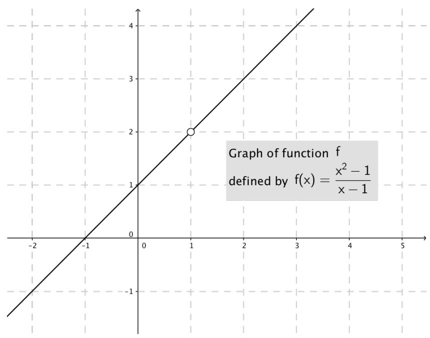
2.2: Definition of Limits

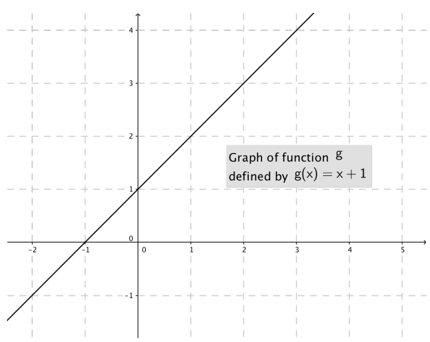
Problem 1

(a) *True or False: To find* $\lim_{x \to 2} f(x)$*, it's enough to know the values of* $f(2.1)$*,* $f(2.01)$*,* $f(2.001)$*, and so on.*

(b) *True or False: If we know the value of* $f(2)$*, then can we conclude that* $\lim_{x \to 2} f(x) = f(2)$*?*

Problem 2 *Use the graphs and the given definitions of the following two functions to answer the questions below.*



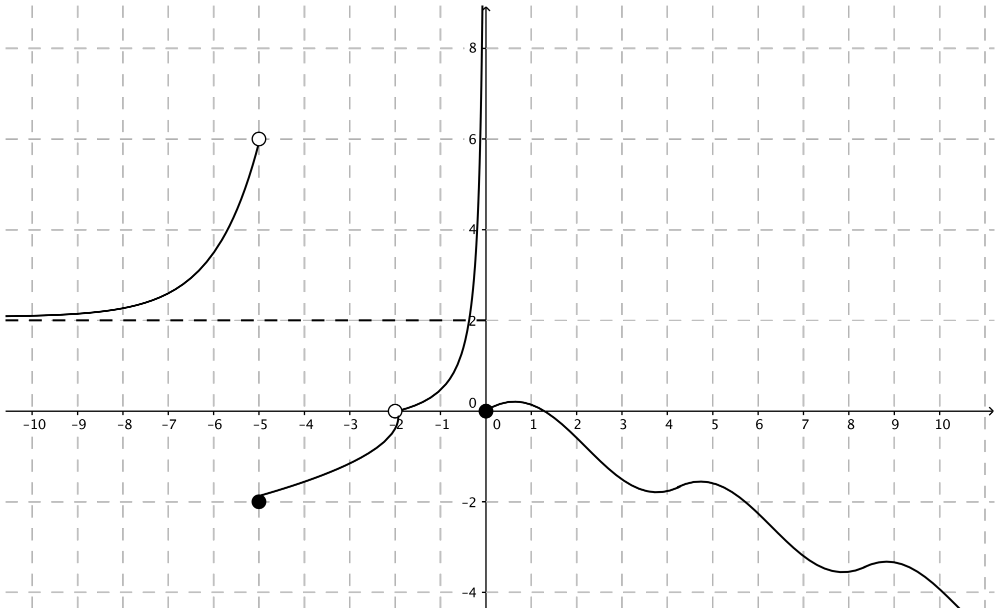


(a) *Find the domain of* f *and the domain of* g*.*

(b) *Is* $f = g$*? (Why or why not?)*

(c) *Looking at the graphs, find* $\lim_{x \to 1} f(x)$ *and* $\lim_{x \to 1} g(x)$*.*

Problem 3 *This problem was on the AU15 Midterm.* *The graph of a function* f *is given below. Use this graph to answer the following questions.*



(a) *Find the domain of f.*

(b) *Find the range of f.*

(c) *Find the following values.*

(i) $\lim_{x\to -2} f(x) =$

(ii) $f(-2) =$

(iii) $f(-5) =$

(iv) $\lim_{x\to 0^+} f(x) =$

(v) $\lim_{x\to 0} f(x) =$

Problem 4 *Sketch a possible graph of a function that satisfies all of the given properties. (You* do not *need to find a formula for the function.)*

Domain: $[-4,3)\cup(3,5)$	$f(1)=3$	$f(-1)=1$	$f(-4)=1$
$\lim_{x\to -1^-} f(x) = -2$	$\lim_{x\to -1^+} f(x) = 1$	$\lim_{x\to 1} f(x) = 2$	$\lim_{x\to 3^+} f(x) = -1$
$\lim_{x\to 3^-} f(x) = 1$		$\lim_{x\to -4^+} f(x) = 1$	$\lim_{x\to 5^-} f(x) = 1$

Problem 5 *True/False: Give an explanation or counterexample. Assume a and L are finite numbers.*

(a) *If $\lim_{x\to a} f(x) = L$, then $f(a) = L$.*

(b) *If $\lim_{x\to a^-} f(x) = L$, then $\lim_{x\to a^+} f(x) = L$.*

(c) *If* $\lim_{x \to a} f(x) = L$ *and* $\lim_{x \to a} g(x) = L$*, then* $f(a) = g(a)$.

(d) $\lim_{x \to a} \frac{f(x)}{g(x)}$ *does not exist if* $g(a) = 0$.