

Concepts of graphing functions (COGF) - Solutions

Problem 1 Fill in the following blanks with the correct choice of the words from this list:

Increasing, decreasing, positive, negative, concave up, concave down

- (a) If you know $f''(x) > 0$, then you know $f'(x)$ is _____ and $f(x)$ is _____.

Solution: increasing, concave up

- (b) If you know $g'(x) < 0$ and decreasing, then you know $g(x)$ is _____ and _____.

Solution: decreasing, concave down

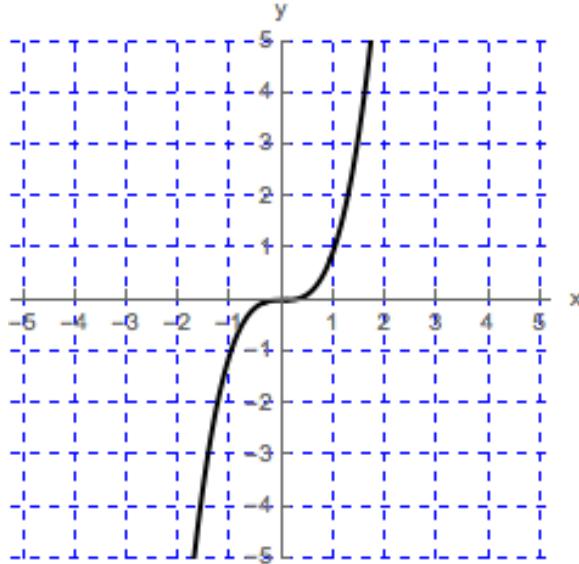
- (c) If you know $h(x)$ is positive, increasing, and concave down, then you know $h'(x)$ is _____ and _____ and that $h''(x)$ is _____.

Solution: positive, decreasing, negative

Problem 2 Sketch a graph of a function that is continuous on $(-\infty, \infty)$ that has the following properties.

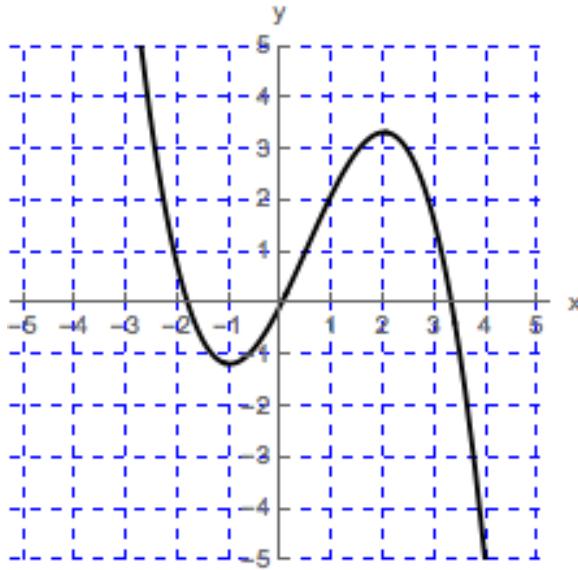
- (a) Function f does not have a local maximum or minimum. f contains a point where $f'(x) = 0$

Solution:



- (b) $g'(x) < 0$ on $(-\infty, -1)$; $g'(x) > 0$ on $(-1, 2)$; $g'(x) < 0$ on $(2, \infty)$.

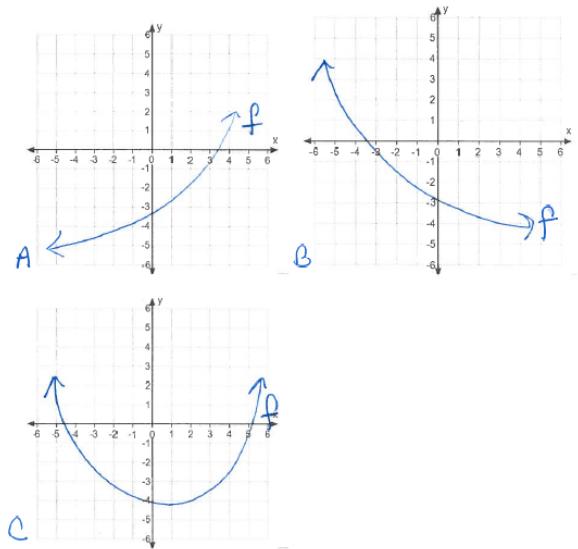
Solution:



Problem 3 Give an example or sketch of a function that is continuous on $(-\infty, \infty)$ and satisfies given properties. If such a function does not exist, explain why.

- (a) A function f is concave up and negative everywhere.

Solution: This is not possible. If a function is always concave up then at some point, the function must cross the x -axis and become positive. See the three example figures below.

