Determine the intervals on which a function has an inverse.
- Relate the domain and range of f and the domain and range of f^{-1}.
- Use a sign-chart to determine where the function is positive/negative.

Recitation Questions

Problem 1 *Find a formula for the function f whose graph is given in the figure below. What is the domain of f? What is the range of f? Is the function f linear?*



Problem 2 *A mirror has the shape of a rectangle surmounted by a semicircle (see figure). The area of the mirror is 32 in^2. Let x be the radius of the semicircle (that lies on top of the rectangle). Express the perimeter of the mirror P (in inches) as a function of x (in inches). Is P a polynomial, a rational function or a transcendental function?*



Problem 3 *Define* $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 0 \\ ? & \text{if } x > 0 \end{cases}$

(a) *Find an expression for "?" such that f will be even.*

(b) *Find an expression for "?" such that f will be odd.*

Problem 4 *Given* $y(t) = t - \dfrac{\pi}{3}$ *and* $w(t) = \sin(t)$. *Find:*

(a) $y(w(t))$

(b) $w(y(t))$

(c) $w\left(y\left(\dfrac{4\pi}{3}\right)\right)$

(d) $y(w(\dfrac{4\pi}{3}))$

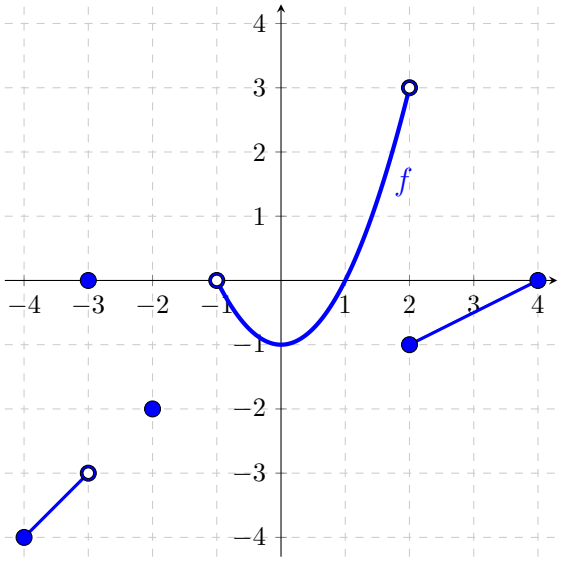
Problem 5 *Define* $g(x) = \begin{cases} |x-2| & \text{if } x < 0 \\ \cos(x) & \text{if } x \geq 0 \end{cases}$

(a) *Sketch a graph of* g

(b) *Find the domain and range of* g

(c) *Find the values of* $g(\pi)$ *and* $g(-\pi)$

Problem 6 *The entire graph of $f(x)$ is given below.*



(a) *Find the domain and range of f*

(b) *Find the values of $f(-3), f(-2), f(-1), f(2)$*

(c) *Find the intervals on which $f(x)$ is positive. Find the intervals on which $f(x)$ is negative.*

(d) *Find the intervals on which f is increasing. Find the intervals on which f is decreasing.*

(e) *True or False:* $f(1.5) < f(2)$

Problem 7 *Determine if the function is even, odd, or neither.*

(a) $h(x) = x^4 + x^2 - 3$

(b) $s(t) = t^2 - t$

(c) *We know that* $\sin(\theta)$ *is odd and* $\cos(\theta)$ *is even. Is* $g(\theta) = \tan(\theta)$ *even, odd, or neither?*

Problem 8 *Using the known graphs of* $y = \sqrt{x}$ *and* $y = \dfrac{1}{x}$*, sketch the graphs of the following using transformations.*

(a) $y = \sqrt{x+2} - 3$

(b) $y = \dfrac{1}{x-4} + 1$

Problem 9 *Find the domain of the function. Determine whether the function is odd, even, or neither.*

(a) $f(x) = \dfrac{x}{\sqrt{x^2 - 9}}$

(b) $g(x) = \dfrac{\sin(x)}{x}$

(c) $h(t) = \ln(t^3 - 1)$

Problem 10 *Let g be a one-to-one function and let g^{-1} be its inverse.* ***True or False:*** *If the point $(2, 1/5)$ lies on the graph of g, then the point $(2, 5)$ lies on the graph of g^{-1}.*

Problem 11 *Each of the following functions are invertible on their given domains. For each one find a formula for its inverse and give the domain and range of the inverse.*

(a) *The function f defined by $f(x) = x^2 - 4x - 5$ for every $x \geq 2$.*

(b) *The function g defined by $g(u) = \sqrt[4]{u+2}$.*

(c) *The function h defined by $h(t) = 1/(t+2)^2$ for every $t > -2$.*

Problem 12 *Explain what each of the following means:*

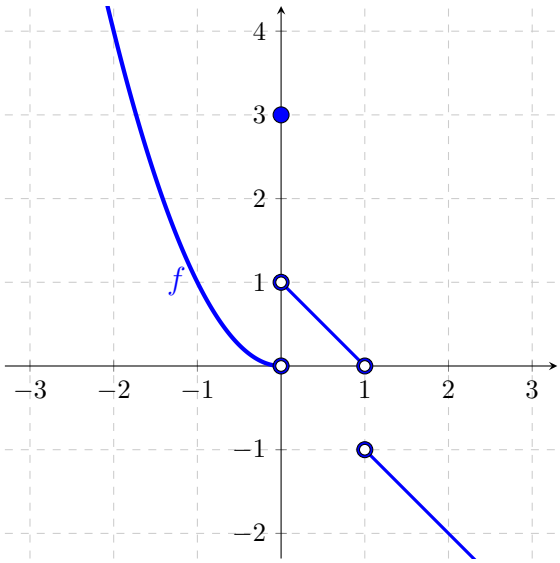
(a) $f^{-1}(x)$

(b) $f(x^{-1})$

(c) $(f(x))^{-1}$

Problem 13 *If $f(x)$ represents the number of packages of buns needed for x packages of hotdogs, what does $f^{-1}(x)$ represent?*

Problem 14 *We're given the following graph of a function:*



Use this graph to answer the following questions:

(a) *What is the domain of this function?*

(b) *What is the range of this function?*

(c) *What is the value of* $f(0)$, $f(1)$, *and* $f(2)$*?*

(d) *Does this function have an inverse? (Why or why not?)*

(e) *Find at least two intervals on which the function is one-to-one.*

(f) *Find* $f^{-1}(3)$ *on a restricted domain of* f*.*

(g) *Using the restricted domain for f of $(-\infty, 0)$, sketch a graph of f^{-1}.*

(h) *Using the restricted domain for f of $[0, 1) \cup (1, \infty)$, sketch a graph of f^{-1}.*