Maximums and minimums (MAM)
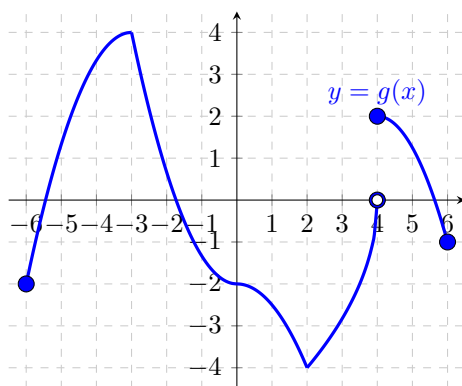
Recitation Questions

Problem 1 *Determine whether the following statements are true or false and give either an explanation or a counterexample.*

(a) *The function $f(x) = \sqrt{x}$ has a local maximum on the interval $[0, 1]$.*

(b) *If $f'(2) = 0$, then $x = 2$ is either a local maximum or local minimum of f.*

Problem   2   *The entire graph of a function g is given below.*

Based on the graph of g, answer the questions below.

(a) *List the x-coordinates of all critical points of g.*

(b) *List the x-coordinates of all critical points of g where $g'(x) = 0$.*

(c) *List the x-coordinates of all critical points of g where $g'(x)$ is **undefined**.*

(d) *List the x-coordinates of all local maximums of g.*

(e) *List the x-coordinates of all local minimums of g.*

(f) *List all intervals where g is both decreasing AND concave down.*

(g) *List all intervals where g is both decreasing AND concave up.*

(h) *List all intervals where g is both increasing AND concave down.*

(i) *List all intervals where g is both increasing AND concave up.*

(j) *List the x-coordinates of all inflection points of g.*

Problem 3 *For each point in the interval (a,b) and identified on the graph below, determine if the function f has a critical point, a local max or min at that point.*
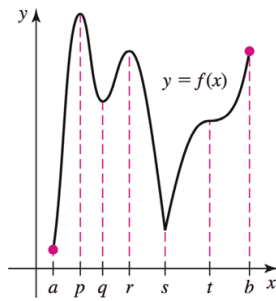
Problem 4 *Locate the critical points and use the second derivative test to determine whether they corre-spond to local maxima or local minima.* **EXPLAIN**.

$f(x) = (x + c)^4$ *where c is a positive constant*

Problem 5 *Let f be a function that is continuous on its domain $(-6, 4)$. The graph of f', the derivative of f, is given below.*
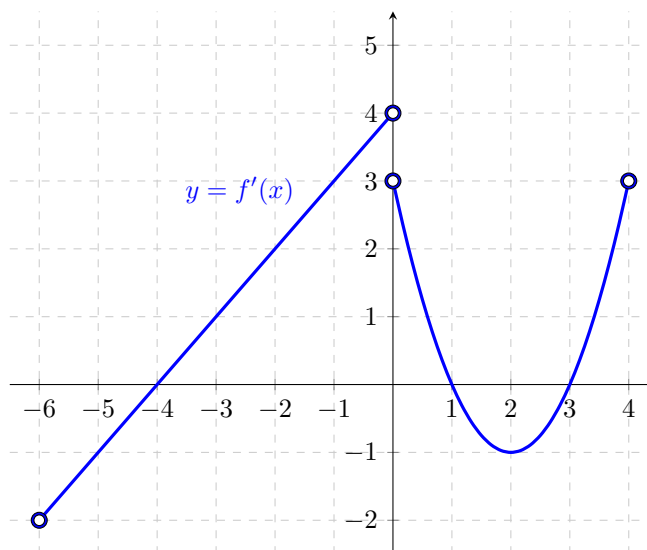
Based on the graph of f', answer the question below.

(a) *List x-coordinates of all critical points of f.*

(b) *List x-coordinates of all critical points of f where $f'(x) = 0$.*

(c) *List x-coordinates of all critical points of f where $f'(x)$ is* **undefined**.

(d) *List x-coordinates of all local minimums of f.*

(e) *List x-coordinates of all local maximums of f.*

(f) *List all intervals where f is decreasing and concave down.*

(g) *List all intervals where f is decreasing and concave up.*

(h) *List all intervals where f is increasing and concave down.*

(i) *List all intervals where f is increasing and concave up.*

(j) *List x-coordinates of all inflection points of f.*

Problem 6 *Find the critical points of f on the given interval. Determine whether the function f has a local maximum, local minimum or no local extremum at each critical point.* **EXPLAIN**.

(a) $f(x) = x\sqrt{2 - x^2}$ on $(-\sqrt{2}, \sqrt{2})$.

(b) $f(x) = x^3 e^{-x}$ on $(-1, 5)$.

(c) $f(x) = x \ln\left(\dfrac{x}{5}\right)$ on $(0, 5)$.

Problem 7 Let $f(x) = \dfrac{1}{1+x^2}$. Find the following for f:

(a) f' and f''

(b) Critical points

(c) *Local extrema (and check your answers with both the first and second derivative tests)*

(d) *Inflection points.* **EXPLAIN**.

Problem 8 *Sketch a possible graph of a function f that has the following properties:*

(a) *f is defined on the interval $(0, 6)$.*

(b) *f has no local maximums.*

(c) *f has exactly two local minimums.*

Problem 9 *Consider the parabola* $f(x) = ax^2 + bx + c$ *where* a, b, c *are constants. For what values of* a, b, c *is* f *concave up? For what values of* a, b, c *is* f *concave down?*