Max-stable processes are the extension of the multivariate extreme value theory to the infinite dimensional setting. More precisely consider a stochastic process $\{Y(x)\}$, $x \in \mathbb{R}^d$, having continuous sample paths. Then the limiting process

$$\left\{ \max_{i=1,\dots,n} \frac{Y_i(x) - b_n(x)}{a_n(x)} \right\}_{x \in \mathbb{R}^d} \longrightarrow \{Z(x)\}_{x \in \mathbb{R}^d}, \qquad n \to \infty \tag{1}$$

where Y_i are independent replications of Y, $a_n(x) > 0$ and $b_n(x) \in \mathbb{R}$ are sequences of continuous functions and the limiting process Z is assumed to be non degenerate. de Haan [1994] shows that the class of the limiting processes $Z(x)$ corresponds to that of max-stable processes and hence emphasizes on their use to model spatial extremes.

Interestingly there are two different ways of charactaresing max-stable processes: these are known as the spectral characterisations. An especially useful special case of the characterisation of de Haan [1984], is

$$Z(x) = \max_{i \geq 1} \xi_i f(x - U_i) \tag{2}$$

where $\{(\xi_i, U_i)\}_{i \geq 1}$ are the points of a Poisson point process on $(0, \infty] \times \mathbb{R}^d$ with intensity measure $\mathrm{d}\Lambda(\xi, u) = \xi^{-2} \mathrm{d}\xi \mathrm{du}$, f is a probability density function on $\mathbb{R}^d$. The process Z defined above is a max-stable process with unit Frechet margins. Taking f as the multivariate Normal density with zero mean and covariance matrix Σ gives the Smith model [Smith, 1990] for which the bivariate distribution function is

$$\Pr[Z(x_1) \leq z_1, Z(x_2) \leq z_2] = \exp\left\{ -\frac{1}{z_1} \Phi\left(\frac{a}{2} + \frac{1}{a} \log \frac{z_1}{z_2} \right) - \frac{1}{z_2} \Phi\left(\frac{a}{2} + \frac{1}{a} \log \frac{z_2}{z_1} \right) \right\}, \tag{3}$$

where Φ is the standard normal distribution function and $a^2 = (x_1 - x_2)^T \Sigma^{-1} (x_1 - x_2)$, $x_1, x_2 \in \mathbb{R}^d$.

Another very useful spectral characterisation for unit Frechet max-stable processes is [Schlather, 2002]

$$Z(x) = \max_{i \geq 1} \xi_i Y_i(x), \tag{4}$$

where $\{\xi_i\}_{i \geq 1}$ are the points of a Poisson point process on $(0, \infty]$ with intensity measure $\mathrm{d}\Lambda(\xi) = \xi^{-2} \mathrm{d}\xi$ and Y_i are independent replications of a positive stochastic process having continuous sample paths Y such that $\mathbb{E}[Y(x)] = 1$ for all $x \in \mathbb{R}^{d}$".

Currently there are several useful models based on Schlather's characterisation. The first model, the Schlather's model, is to use $Y(x) = \sqrt{2\pi} \max\{0, \varepsilon(x)\}$, where ε is a standard Gaussian process. This leads to the bivariate distribution function

$$\Pr[Z(x_1) \leq z_1, Z(x_2) \leq z_2] = \exp\left[-\frac{1}{2} \left(\frac{1}{z_1} + \frac{1}{z_2} \right) \left(1 + \sqrt{1 - \frac{2\{1 + \rho(h)\} z_1 z_2}{(z_1 + z_2)^2}} \right) \right], \tag{5}$$

Another possibility is to take $Y(x) = \exp\{\sigma\varepsilon(x) - \sigma^2/2\}$, where $\sigma > 0$. This is known as the Geometric gaussian model for which the bivariate distribution is similar to the Smith model with $a^2 = 2\sigma^2\{1 - \rho(h)\}$. A last possibility, which is a generalisation of the geometric Gaussian model, is to take $Y(x) = \exp\{\varepsilon(x) - \gamma(x)\}$, where ε is a zero mean Gaussian process having stationary increments and (semi)variogram γ such that $\varepsilon(o) = 0$ almost surely. Its bivariate distribution function is again similar to the Smith model with $a^2 = 2\gamma(x_1 - x_2)$.