Derivatives as functions (DAF)

- The derivative of a function f is a function f' whose domain consists of all points in the domain of f where the function f is differentiable. The values of f' are given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

  where this value exists.

- If f is differentiable at $x = a$, then f is continuous at $x = a$.

Recitation Questions

Problem 1 (a) *Suppose $f'(2)$ exists. Which of the following must be true?*

(i) $\lim_{x \to 2} f(x)$ *must exist, but* $\lim_{x \to 2} f(x) \neq f(2)$

(ii) $\lim_{x \to 2} f(x) = f(2)$.

(iii) $\lim_{x \to 2} f(x) = f'(2)$

(iv) $\lim_{x \to 2} f(x)$ *need not exist.*

(b) *Assuming that* $\lim_{x \to 0} \dfrac{\sin(x)}{x} = 1$, *which of the following is true?*

(i) $\dfrac{0}{0} = 1$

(ii) *the tangent line to* $y = \sin(x)$ *at* $(0, 0)$ *has slope* 1.

(iii) *you can cancel the* x*'s.*

(iv) *for all* x *near* 0, $\sin(x) = x$.

(v) *for all* x *near* 0, $\sin(x) \approx x$.

Problem 2

(a) *Fill in the blanks*

$$f'(x) = \lim_{???} \frac{????}{h}$$

if the limit exists.

(b) *Let*

$$f(x) = \frac{1}{x+4}.$$

Use the (limit) definition *of derivative in* (a) *to find* $f'(x)$.

Problem 3 *Let* $f(x) = |5 - x|$.

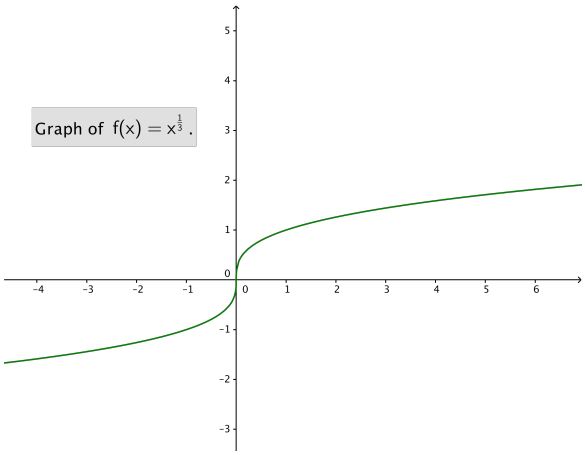
(a) *For* $a < 5$, *find* $f'(a)$.

(b) *For* $a > 5$, *find* $f'(a)$.

(c) *Determine whether* $f'(5)$ *exists.*

(d) *Sketch a graph of the function* $f(x)$ *and its derivative* $f'(x)$

Problem 4 *Define the function f by $f(x) = x^{1/3}$ and consider the graph of this function:*

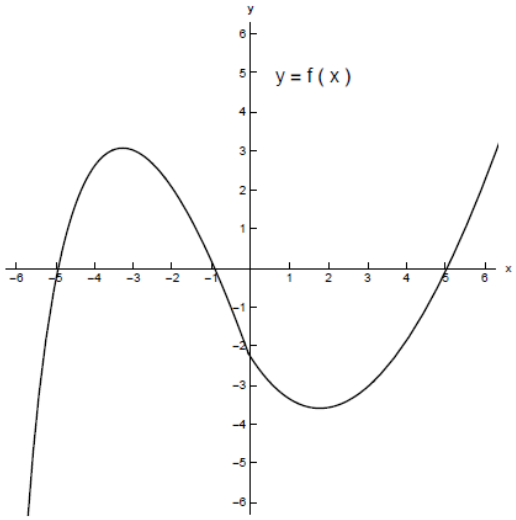


Which of the following two statements are true:

(a) *The graph of f has a tangent line at $x = 0$.*

(b) *The derivative $f'(0)$ is defined.*

Problem 5 *Suppose we are given the graph of a function f:*



(a) *Use this graph to find the following: (Assume all values will be integers or $+\infty$ or $-\infty$)*

(i) *all x where $f(x) = 0$,*

(ii) *all x where $f(x) > 0$,*

(iii) *all x where $f(x) < 0$, and*

(iv) *all x where $f(x)$ attains a local maximum and all x where f attains a local minimum.*

Without sketching the graph of f' find

(b) (i) *all x where $f'(x) = 0$,*

(ii) *all x where $f'(x) > 0$,*

(iii) *all x where $f'(x) < 0$, and*

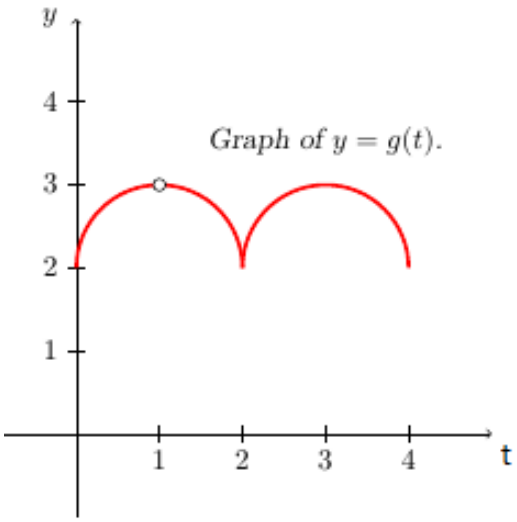
(iv) *On the following intervals, does $f'(x)$ seem to be increasing or decreasing?*

i. $(-\infty, 0)$

ii. $(0, \infty)$

(c) *Sketch a graph of f'.*

Problem 6 *Use the graph of g*



(a) *Find the values of t in $(0, 4)$ at which g is not continuous.*

(b) *Find the values of t in $(0, 4)$ at which g is not differentiable.*

Problem 7 *Given the following graph of a function h sketch a graph of the derivative h'.*