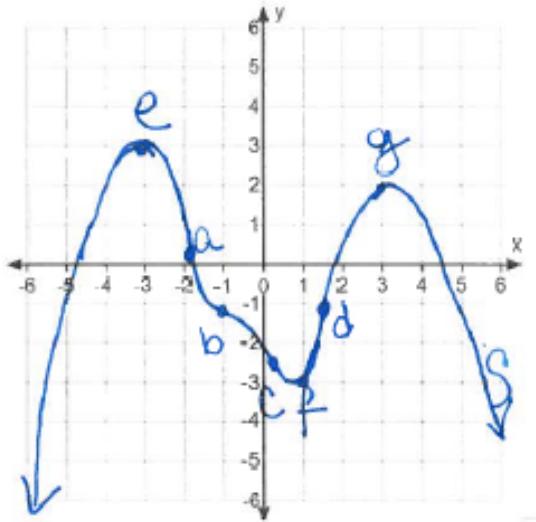


- (b) A function f is decreasing and concave up everywhere.

Solution: This is possible. See figure B above.

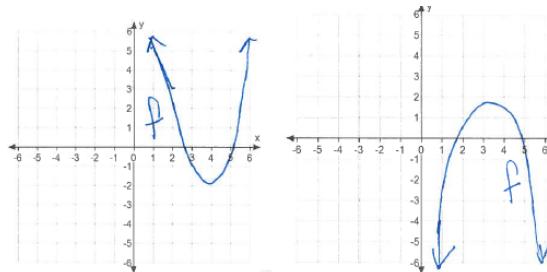
- (c) A function s has exactly 3 local extrema and four inflection points.

Solution: This is possible. See the figure below. The graph of s has inflection points at a, b, c, d and local extrema at e, f, g .



- (d) A function f has exactly 2 zeros and one local extrema.

Solution: This is possible. See the figures below for examples.

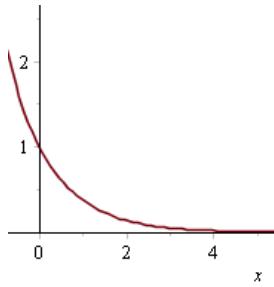


Problem 4

- (a) You are given that $f''(x) > 0$ for all x . Which of the following must be true about $f(x)$ on the region $0 \leq x \leq 2$?

- (i) There is a critical point between 0 and 2.
- (ii) There is a local maximum, but not enough information is given to determine where.
- (iii) f need not have a local maximum.

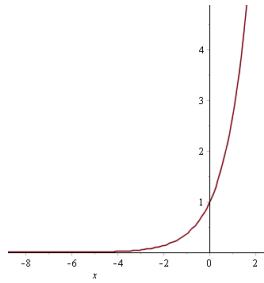
Solution: Only (iii) must be true. The following picture provides a counterexample for both (i) and (ii):



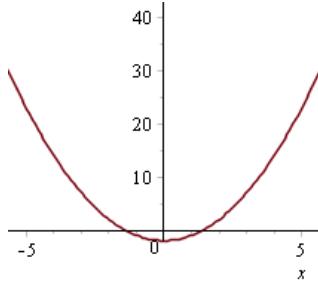
(b) You are told that $f''(x) > 0$ for all x . Which of the following must be true about the graph of $y = f(x)$?

- (i) The graph is a straight line.
- (ii) The graph crosses the x -axis at most once.
- (iii) The graph is concave down.
- (iv) The graph crosses the y -axis more than once.
- (v) The graph is concave up.

Solution: Only (v) must be true. For (i), the function $f(x) = e^x$ provides a counterexample:



For part (ii), the function $f(x) = x^2 - 2$ provides a counterexample:



Part (iii) is clearly false since $f''(x) > 0$ means that f is concave up. Part (iv) is false for any function.

Problem 5 Suppose a function f satisfies the following conditions:

(a) $f(0) = 0$ and $f'(-4) = f'(2) = f'(10) = 0$

(b) $\lim_{x \rightarrow 6} f(x) = -\infty$, and $\lim_{x \rightarrow +\infty} f(x) = 6$

(c) $f'(x) < 0$ on $(-\infty, -4)$, $(2, 6)$, and $(10, +\infty)$

(d) $f'(x) > 0$ on $(-4, 2)$, and $(6, 10)$

(e) $f''(x) > 0$ on $(-\infty, 0)$, and $(14, +\infty)$

(f) $f''(x) < 0$ on $(0, 6)$, and $(6, 14)$

(a) List the **interval(s)** where the function f is **both increasing** and **concave UP**.

Solution: $(-4, 0)$

(b) List the **interval(s)** where the function f is **both increasing** and **concave DOWN**.

Solution: $(0, 2)$, $(6, 10)$

(c) List the **interval(s)** where the function f is **both decreasing** and **concave UP**.

