

# Rules of differentiation (ROD)

- **The Power Rule:** For any real number  $n$ ,  $\frac{d}{dx}x^n = nx^{n-1}$ .
- **The Natural Exponential Function:**  $\frac{d}{dx}e^x = e^x$ .
- **The Sine Function:**  $\frac{d}{dx}\sin(x) = \cos(x)$ .
- **The Sum Rule:** If  $f(x)$  and  $g(x)$  are differentiable functions and  $c$  is a constant, then:
  - $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$ ,
  - $\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$ ,
  - $\frac{d}{dx}(c \cdot f(x)) = c \cdot f'(x)$ .

## Recitation Questions

**Problem 1** For each of the following functions, use the "short cut derivative rules" to compute their derivative.

(a)  $f(x) = \sqrt{x}$

(b)  $s(u) = \frac{5}{u^2}$

(c)  $p(t) = t^5 + 4t^3 + \pi$

**Problem 2** Given the polynomial function  $q$  defined by  $q(v) = 2v^3 - 5v^2 + 7v - 9$  find:

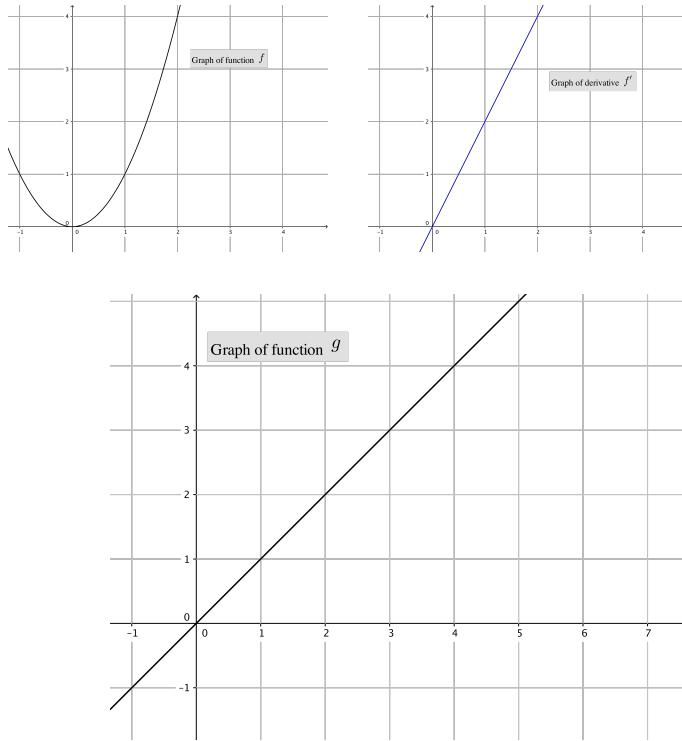
- (a) The slope of the tangent line to the graph of  $q$  at the point where  $v = 3$  using the limit definition of a derivative.
- (b) The slope of the tangent line to the graph of  $q$  at the point where  $v = 3$  using the “short-cut derivative rules” to find a formula for  $q'$  and evaluating  $q'(3)$ .
- (c) The equation of the tangent line to the graph of  $q$  at  $v = 3$ .

---

**Problem 3** Find  $s'$  of the function  $s$  defined by  $s(t) = 3t^2 + 5e^t - \frac{1}{t}$ .

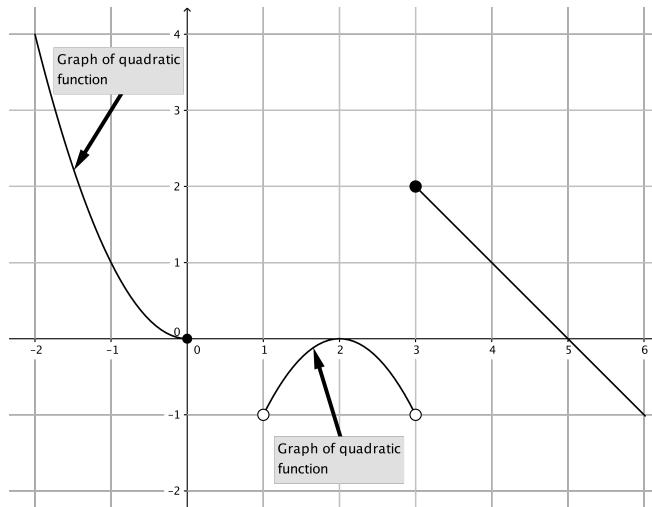
---

**Problem 4** Use the given graphs of  $f$  and  $g$  and their accompanying derivatives to answer the following questions.



- Write an equation for the tangent line to  $f$  at  $x = 2$ .
- Draw the graph of  $g'$
- Compute the value of the  $(5f + 3g)'(2)$ .
- Find the equation of the tangent line to the graph of  $(5f + 3g)$  at the point where  $x = 2$ .
- Use the given graph of  $f'$  and  $g'$  to find the following:
  - A formula for  $f'$
  - A formula for  $g'$
- Find the expression for  $(5f + 3g)'(x)$ .

**Problem 5** Sketch the graph of the derivative of the given function:



**Problem 6** A company is producing cell phones. The cost of producing  $x$  cell phones is given by  $C(x)$ , defined by

$$C(x) = -0.01x^2 + 40x + 400, \text{ for } 0 \leq x < 1000.$$

AVERAGE cost of producing the first  $x$  cell phones is given by

$$C_{AVG} = \frac{C(x)}{x}$$

If the company has produced  $x$  cell phones, the cost of producing one more item is given by

$$\text{COST of producing one more item} = C(x+1) - C(x)$$

MARGINAL COST is approximation of the cost of producing one more cell phone

$$\text{MARGINAL COST} = C'(x)$$

- (a) Compute the average cost of the first 300 cellphones that the company produces.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
- (b) Compute the cost of producing one more cell phone, if the company has produced 300 cell phones.
- (c) Compute the marginal cost, if 300 cell phones have been produced.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
- (d) Why is the marginal cost a good approximation of the cost of producing one more item? Explain!