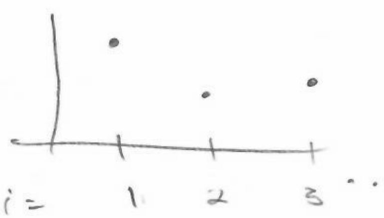
Gaussian Processes - Part II

Visualizing Gaussians:

- Visualize some 2d Gaussian as scatter plots. How does changing μ, Σ affect things?

How to visualize in higher dimensions? Lets plot points like



- Visualize this in 2 dimensions.
- Pick a covariance matrix form, like $Var(x_i, x_j) = \exp(-c\,|x_i - x_j|^2)$ and visualize some higher dim Gaussians. Play w/ parameters in the cov matrix & see how it changes things.

Intuitively, will get to GPs by considering a multivariate Gaussian w/ infinite index set rather than $i = 1, \dots, n$.

__Def__ A stochastic process is a collection of RVs indexed by some variable $x \in \mathcal{X}$,

$$Y = \{ y(x) : x \in \mathcal{X} \}$$

- often, $\mathcal{X} = \mathbb{N}$
- If $|\mathcal{X}| = \infty$, Y is an infinite dimensional stoch proc.

Claim: The characteristics of a stochastic process are uniquely defined by specifying the characteristics of all finite dimensional distributions,

$$P(y(x_1) \le c_1, \ldots, y_n(x_n) \le c_n)$$

for all $n \in \mathbb{N}$ and $x_1, \ldots, x_n$.

Def A GP is a stochastic process with Gaussian finite dimensional distributions, i.e.,

$$(y(x_1), \ldots, y(x_n)) \sim \mathcal{N}(\mu_n, \Sigma_n)$$

Notation we write $Y(\cdot) \sim GP$ to denote Y is a GP.

To fully specify a GP, we only need to specify the mean and cov functions

$$y(\cdot) \sim GP(m(\cdot), k(\cdot, \cdot))$$

where

$$\mathbb{E}\, y(x) = m(x)$$
$$\mathrm{Cov}(y(x), y(x')) = k(x, x').$$

- Mean $m(\cdot)$: pick to be whatever you want (basically)
- cov: pick $k(\cdot, \cdot)$ so that you always get a valid cov matrix. (Pos semi def)

Common choices

Stationary process : $\operatorname{Cov}(Y(x), Y(x')) = k(x - x')$

Isotropic : $\operatorname{Cov}(\cdot, \cdot) = k(\|x - x'\|)$

Note: The choice of k determines "nature" of GP / hypothesis space / space of functions

Examples