An application of limits (AAOL)

Suppose an object is traveling along a straight line, with displacement given by a function $s(t)$.

- The *average velocity*, v_{av}, of the object between time $t = a$ and time $t = b$ is given by

$$\begin{aligned} v_{av} &= \frac{\Delta s}{\Delta t} \\ &= \frac{s(b) - s(a)}{b - a} \end{aligned}$$

- The *instantaneous velocity* of the object at time $t = a$ is given by

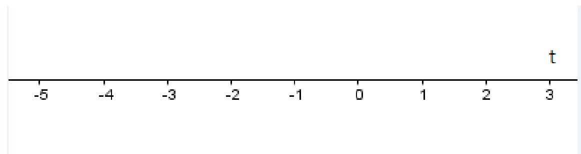
$$\begin{aligned} v(a) &= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \\ &= \lim_{t \to a} \frac{s(t) - s(a)}{t - a} \\ &= \lim_{t \to a} v_{av}(t) \end{aligned}$$

  where $v_{av}(t)$ denotes the average velocity on the interval $[a, t]$ if $t > a$, and $[t, a]$ if $t < a$.

Recitation Questions

Problem 1 *The position, $s(t)$, of an object moving along a horizontal line is given by $s(t) = t^2 - 4$, where s is in meters and t is in seconds, $0 \leq t < 5$.*

(a) *Mark the position of the object on the line at time $t = 1$:*



(b) *Find the average velocity, v_{AV}, of the object during the time interval $[1, 3]$.*

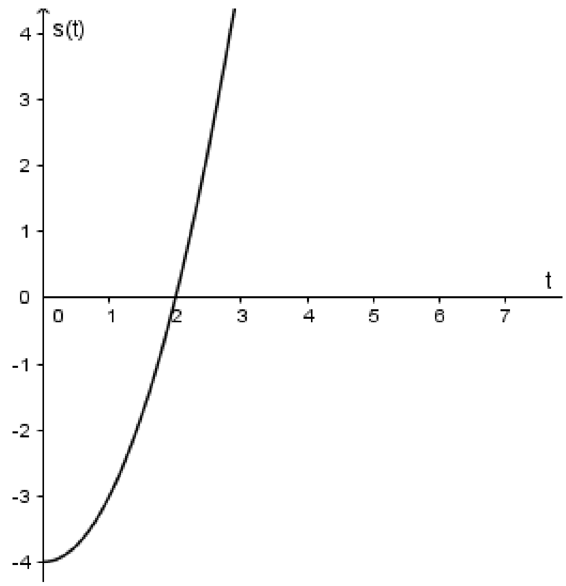
(c) *Compute the average velocity, $v_{\mathrm{AV}}(t)$, of the object during the time interval*

(i) $[1, t]$, *for $1 < t < 5$;*

(ii) $[t, 1]$, *for $0 \leq t < 1$.*

(d) *Find the instantaneous velocity, v_{inst}, of the object at $t = 1$. Justify your answer.*

(e) *The position-time graph of the function s is given in the figure below.*



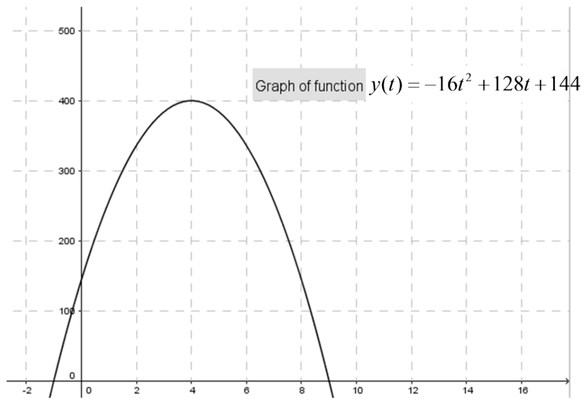
(i) *Assume P is a point on the graph of s. Fill in the blank.*

$$P = (1, \_\_\_\_).$$

(ii) *Plot the point P and draw the tangent line at this point in the figure above.*

(iii) *Find the slope, $m_{\tan}$, of the tangent line in part (ii). Explain.*

Problem 2 *Part of the given parabola can be used to model the "position-time" graph of a ball thrown straight up into the air. The graph gives the height of the ball in feet t seconds after being thrown into the air.*
Let the function f be defined by $f(t) = -16t^2 + 128t + 144$.
Use this graph, and the given function, f, to answer the following questions.



(a) *Mark the part of the parabola that can be used to model the position of the ball.*

(b) *What are the units on the t axis? What are the units on the y axis?*

(c) *If you were watching a movie of the ball being thrown, is the graph a picture of the path that the ball follows? Why or why not?*

(d) *Let $f(t)$ denote the height of the ball at time $t, t \geq 0$. What is the height of the ball at time $t = 0$?*

(e) *When will the ball hit the ground?*

(f) *What is the domain of the position function, f, of the ball?*

(g) *Use the table of values to find the average velocity of the ball between $t = 8.9$ and $t = 9$ seconds.*

t	$\approx f(t)$
8.9	15.84
8.99	1.6
8.999	0.159984
8.9999	0.015998
9	0

(h) *Use the table of average velocities to approximate the instantaneous velocity of the ball when it hits the ground.*

Time Interval	*Average Velocity*
$[8.9, 9]$	$\dfrac{f(9) - f(8.9)}{.1} = \dfrac{0 - 15.84}{.1} = -158.4$
$[8.99, 9]$	$\dfrac{f(9) - f(8.99)}{.01} = \dfrac{0 - 1.5984}{.01} = -159.84$
$[8.999, 9]$	$\dfrac{f(9) - f(8.999)}{.001} = \dfrac{0 - .159984}{.001} = -159.984$
$[8.9999, 9]$	$\dfrac{f(9) - f(8.999)}{.0001} = \dfrac{0 - .0159998}{.0001} = -159.998$

(i) *Compute* $v_{AV(t)}$ *the average velocity of the ball on the time interval* $[t, 9]$*, where* $t < 9$.

(j) *Compute* $v(9)$*, the* ***instantaneous*** *velocity of the ball at* $t = 9$.