Definition of the derivative (DOTD)

- The *average rate of change*, $AvRCh$, of a function f across an interval $[a, b]$ is given by

$$\begin{aligned} AvRCh &= \frac{\Delta f}{\Delta x} \\ &= \frac{f(b) - f(a)}{b - a} \end{aligned}$$

- The slope, m_{sec}, of the line secant to the graph of a function f at the points $(a, f(a))$ and $(b, f(b))$ is given by

$$\begin{aligned} m_{sec} &= \frac{\Delta y}{\Delta x} \\ &= \frac{f(b) - f(a)}{b - a} \end{aligned}$$

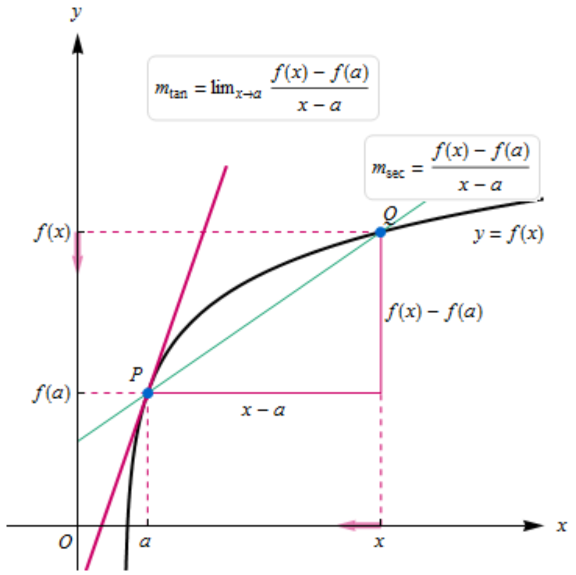
- The *derivative* of the function f at a, denoted $f'(a)$, is given by

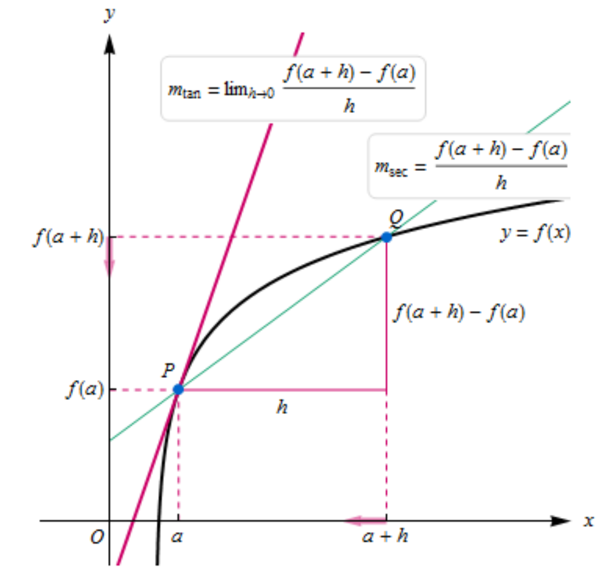
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

  and represents the instantaneous rate of change of f with respect to x at $x = a$ and also the slope of the line tangent to the graph of $y = f(x)$ at the point $(a, f(a))$.

Recitation Questions

Problem 1 *Consider the following two figures depicting the same graph $y = f(x)$ and the same two lines, and the same two points P and Q:*





(a) *In Figure 1 on the left, what are the coordinates of P and Q? In Figure 2 on the right, what are the expressions for the coordinates of P and Q?*

(b) *Express the slope of the secant line through P and Q in terms of the above coordinates for each Figure 1 and Figure 2.*

(c) *Express the slope of the tangent line at the point P in terms of the above coordinates for each Figure 1 and Figure 2.*

(d) *What is the difference?*

(e) *For each of the two graphs, which lines are the secant lines?*

(f) *For each of the two graphs, which lines are the tangent lines?*

Problem 2 *For each of the following functions find an equation of the tangent line at the given point.*

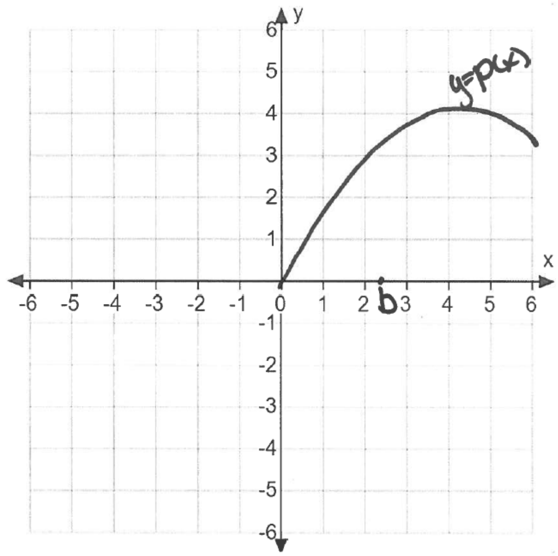
(a) $f(x) = -5x^2 + 7x - 9 \quad \text{at } x = 3.$

(b) $g(u) = \sqrt{5u - 4} \quad \text{at } u = 3.$

(c) $s(z) = \dfrac{z}{z - 5} \quad \text{at } z = 3.$

Problem 3 *Find an equation of the tangent line at the given point. Then graph the function and the tangent line on the same plot.* $f(x) = \sqrt{x+1}$ *at* $x = 3$

Problem 4 *The graph of a function p and a point in its domain, b, are shown in the figure below.*



(a) *In the figure above, draw and mark clearly the quantity $\Delta y = p(b) - p(1)$ and the quantity $\Delta x = b - 1$.*

(b) *Complete the sentence. The quotient $\dfrac{p(b) - p(1)}{b - 1}$ is the slope...*

(c) *Complete the sentence. Provided it exists, the limit $\lim_{x \to 1} \dfrac{p(x) - p(1)}{x - 1}$ is the slope...*

Problem 5 *An object moving along a straight line has a position given by* $s(t) = \dfrac{1}{t-4}$, *where* s *is measured in meters and* t *in seconds. Find the velocity of the object at time* $t = 6$.