

Computations for graphing functions (CFGF)

Recitation Questions

Problem 1 Let f be a function given by: $f(x) = (\tan^{-1}(x))^2$.

(a) State the domain of f . Write your answer in interval notation.

(b) Find the critical points of f . Show your work!

(c) Show that $f''(x) = \frac{2 - 4x \tan^{-1}(x)}{(x^2 + 1)^2}$.

(d) Use the **second derivative test** to determine whether critical point(s) of f correspond to local minima, local maximums, or whether the test is inconclusive.

Problem 2 Determine the following information about the given function and then graph the function:

$$f(x) = \frac{x^2 + x + 1}{x^2}$$

Domain

x, y-intercepts

Symmetry

Asymptotes

Intervals of increasing/decreasing

Maxima/Minima

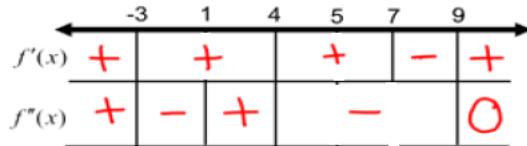
Intervals of concavity

Inflection points

Problem 3 Given that:

$$\begin{array}{llll} \lim_{x \rightarrow -\infty} f(x) = 0 & \lim_{x \rightarrow -3^-} f(x) = \infty & \lim_{x \rightarrow -3^+} f(x) = -\infty & \lim_{x \rightarrow 4^-} f(x) = \infty \\ \lim_{x \rightarrow 4^+} f(x) = -\infty & f(1) = 1 & f(5) = -2 & \lim_{x \rightarrow 9} f(x) = 3 \\ f'(1) \neq 0 & f'(7) = 0 & f'(x) = 2 \text{ for } x > 9 & f''(1) = 0 \end{array}$$

the domain of f is: $(-\infty, -3) \cup (-3, 4) \cup (4, 9) \cup (9, \infty)$ and f is continuous on its domain. The following sign chart for the first and second derivatives of f :



find the following:

- (a) Critical points.
- (b) Intervals where f is increasing/decreasing.
- (c) Local extrema.
- (d) Inflection points.
- (e) Intervals of concavity.
- (f) Sketch the graph of f .

Problem 4 Determine the following information about the given function and then graph the function:

$$f(x) = 3 \sin\left(\frac{x}{2}\right), [-2\pi, 2\pi]$$

Domain

x, y -intercepts

Symmetry

Asymptotes

Intervals of increasing/decreasing

Maxima/Minima

Inflection points

Intervals of concavity

Sketch a graph of f

Problem 5 Determine the following information about the given function and then graph the function:

$$f(x) = x \ln(x)$$

Domain

x, y-intercepts

Symmetry

Intervals of increasing/decreasing

Maxima/Minima (including absolute)

Intervals of concavity

Inflection points

Given $\lim_{x \rightarrow 0^+} f(x) = 0$, sketch a graph f .

Problem 6 Sketch a graph of the function f defined by

$$f(x) = \begin{cases} 2x - 2 \tan^{-1}(x) - x \ln(x^2 + 1) & \text{for } x \leq 1 \\ \sqrt{x-2} & \text{for } x \geq 2 \end{cases}$$

(a) Find the domain of f .

(b) Show that $f'(x) = -\ln(x^2 + 1)$ on the interval $(-\infty, 1)$.

(c) Compute a formula for f' on the interval $(2, \infty)$.

(d) Compute a formula for f'' on the interval $(-\infty, 1)$.

(e) Compute a formula for f'' on the interval $(2, \infty)$.

(f) List all the asymptotes (with limit justifications, if appropriate). (If f does not have a particular asymptote write “NONE”.)

(i) Vertical asymptotes:

(ii) Horizontal asymptotes:

(g) In the following sign chart for f' , list the points (in the domain of f) where $f'(x) = 0$ or $f'(x)$ is undefined, indicate the sign of f' on each subinterval, and identify any local extrema (with “local max” or “local min”). In the following sign chart for f'' , list the points (in the domain of f) where $f''(x) = 0$ or $f''(x)$ is undefined, indicate the sign of f'' on each subinterval, and identify any inflection points. Use this information to sketch a graph of f in the blank grid. (Be sure to indicate any x - or y -intercepts.)