Solution: $(-\infty, -4)$, $(14, +\infty)$

(d) List the **interval(s)** where the function f is **both decreasing** and **concave DOWN**.

Solution: $(2, 6)$, $(10, 14)$

(e) List the **x-coordinates** at which f has a **local minimum**. Write "none" if appropriate.

Solution: $x = -4$

(f) List the **x-coordinates** at which f has a **local maximum**. Write "none" if appropriate.

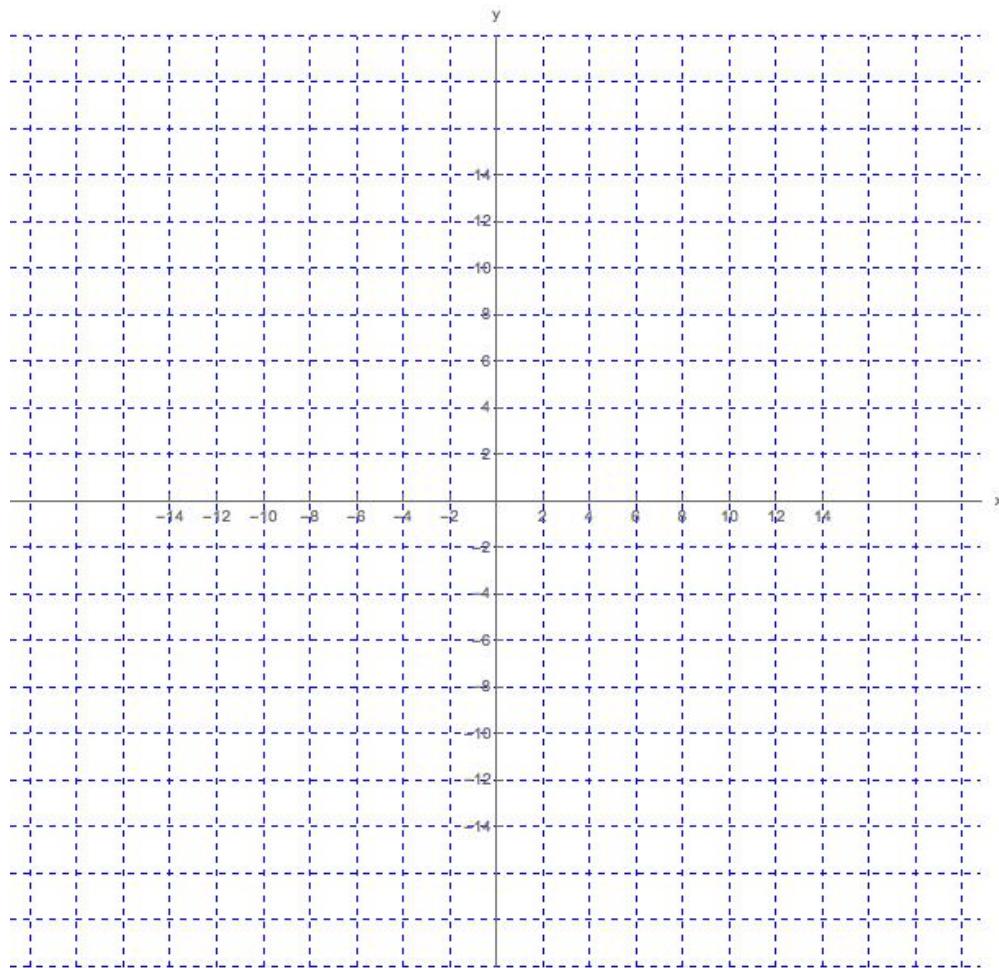
Solution: $x = 2$, $x = 10$
(f can also have a local maximum at $x = 6$, if we include $x = 6$ in the domain)

(g) List the **x-coordinates** of all **inflection points** of f . Write "none" if appropriate.

Solution:

