Product Rule and Quotient Rule (PRAQR)

- **The Product Rule**: If f and g are differentiable functions, then $\dfrac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$.

- **The Quotient Rule**: If f and g are differentiable functions, then $\dfrac{d}{dx}\dfrac{f(x)}{g(x)} = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$.

Recitation Questions

Problem 1 *Differentiate the functions using product or quotient rule.*

(a) $h(u) = 7ue^u$

(b) $s(t) = \dfrac{\sqrt{t}}{e^t}$

Problem   2   *Differentiate the function f defined by $f(x) = 1/x^8$ in two different ways.*

Problem 3 *Suppose that $f(5) = 7$, $f'(5) = 8$, $g(5) = 3$, and $g'(5) = -4$. Find:*

(a) $(fg)'(5)$.

(b) $\left[\dfrac{d}{dx}\left(\dfrac{f}{g}\right)\right]_{x=5}$

(c) $\left(\dfrac{g}{f}\right)'(5)$

(d) $\left[\dfrac{d}{dx}\left(\dfrac{g(x)}{x+2}\right)\right]_{x=5}$

(e) $\left[\dfrac{d}{dx}\left(\dfrac{xf(x)}{g(x)}\right)\right]_{x=5}$

(f) $\displaystyle\lim_{x\to 5}\dfrac{x^2 g(x) - 75}{x - 5}$. ***EXPLAIN.***

(g) $\displaystyle\lim_{x\to 5}\dfrac{\frac{x+2}{f(x)} - 1}{x - 5}$. ***EXPLAIN.***

Problem 4 *Use the given information to find the equation of the tangent line.*

(a) Given $g(x) = x^3 f(x)$, $f(2) = 4$, and $f'(2) = 7$, find the equation of the tangent line to the graph of g at $x = 2$.

(b) Given $h(z) = \dfrac{zs(z)}{z-3}$, $s(2) = 4$, and $s'(2) = 7$, find the equation of the tangent line to the graph of h at $z = 2$.

(c) Given

x	1	2	3	4	5
$f(x)$	5	3	0	−4	3
$f'(x)$	−3	−5	−2	6	−4
$g(x)$	6	9	−8	13	15
$g'(x)$	8	5	−10	7	6

find the equation of the tangent line of
$$\frac{f(x)}{e^x g(x)}$$
at $x = 2$.

Problem 5 *Differentiate the following functions:*

(a) *Given* $f(x) = (x^2 + 4x - 7)e^{-x}$, *show* $f'(x) = (11 - x^2 - 2x)e^{-x}$.

(b) *Given* $g(x) = \dfrac{x^2 + 4x - 7}{e^{-x}}$, *show* $g'(x) = \dfrac{x^2 + 6x - 3}{e^{-x}}$

Problem 6 *The graph of a function g is given below. Using the graph, estimate the derivative at the given point:*

$$\frac{d}{dx}\left(\frac{xg(x)}{x+3}\right) \text{ at } x = 1$$

$y = g(x)$