Antiderivatives and area (AAA)

SUMMARY: Antiderivatives and Area

We know, from the Antiderivatives section, that
(1) the position function, s(t), of the object moving along a straight line with velocity $v(t)$ is an antiderivative of the velocity, i.e.

$$\int v(t)\,dt = s(t) + C$$

(2) the displacement of the object over the time interval $[a, b]$ is given by

$$\text{Displacement} = \Delta s = s(b) - s(a)$$

In this section, we learned that
(3) the displacement of the object over the time interval $[a, b]$ can be represented as the area under the curve $y = v(t)$, (if $v(t) \geq 0$ on $[a, b]$), which means that

$$\text{Displacement} = \int_a^b v(t)\,dt$$

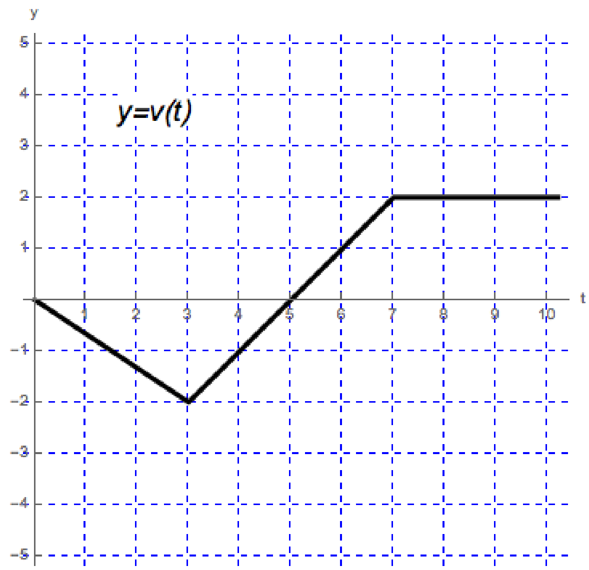
(4) Combining the last two equations, we get

$$s(b) - s(a) = \int_a^b v(t)\,dt$$

Note: The last formula states that a definite integral of a *velocity* function can be computed by using an antiderivative of the function and evaluating it at the and points of the interval. This the first time that we make a connection between definite integrals and antiderivatives.

Recitation Questions

Problem 1 *The graph of the velocity (t in min, v in ft/min) of a particle moving along a straight line is given in the figure. Assume that the particle was at the origin initially.*



(a) *Evaluate the displacement of the particle over the following intervals.*

(i) $[0, 10]$

(ii) $[0, 7]$

(iii) $[0, 5]$

(b) *When was the particle farthest from the origin?*

(c) *What was the total distance the particle has travelled over the time interval* $[0, 10]$*?*

Problem 2 *The velocity of an object moving along a straight line is given by $v(t)$ (in ft/s) and we only know the following:* $\int_0^3 v(t)\,dt = -3$, *and* $\int_3^4 v(t)\,dt = 5$.

Compute the following displacements, if possible. If it is not possible, give examples explaining why not.

(a) $s(3) - s(0)$

(b) $s(4) - s(0)$

(c) *Find the displacement during the interval $[0,4]$ if the velocity at the time t, $0 \le t \le 4$, was $v(t) + 2$ ft/s instead?*

(d) *Find the displacement during the interval $[0,4]$ if the velocity at the time t, $0 \le t \le 4$, was $5v(t)$ ft/s instead?*

Problem 3 *The velocity of the object moving along a straight line is given by* $v(t) = t - 5$, $0 \leq t \leq 10$, *where* t *is in seconds and* v *in ft/s.*

(a) *Use geometry to evaluate the displacement of the object on the time interval* $[0, 10]$. *Sketch the graph of the velocity and shade the relevant region.*

(b) *Compute the displacement on the interval* $[0, 10]$ *using antiderivatives of the function* v.

(c) *Use geometry to evaluate the displacement of the object on the time interval* $[0, 8]$. *Sketch the graph of the velocity and shade the relevant region.*

(d) *Compute the displacement on the interval* $[0, 8]$ *using antiderivatives of the function* v.

Problem 4 *The **velocity** function for a man walking along a straight road which runs east and west is given by* $v(t) = -t^2 + 4t - 3$ *ft/min.*

(a) *Set up a definite integral for the man's **displacement** during the time interval from 2 minutes to 6 minutes after he began running.*

(b) ***At home:*** *Evaluate the definite integral using the limit of a right Riemann sum.*

(c) *Is this the same as the total **distance** the man walked from 2 minutes to 6 minutes? Why or why not?*