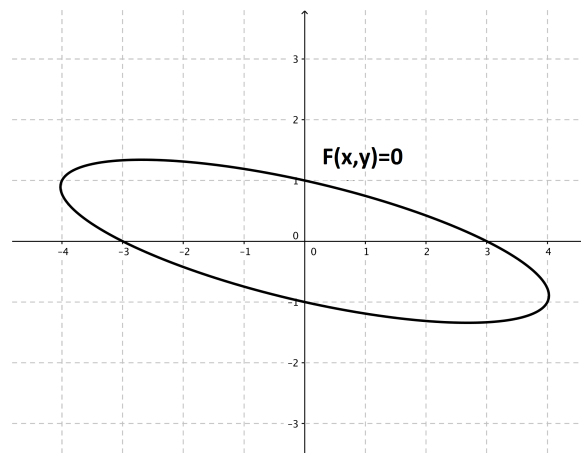


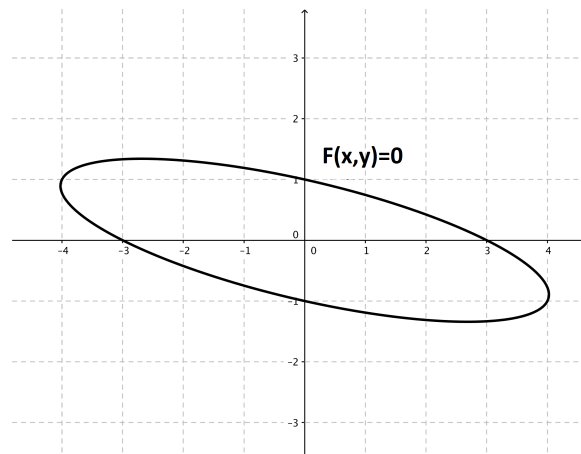
Implicit differentiation (ID)

Recitation Questions

Problem 1 On the graph below, sketch the tangent lines at $x = 0$. Then, explain why both the x -coordinate and the y -coordinate are generally needed to find the slope of the tangent line at a point on the graph of an equation of the form $F(x, y) = 0$



Problem 2 Consider the equation $x^2 + 4xy + 9y^2 = 9$. Note: This equation is equivalent to $x^2 + 4xy + 9y^2 - 9 = 0$. Therefore it has a form $F(x, y) = 0$

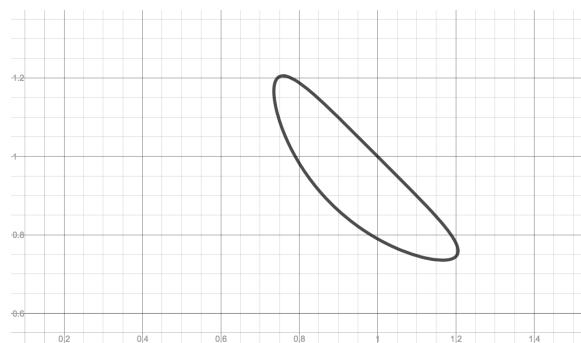


(a) Find $\frac{dy}{dx}$.

(b) Find the equation(s) of the tangent line(s) when $x = 0$. Draw the tangent line(s) on the above picture.

- (c) Find the point(s) where the tangent line is horizontal. Draw the point(s) and line(s) on the above picture.

Problem 3 A part of the curve with equation $\cos(\pi xy) + x + y = 1$ is sketched below.



(a) Use the implicit differentiation to find the derivative dy/dx .

(b) Consider the point $(1, 1)$. Show (algebraically) that this point lies on the curve.

(c) Find the equation of the line tangent to the curve at $(1, 1)$. Draw this line in the figure above.



Problem 4 For each of the curves given by the following equations, find a formula for the slope of the tangent line at a point (x, y) .

(a) $e^{x^2y^3} - 5x + 7y = 36$

(b) $7 = 22 \tan(y) + \frac{4}{x} - \frac{7}{y}$

(c) $\cos(xy) - x^3 = 5y^3$

Problem 5 The volume of a doughnut with an inner radius of a and an outer radius of b is

$$V = \pi^2 \frac{(b+a)(b-a)^2}{4}.$$

Find db/da if the volume of a doughnut is $64\pi^2$ and does not change.

Problem 6 The curve is given by the equation $x^{1/3} + y^{2/3} = 2$. Find $\frac{d^2y}{dx^2}$.

Problem 7 Sketch both the curve $y = \ln(x)$ and the tangent line to the curve at the point where $x = 1$. Then, write an equation of the tangent line to the curve $y = \ln(x)$ at the point where $x = 1$.

Problem 8 (a) Let f be a positive differentiable function, defined on an open interval I . Find the formula for the derivative of the function $\ln(f(x))$.

(b) Using the formula obtained in part (a), compute the derivatives of the following functions.

(i) $f(x) = \ln(x^2 + x + 1)$

(ii) $f(x) = \ln(\sec(x) + \tan(x))$

(iii) $f(x) = \ln(\ln(x))$

Problem 9 Compute $f'(x)$.

(a) $f(x) = x \ln(x)$

(b) $f(x) = \sin(x) \left(\ln(\sec(x) + 1) \right)$

(c) $f(x) = 2^x \sqrt{\ln(5x + 7)}$