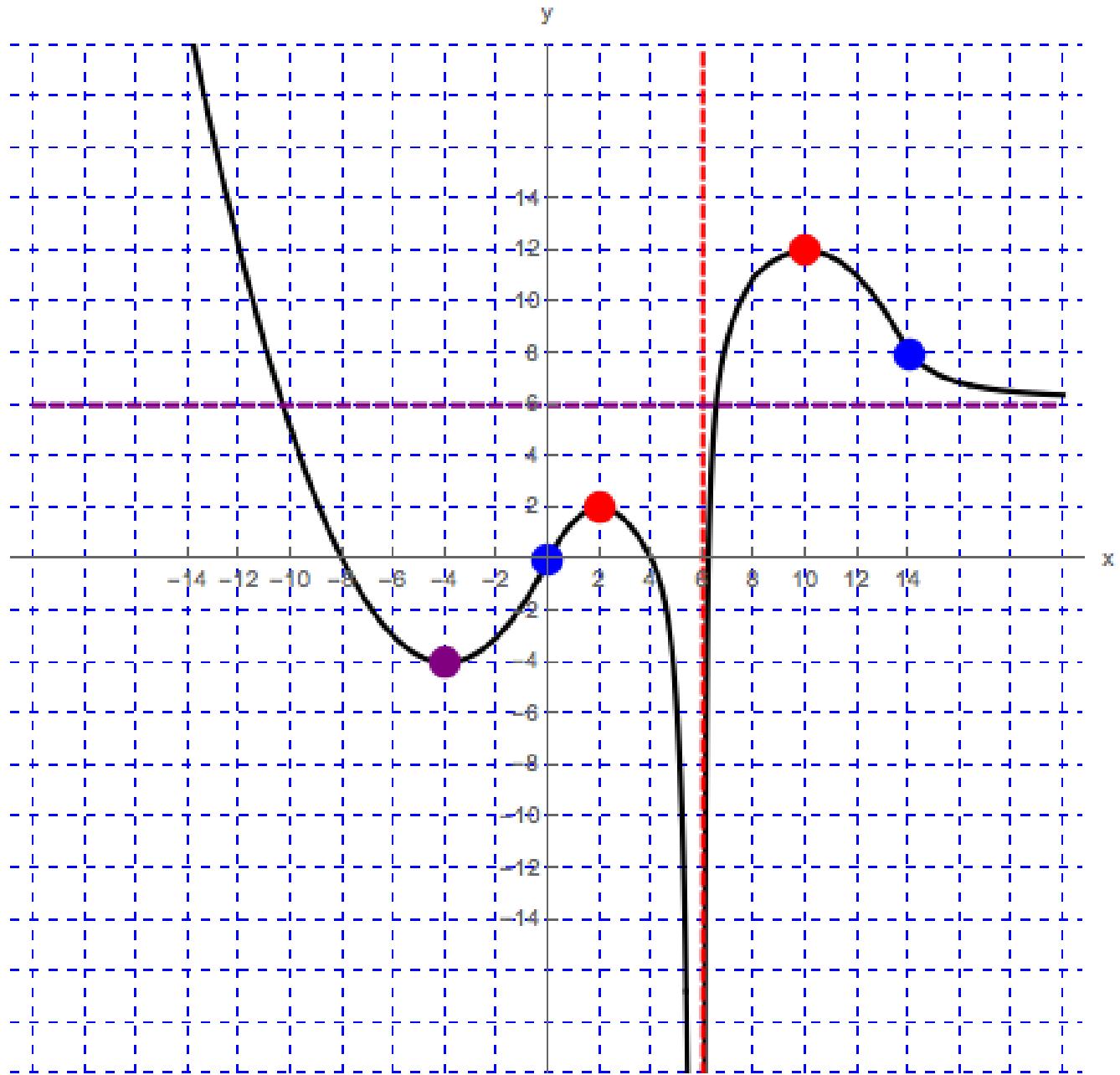
$$x = 0, \quad x = 14$$

(h) Sketch the graph of f .



Solution: This is just one of many possible graphs.

The domain of f could be $(-\infty, +\infty)$, and f could satisfy the given conditions.



Purple point : local minimum

Red points : local maxima

Blue points : inflection points

Problem 6 Sketch the graph of a function f satisfying all of the conditions:

(a) f is continuous and odd, $f(0) = 0$,

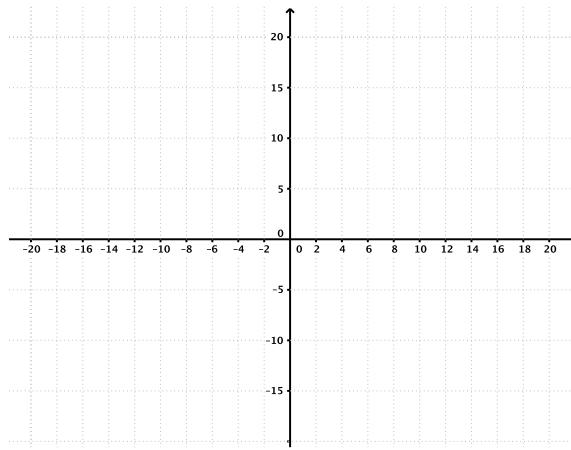
(b) $\lim_{x \rightarrow \infty} f(x) = -5$,

(c) $f'(x) > 0$ on $(6, \infty)$,

(d) $f'(x) < 0$ on $(0, 6)$,

(e) $f''(x) > 0$ on $(0, 12)$, and

(f) $f''(x) < 0$ on $(12, \infty)$.



Solution:

